

# 12.540 Principles of the Global Positioning System Lecture 04

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## Review

- So far we have looked at measuring coordinates with conventional methods and using gravity field
- Today lecture:
  - Examine definitions of coordinates
  - Relationships between geometric coordinates
  - Time systems
  - Start looking at satellite orbits

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## Coordinate types

- Potential field based coordinates:
  - Astronomical latitude and longitude
  - Orthometric heights (heights measured about an equipotential surface, nominally mean-sea-level (MSL))
- Geometric coordinate systems
  - Cartesian XYZ
  - Geodetic latitude, longitude and height

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## Astronomical coordinates

- Astronomical coordinates give the direction of the normal to the equipotential surface
- Measurements:
  - Latitude: Elevation angle to North Pole (center of star rotation field)
  - Longitude: Time difference between event at Greenwich and locally

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## Astronomical Latitude

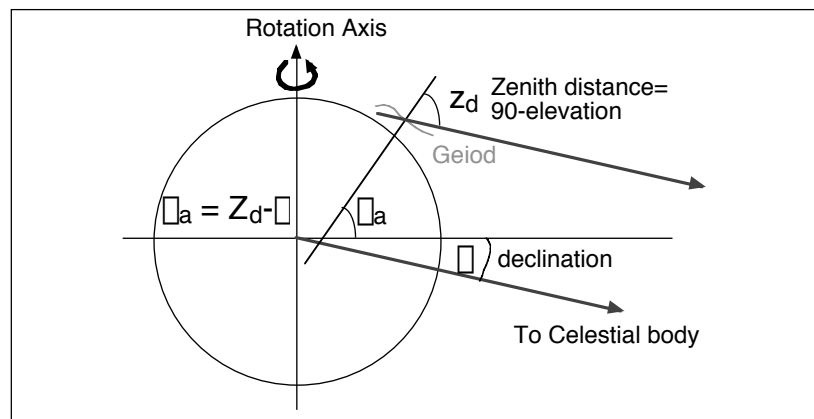
- Normal to equipotential defined by local gravity vector
- Direction to North pole defined by position of rotation axis. However rotation axis moves with respect to crust of Earth!
- Motion monitored by International Earth Rotation Service IERS <http://www.iers.org/>

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## Astronomical Latitude



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## Astronomical Latitude

- By measuring the zenith distance when star is at minimum, yields latitude
- Problems:
  - Rotation axis moves in space, precession nutation. Given by International Astronomical Union (IAU) precession nutation theory
  - Rotation moves relative to crust

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## Rotation axis movement

- Precession Nutation computed from Fourier Series of motions
- Largest term 9" with 18.6 year period
- Over 900 terms in series currently (see [http://bowie.mit.edu/~tah/mhb2000/JB000165\\_online.pdf](http://bowie.mit.edu/~tah/mhb2000/JB000165_online.pdf))
- Declinations of stars given in catalogs
- Some almanacs give positions of "date" meaning precession accounted for

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## Rotation axis movement

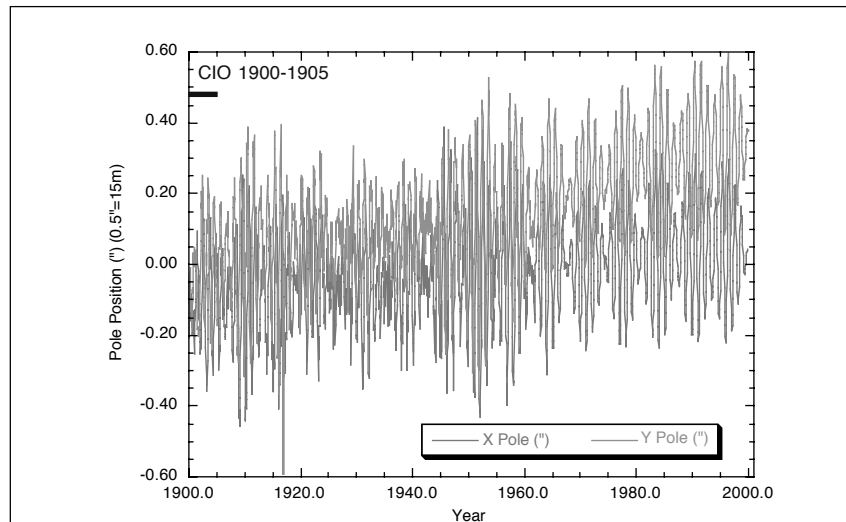
- Movement with respect crust called “polar motion”. Largest terms are Chandler wobble (natural resonance period of ellipsoidal body) and annual term due to weather
- Non-predictable: Must be measured and monitored

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## Evolution (IERS C01)

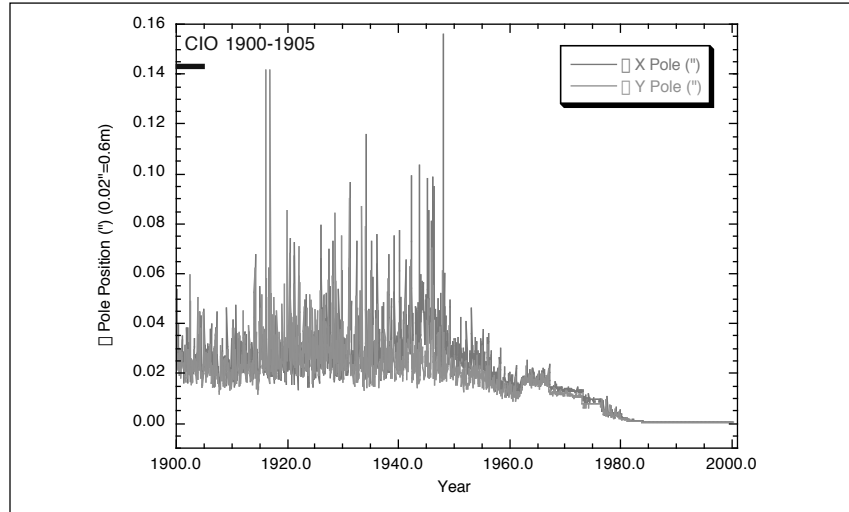


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## Evolution of uncertainty

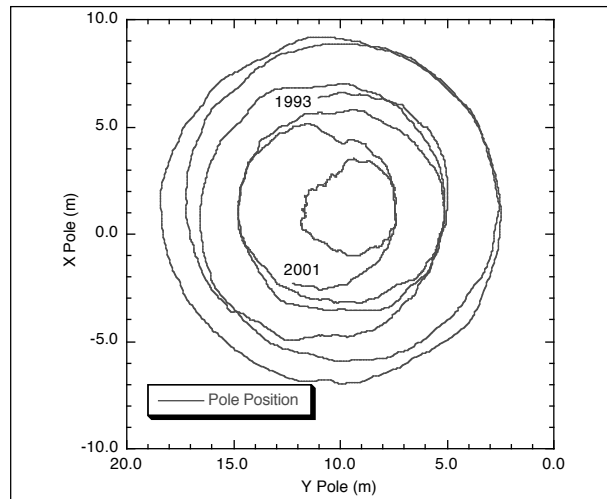


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## Behavior 1993-2001



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## Astronomical Longitude

- Based on time difference between event in Greenwich and local occurrence
- Greenwich sidereal time (GST) gives time relative to fixed stars

$$GST = 1.0027379093UT1 + \underbrace{\theta_0}_{\text{GMST}} + \underbrace{\theta \cos \epsilon}_{\text{Precession}}$$

$$\theta_0 = 24110.54841 + 8640184.812866 \underbrace{T}_{\text{Julian Centuries}} + 0.093104T^2 - 6.2 \times 10^{-6}T^3$$

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□

## Universal Time

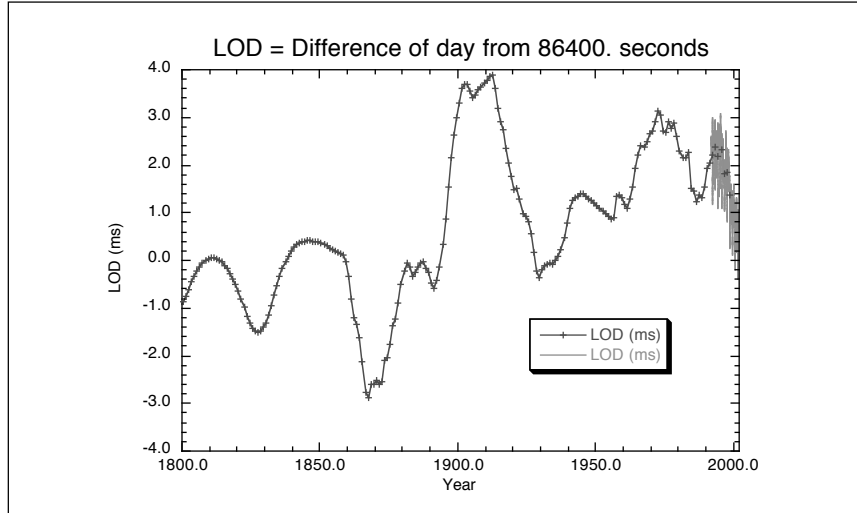
- UT1: Time given by rotation of Earth. Noon is “mean” sun crossing meridian at Greenwich
- UTC: UT Coordinated. Atomic time but with leap seconds to keep aligned with UT1
- UT1-UTC must be measured

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# Length of day (LOD)

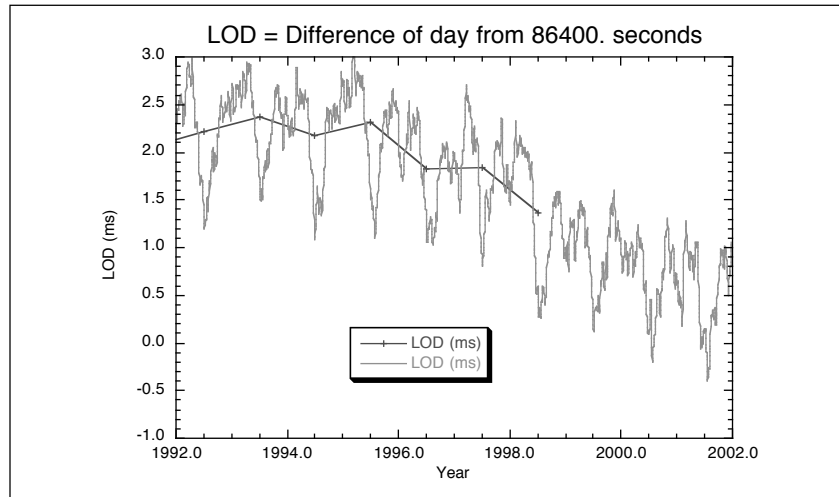


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# Recent LOD



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## LOD to UT1

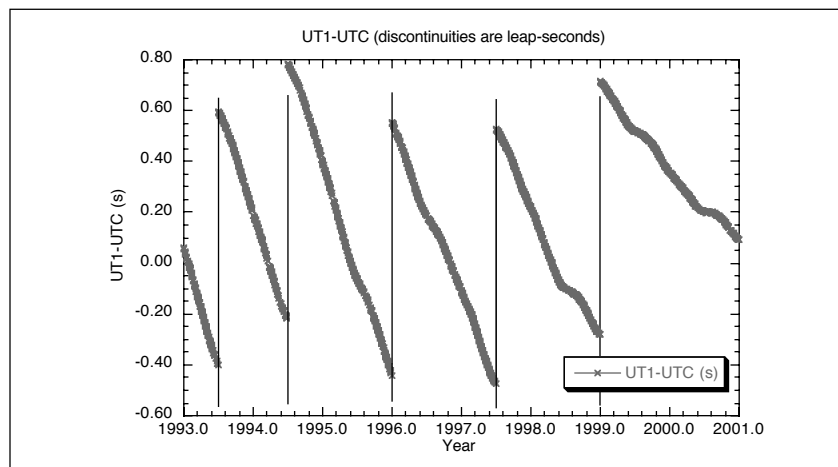
- Integral of LOD is UT1 (or visa-versa)
- If average LOD is 2 ms, then 1 second difference between UT1 and atomic time develops in 500 days
- Leap second added to UTC at those times.

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## UT1-UTC



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## Transformation from Inertial Space to Terrestrial Frame

- To account for the variations in Earth rotation parameters, as standard matrix rotation is made

$$\underbrace{x_i}_{\text{Inertial}} = \underbrace{P}_{\text{Precession}} \underbrace{N}_{\text{Nutation}} \underbrace{S}_{\text{Spin}} \underbrace{W}_{\text{Polar Motion}} \underbrace{x_t}_{\text{Terrestrial}}$$

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## Geodetic coordinates

- Easiest global system is Cartesian XYZ but not common outside scientific use
- Conversion to geodetic Lat, Long and Height

$$\begin{aligned} X &= (N + h) \cos \varphi \cos \lambda \\ Y &= (N + h) \cos \varphi \sin \lambda \\ Z &= \left(\frac{b^2}{a^2} N + h\right) \sin \varphi \\ N &= \frac{a^2}{\sqrt{a^2 \cos^2 \varphi + b^2 \sin^2 \varphi}} \end{aligned}$$

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## Geodetic coordinates

- WGS84 Ellipsoid:
  - $a=6378137$  m,  $b=6356752.314$  m
  - $f=1/298.2572221$  ( $=[a-b]/a$ )
- The inverse problem is usually solved iteratively, checking the convergence of the height with each iteration.
- (See Chapters 3 &10, Hofmann-Wellenhof)

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## Heights

- Conventionally heights are measured above an equipotential surface corresponding approximately to mean sea level (MSL) called the geoid
- Ellipsoidal heights (from GPS XYZ) are measured above the ellipsoid
- The difference is called the geoid height

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## Geoid Heights

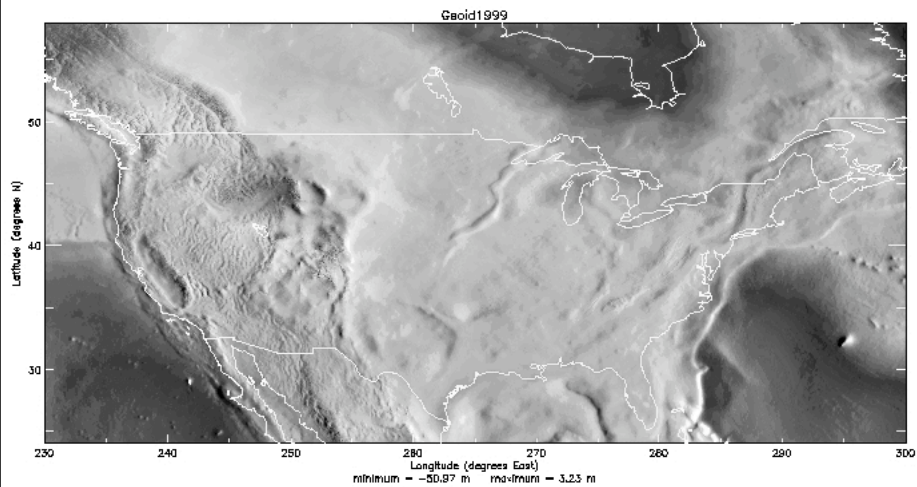
- National geodetic survey maintains a web site that allows geoid heights to be computed (based on US grid)
- [http://www.ngs.noaa.gov/cgi-bin/GEOID\\_STUFF/geoid99\\_prompt1.pl](http://www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/geoid99_prompt1.pl)
- New Boston geoid height is -27.688 m

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NGS Geoid 99 <http://www.ngs.noaa.gov/GEOID/GEOID99/>



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## Spherical Trigonometry

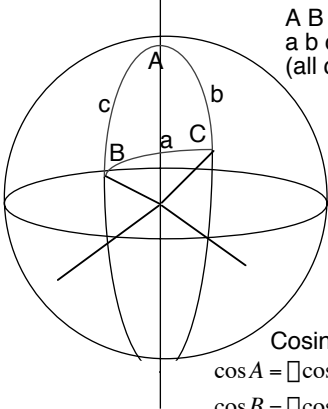
- Computations on a sphere are done with spherical trigonometry. Only two rules are really needed: Sine and cosine rules.
- Lots of web pages on this topic (plus software)
- <http://mathworld.wolfram.com/SphericalTrigonometry.html> is a good explanatory site

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## Basic Formulas



A B C are angles  
a b c are sides  
(all quantities are angles)

Sine Rule  

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Cosine Rule sides

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$

$$\cos c = \cos b \cos a + \sin a \sin b \cos C$$

Cosine Rule angles

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

$$\cos B = -\cos A \cos C + \sin A \sin C \cos b$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

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## Basic applications

- If  $b$  and  $c$  are co-latitudes,  $A$  is longitude difference,  $a$  is arc length between points (multiply angle in radians by radius to get distance),  $B$  and  $C$  are azimuths (bearings)
- If  $b$  is co-latitude and  $c$  is co-latitude of vector to satellite, then  $a$  is zenith distance (90-elevation of satellite) and  $B$  is azimuth to satellite
- (Colatitudes and longitudes computed from  $\square XYZ$  by simple trigonometry)

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## Summary of Coordinates

- While strictly these days we could realize coordinates by center of mass and moments of inertia, systems are realized by alignment with previous systems
- Both center of mass (1-2cm) and moments of inertia (10 m) change relative to figure
- Center of mass is used based on satellite systems
- When comparing to previous systems be cautious of potential field, frame origin and orientation, and ellipsoid being used.

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