14.54 International Economics
Handout 1

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1 An Exchange Economy

Consider two countries, Home and Foreign. Consumers in each country are endowed with different amounts of two goods, Apples and Bananas.

The two countries are characterized by the following preferences. Home consumers' utility function is:

$$U(x_A, x_B),$$

while Foreign consumers' utility function is:

$$U^*(x_A^*, x_B^*).$$

Consumers' endowments are the following: a home consumer has e_A Apples and e_B Bananas while a foreign consumer has e_A^* Apples and e_B^* Bananas. There are a continuum of measure 1 of home consumers and of foreign consumers.

Let's start by looking at what happens if the two countries are in autarky. Then we will analyze what happens if the two countries open to international trade.

1.1 Demand function

The demand function can be derived from:

$$\begin{array}{ll} \max & U(x_A, x_B) \\ & p_A x_A + p_B x_B \leq p_A e_A + p_B e_B \end{array}$$

$$\begin{aligned} \frac{\partial U}{\partial x_A} &= \lambda p_A \\ \frac{\partial U}{\partial x_B} &= \lambda p_B \end{aligned}$$

$$\frac{\frac{\partial U}{\partial x_A}}{\frac{\partial U}{\partial x_B}} = \frac{p_A}{p_B} \tag{1}$$

$$p_A x_A + p_B x_B = p_A e_A + p_B e_B \tag{2}$$

These two equations define, implicitly, the *marshallian* demand function for good A of the Home consumer $X_A(.,.)$:

$$x_A = X_A\left(\frac{p_A}{p_B}, \frac{p_A}{p_B}e_A + e_B\right),$$

that depends on the relative prices and on real income. Notice that the consumers real income $\frac{p_A}{p_B}e_A + e_B$ is determined once we know the relative prices. So, for given e_A and e_B we have a function that depends only on the relative price $\frac{p_A}{p_B}$ and we can write:

$$x_A = f_A\left(\frac{p_A}{p_B}\right).$$

Home consumers will buy or sell good A depending on whether we have $x_A > e_A$ or $x_A < e_A$.

- Why marshallian demand function depends only on the relative price and not on p_A and p_B separately?
- What do we know about $X_A(.,.)$? (review substitution effect, income effect)
- Review graphical representation with indifference curves and budget constraint.

Similar derivations and we get the demand for the foreign consumer

$$x_A^* = f_A^*\left(\frac{p_A}{p_B}\right).$$

1.2 Autarky

Suppose the home country is closed to international trade.

Competitive equilibrium: find the relative price $\frac{p_A}{p_B}$ that solve:

$$f_A\left(\frac{p_A}{p_B}\right) = e_A.$$

Market clearing condition.

Why not look at the other good?

$\longrightarrow$If the budget constraint is satisfied and $x_A = e_A$ then it has to be true that $x_B = e_B$, because:

$$x_B - e_B = \frac{p_A}{p_B}(x_A - e_A)$$

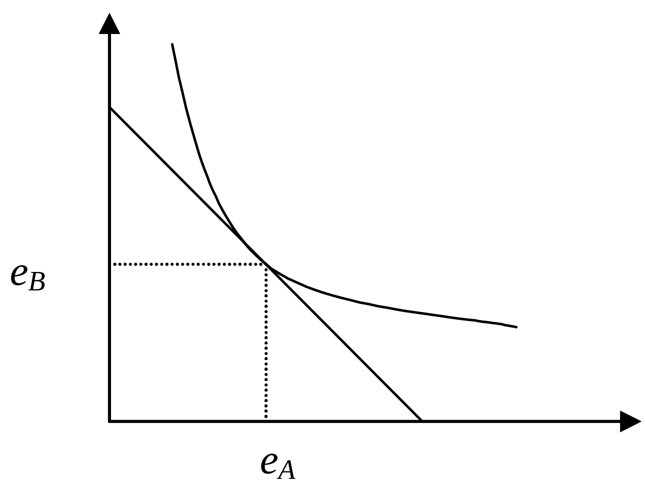
So one equilibrium condition is enough. In general with n goods there are $n-1$ market clearing conditions that are independent (this is called Walras Law).

Claim 1 *At autarky competitive equilibrium the relative prices are equal to the MRS computed at the endowment:*

$$\left.\frac{\frac{\partial U}{\partial x_A}}{\frac{\partial U}{\partial x_B}}\right|_{\substack{x_A=e_A \\ x_B=e_B}} = \frac{p_{A,a}}{p_{B,a}}$$

where the a stands for autarky prices.

To check that this is an equilibrium just check that conditions (1) and (2) are satisfied.



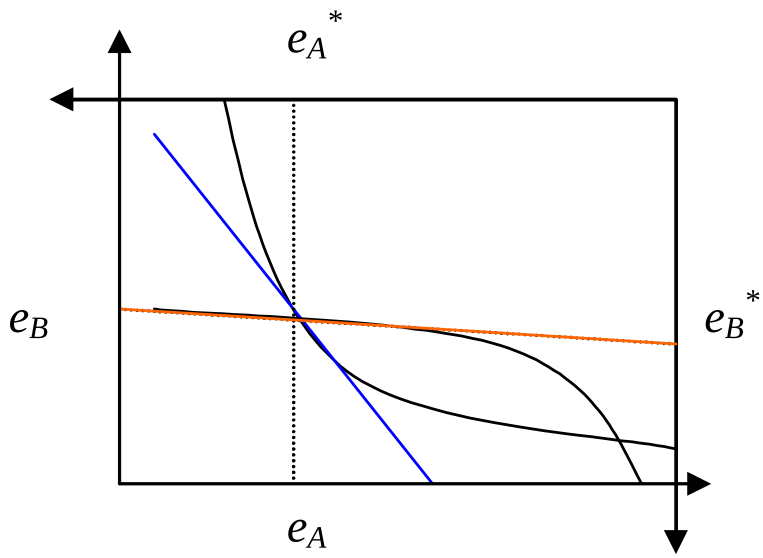
The MRS represents the rate at which consumers are willing to trade apples for bananas, i.e. how many bananas are they willing to give away in exchange for one apple.

1.3 Trade

Suppose the two countries are in autarky and

$$\frac{p_{A,a}}{p_{B,a}} > \frac{p^*_{A,a}}{p^*_{B,a}}$$

Edgeworth box. When they consume (e_A, e_B) and (e^*_A, e^*_B) the home consumer assigns a higher "price" to apples than the foreign consumer, i.e. his MRS is higher, i.e. the blue line tangent to the home consumer's indifference curve is steeper than the orange line tangent to the foreign consumer's indifference curve. The home consumer is willing to part with 3 bananas to obtain one extra apple, the foreign consumer is willing to accept 1/4 of a banana in exchange for one apple. There is room for a Pareto improvement. The home consumer gives bananas to the foreign consumer in exchange for apples. Everybody is better off.



Suppose we open trade between the two countries. Now equilibrium international prices are prices such that:

$$f\left(\frac{p_A}{p_B}\right) + f^*\left(\frac{p_A}{p_B}\right) = e_A + e^*_A$$

Or

$$f\left(\frac{p_A}{p_B}\right) - e_A = e^*_A - f^*\left(\frac{p_A}{p_B}\right)$$

what home consumers buy is equal to what foreign consumers sell.

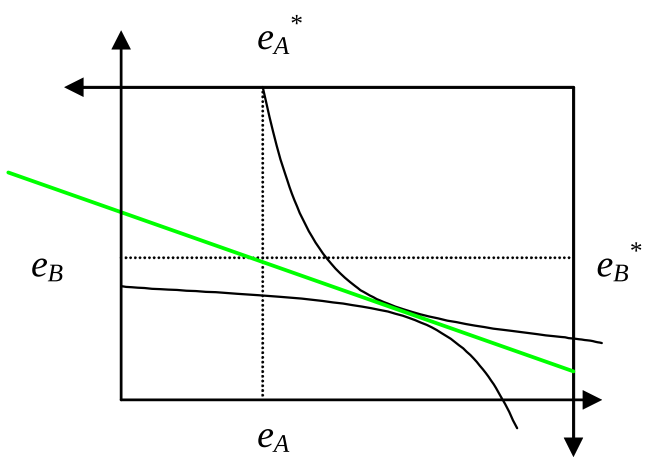
Again, enough to look only at equilibrium in one market (Walras Law again...)

Why? From budget constraints we know that

$$\begin{aligned} e_B - x_B &= \frac{p_A}{p_B}(x_A - e_A) \\ x_B^* - e_B^* &= \frac{p_A}{p_B}(e_A^* - x_A^*) \end{aligned}$$

(the value of their sales must correspond to the value of their purchases). But, the two right hand sides are equal, so the two left hand sides are equal too!

$$e_B - x_B = x_B^* - e_B^*.$$



Now the green line represent the equilibrium relative price. Home consumers are buying apples and selling bananas. The equilibrium relative prices of the exports in terms of the imports is called the *terms of trade* for the home country (here is $\frac{p_B}{p_A}$). Here we have that:

$$\frac{p_{A,a}}{p_{B,a}} < \frac{p_A}{p_B} < \frac{p_{A,a}^*}{p_{B,a}^*}.$$

Both countries gain from trading with the other country, home country buys apples cheaper and country foreign buys banana cheaper.

Claim 2 *There is positive trade in equilibrium as long as*

$$\frac{p_{A,a}}{p_{B,a}} \neq \frac{p^*_{A,a}}{p^*_{B,a}}.$$

Claim 3 *Both home and foreign consumers gain from trade.*

Proof. Suppose (p_A, p_B) are equilibrium with international trade. Then home consumers utility is equal to

$$U^T = \max \quad U(x_A, x_B)$$
$$p_A x_A + p_B x_B \leq p_A e_A + p_B e_B$$

Notice that (e_A, e_B) is feasible so it must be

$$U^T \geq U(e_A, e_B).$$

The same is true for the foreign consumer. But utility in autarky is clearly given by

$$U^A = U(e_A, e_B).$$

So we have

$$U^T \geq U^A, U^{T*} \geq U^{A*}.$$

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