Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.061 Introduction to Power Systems

Problem Set 3 Solutions February 17, 2003

Problem 1: Assuming that sending-end and receiving end voltages are of equal magnitude and differ in phase angle by δ, *real* power flow is

$$P = \frac{|V|^2}{X_L} \sin \delta$$

or, in terms of the values given in the problem:

$$10 \times 10^6 = \frac{(10 \times 10^3)^2}{5} \times \sin \delta$$

Which implies that $\sin \delta = \frac{1}{2}$, or $\delta = 0.5236\text{Radians} = 30^\circ$. Figure 2 below shows, approximately, the phasor diagram that results.

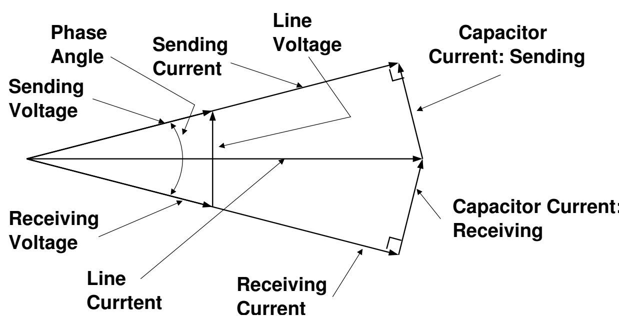

Figure 1: Transmission Line

Voltage across the transmission line is

$$V_L = 2|V| \sin \frac{\delta}{2} = 5176V$$

Reactive power absorbed by the line is

$$Q_L = \frac{|V_L|^2}{X_L} = 5.359\text{MVAR}$$

Reactive power required of the capacitance at each end is half of this, so that

$$Q_C = \frac{1}{2} Q_L = \frac{|V|^2}{X_C} = |V|^2 \omega C$$

Or:

$$C = \frac{Q_L}{2\omega |V|^2} \approx 71\mu F$$

Problem 2: As seen from the source, line plus load impedance are

$$Z = j\omega L + R$$

Since $\omega L = 377 \times .0265 \approx 10\Omega$, the total impedance has magnitude of 75.66Ω and a phase angle of very nearly $7.6°$

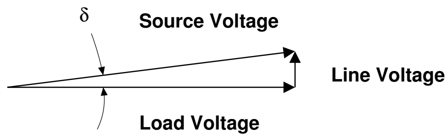

Figure 2: Phasor Diagram for Problem 2

Load current is

$$|I_L| = \frac{8000}{75.66} \approx 105.7A$$

and power in the load is

$$P_L = |I_L|^2 R \approx 838.4kW$$

For the second part, including the parallel capacitance, we may write the expression for load voltage, a simple voltage divider:

$$V_L = V_S \frac{R||\frac{1}{j\omega C}}{R||\frac{1}{j\omega C} + j\omega L} = \frac{V_S}{(1 - \omega^2 LC) + j\omega \frac{L}{R}}$$

To make $V_L = V_S$ we need to make

$$\left(1 - \omega^2 LC\right)^2 + \left(\omega \frac{L}{R}\right) = 1$$

Which, after a little algebra, boils to

$$\omega C = \frac{1}{\omega L} \pm \sqrt{\left(\frac{1}{\omega L}\right)^2 - \left(\frac{1}{R}\right)^2}$$

The smaller capacitance results from taking the minus sign, for which the value of $C \approx 2.37\mu F$.

It is possible to solve, using MATLAB, the magnitude of receiving end voltage vs. capacitance (shown in Figure 1).

Problem 3: If the capacitor is really holding voltage constant, maximum and minimum currents in the inductor are:

$$\begin{aligned} I_m &= I_\ell + \frac{V_s - V_o}{L} t_{on} \\ I_\ell &= I_m - \frac{V_o}{L} t_{off} \end{aligned}$$

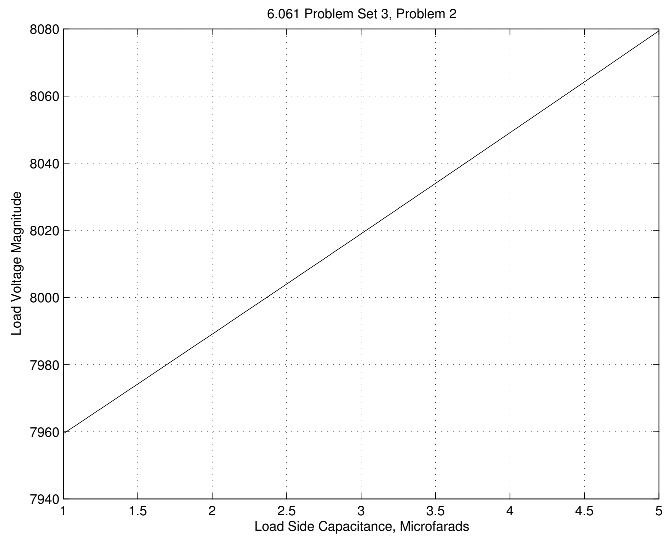

Figure 3: Receiving End Voltage vs. Compensating Cap

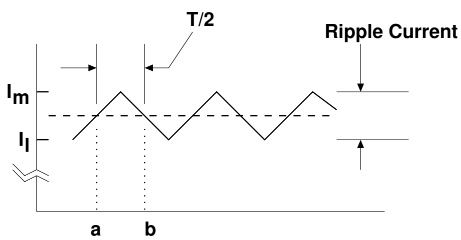

Figure 4: Cartoon of ripple current

which may be handily solved for, as expected:

$$V_o = V_s \frac{t_{on}}{t_{on} + t_{off}}$$

Now, if output voltage is indeed constant, we may also conclude that average inductor current is the same as output current:

$$\frac{I_\ell + I_m}{2} = \frac{V_o}{R}$$

Combining these relationships we find:

$$\begin{aligned} I_m &= \frac{V_o}{R} + \frac{1}{2}\frac{V_o}{L}t_{off} = 5.625A \\ I_\ell &= \frac{V_o}{R} = \frac{1}{2}\frac{V_o}{L}t_{off} = 4,375A \end{aligned}$$

Now, to get ripple voltage, note that the area of current ripple as shown in Figure 4 (the area above the average current line) represents charge added to the capacitor during the positive ripple excursion. This is:

$$\Delta Q = \frac{1}{2}\Delta I \frac{T}{2} = \frac{1}{2} \times \frac{1}{2} \times 10^{-4} \times .625A \approx 1.56 \times 10^{-5}\text{coulombs}$$

With $C = 100\mu F$, this implies a voltage excursion of about 156 mV.

The script for simulating this circuit is appended. What it does is shown in Figure 5, which shows voltage buildup and Figure 6 which shows a couple of cycles near steady state to show ripple.

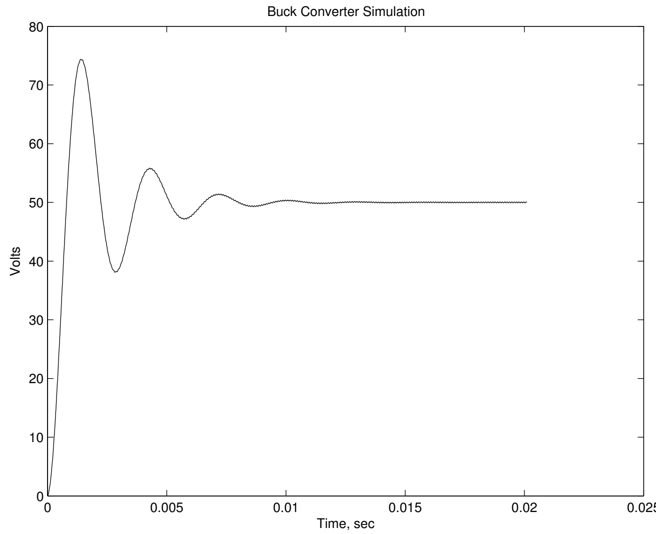

Figure 5: Voltage Buildup in Capacitor Filtered Buck Converter

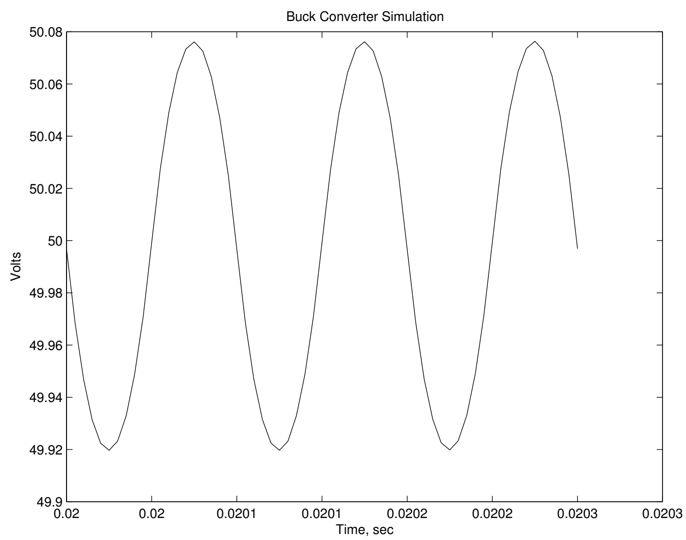

Figure 6: Ripple in Capacitor Filtered Buck Converter

Problem 4: In the Boost Converter, with $t_{on} = t_{off}$, output voltage should be just twice input voltage, or 100 V. With a load of 10Ω, this would draw 10 A. Voltage change is then:

$$\Delta V = \frac{It_{off}}{C}$$

which, for 10 A, $t_{off} = \frac{1}{2} \times 10^{-4}$ seconds and $C = 10^{-4}$ F, is just about 5 V.

This circuit, too, has been simulated, and the scripts are appended. The results are shown in Figure 7 (voltage buildup) and Figure 8 (steady state ripple).

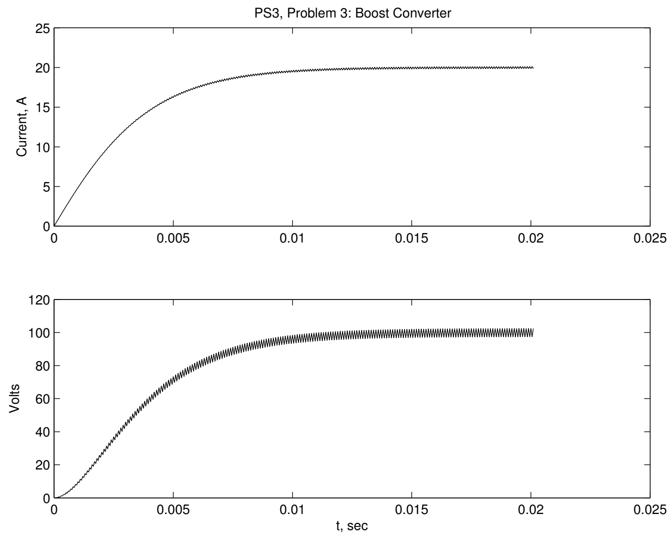

Figure 7: Boost Converter Voltage Buildup

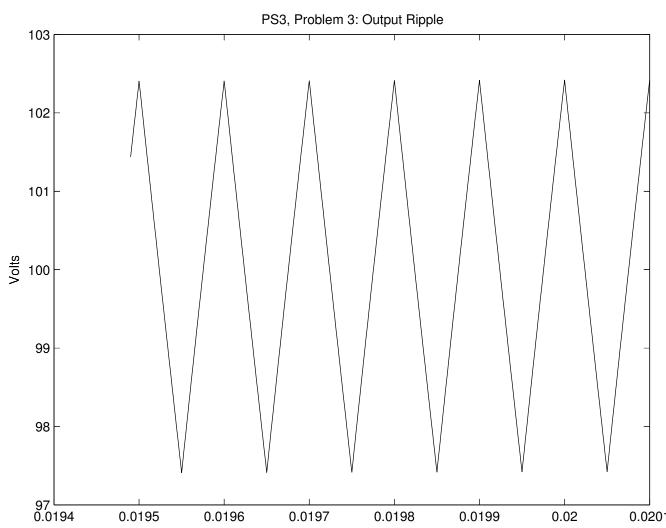

Figure 8: Boost Converter Voltage Ripple

Here are the buck and boost converter scripts:

```
--------------------------------------------
% Problem set 3, PRoblem 3 simulation
% buck converter with output capacitor
global Vs L R C
Vs=100;
L=.002;
R=10;
C=100e-6;
X=[];
t = [];
T = 1e-4;
d = .5;
ton = T*d;
toff = T*(1-d);
S0=[0; 0];
for n = 0:200
    [tt, S] = ode23('bon', [n*T n*T+ton], S0);
    t = [t' tt']';
    X = [X S'];
    S0 = S(length(tt),:);
    [tt, S] = ode23('boff', [n*T+toff (n+1)*T], S0);
    t = [t' tt']';
    X = [X S'];
    S0 = S(length(tt),:);
end
Il = X(1,:);
vo = X(2,:);
figure(1)
plot(t, vo)
title('Buck Converter Simulation')
ylabel('Volts');
xlabel('Time, sec');
% now just a couple of cycles to get ripple
tf = [];
Xf = [];
for n = 200:202
    [tt, S] = ode23('bon', [n*T n*T+ton], S0);
    tf = [tf' tt']';
    Xf = [Xf S'];
    S0 = S(length(tt),:);
    [tt, S] = ode23('boff', [n*T+toff (n+1)*T], S0);
    tf = [tf' tt']';
    Xf = [Xf S'];
    S0 = S(length(tt),:);
```

```
end
Ilf = Xf(1,:);
vof = Xf(2,:);
figure(2)
plot(tf, vof)
title('Buck Converter Simulation')
ylabel('Volts');
xlabel('Time, sec');
--------------------------------------------
function DS = bon(t, X)
global Vs L R C
il = X(1);
vc = X(2);
didt = (Vs-vc)/L;
dvdt = (il-vc/R)/C;
DS = [didt dvdt]';
--------------------------------------------
function DS = boff(t, X)
global Vs L R C
il = X(1);
vc = X(2);
didt = -vc/L;
dvdt = (il-vc/R)/C;
DS = [didt dvdt]';
--------------------------------------------
```

```
---------------------------------------------
% trivial boost converter model
global vs L C R
vs = 50;
f = 10000;
alf = .5;
L = 10e-3;
C = 100e-6;
R = 10;
T=1/f;
Dton = alf/f;
Dtoff = (1-alf)/f;
dton = Dton/10;
dtoff = Dtoff/10;
v0 = 0;
i0 = 0;
t = [];
i = [];
v = [];
for n = 0:200;
[tc, S] = ode45('upon',[n*T n*T+Dton] , [i0 v0]');
t = [t tc'];
ic = S(:,1);
vc = S(:,2);
i = [i ic'];
v = [v vc'];
i0 = ic(length(tc));
v0 = vc(length(tc));
[tc, S] = ode45('upoff',[n*T+Dton (n+1)*T] , [i0 v0]);
ic = S(:,1);
vc = S(:,2);
t = [t tc'];
i = [i ic'];
v = [v vc'];
i0 = ic(length(tc));
v0 = vc(length(tc));
end
figure(1)
subplot 211
plot(t, i)
title('PS3, Problem 3: Boost Converter')
ylabel('Current, A');
subplot 212
plot(t, v)
ylabel('Volts');
```

```
xlabel('t, sec');
N = length(t);
figure(2)
plot(t(N-500:N), v(N-500:N))
title('PS3, Problem 3: Output Ripple')
ylabel('Volts');
xlabel('t, sec');
-----------------------------------
function Sdot = up(t, S)
global vs L C R
     il = S(1);
     vc = S(2);
     vdot = (1/C) * ( - vc/R);
     idot = (1/L) * (vs);
     Sdot = [idot vdot]';
-------------------------------------
function Sdot = up(t, S)
global vs L C R
     il = S(1);
     vc = S(2);
     vdot = (1/C) * (il - vc/R);
     idot = (1/L) * (vs - vc);
     Sdot = [idot vdot]';
```