

# 12.540 Principles of the Global Positioning System Lecture 18

Prof. Thomas Herring

# Mathematical models in GPS

- Review assignment dates (updated on class web page)
  - Paper draft due Mon April 29
  - Homework 3 due Fri May 03
  - Final class is Wed May 15. Oral presentations of papers. Each presentation should be 15-20 minutes, with additional time for questions.
- Next three lectures:
  - Mathematical models used in processing GPS
  - Processing methods used

# Mathematical models used in GPS

- Models needed for millimeter level positioning
- Review of basic estimation frame:
  - Data (phase and pseudorange) are collected at a sampling interval (usually 30-sec) over an interval usually a multiple of 24-hours. Typically 6-8 satellites are observed simultaneously
  - A theoretical model is constructed to model these data. This model should be as complete as necessary and it uses apriori values of the parameters of the model.
  - An estimation is performed in which new values of some of the parameters are determined that minimize some cost function (e.g., RMS of phase residuals).
  - Results in the form of normal equations or covariance matrices may be combined to estimate parameters from many days of data (Dong D., T. A. Herring, and R. W. King, Estimating Regional Deformation from a Combination of Space and Terrestrial Geodetic Data, *J. Geodesy*, 72, 200–214, 1998.)

# Magnitude of parameter adjustments

- The relative size of the data noise to effects of a parameter uncertainty on the observable determines in general whether a parameter should be estimated.
- In some cases, certain combinations of parameters can not be estimated because the system is rank deficient (discuss some examples later)
- How large are the uncertainties in the parameters that effect GPS measurements?

# Magnitudes of parameter adjustments

- Major contributions to GPS measurements:
  - Pseudorange data: Range from satellite to receiver, satellite clock and receiver clock ( $\pm 10$  cm)
  - Phase data: Range from satellite to receiver, satellite clock oscillator phase, receiver clock oscillator phase and number of cycles of phase between satellite and receiver ( $\pm 2$  mm)
- Range from satellite to receiver depends on coordinates of satellite and ground receiver and delays due to propagation medium (already discussed).
- How rapidly do coordinates change? Satellites move at 1 km/sec; receivers at 500 m/s in inertial space.
- To compute range coordinates must in same frame.

# Parameter adjustment magnitudes

- Already discussed satellite orbital motion: Parameterized as initial conditions (IC) at specific time and radiation model parameters.
- For pseudo range positioning, broadcast ephemeris is often adequate. Post-processed orbits (IGS)  $\pm 3-5$  cm (may not be adequate for global phase processing).
- Satellites orbits are easiest integrated in inertial space, but receiver coordinates are nearly constant in an Earth-fixed frame.
- Transformation between the two systems is through the Earth orientation parameters (EOP). Discussed in Lecture 4.

# EOP variations

- If analysis is near real-time, variations in polar motion and UT1 will need to be estimated.
- After a few weeks, these are available from IERS ( $\pm 0.2$  mas of pole position, 0.05 ms UT1) in the ITRF2000 no-net-rotation system.
- For large networks, normally these parameters are re-estimated. Partials are formed by differentiating the arguments of the rotation matrices for the inertial to terrestrial transformation.

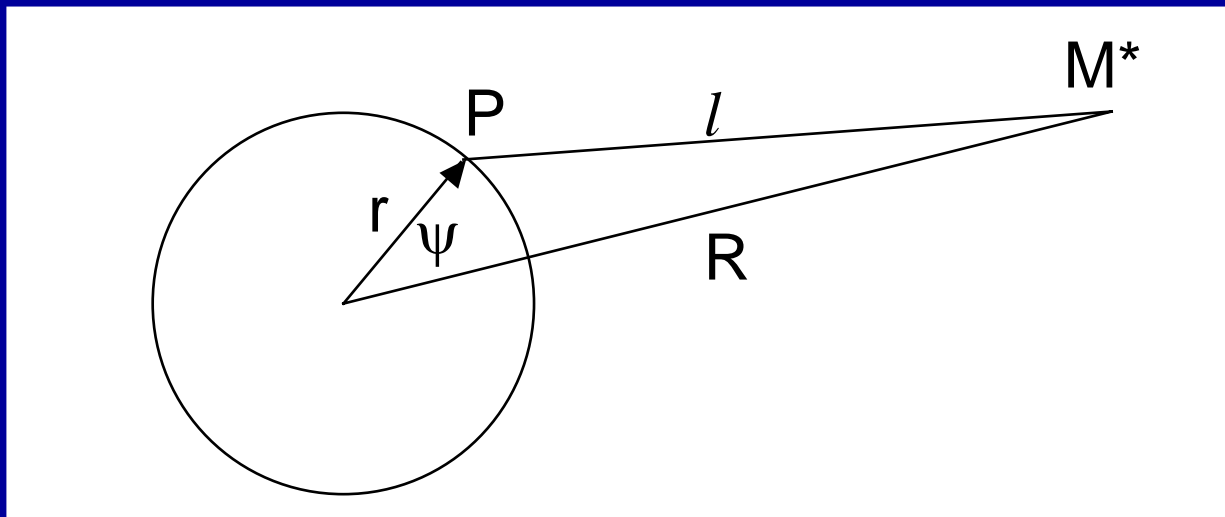
# Position variations in ITRF frame

- The International Terrestrial Reference Frame (ITRF) defines the positions and velocities of ~1000 locations around the world (GPS, VLBI, SLR and DORIS).
- Frame is defined to have no net rotation when motions averaged over all tectonic plates.
- However, a location on the surface of the Earth does not stay at fixed location in this frame: main deviations are:
  - Tectonic motions (secular and non-secular)
  - Tidal effects (solid Earth and ocean loading)
  - Loading from atmosphere and hydrology
- First are normally accounted for.



# Solid Earth Tides

- Solid Earth Tides are the deformations of the Earth caused by the attraction of the sun and moon. Tidal geometry



# Solid Earth Tide

- The potential at point P  $U=GM^*/l$
- We can expand  $1/l$  as:

$$\frac{1}{l} = \frac{1}{\sqrt{R^2 - 2Rr \cos \psi + r^2}} = \sum_{n=0}^{\infty} \frac{r^n}{R^{n+1}} P_n(\cos \psi)$$

- For  $n=0$ ;  $U_0$  is  $GM^*/R$  and is constant for the whole Earth
- For  $n=1$ ;  $U_1=GM^*/R^2[r \cos \psi]$ . Taking the gradient of  $U_1$ ; force is independent of position in Earth. This term drives the orbital motion of the Earth

# Solid Earth Tide

- The remaining terms are the tidal potential,  $U_T$ .

$$U_T = \frac{GM}{R} \sum_{n=2}^{\infty} \left(\frac{r}{R}\right)^n P_n(\cos \psi)$$

$$U_T = GM \left( \frac{1}{l} - \frac{1}{R} - \frac{\mathbf{R} \cdot \mathbf{r}}{R^3} \right)$$

- Second form is often referred to as the “vector” tide model (convenient if planetary ephemerides are available)

# Solid Earth Tide

- The work of Love on tides showed that the response of the (spherical) Earth is dependent on the degree  $n$  of the tidal deformation and that:

$$d_r = \frac{h_n}{g} U_T \hat{e}_r \quad \text{Radial}$$

$$d_t = \frac{l_n}{g} \nabla U \hat{e}_t = \frac{l_n}{g} \frac{\partial U_T}{\partial \theta} \hat{\theta} + \frac{l_n}{g \sin \theta} \frac{\partial U_T}{\partial \lambda} \hat{\lambda} \quad \text{Tangential}$$

# Solid Earth Tide

- $h_n$  and  $l_n$  are called Love numbers (also a  $k_n$  for the change in potential, needed for orbit integration).
- For the moon  $r/R=1/60$  and for the Sun  $r/R=1/23,000$ : Most important tidal terms are 2nd degree harmonics:  $k_2=0.3$ ;  $h_2=0.609$ ;  $l_2=0.085$
- Expand the second harmonic term in terms of  $\theta, \lambda$  of point and  $\theta', \lambda'$  extraterrestrial body

# Solid Earth Tide:

- Resultant expansion gives characteristics of tides:

$$U_T = \frac{GM^* r}{R} \left\{ \left[ \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right] \left[ \frac{3}{2} \cos^2 \theta' - \frac{1}{2} \right] \rightarrow \text{Long period} \right. \\ \left. + \cos \theta \sin \theta \cos \theta' \sin \theta' [\cos(\lambda - \lambda')] \rightarrow \text{Diurnal} \right. \\ \left. + \frac{1}{4} \sin^2 \theta \sin^2 \theta' [\cos 2(\lambda - \lambda')] \rightarrow \text{Semidiurnal} \right.$$

# Solid Earth Tide

- Magnitude:  $h_2(GM^*/R)r=26.7$  cm.
- Long period tides: 0 at  $\pm 35$  degree latitude
- Diurnal tides: Max at mid-latitudes
- Semidiurnal tides: zero at poles
- The planetary positions  $\theta'$ ,  $\lambda'$  have periodic variations that set the primary tidal frequencies.
- Major lunar tide M2 has a variation with period of 13.66 days (1/2 lunar period)
- Additional consideration: Presence of fluid core affects the tides. Largest effect is  $\Delta h_2 = -0.089$  at 1 cycle/sidereal day

# Series expansion

- The tidal potential can be expanded in series in terms of:

$l_m, l_s$  - Longitude of moon and sun,

$\omega$  - Argument of lunar perigee,

GST - Greenwich sidereal time

- The other system used with tides is Doodson's arguments:

$\tau$  - Time angle in lunar days;

$s, h$  - Mean longitude of Sun and Moon

$p, p_1$  - Long of Moon's and Sun's perigee

$N'$  - Negative of long of Moon's Node



## Earth tides:

- The Fortran routines `earth_tide.f` and `gst_jd.f` compute the tidal displacement at any location on the Earth. (This routine uses numerical derivatives for the tangential components. Analytic derivatives are not that difficult to derive.)
- (The `const_param.h` file contains quantities such as  $\pi$ ).

# Ocean tides

- The ocean tides also load the solid Earth causing an additional tidal like signal in the Earth.
- At the temporal frequencies of the tides, both systems behave linearly and so the temporal frequencies of the response is the same.
- For the solid Earth tides, the spatial frequency response is also linear but not so for the ocean tides.
- The  $P_2$  forcing of the ocean tides, generates many spherical harmonics in the ocean response and thus the solid earth response has a complex spatial pattern.

# Ocean tides

- The solution to the ocean tidal loading problem requires knowledge of the ocean tide potential (the level of the tides) and the loading response of the Earth.
- The loading problem has a similar solution to the standard tidal problem but in this case load Love numbers, denoted  $k_n'$ ,  $h_n'$  and  $l_n'$  are used.

# Load Love numbers

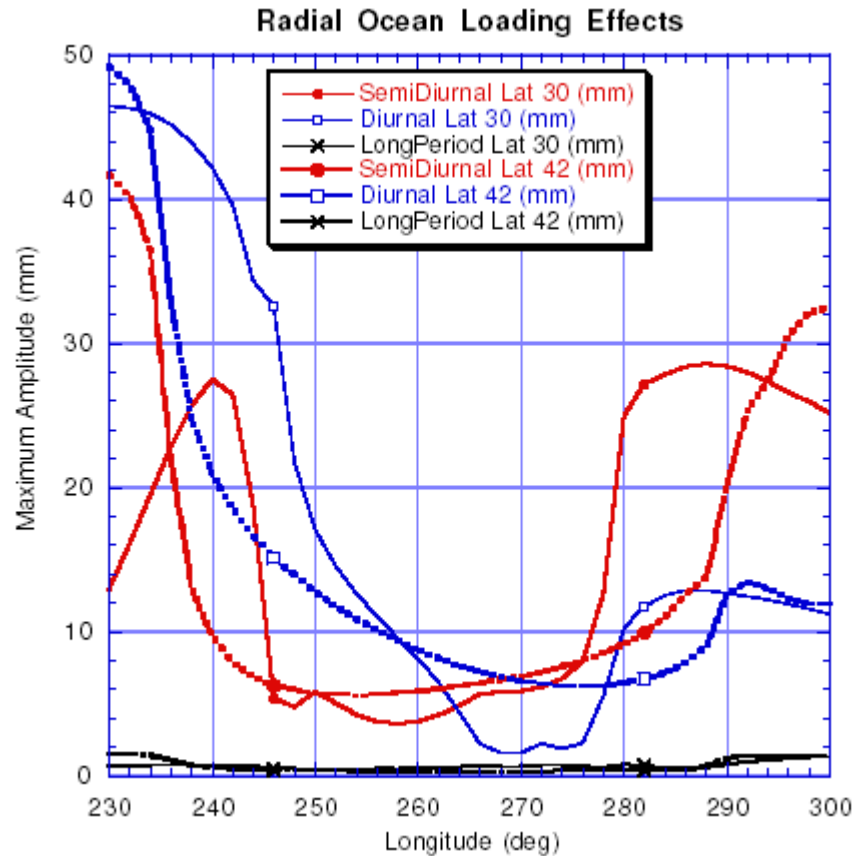
- The load love numbers depend on the elastic properties of the Earth (deduced from seismic velocities)

n	$-h_n'$	$nl_n'$	$-nk_n'$
1	0.290	0.113	0
2	1.001	0.059	0.615
3	1.052	0.223	0.585
4	1.053	0.247	0.527

# Load signal calculations

- For simple homogenous Earth, the Love numbers depend on rigidity of the Earth
- Load signals can be computed by summing all the spherical harmonics.
- An alternative is a Green's function approach (Farrell, 1972) in which the response to a point load is computed (the point load is expanded in spherical harmonics)
- The Green's function can then be convolved with a surface load to compute the amount of deformation.

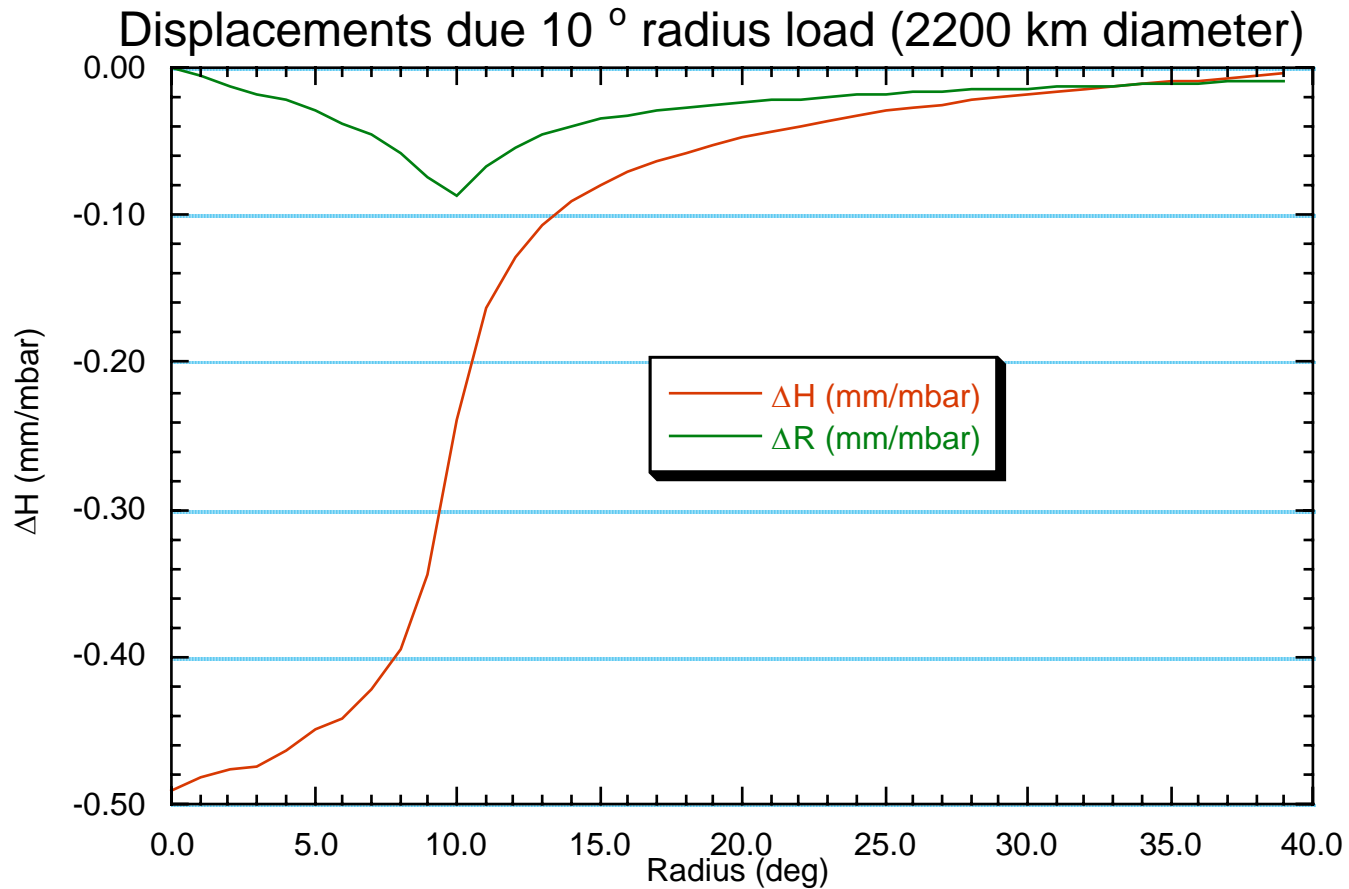
# Ocean loading magnitudes



# Loading signals

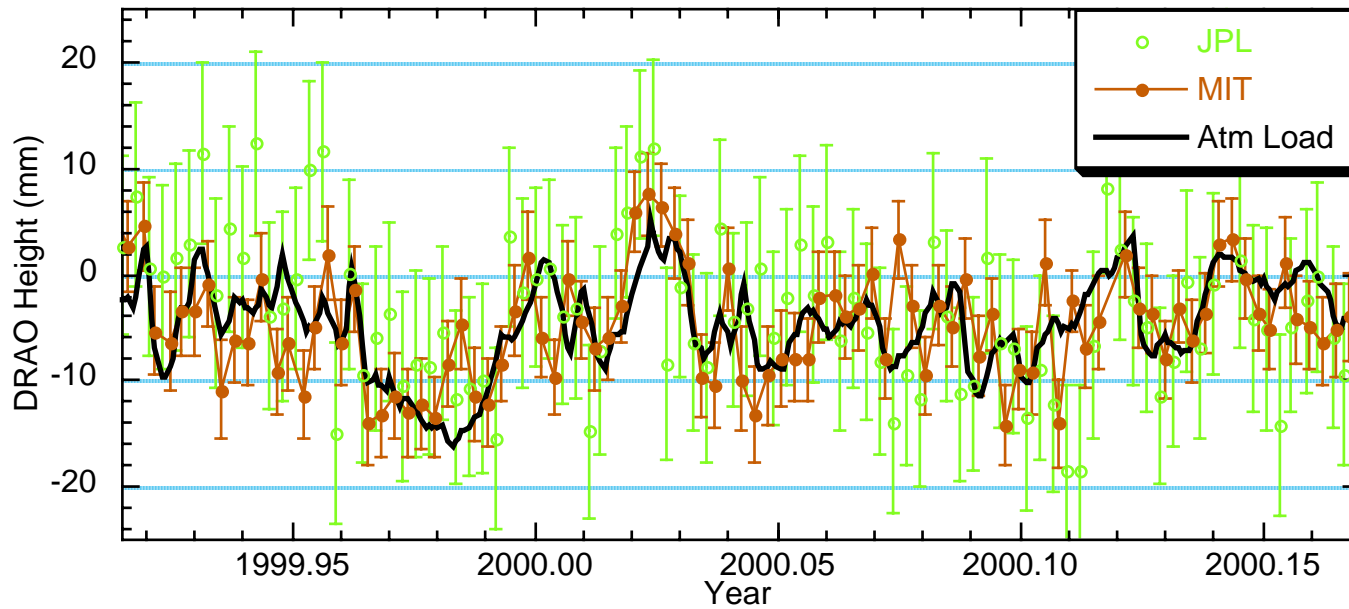
- In addition to ocean tidal loading, any system that loads the surface will cause loading deformations.
- Main sources are:
  - Atmospheric pressure loading ( $\sim 0.5$  mm/mbar). Often short period, but annual signals in some parts of the world.
  - Surface water loading ( $\sim 0.5$  mm/cm of water). More difficult to obtain load data
- In some locations, sediment expansion when water added (eg. LA basin)

# Basic loading effect





# Example: Penticton Canada



# Summary

- Tides and loading
  - All high-precision GPS analyses account for solid-Earth tides; most account for ocean tidal loading
  - Loading effects for the atmosphere, surface water and non-tidal ocean loading are not commonly directly applied because inputs are uncertain.
  - Atmospheric pressure loading could be routinely applied soon (data sets are high quality)
  - Gravity mission GRACE might recover surface loads well enough to allow these to be applied routinely (current research topic).