

**Massachusetts Institute of Technology**  
**Department of Electrical Engineering and Computer Science**  
6.061/6.979 Introduction to Power Systems

Problem Set 10 Solutions

April 20, 2003

The bulk of this problem set solution is contained in the single (fairly long) MATLAB script that is appended.

**Problem 1:** The torque-speed curve is generated in a straightforward way by first finding the input impedance of the machine, finding terminal current, doing the current divider thing to find rotor current and using that to find torque. The resulting curve is shown in Figure 1

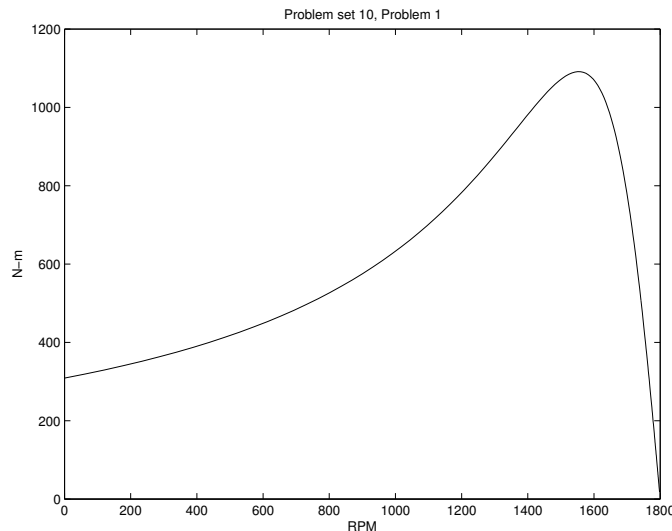


Figure 1: Torque-Speed Curve

**Problem 2:** Terminal current comes essentially for free and is shown in Figure 2

**Problem 3:** To find breakdown torque, recognize that is produced when the rotor equivalent circuit is dissipating maximum power, and that condition is when rotor resistance  $\frac{R_2}{s}$  is equal to its source impedance. So we first calculate source impedance, set  $\frac{R_2}{s}$  equal, which means setting  $s$ , and finding what the machine does. Here is a snippet of output from the script:

```
6.061 Problem Set 10, Problem 3
Breakdown Torque = 1091.17
Slip at Breakdown = 0.136681
Speed at Breakdown = 1553.97
Current at Breakdown Torque = 386.776
Power Factor at Breakdown Torque = 0.696053
```

**Problem 4:** Running 'light', the only power output is windage, and we have to do a little searching to find the correct slip. Fortunately, in the low slip region, torque and power are both very

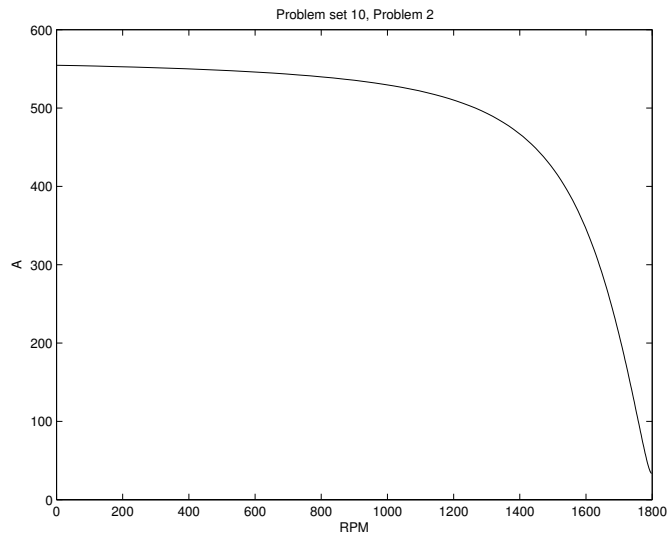


Figure 2: Current-Speed Curve

nearly proportional to slip so the search procedure is simple. The resulting input power is very little more than core plus windage loss, but the power factor is, as expected, quite low.

**Problem 4: Running Light**  
 Rotational Speed = 1798.89 RPM  
 Real Power = 3071.34  
 Reactive Power = 27385

**Problem 5** Blocked rotor is easier as current is defined. So we find voltage along with torque.

**Problem 5: Blocked Rotor at 120 A**  
 Terminal Voltage = 59.9627  
 Real Power = 4429.59  
 Reactive Power = 21127.2  
 Blocked Rotor Torque = 14.4539

**Problem 6: For 6.979** The trick here is to find the values of slip corresponding with five and one hundred percent of rated torque. This is done using a technique very similar to that in finding the 'running light' condition, but in this case we must add rated and five percent torque. Just to check, I calculated output power for the rated case. Once the two values of slip are calculated we can do a sweep between them to get efficiency and power factor, which are in Figures 3 and 4 respectively.

**Problem 7: For 6.979** This is straightforward: all that we change is frequency and voltage between curves. The results are plotted in Figure 5

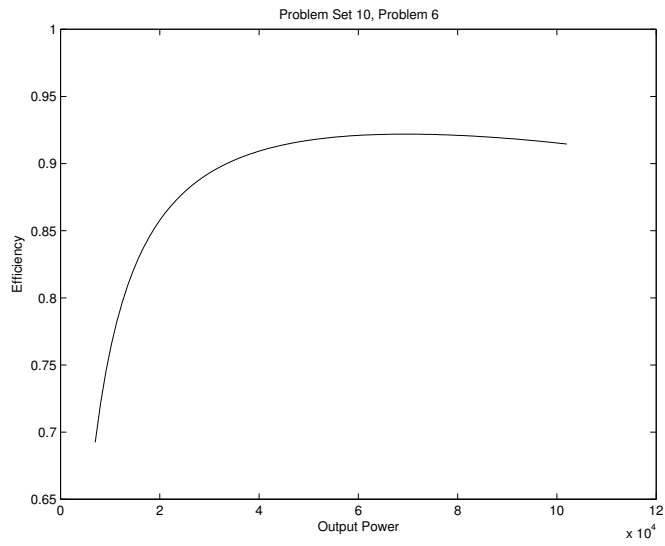


Figure 3: Rated Frequency Efficiency

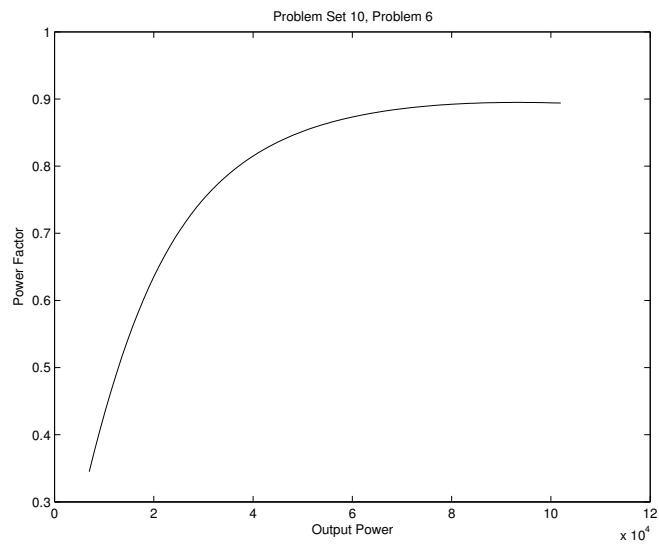


Figure 4: Rated Frequency Power Factor

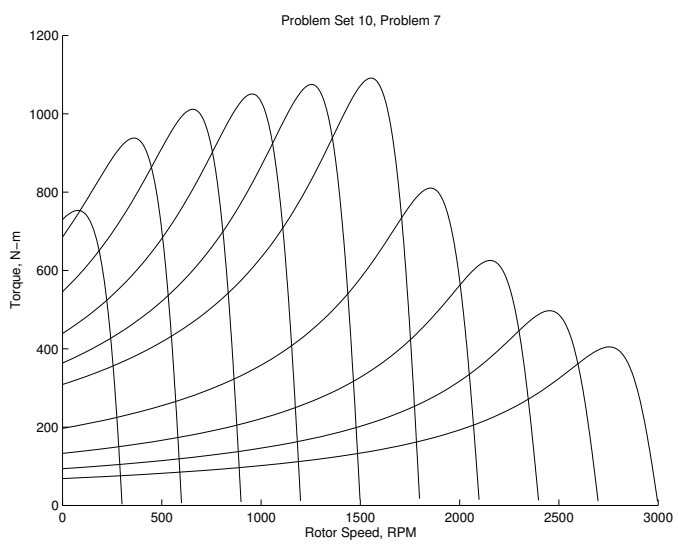


Figure 5: Torque Curves: Multiple Frequencies

```

% 6.061/6,979 Spring 2003 Problem Set 10
% Parameters:
Vll = 480;           % line-line voltage
p = 2;             % four pole machine
f = 60;           % line frequency, Hz
R1 = .0392;
R2 = .0670;
X1 = .2480;
X2 = .2480;
Xm = 8.16;
Pc = 1000;
Pw = 2000;
Prat = 100000;     % this is rated power
tol = 1e-14;      % tolerance for iterative stuff
% find core loss element
Vlg = Vll/sqrt(3);
Rc = Vlg^2/(Pc/3); % only 1/3 per phase!
% Problem 1
s = logspace(-3, 0, 200); % trick to spacing points
ome = 2*pi*f;      % electrical frequency
omm = (ome/p) .* (1 - s); % mechanical frequency
N = (f/(2*pi)) .* omm; % speed in RPM
Zr = j*X2 + R2 ./ s; % rotor circuit branch impedance
Zg = j*Xm*Rc / (j*Xm + Rc); % gap and core elements
Zag = Zg .* Zr ./ (Zg + Zr); % air-gap impedance
Zt = Zag + j*X1 + R1; % terminal impedance
It = Vlg ./ Zt;    % terminal current
I2 = It .* Zg ./ (Zg + Zr); % current divider ratio
T = (3*p/ome) .* abs(I2) .^2 .* R2 ./ s;

figure(1)
plot(N, T)
title('Problem set 10, Problem 1')
ylabel('N-m')
xlabel('RPM')

% problem 2
figure(2)
plot(N, abs(It))
title('Problem set 10, Problem 2')
ylabel('A')
xlabel('RPM')

% Problem 3: To find breakdown torque
Za = R1 + j*X1; % armature branch

```

```

Zs = Za*Zg/(Za+Zg)+j*X2;           % source impedance
sM = R2/(abs(Zs));                 % slip at max torque
Zrm = j*X2 + R2 / sM;              % rotor circuit branch impedance
Zagm = Zg * Zrm / (Zg + Zrm);      % air-gap impedance
Ztm = Za + Zagm;                   % terminal impedance
Itm = Vlg/Ztm;                     % terminal current
I2m = Itm *Zg/(Zg+Zrm);            % current divider
Tm = (3*p/ome) * abs(I2m) ^2 * R2 / sM;
Nb = (60/(2*pi))*(ome/p)*(1-sM);
pfm = real(Itm)/abs(Itm);          % power factor
fprintf('6.061 Problem Set 10, Problem 3\n');
fprintf('Breakdown Torque = %g\n',Tm);
fprintf('Slip at Breakdown = %g\n',sM);
fprintf('Speed at Breakdown = %g\n',Nb);
fprintf('Current at Breakdown Torque = %g\n',abs(Itm));
fprintf('Power Factor at Breakdown Torque = %g\n',pfm);

%problem 4: running light
not_done=1;                         % we need to iterate here
s1 = .001;                           % starting guess
while not_done==1,
    Zr = j*X2 + R2 / s1;              % rotor circuit branch impedance
    Zg = j*Xm*Rc / (j*Xm + Rc);      % gap and core elements
    Zag = Zg * Zr / (Zg + Zr);       % air-gap impedance
    Zt = Zag + j*X1 + R1;            % terminal impedance
    It = Vlg / Zt;                   % terminal current
    I2 = It .* Zg ./ (Zg + Zr);      % current divider ratio
    Pm = 3*I2^2*(R2/s1)*(1-s1);
    if abs(1-Pm/Pw)^2 <tol
        not_done=0;                 % slip is about right
    else
        s1 = s1*(Pw/Pm);            % new guess for slip
        fprintf('.')
    end
end
end
P1 = real(3*Vlg*conj(It));
Q1 = imag(3*Vlg*conj(It));
N1 = 60*(f/p)*(1-s1);
fprintf('\nProblem 4: Running Light\n');
fprintf('Rotational Speed = %g RPM\n',N1);
fprintf('Real Power = %g\n',P1);
fprintf('Reactive Power = %g\n',Q1);

% Problem 5: Blocked Rotor
Zr = j*X2 + R2;                      % rotor circuit branch impedance (s=1)

```

```

Zg = j*Xm*Rc / (j*Xm + Rc);      % gap and core elements
Zag = Zg * Zr / (Zg + Zr);      % air-gap impedance
Zt = Zag + j*X1 + R1;          % terminal impedance
It = 120;                       % terminal current is fixed
Vb = It*Zt;                     % required terminal voltage
I2 = It .* Zg ./ (Zg + Zr);     % current divider ratio
Pb = real(3*Vb*conj(It));
Qb = imag(3*Vb*conj(It));
Tb = 3*abs(I2)^2*R2*p/ome;      % torque

fprintf('\nProblem 5: Blocked Rotor at %g A\n', It);
fprintf('Terminal Voltage = %g\n',abs(Vb));
fprintf('Real Power = %g\n',Pb)
fprintf('Reactive Power = %g\n',Qb);
fprintf('Blocked Rotor Torque = %g\n',Tb);
% Problem 6
% Now we are going to get power factor and efficiency: first
% we need to find slip for the full power point
Ptarget = Prat+Pw;              % gonna adjust slip to get this out
notdone = 1;
s0 = .01;
while notdone == 1,
    Zr = j*X2 + R2 / s0;        % rotor circuit branch impedance
    Zag = Zg * Zr / (Zg + Zr);  % air-gap impedance
    Zt = Zag + j*X1 + R1;      % terminal impedance
    It = Vlg / Zt;             % terminal current
    I2 = It * Zg / (Zg + Zr);  % current divider ratio
    Pag = 3*abs(I2)^2*R2/s0;   % air-gap power
    Pm = Pag*(1-s0)-Pw;
    if abs(Pm/Ptarget -1)^2 < tol, % close enough
        notdone = 0;
    else
        s0 = s0*Ptarget/Pm;    % try new value for slip
        fprintf(' ');
    end
end
end
Sr = s0;                        % max value of slip
% just to check...
Tm = (p/ome)*(Pm-Pw)/(1-s0);    % output torque
Nm = 60*ome*(1-s0)/(2*pi*p);    % running speed
Pout = Tm*2*pi*Nm/60;          % check output power
fprintf('Check: Full Power Running\n')
fprintf('Speed = %g RPM (slip = %g)\n',Nm, s0);

```

```

fprintf('Torque = %g Power = %g\n',Tm, Pout);

% now to find the value for five percent of power
Ptarget = .05*Prat+Pw;
notdone = 1;
s0 = .01;
while notdone == 1,
    Zr = j*X2 + R2 / s0;           % rotor circuit branch impedance
    Zag = Zg * Zr / (Zg + Zr);    % air-gap impedance
    Zt = Zag + j*X1 + R1;        % terminal impedance
    It = Vlg / Zt;               % terminal current
    I2 = It * Zg / (Zg + Zr);    % current divider ratio
    Pag = 3*abs(I2)^2*R2/s0;     % air-gap power
    Pm = Pag*(1-s0)-Pw;
    if abs(Pm/Ptarget -1) < tol, % close enough
        notdone = 0;
    else
        s0 = s0*Ptarget/Pm;      % try new value for slip
        fprintf(' ');
    end
end
Sm = s0;                         % min value of slip

% now we can do this against slip

s = Sm:(Sr-Sm)/100:Sr;          % slips to use
Zr = j*X2 + R2 ./ s;           % rotor circuit branch impedance
Zg = j*Xm*Rc / (j*Xm + Rc);    % gap and core elements
Zag = Zg .* Zr ./ (Zg + Zr);   % air-gap impedance
Zt = Zag + j*X1 + R1;         % terminal impedance
It = Vlg ./ Zt;               % terminal current
I2 = It .* Zg ./ (Zg + Zr);    % current divider ratio
Pag = 3 .* abs(I2) .^2 .* R2 ./ s;
Pt = 3 .* real(Vlg .* conj(It));
Pm = Pag .* (1 - s) - Pw;
eff = Pm ./ Pt;
pf = Pt ./ (3*Vlg .* abs(It));

figure(3)
    plot(Pm, eff)
    title('Problem Set 10, Problem 6')
    xlabel('Output Power')
    ylabel('Efficiency')

figure(4)

```



```

    plot(Pm, pf)
    title('Problem Set 10, Problem 6')
    xlabel('Output Power')
    ylabel('Power Factor')

% Problem 7: voltgs per Hz control
F_r = [1/6 2/6 3/6 4/6 5/6 1 7/6 8/6 9/6 10/6]; % for these relative frequencies
V_r = [1/6 2/6 3/6 4/6 5/6 1 1 1 1 1]; % and voltages

figure(5)
clf
hold on

% now it is just like the very first part of the problem

for i=1:length(F_r);
    fa = f*F_r(i); % here is our absolute frequency
    vlga = Vlg*V_r(i); % need to adjust reactances for frequency
    X2a = X2*F_r(i);
    Xma = Xm*F_r(i);
    X1a = X1*F_r(i);

    s = logspace(-3, 0, 200); % trick to spacing points
    ome = 2*pi*fa; % electrical frequency
    omm = (ome/p) .* (1 - s); % mechanical frequency
    N = (60/(2*pi)) .* omm; % speed in RPM
    Zr = j*X2a + R2 ./ s; % rotor circuit branch impedance
    Zg = j*Xma*Rc / (j*Xma + Rc); % gap and core elements
    Zag = Zg .* Zr ./ (Zg + Zr); % air-gap impedance
    Zt = Zag + j*X1a + R1; % terminal impedance
    It = vlga ./ Zt; % terminal current
    I2 = It .* Zg ./ (Zg + Zr); % current divider ratio
    T = (3*p/ome) .* abs(I2) .^2 .* R2 ./ s;

    plot (N, T)
end
hold off
title('Problem Set 10, Problem 7')
ylabel('Torque, N-m')
xlabel('Rotor Speed, RPM')

```