12.540 Principles of the Global Positioning System Lecture 02

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Coordinate Systems

- Today we cover:
  - Definition of coordinates
  - Conventional “realization” of coordinates
  - Modern realizations using spaced based geodetic systems (such as GPS).

Coordinate system definition

- To define a coordinate system you need to define:
  - Its origin (3 component)
  - Its orientation (3 components, usually the direction cosines of one axis and one component of another axes, and definition of handed-ness)
  - Its scale (units)

Coordinate system definition

- In all 7 quantities are needed to uniquely specify the frame.
- In practice these quantities are determined as the relationship between two different frames
- How do we measure coordinates
- How do we define the frames

Measuring coordinates

- Direct measurement (OK for graph paper)
- Triangulation: Snell 1600s: Measure angles of triangles and one-distance in base triangle
- Distance measured with calibrated “chain” or steel band (about 100 meters long)
- “Baseline” was about 1 km long
- Triangles can build from small to larges ones.
- Technique used until 1950s.

Measuring coordinates

- Small errors in the initial length measurement, would scale the whole network
- Because of the Earth is “nearly” flat, measuring angles in horizontal plane only allows “horizontal coordinates” to be determined.
- Another technique is needed for heights.

Measuring coordinates

- In 1950s, electronic distance measurement (EDM) became available (out growth of radar)
- Used light travel times to measure distance (strictly, travel times of modulation on either radio, light or near-infrared signals)

Measuring coordinates

- Advent of EDM allowed direct measurements of sides of triangles
- Since all distances measured less prone to scale errors.
- However, still only good for horizontal coordinates

Accuracies

- Angles can be measured to about 1 arc second (5×10^{-6} radians)
- EDM measures distances to 1×10^{-6} (1 part-per-million, ppm)
- Atmospheric refraction 300 ppm
- Atmospheric bending can be 60” (more effect on vertical angles)

Height coordinates

- Two major techniques:
  - Measurement of vertical angles (atmospheric refraction)
  - “Leveling” measurement of height differences over short distances (<50 meters).
  - Level lines were used to transfer height information from one location to another.

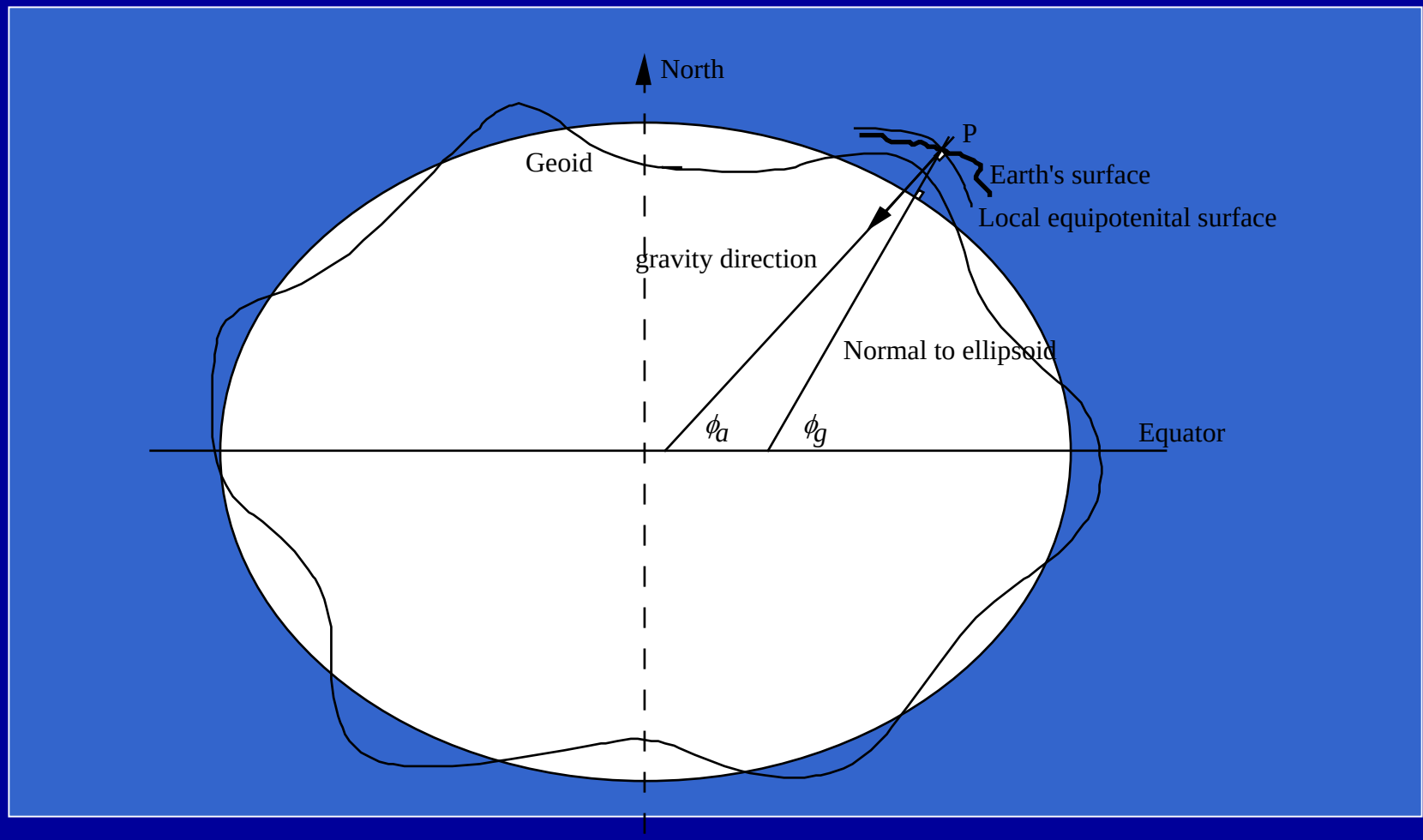
Other methods

- Maps were made with “plotting tables” (small telescope and angular distance measurements-angle subtended by a known distance
- Aerial photogrammetry coordinates inferred from positions in photographs. Method used for most maps

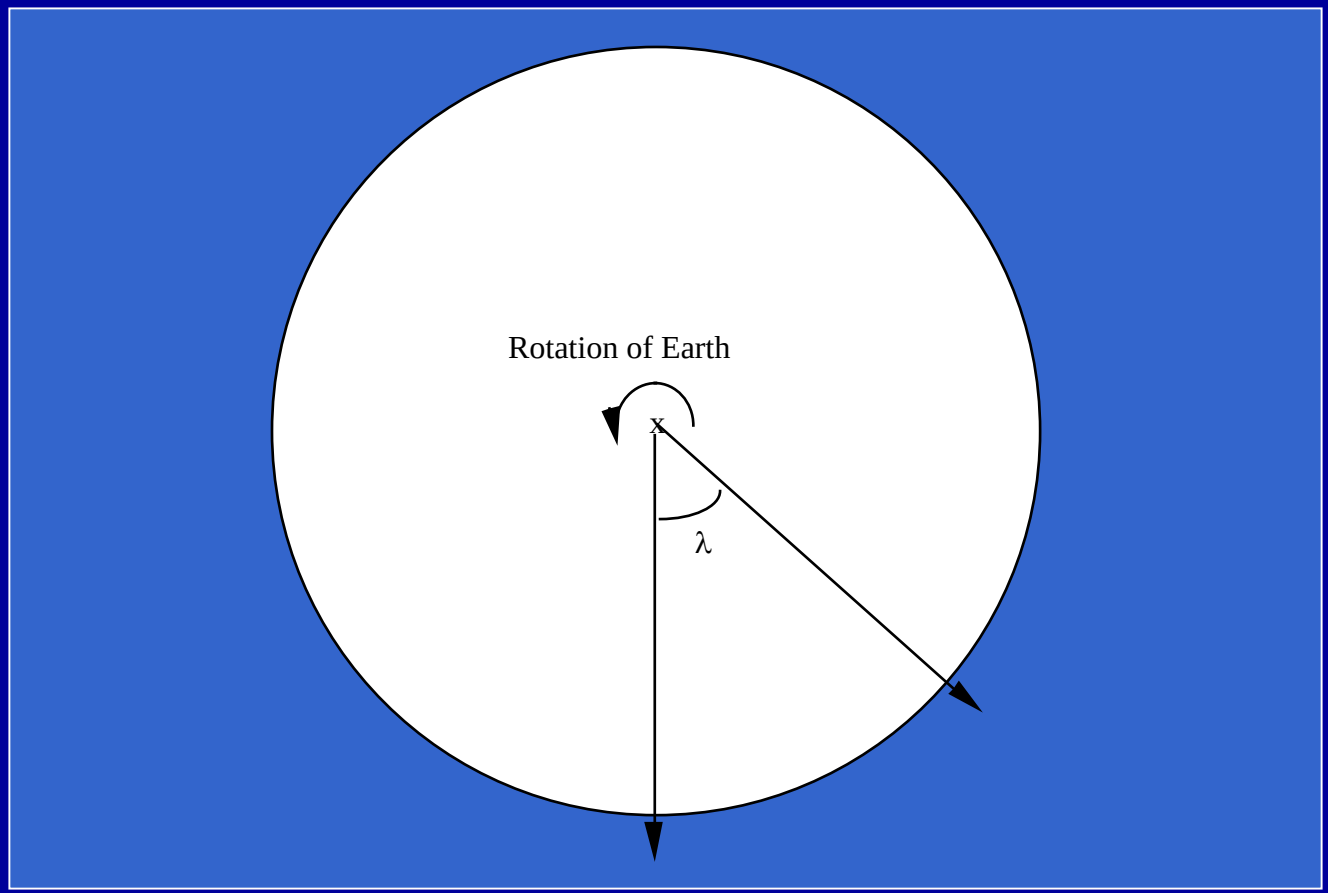
Other methods

- What is latitude and longitude
- Based on spherical model what quantities might be measured
- How does the rotation of the Earth appear when you look at the stars?
- Concept of astronomical coordinates

Geodetic coordinates: Latitude



Longitude



Longitude measured by time difference of astronomical events

Astronomical coordinates

- Return to later but on the global scale these provide another method of determining coordinates
- They also involve the Earth's gravity field
- Enters intrinsically in triangualtion and trilateration through the planes angles are meaured in

Web sites about geodetic measurements

- http://www.geography.wisc.edu/sco/geodetic/horizontal.html Geodetic control for Wisconsin
- http://www.ngs.noaa.gov/ is web page of National Geodetic Survey which coordinates national coordinate systems

Earth's Gravity field

- All gravity fields satisfy Laplace's equation in free space or material of density ρ. If V is the graviational potential then

$$\nabla^2 V = 0$$

$$\nabla^2 V = 4\pi G\rho$$

Solution to gravity potential

- The homogeneous form of this equation is a “classic” partial differential equation.
- In spherical coordinates solved by separation of variables, r=radius, λ=longitude and θ=co-latitude

$$V(r,\theta,\lambda)=R(r)g(\theta)h(\lambda)$$