Handout 3 - Notes on the Current Account

Guido Lorenzoni

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1 The Current Account

A country has GDP Y_t, consumption C_t and investment I_t. We can write the trade balance in a given period as

$$Y_t - C_t - I_t$$

where all variables are in dollars. This is the amount of dollars that enters the country to pay for goods.

The country assets are denoted by A_t and the liabilities by L_t.

Suppose assets and liabilities receive the same rate of return r_t. Each period $r_t A_t$ dollars enter the country to pay for the services of the assets that we hold abroad, and $r_t L_t$ exit the country to pay for the services of the assets that foreigners hold at home.

For now, we omit the public sector for simplicity. Then the net amount of dollars entering the country is:

$$CA_t = Y_t - C_t - I_t + r_t A_t - r_t L_t$$

this is the current account surplus.

The current account surplus must go to finance either the accumulation of assets or the decumulation of liabilities:

$$(A_{t+1} - A_t) - (L_{t+1} - L_t) = Y_t - C_t - I_t + r_t A_t - r_t L_t.$$

This is the basic *current account identity* that has to

Now we can decompose GDP into two parts: non-tradable goods and tradable home goods:

$$Y_t = Y_t^{NT} + Y_t^H.$$

We can decompose consumption and investment in three parts:non-tradable goods, tradable home goods and foreign goods:

$$\begin{aligned} C_t &= C_t^{NT} + C_t^H + C_t^F, \\ I_t &= I_t^{NT} + I_t^H + I_t^F. \end{aligned}$$

For non-tradable goods the following accounting identity holds

$$Y_t^{NT} = C_t^{NT} + I_t^{NT}$$

this means that all non-tradables are used at home, by definition.

Then we can rewrite the trade balance as:

$$\begin{aligned} &\left(Y_t^H - C_t^H - I_t^H\right) - \left(C_t^F + I_t^F\right) = \\ &EXP_t - IMP_t \end{aligned}$$

and see that, indeed, the trade balance is equal to exports minus imports.

2 A Two Period, One Commodity Example

Consider two countries, Home and Foreign.

The world lasts two periods.

There is one commodity, wheat.

There are n identical consumers in country Home and n in country Foreign. Consumption of wheat by Home consumers in periods 1 and 2 is denoted c_1 and c_2 and will be determined in equilibrium. The preferences of consumers are described by the utility function

$$u(c_1) + \beta u(c_2), \quad \beta \in [0,1]$$

Consumption by Foreign consumers is denoted c_1^* and c_2^*. Their preferences are identical to Home consumer preferences.

Each consumer at Home owns a farm that produces a given amount of wheat in each period, y_1 and y_2. For the Foreign country we have y_1^* and y_2^*. Wheat cannot be stored. Wheat can be transported at no cost from country Home to Foreign.

The countries can borrow and lend from each other on the international capital market at the interest rate r.

Let b denote the savings of home consumers. Consumers in country Home determine their optimal savings solving

$$\begin{aligned} & \max u(c_1) + \beta u(c_2) \qquad \text{(P)} \\ s.t.\ b &= y_1 - c_1 \\ c_2 &= y_2 + (1+r)\,b \end{aligned}$$

if $b < 0$ the consumers are net borrowers, if $b > 0$ they are net lenders.

(Exercise: check that the current account identity

The budget constraints in periods 1 and 2 can be collapsed into the *intertemporal budget constraint*

$$c_1 + \frac{1}{1+r}c_2 = y_1 + \frac{1}{1+r}y_2$$

Consumer Choice

We will consider the case where

$$u(c) = \log c$$

Then optimal consumption entails

$$\begin{aligned} c_1 &= \frac{1}{1+\beta}\left(y_1 + \frac{1}{1+r}y_2\right) \\ c_2 &= \frac{\beta(1+r)}{1+\beta}\left(y_1 + \frac{1}{1+r}y_2\right) \end{aligned}$$

Consumers take their total lifetime income, in net present value:

$$y_1 + \frac{1}{1+r}y_2$$

and spend a fraction $\frac{1}{1+\beta}$ in the first period of their life.

Autarky (Closed Economy Case)

The market clearing condition is:

$$\begin{aligned} b &= 0 \;\; or \;\; c_1 = y_1 \\ c_2 &= y_2 \end{aligned}$$

The savings of country home is

$$\begin{aligned} b(r) &= y_1 - c_1 = \\ &= \frac{1}{1+\beta}\left[\beta y_1 - \frac{1}{1+r}y_2\right]. \end{aligned}$$

So if we set $b = 0$ we obtain the following condition:

$$\frac{y_2}{y_1} = \beta(1 + r^a)$$

Market Equilibrium

Put together the consumption choice of Home and Foreign consumers and write the market clearing condition for wheat in period 1

$$b(r) + b^*(r) = 0$$

Which is the same as:

$$CA_1 + CA_1^* = 0$$

(since they start with zero assets the current account is equal to the assets accumulated at the end of period 1).

You obtain

$$\begin{aligned} b(r) &= -b^*(r) \\ y_1 - \frac{1}{1+\beta}\left(y_1 + \frac{1}{1+r}y_2\right) &= y_1^* - \frac{1}{1+\beta}\left(y_1^* + \frac{1}{1+r}y_2^*\right) \end{aligned}$$

Let y_t^w denote the world per capita endowment of wheat, that is

$$y_t^w = \frac{1}{2}(y_t + y_t^*)$$

and let

$$1 + g = \frac{y_2^w}{y_1^w}$$

be the world groth rate.

Then we obtain

$$\frac{1}{1+\beta}\left(y_1^w + \frac{1}{1+r}y_2^w\right) = y_1^w$$

and finally we obtain the equilibrium interest rate on the world market

$$1 + r = \frac{1}{\beta}(1+g)$$

The equilibrium interest rate *does not* depend on the income profile of individual countries, only on the common discount factor β and on the world-wide growth rate g.

Graphically:

Implications

Suppose

$$y_1 < \frac{1}{1+\beta}\left(y_1 + \frac{\beta}{1+g}y_2\right)$$

$$\frac{y_2}{y_1} > 1 + g$$

then the Home country will borrow in period 1 and repay in period 2.

Proof:

$$\frac{c_1}{y_1} = \frac{1}{1+\beta}\left(1 + \frac{1}{1+r}\frac{y_2}{y_1}\right) > \frac{1}{1+\beta}\left(1 + \frac{1+g}{1+r}\right) = 1$$

and we obtain

$$\frac{c_1}{y_1} > 1 \Rightarrow b < 0$$

- The country growing faster borrows from the country growing slower, to finance early consumption.

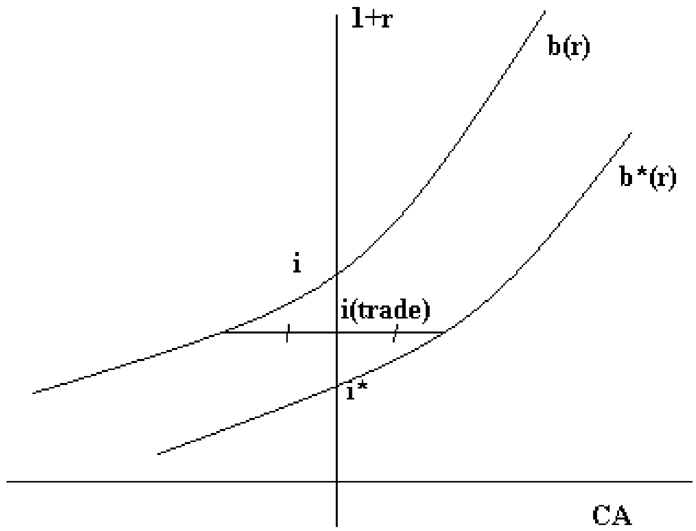

Figure 1:

Notice that writing

$$\frac{y_2}{y_1} > 1 + g$$

is the same as writing

$$y_1 < \frac{1}{1+\beta}\left(y_1 + \beta\frac{y_2}{1+g}\right)$$

the right hand side is a weighted average of income in periods 1 and 2 (corrected for world growth). We can call it "permanent income". So this means that:

- The country with income in period 1 lower than its "permanent income" borrows from the country with income in period 1 higher than its "permanent income".

The growth rate of consumption will be equal for the two countries and equal to the world growth rate. You can get it straight from the first order condition

$$\frac{1}{c_1} = \beta\,(1+r)\,\frac{1}{c_2}$$

and from the equilibrium interest rate. This gives us

$$\frac{y_2^*}{y_1^*} < \frac{c_2^*}{c_1^*} = 1 + g = \frac{c_2}{c_1} < \frac{y_2}{y_1}$$

- International capital markets induce a more similar consumption path across countries, i.e. they make the path of consumption less responsive to local changes in output and more responsive to world changes in output.

Exercise 1 *Suppose that the two countries have different discount rates $\beta \neq \beta^*$.*
(i) Derive the equation that determines market clearing in the goods market at date 1;
(ii) Derive the equilibrium interest rate in the case $y_1 = y_1^* = y_2 = y_2^*$;
(iii) Derive the pattern of borrowing and lending in this case;
(iv) In the general case show that if $\frac{1}{\beta^*}\frac{y_2^*}{y_1^*} < \frac{1}{\beta}\frac{y_2}{y_1}$ the equilibrium interest rate has to satisfy

$$\frac{1}{\beta^*}\frac{y_2^*}{y_1^*} < 1 + r < \frac{1}{\beta}\frac{y_2}{y_1}$$

(Hint: a graphical argument should work fine).

3 Adjustment and real exchange rates

Now we enrich the model with 2 goods: a Home good and Foreign good.

Let the countries be US and Europe.

Consumption of the two goods is C_H, C_F at home and C_H^*, C_F^* abroad.

Production of home good is Y_H. Their prices are P_H and P_F. The nominal exchange rate is e. The price of the foreign good is given in euros, so its price in dollars is eP_F.

The current account (in dollars) is

$$\begin{aligned} CA_t &= EXP_t - IMP_t \\ &= P_{H,t}\left(Y_{H,t} - C_{H,t}\right) - e_t P_{F,t} C_{F,t} + r_t B_t \end{aligned}$$

where r_t is the return on net foreign assets and $B_t = A_t - L_t$ is net foreign assets.

To avoid too much notation suppose for now that the prices P_H and P_F are constant and equal to 1.

The consumer problem is now to choose how much to spend in periods 1 and 2 in each of the goods, home and foreign, that is to choose the vector $(C_{H,1}, C_{F,1}, C_{H,2}, C_{F,2})$. Suppose the utility function of the consumer is of the form $U\left(C_{H,1}, C_{F,1}\right) + \beta U\left(C_{H,2}, C_{F,2}\right)$, then the general problem can be stated as:

$$\begin{aligned} \max \quad & U\left(C_{H,1}, C_{F,1}\right) + \beta U\left(C_{H,2}, C_{F,2}\right) \\ s.t. \quad & B_1 = Y_{H,1} - \left(C_{H,1} + e_1 C_{F,1}\right) \\ & C_{H,2} + e_2 C_{F,2} = Y_{H,2} + (1+r) B_1 \end{aligned} \tag{P'}$$

where r is the interest rate (in terms of good H) and B_1 is the financial position of country 1 (also, in terms of good H).

The general approach to this problem is the usual one: now we have 3 prices (e_1, e_2, r), and we can find the demand for the 4 goods $(C_{H,1}, C_{F,1}, C_{H,2}, C_{F,2})$. Then we could study the autarky equilibrium and find the prices (e_1^a, e_2^a, r^a) such that the economy does not trade:

$$(C_{H,1}, C_{F,1}, C_{H,2}, C_{F,2}) = (Y_{H,1}, 0, Y_{H,2}, 0).$$

Then we could put together the two countries and find the world equilibrium prices (e_1, e_2, r) such that the four markets are clearing:

$$(C_{H,1}, C_{F,1}, C_{H,2}, C_{F,2}) + (C_{H,1}^*, C_{F,1}^*, C_{H,2}^*, C_{F,2}^*) = (Y_{H,1}, 0, Y_{H,2}, 0) + (0, Y_{F,1}^*, 0, Y_{F,2}^*).$$

However, this problem is not very easy to manage. To make the problem more manageable and derive useful expressions to think about the world equilibrium the idea is to split the problem in two: the financial side and the real side.

Think that the consumer solves problem $P\prime$ in two stages.

First he chooses how much to spend in each period, i.e. the expenditure levels EX_1 and EX_2, where

$$\begin{aligned} EX_1 &= C_{H,1} + e_1 C_{F,1}, \\ EX_2 &= C_{H,2} + e_2 C_{F,2}. \end{aligned}$$

Then he decides how much to spend in each of the two goods in each separate period.

The second problem in period t gives us:

$$\begin{aligned} V\left(EX_t, e_t\right) = \max_{C_{H,t}, C_{F,t}} \quad & U\left(C_{H,t}, C_{F,t}\right) \\ s.t. \quad & C_{H,t} + e_1 C_{F,t} = EX_t. \end{aligned} \tag{P2}$$

where $V\left(EX_t, e_t\right)$ is an "indirect utility function", that gives the utility of a consumer that can spend EX_t if the exchange rate is e_t.

The first problem is then

$$\begin{aligned} & \max V\left(EX_1, e_1\right) + \beta V\left(EX_2, e_2\right) \\ s.t.\ B_1 &= Y_{1,H} - EX_1, \\ EX_2 &= Y_{2,H} + (1+r) B_1. \end{aligned} \tag{P1}$$

Note one thing: problem $P1$ looks a lot like problem P that we solved in the case of one good.

A special case that is very useful is the case where

$$U\left(C_{H,t}, C_{F,t}\right) = \alpha \log C_{H,t} + (1-\alpha) \log C_{F,t}$$

in this case

$$V\left(EX_t, e_t\right) = \alpha \log \alpha + (1-\alpha) \log (1-\alpha) + \log\left(EX_t\right) - (1-\alpha) \log e_t$$

and the problem $P1$ is simply given by:

$$\begin{aligned} \max \quad & \log\left(EX_1\right) + \beta \log\left(EX_2\right) \\ s.t. \quad & B_1 = Y_{1,H} - EX_1, \\ & EX_2 = Y_{2,H} + (1+r) B_1. \end{aligned}$$

(remember: the constants in the utility function don't matter) and this is exactly the probelm we solved above.

From now on we will concentrate on the simple case of log utility.

Exercise 2 *Check that this expression is correct. Solve problem P2 and find* $V(EX_t, e_t)$ *in the case where the utility function is instead given by:*

$$U(C_{H,t}, C_{F,t}) = \left(1 - \frac{1}{\eta}\right)\left(C_{H,t}^{1-\frac{1}{\eta}} + C_{F,t}^{1-\frac{1}{\eta}}\right),$$

where η *is a positive constant.*

The financial side

The problem $P1$ gives us the optimal expenditure path

$$\begin{aligned} EX_1 &= \frac{1}{1+\beta}\left[Y_{1,H} + \frac{1}{1+r}Y_{2,H}\right] \\ EX_2 &= \frac{1}{1+r}\frac{\beta}{1+\beta}\left[Y_{1,H} + \frac{1}{1+r}Y_{2,H}\right] \end{aligned}$$

This is very similar to the problem we solved before.

The current account (in dollars) of the home country is given by

$$CA_1 = B_1 = Y_{1,H} - \frac{1}{1+\beta}\left[Y_{1,H} + \frac{1}{1+r}Y_{2,H}\right].$$

As in the case with one good, if country home is growing fast relative to the world interest rate so that

$$\frac{Y_{2,H}}{Y_{1,H}} > \beta(1+r)$$

then the current account will be in *deficit*.

The real side (the return of the Transfer Problem)

Once we have figured the optimal path for expenditure we want to figure out the demand for the two goods in period 1.

Remember that

$$EX_1 = Y_{H,1} + CA_1.$$

Now we have the demand of the home good

$$C_{H,1} = \alpha(Y_{H,1} + CA_1)$$

Now we can do the same for the foreign country and find

$$C_{H,1}^* = (1-\alpha)\left(e_1 Y_{F,1}^* + CA_1^*\right).$$

Equilibrium in the financial markes requires, as usual, that

$$CA_1 + CA_1^* = 0,$$

so $CA_1 > 0$ is very much like a "transfer".

Assume now that there is home bias in consumption, that is

$$\alpha > 1/2.$$

then the equilibrium in the goods markets in period 1 requires

$$\alpha Y_{H,1} + (1-\alpha)\, e_1 Y_{F,1}^{*} + (2\alpha - 1)\, CA_1 = Y_{H,1}.$$

From financial autarky to open world capital markets.

Now we make the following experiment. Start from a situation of *financial autarky*. Countries cannot borrow or lend from each other. However, there is free trade of goods. The financial markets are closed but the goods markets are open.

This means that the current account has to be zero in all periods: $CA_1 = 0$, that is we want to have trade balance in each period. To put it another way the home country has to finance all its imports with exports:

$$e_1 C_{F,1} = Y_{H,1} - C_{H,1}.$$

To put it yet another way, this means that expenditure always has to equal income

$$EX_1 = Y_{H,1}.$$

Then we obtain the equilibrium exchange rate in periods 1 and 2:

$$\alpha Y_{H,t} + (1-\alpha) e_t^{fa} Y_{F,t} = Y_{H,t}$$

$$e_t^{fa} = \frac{Y_{H,t}}{Y_{F,t}}$$

the exchange rate depends on the relative supply of the two goods period by period (fa stands for financial autarky).

Now suppose the financial markets open. Suppose that in the new equilibrium country Home is borrowing at date 1 and repaying at date 2. This means that $CA_1 = B_1 < 0$ and

$$\begin{aligned}
EX_1 &= Y_{1,H} + (-CA_1)\,, \\
EX_2 &= Y_{2,H} - (1+r)\,(-CA_1)\,, \\
EX_1^{*} &= e_1 Y_{1,F}^{*} - (-CA_1)\,, \\
EX_2^{*} &= e_2 Y_{2,F}^{*} + (1+r)\,(-CA_1)\,.
\end{aligned}$$

Now we can derive the equilibrium in the two periods and get

$$\begin{aligned}
\alpha Y_{H,1} + (1-\alpha) e_1 Y_{F,1} + (2\alpha - 1)\,(-CA_1) &= Y_{H,1} \\
\alpha Y_{H,2} + (1-\alpha) e_2 Y_{F,2} - (2\alpha - 1)\,(1+r)\,(-CA_1) &= Y_{H,2}
\end{aligned}$$

$$e_1 = \frac{Y_{H,1}}{Y_{F,1}} - 2\left(\alpha - \frac{1}{2}\right)\frac{-CA_1}{Y_{F,1}} < e_1^{fa} \quad (1)$$

$$e_2 = \frac{Y_{H,2}}{Y_{F,2}} + 2\left(\alpha - \frac{1}{2}\right)(1+r)\frac{-CA_1}{Y_{F,1}} > e_2^{fa} \quad (2)$$

We can summarize this as follows:

- When we open capital markets and country Home runs a current account deficit ($CA_1 < 0$) this brings about:

  (i) An appreciation of the currency in the short run.

  (ii) A depreciation in the long run.

Exercise 3 *Suppose that the countries have identical endowments over time* $Y_{H,1} = Y_{H,2} = Y_{F,1}^* = Y_{F,2}^* = 1$ *and suppose that* $\beta < \beta^*$.
(i) Show that the savings of country foreign are given by

$$B_1^* = e_1 - \frac{1}{1+\beta^*}\left[e_1 + \frac{1}{1+r}e_2\right].$$

(ii) Substitute (1) and (2) and use the fact that $B_1 = -B_1^*$ *to write* B_1^* *as a function of* r *alone (Hint: you will get something like*

$$B_1^* = \phi\frac{1}{1+\beta^*}\left[\beta^* - \frac{1}{1+r}\right]$$

where ϕ *is a constant that depends on* α*, if* $\alpha = 1/2$ *you should check that* $\phi = 1$*.)*
(iii) Show that the savings of country home are given by

$$B_1 = \frac{1}{1+\beta}\left(\beta - \frac{1}{1+r}\right).$$

(iv) Write down the equilibrium condition for international capital markets and show that it is given by an equation like

$$\phi\frac{1}{1+\beta^*}\left[\beta^*(1+r) - 1\right] = -\frac{1}{1+\beta}\left[\beta(1+r) - 1\right],$$

show that in a world equilibrium $r^{fa*} < r < r^{fa}$.
(v) Derive the equilibrium level of r*, show that a larger* α *implies that the interest rate is higher (i.e. closer to* r^{fa}*) and* $-B_1$ *is smaller.*