Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.061 Introduction to Power Systems

Quiz 1 March 19, 2003

Problem 1: First, get the actual transformer ration:

$$\frac{N_\Delta}{N_Y} = \sqrt{3} \times \frac{480}{240} = 3.464$$

Now the voltages on the secondary side must be:

$$\begin{aligned} V_a &= \frac{2400}{\sqrt{3}} e^{j\frac{\pi}{6}} \\ V_b &= \frac{2400}{\sqrt{3}} e^{-j\frac{\pi}{2}} \\ V_c &= \frac{2400}{\sqrt{3}} e^{j\frac{5\pi}{6}} \end{aligned}$$

Then the voltage across the resistor is:

$$V_b - V_c = 240 e^{-j\frac{\pi}{3}}$$

Current in the resistor is thus

$$I = \frac{1}{R} \times \left(V_b - V_c = 240 e^{-j\frac{\pi}{3}} \right) = 10 e^{-j\frac{\pi}{3}}$$

On the left-hand side of the transformer:

$$\begin{aligned} I_A &= \frac{10}{3.464} e^{-j\frac{\pi}{3}} \\ I_B &= \frac{10}{3.464} e^{-j\frac{\pi}{3}} \\ I_C &= -2 \times \frac{10}{3.464} e^{-j\frac{\pi}{3}} \\ |I_A| = |I_C| &= 2.89A \\ |I_C| &= 2 \times 2.89 = 5.78A \end{aligned}$$

Problem 2: See the figure: the various voltages are simply constructed

- From Phase Terminal a to ground? 208 V

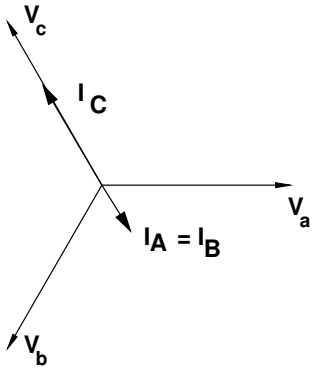

Figure 1: Template for your answer to Problem 1

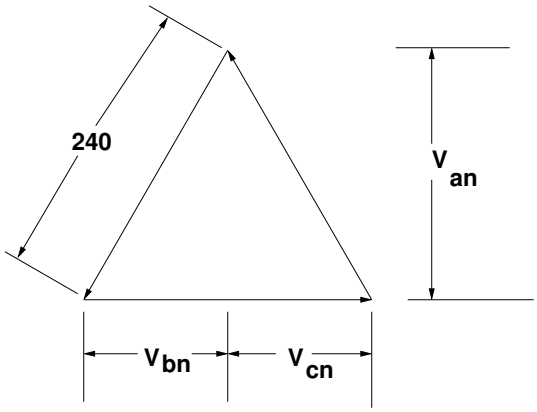

Figure 2: Transformer Voltages

- From Phase Terminal b to ground? 120 V
- From Phase Terminal c to ground? 120 V

Problem 3 1. To start we need to find the phase angle δ across the line: Power transfer is:

$$P = \frac{1000^2}{10} \sin\delta = 50,000$$

which implies that

$$\sin\delta = \frac{1}{3}$$

or

$$\delta = \frac{\pi}{6} = 30^\circ$$

Then

$$Q = \frac{1000^2}{10}(1 - \cos\delta) = 10^5\left(1 - \frac{\sqrt{3}}{2}\right) = 13.4\text{kVAR}$$

2. Now we add capacitors.To make the power factor at the sending end unity we must *supply* as much reactive power as was being supplied by the source:

$$X_c = \frac{10^6}{13,400} = 74.6\Omega$$

3. The currents must line up with the voltages. Since the voltages have equal magnitude and a 30° phase shift, they will lie along the ±15° lines on the chart. The currents contributed by the capacitors are perpendicular to the voltages.Note that the sign of the currents called out here is the opposite of what you would usually use. Also see that $I_S + I_{CS} = I_L$ and $I_L + I_{CR} = I_R$.

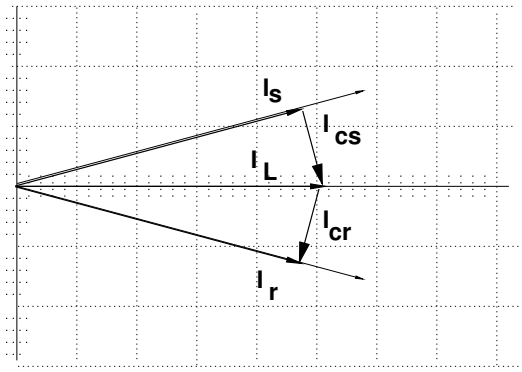

Figure 3: Answer to Problem 3

Problem 4 This is part of an old homework problem! You should already know that the answer is:

$$V_o(t) = 1000e^{-1000t}$$

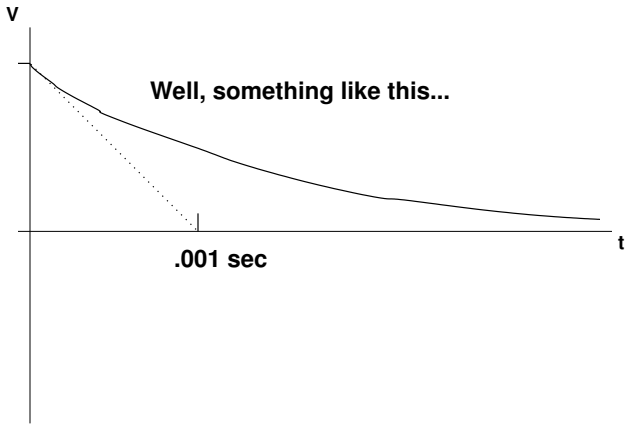

Figure 4: Space for your answer to Problem 4