12.540 Principles of the Global Positioning System
Lecture 05

Prof. Thomas Herring

Satellite Orbits

- Treat the basic description and dynamics of satellite orbits
- Major perturbations on GPS satellite orbits
- Sources of orbit information:
  - SP3 format from the International GPS service
  - Broadcast ephemeris message
- Accuracy of orbits and health of satellites
- Logistics: Who can attend lecture on Fridays at 11-12:30?

Dynamics of satellite orbits

- Basic dynamics is described by F=Ma where the force, F, is composed of gravitational forces, radiation pressure (drag is negligible for GPS), and thruster firings (not directly modeled).
- Basic orbit behavior is given by

$$\ddot{\mathbf{r}} = -\frac{GM_e}{r^3}\mathbf{r}$$

Simple dynamics

- $GM_e = \mu = 3986006\text{x}10^8\ m^3s^{-2}$
- The analytical solution to the central force model is a Keplerian orbit. For GPS these are elliptical orbits.
- Mean motion, n, in terms of period P is given by

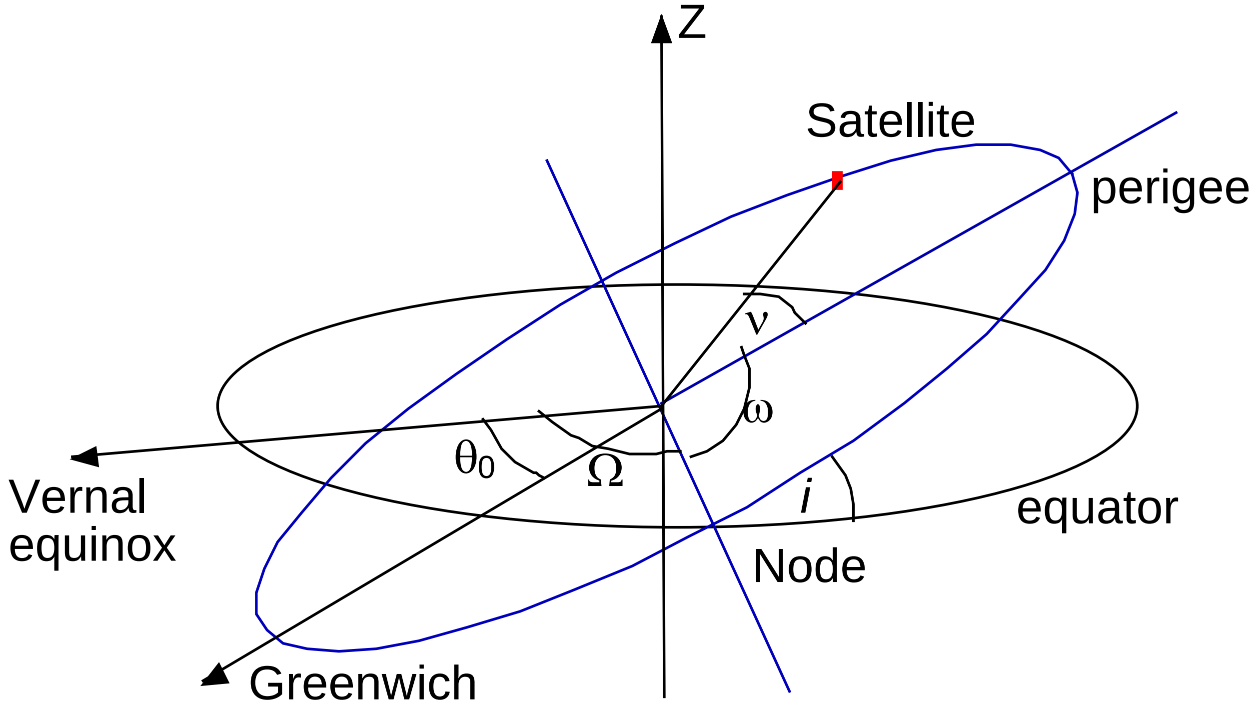
$$n = \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}}$$

- For GPS semimajor axis a ~ 26400km

Solution for central force model

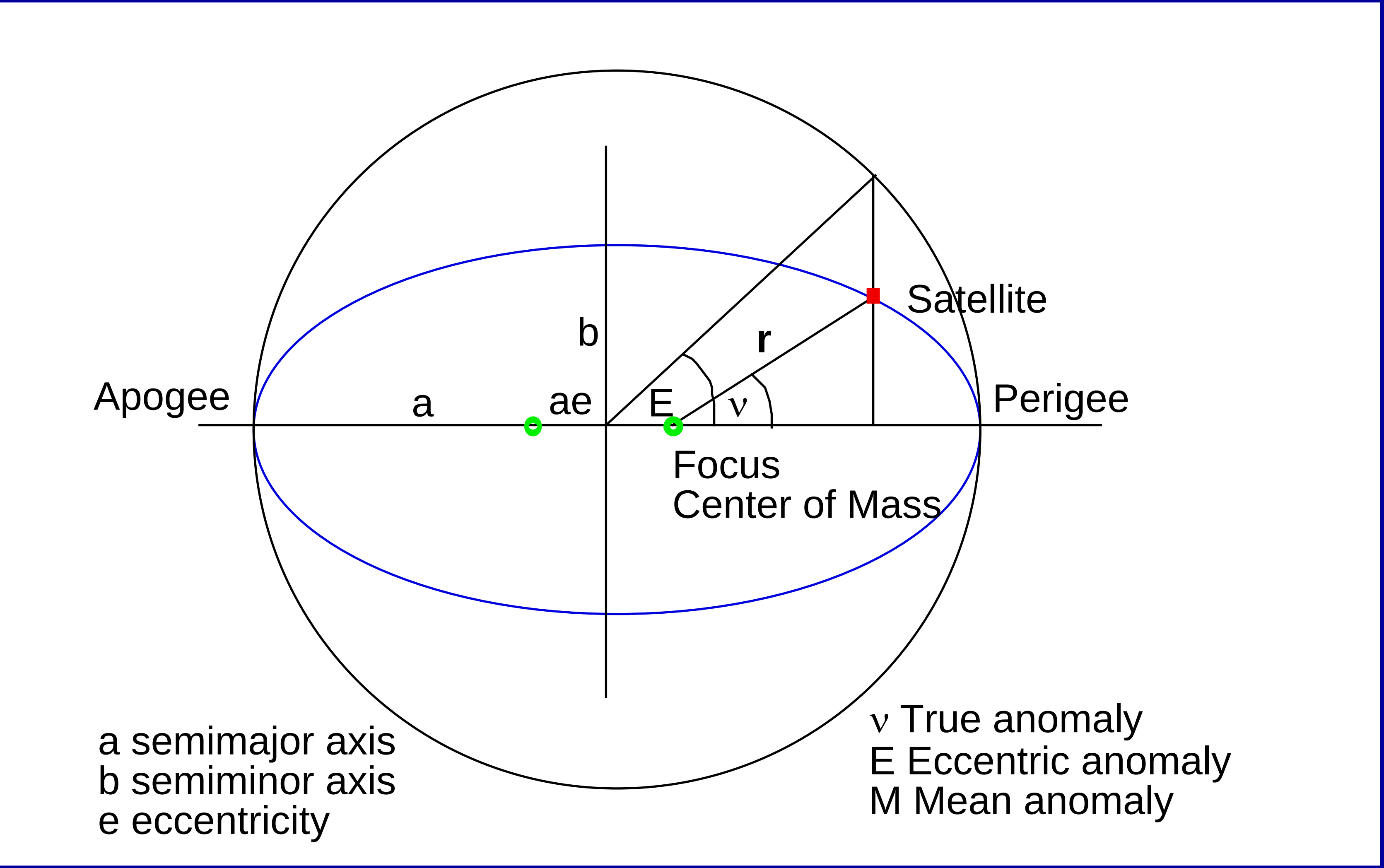
- This class of force model generates orbits that are conic sections. We will deal only with closed elliptical orbits.
- The orbit plane stays fixed in space
- One of the foci of the ellipse is the center of mass of the body
- These orbits are described Keplerian elements

Keplerain elements: Orbit plane



i Inclination
Ω Right Ascension of ascending node
ω Argument of perigee
ν True anomaly

Keplerain elements in plane



Satellite motion

- The motion of the satellite in its orbit is given by

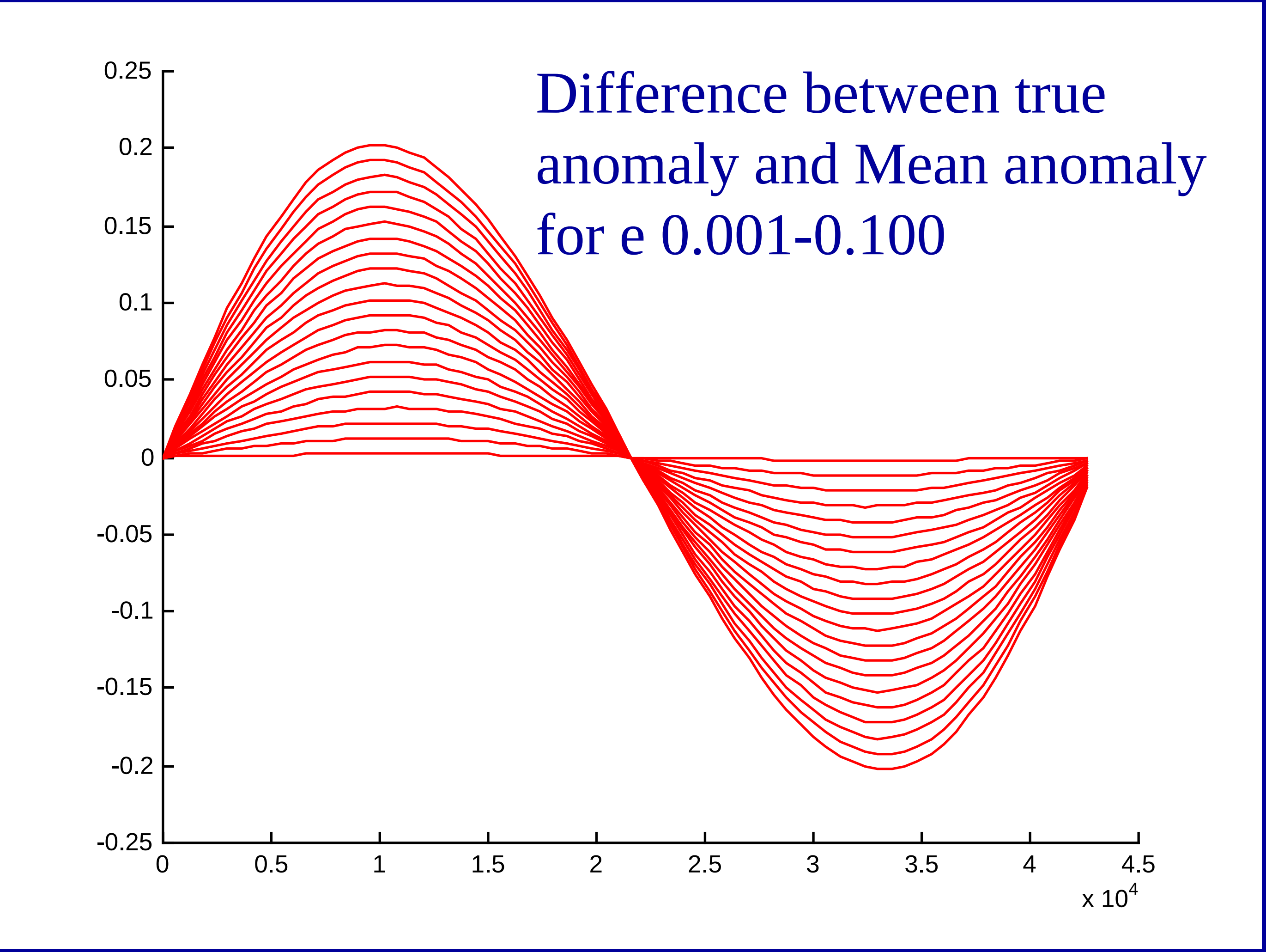
$$M(t) = n(t - T_0)$$

$$E(t) = M(t) + e \sin E(t)$$

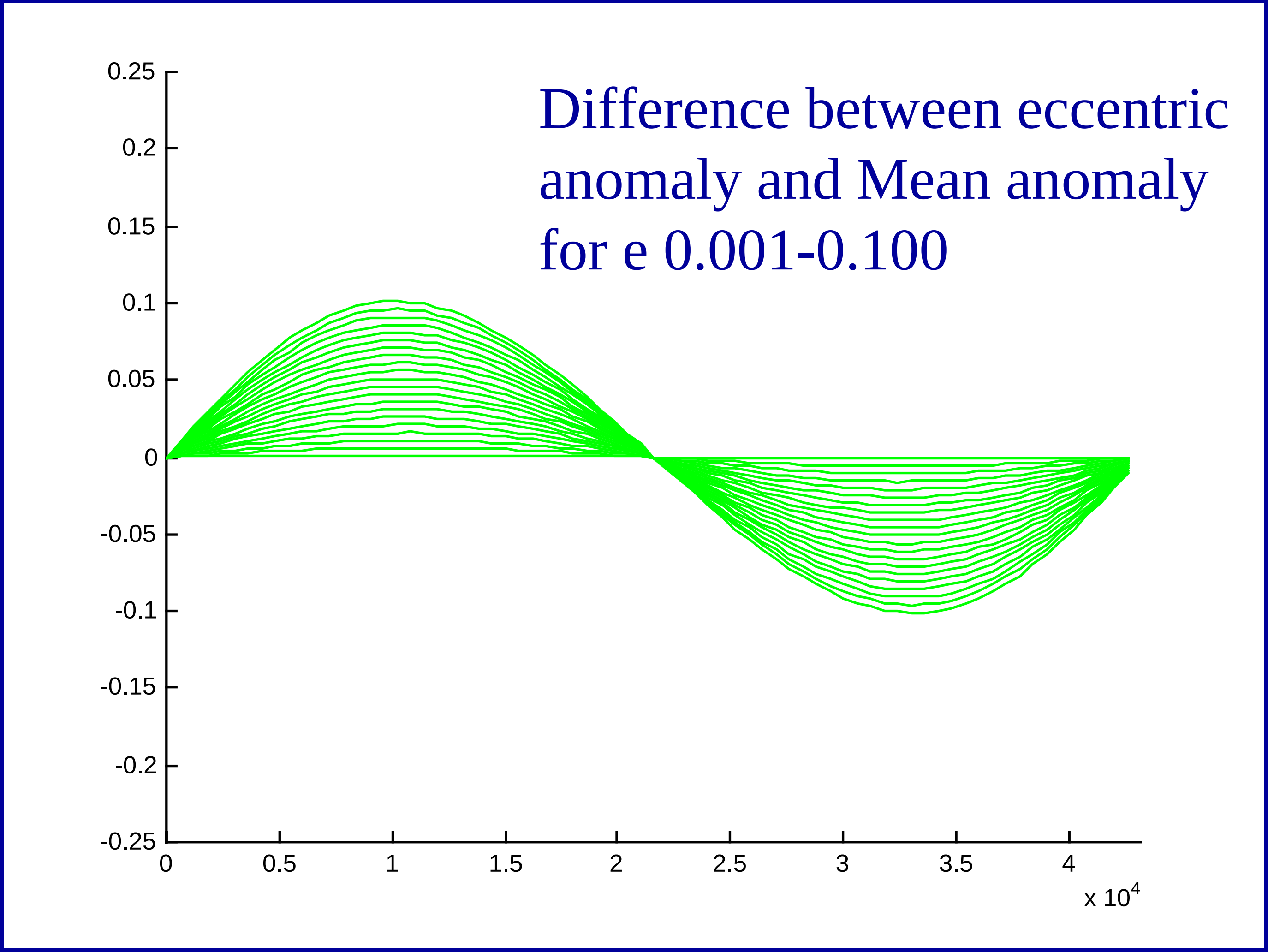
$$\nu(t) = \tan^{-1}\left[\frac{\sqrt{1-e^2}\sin E(t)/(1-e\cos E(t))}{(\cos E(t)-e)/(1-e\cos E(t))}\right]$$

- T_o is time of perigee

True anomaly



Eccentric anomaly



Vector to satellite

- At a specific time past perigee; compute Mean anomaly; solve Kepler's equation to get Eccentric anomaly and then compute true anomaly. See Matlab/truea.m
- Vector **r** in orbit frame is

$$\mathbf{r} = a\begin{bmatrix} \cos E - e \\ \sqrt{1-e^2}\sin E \end{bmatrix} = r\begin{bmatrix} \cos\nu \\ \sin\nu \end{bmatrix}$$

$$r = a(1 - e\cos E) = \frac{a(1-e^2)}{1+e\cos\nu}$$

Final conversion to Earth Fixed XYZ

- Vector r is in satellite orbit frame
- To bring to inertial space coordinates or Earth fixed coordinates, use

$$\mathbf{r_i} = R_3(-\Omega)R_1(-i)R_3(-\omega)\mathbf{r}$$

$$\mathbf{r_e} = R_3(-\Omega+\theta)R_1(-i)R_3(-\omega)\mathbf{r}$$

- This basically the method used to compute positions from the broadcast ephemeris

Perturbed motions

- The central force is the main force acting on the GPS satellites, but there are other significant perturbations.
- Historically, there was a great deal of work on analytic expressions for these perturbations e.g. Lagrange planetary equations which gave expressions for rates of change of orbital elements as function of disturbing potential
- Today: Orbits are numerically integrated although some analytic work on form of disturbing forces.

Perturbation from Flattening J_2

- The J_2 perturbation can be computed from the Lagrange planetary equations

$$\dot{\Omega} = -\frac{3}{2} n a_e^2 \frac{\cos i}{a^2(1-e^2)^2} J_2$$

$$\dot{\omega} = \frac{3}{4} n a_e^2 \frac{5\cos^2 i - 1}{a^2(1-e^2)^2} J_2$$

$$\dot{M} = n + \frac{3}{4} n a_e^2 \frac{3\cos^2 i - 1}{a^2\sqrt{(1-e^2)^3}} J_2$$

J_2 Perturbations

- Notice that only Ω ω and n are effected and so this perturbation results in a secular perturbation
- The node of the orbit precesses, the argument of perigee rotates around the orbit plane, and the satellite moves with a slightly different mean motion
- For the Earth, $J_2 = 1.08284\times10^{-3}$

Gravitational perturbation styles

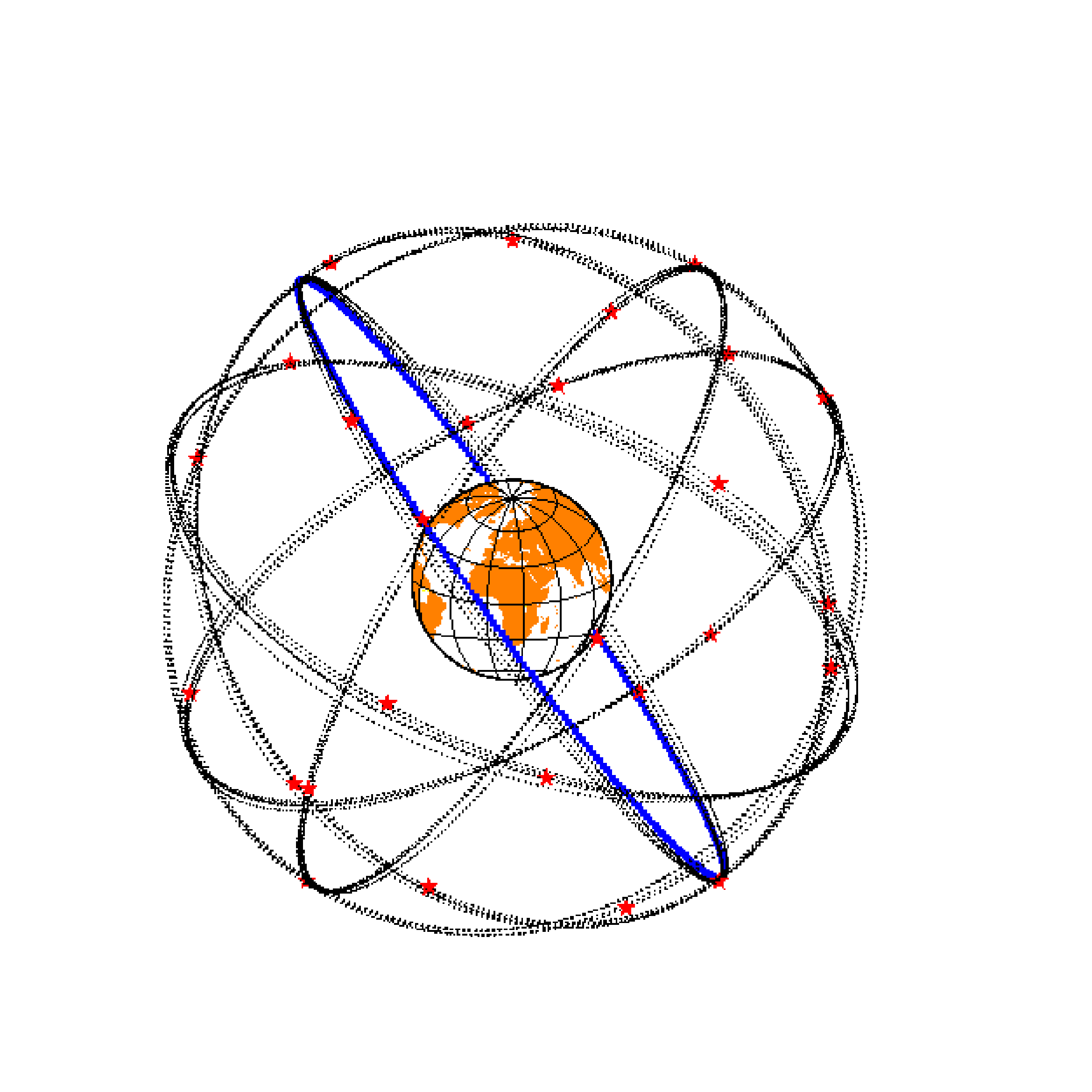
Parameter	Secular	Long period	Short period
a	No	No	Yes
e	No	Yes	Yes
i	No	Yes	Yes
Ω	Yes	Yes	Yes
ω	Yes	Yes	Yes
M	Yes	Yes	Yes

Other perturbation on orbits and approximate size

Term	**Acceleration (m/sec^2)**
Central	0.6
J_2	5×10^{-5}
Other gravity	3×10^{-7}
Third body	5×10^{-6}
Earth tides	10^{-9}
Ocean tides	10^{-10}
Drag	~0
Solar radiation	10^{-7}
Albedo radiation	10^{-9}

GPS Orbits

- Orbit characteristics are
  - Semimajor axis 26400 km (12 sidereal hour period)
  - Inclination 55.5 degrees
  - Eccentricity near 0 (largest 0.02)
  - 6 orbital planes with 4-5 satellites per plan
- Design lifetime is 6 years, average lifetime 10 years
- Generations: Block II/IIA 9729 kg, Block IIR 11000 kg



Basic Constellation

Orbits shown in inertial space and size relative to Earth is correct

Broadcast Ephemeris

- Satellites transmit as part of their data message the elements of the orbit
- These are Keplerian elements with periodic terms added to account for solar radiation and gravity perturbations
- Periodic terms are added for argument of perigee, geocentric distance and inclination
- The message and its use are described in the ICD-GPS-200 icd200c123.pdf (page 105 in PDF)

Distribution of Ephemerides

- The broadcast ephemeris is decoded by all GPS receivers and for geodetic receivers the software that converts the receiver binary to an exchange format outputs an ASCII version
- The exchange format: Receiver Independent Exchange format (RINEX) has a standard for the broadcast ephemeris.
- Form [4-char][Day of year][Session].[yy]n
  e.g. brdc0120.02n

RINEX standard

- Description of RINEX standard can be found at ftp://igscb.jpl.nasa.gov/igscb/data/format/rinex2.txt
- Homework number 1 also contains description of navigation file message (other types of RINEX files will be discussed later)
- 12.540.HW1.PDF is first homework: Due Fri March 8.