Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.061/6.979 Introduction to Power Systems

Solution To Problem Set 1 February 8, 2003

Problem 1: Domestic circuits in the United States have a nominal voltage of 120V, RMS and come in two current ratings: 15A and 20A. It will have taken you a little bit of lookup in handbooks, but you should have found that 1HP = 746W and 1BTU = 1054J. Since one watt-hour is 3,600 J, we have 1BTU/hour = .293W. The resulting capability, assuming unity power factor, is:

Circuit	15A	20A
P (W)	1800	2400
P (HP)	2.4	3.2
P(BTU/hr)	6164	8219

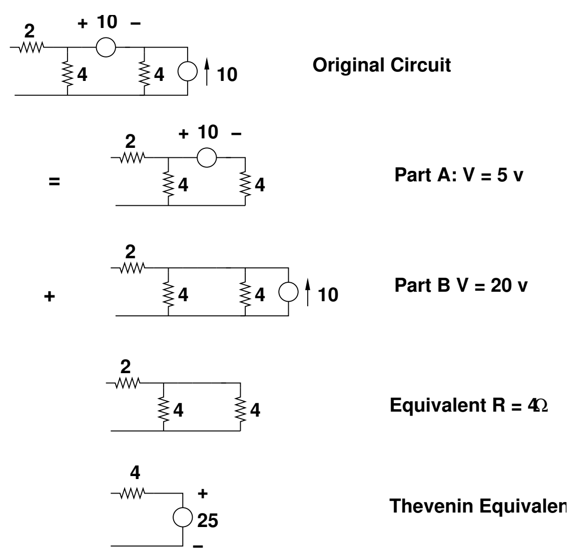

Figure 1: Circuit

Problem 2: Figure 1 shows the steps in finding the equivalent circuit. Superposition of the voltages computed using 'Part A' and 'Part B' yields open circuit voltages of 5 V and 20 V, respectively, and the equivalent input impedance is $R_{th} = 2 + 4||4 = 2 + 2 = 4$.

Problem 3: The two-port resistance matrix representation of the circuit of the first part of Figure 2 is:

$$\underline{\underline{R}} = \begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix}$$

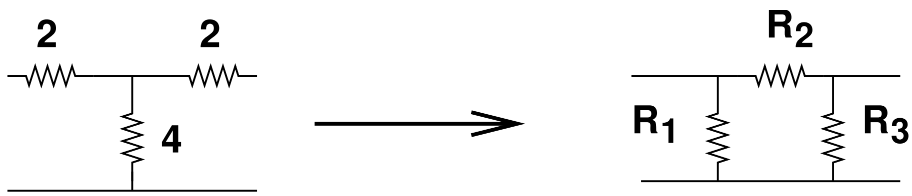

Figure 2: Circuit

The inverse of this is

$$\underline{\underline{Y}} = \underline{\underline{R}}^{-1} = \begin{bmatrix} \frac{6}{20} & -\frac{4}{20} \\ -\frac{4}{20} & \frac{6}{20} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_3} + \frac{1}{R_2} \end{bmatrix}$$

Thus it is clear that $R_2 = 5$ and

$$\frac{1}{R_1} + \frac{1}{5} = \frac{1}{R_3} + \frac{1}{5} = \frac{6}{20}$$

which is easily solved to be $R_1 = R_3 = 10$

As a check, note that the driving point and transfer resistances of the second circuit are:

$$\begin{aligned} R_{11} &= R_1 || R_2 = 10||15 = \frac{150}{25} = 6 \\ R_{12} &= R_1 \frac{R_3}{R_1 + R_2 + R_3} = \frac{100}{25} = 4 \end{aligned}$$

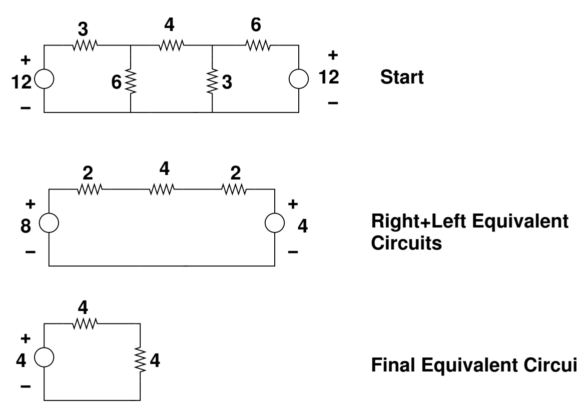

Figure 3: Loaded Bridge

Problem 4: With reference to Figure 3, note that the circuit shown on the problem set can be re-drawn as shown opposite 'Start'. That has split the voltage source into two, but it is still the same situation. Then find the Thevenin equivalent circuits as shown in the 'Right+Left...'

part of the picture. The Thevenin equivalent voltages are $12 \times \frac{6}{6+3} = 8$ and $12 \times \frac{3}{6+3} = 4$ and the equivalent resistances are both $3||6 = 2$. Finally, note the two Thevenin equivalent circuits are in series (and in series with the output resistance). The equivalent voltages subtrace and the equivalent resistances add, forming the 'Final Equivalent Circuit'. The voltage divider gives $4 \times \frac{4}{4+4} = 2$.

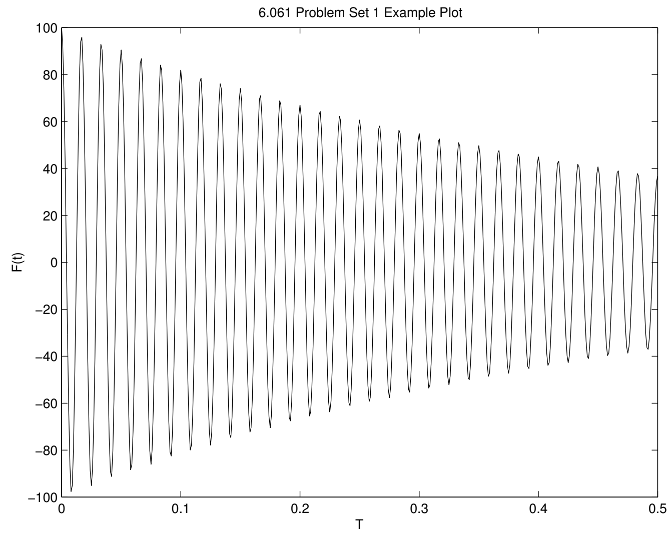

Figure 4: Answer for Problem 5

Problem 5: This is approximately the picture you should get. I have reduced it in size and moved it to fit onto this page.

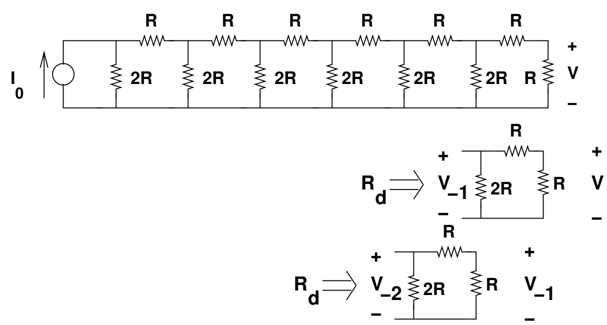

Figure 5: Magic Ladder Circuit

Problem 6: For 6.979 only The key to doing this easily is shown in Figure 5, where one of the cells of the ladder is called out. It is easy to see that the driving point impedance of that cell of the ladder is simply $2R||2R = R$ and the transfer relationship between input and output is $V = \frac{1}{2}V_{-1}$. Moving left one cell, it is also clear the the driving point resistance looking into the cell is still just R and the transfer is $V_{-1} = \frac{1}{2}V_{-2}$. This is true of each successive cell until we reach the source. At the current source the driving point resistance is $R = 1k\Omega$ so the driving point voltage is $V_d = 10mA \times 1k\Omega = 10v$. There are six divider cells so the output voltage is: $V = \frac{10}{2^6} = \frac{10}{64}v$