

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
 6.061 Introduction to Power Systems

Problem Set 6 Solutions

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Problem 1: In this problem we take advantage of the fact that it is easy for us to calculate the phase admittance matrix, so that phase currents are, in terms of phase voltage:

$$\underline{I}_{ph} = \underline{Y}_{ph} \underline{V}_{ph}$$

Then symmetrical component currents are easily calculated as:

$$\underline{I}_s = \underline{I}_s \underline{V}_s$$

where

$$\underline{Y}_s = \underline{T} \underline{Y}_{ph} \underline{T}^{-1}$$

Noting the admittance matrices are, in the various cases:

1. The wye connected load is solidly grounded,

$$\underline{Y}_{ph} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{20} & 0 \\ 0 & 0 & \frac{1}{20} \end{bmatrix}$$

2. The wye connected load is ungrounded,

$$\underline{Y}_{ph} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{40} & -\frac{1}{40} \\ 0 & -\frac{1}{40} & \frac{1}{40} \end{bmatrix}$$

3. The wye connected load is grounded through a 5 ohm resistor.

$$\underline{Y}_{ph} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{24} & -\frac{1}{120} \\ 0 & -\frac{1}{120} & \frac{1}{24} \end{bmatrix}$$

Note here that:

$$y_{bb} = y_{cc} = \frac{1}{20 + 20 \parallel 5} = \frac{1}{24}$$

$$y_{bc} = y_{cb} = -\frac{5}{5 + 25} \times \frac{1}{20 + 20 \parallel 5} = \frac{1}{120}$$

At this point we let sloth get the better of us and wrote a MATLAB script to do the work. That script is appended and here is a screen shot of what it said:

Part 1
 Symmetrical Component Currents = $11.547 e^{j7.73256e-17}$, $5.7735 e^{-j-3.14159}$, $5.7735 e^{-j-3.14159}$
 Load Phase Currents = $1.70925e-15 e^{j0.46318}$, $17.3205 e^{-j-2.0944}$, $17.3205 e^{j2.0944}$
 Ground Current = $17.3205 e^{-j-3.14159}$
 Source Phase Currents = $1.44231 e^{-j-1.0472}$, $1.44231 e^{-j-2.0944}$, $2.49815 e^{j1.5708}$
 Check: Sum of Phase Currents = $1.31216e-16$

Part 2
 Symmetrical Component Currents = $8.66025 e^{j9.61552e-17}$, $8.66025 e^{-j-3.14159}$, $0 e^{j0}$
 Load Phase Currents = $1.01305e-15 e^{-j-0.319127}$, $15 e^{-j-1.5708}$, $15 e^{j1.5708}$
 Ground Current = $3.58512e-15 e^{-j-1.43624}$
 Source Phase Currents = $1.24908 e^{-j-1.5708}$, $1.24908 e^{-j-1.5708}$, $2.49815 e^{j1.5708}$
 Check: Sum of Phase Currents = $6.00192e-17$

Part 3
 Symmetrical Component Currents = $10.5848 e^{j8.24563e-17}$, $6.73575 e^{-j-3.14159}$, $3.849 e^{-j-3.14159}$
 Load Phase Currents = $8.49492e-16 e^{j0.494655}$, $16.0728 e^{-j-1.93822}$, $16.0728 e^{j1.93822}$
 Ground Current = $11.547 e^{-j-3.14159}$
 Source Phase Currents = $1.3384 e^{-j-1.20337}$, $1.3384 e^{-j-1.93822}$, $2.49815 e^{j1.5708}$
 Check: Sum of Phase Currents = $9.56266e-17$

Problem 2: Figure 1 shows the symmetrical component networks for this problem.

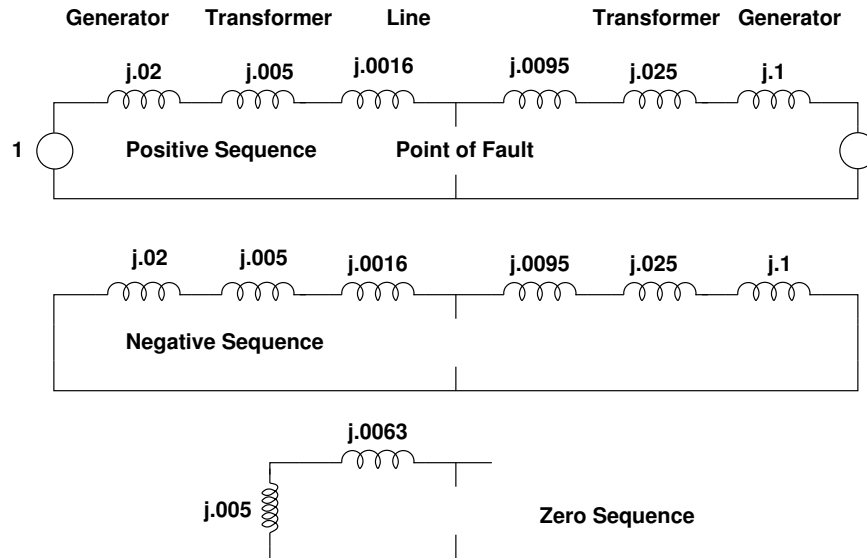


Figure 1: Symmetrical Component Networks

The reactances must all be referred to a single base power. In this case I have chosen to use a common base of 100 MVA. Since the voltage bases are already consistent, translation involves only power. It goes like this, for positive and negative sequence reactance:

$$x_{G1} = \frac{100}{1000} \times .2 = .02$$

$$x_{T1} = \frac{100}{1000} \times .05 = .005$$

$$x_{G2} = \frac{100}{200} \times .2 = .1$$

$$x_{T2} = \frac{100}{200} \times .05 = .025$$

Base impedance for the line is

$$Z_B = \frac{(345 \times 10^3)^2}{1000 \times 10^6} = 1190\Omega$$

And base currents are:

$$\begin{aligned} \text{Line } I_B &= \frac{100 \times 10^6}{\sqrt{3} \times 345 \times 10^3} = 167.3\text{A} \\ \text{Generator } I_B &= \frac{100 \times 10^6}{\sqrt{3} \times 26 \times 10^3} = 2206\text{A} \end{aligned}$$

Impedances of the two line segments are:

$$\begin{aligned} \text{Line 1 } X_1 = X_2 &= 377 \times (.01 - .005) = 1.885\Omega = .0016\text{per-unit} \\ X_0 &= 377 \times (.01 + 2 \times .005) = 7.54\Omega = .0063\text{per-unit} \\ \text{Line 2 } X_1 = X_2 &= 377 \times (.006 - .003) = 1.131\Omega = .00095\text{per-unit} \\ X_0 &= 377 \times (.006 + 2 \times .005) = 4/524\Omega = .0038\text{per-unit} \end{aligned}$$

For the symmetric fault case, only the positive sequence circuit is involved, and it is shorted at the point indicated. Impedances of the left-hand and right-hand parts of the circuit are:

$$\begin{aligned} z_\ell &= j.02 + j.005 + j.0016 = j.0266 \\ z_r &= j.1 + j.025 + j.00095 = j.12595 \end{aligned}$$

Fault current is:

$$i_f = \frac{1}{j.0266} + \frac{1}{j.12595} = -j(37.59 + 7.9397) = -j45.5\text{per-unit} \times 167.3 = -j7620\text{A}$$

In the left-hand generator (G_1) leads it is:

$$i_f = -j37.59\text{per-unit} \times 2206 = 82.9\text{kA}$$

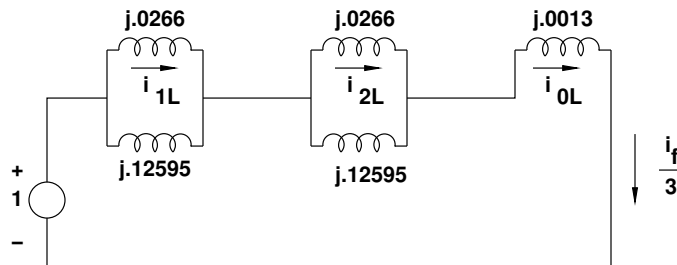


Figure 2: Symmetrical Component Networks

For the single-phase fault case the three symmetrical component networks are connected in series. The situation is shown in Figure 2. The positive and negative sequence impedances

are as described for the symmetric fault. Only the left-hand part of the network carries zero-sequence current and its impedance is:

$$x_z = j.005 + j.0063 = j.0113 \text{ per-unit}$$

Note that the complete impedance of the positive and negative sequence networks is:

$$j.0266 || j.12595 = j.022$$

Then, in the fault:

$$i_1 = i_2 = i_0 = \frac{1}{3} i_f = \frac{1}{2 \times j.022 + j.0113} = \frac{1}{.0552} = -j18.1$$

Then current magnitude in the fault is:

$$I_f = 3 \times 18.1 \times 167.3 = 9076 \text{ A}$$

To get current in the generator leads we must use a current divider:

$$i_{1l} = i_{2l} = i_{1f} = -j18.1 \times \frac{.12595}{.0266 + .12595} = -j14.95 \text{ per-unit}$$

Then, in the actual generator leads, taking into account the thirty degree phase rotation:

$$\begin{aligned} i_1 &= -j14.95 \times e^{-j\frac{\pi}{6}} \\ i_2 &= -j14.95 \times e^{j\frac{\pi}{6}} \end{aligned}$$

And the phase currents can be derived by:

$$\begin{aligned} i_a &= -j14.95 \left(e^{-j\frac{\pi}{6}} + e^{j\frac{\pi}{6}} \right) = -j\sqrt{3} \times 14.95 = -j25.9 \\ i_b &= -j14.95 \left(a^2 e^{-j\frac{\pi}{6}} + a e^{j\frac{\pi}{6}} \right) = -j14.95 \left(e^{-j\frac{5\pi}{6}} + e^{j\frac{5\pi}{6}} \right) = j\sqrt{3} \times 14.95 = j25.9 \\ i_c &= -j14.95 \left(a e^{-j\frac{\pi}{6}} + a^2 e^{j\frac{\pi}{6}} \right) = -j14.95 \left(e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}} \right) = 0 \end{aligned}$$

The magnitude of current in the generator leads is:

$$|I_A| = 2206 \times 25.9 = 57.1 \text{ kA}$$

Problem 3: for 6.979 The way of working this problem is to find the mutual admittance between positive sequence voltage and negative sequence current. There will be positive sequence voltage applied across the combined impedance of the line, transformers and generators because the system is carrying current (in fact it is carrying real power), and then that voltage, applied to the admittance matrix for the system results in current in both positive and negative sequences. The transmission line has phase admittance:

$$\underline{\underline{Z}}_{ph} = j377 \times \begin{bmatrix} .016 & .01 & .008 \\ .01 & .016 & .01 \\ .008 & .01 & .016 \end{bmatrix}$$

That impedance is normalized using the line base impedance (see Problem 2) and then the positive and negative sequence impedances are added. To find the voltage applied to the line we see we are carrying some real power (If you normalized to 100 MVA that would be 2 per-unit). Voltage across the line is just the difference between the sending and receiving ends and that is applied to the admittance of the system.

A script to implement this procedure was written and is appended. The results are:

Problem 6.3

Transmission Line Impedance, per-unit

z1 =

0 + 0.0051i	0 + 0.0032i	0 + 0.0025i
0 + 0.0032i	0 + 0.0051i	0 + 0.0032i
0 + 0.0025i	0 + 0.0032i	0 + 0.0051i

Symmetrical Component Impedances

z1s =

0.0000 + 0.0021i	-0.0004 + 0.0002i	-0.0002 - 0.0001i
0.0004 + 0.0002i	0.0000 + 0.0021i	0.0002 - 0.0001i
0.0002 - 0.0001i	-0.0002 - 0.0001i	-0.0000 + 0.0110i

Total Impedances

zt =

0.0000 + 0.1521i	-0.0004 + 0.0002i	-0.0002 - 0.0001i
0.0004 + 0.0002i	0.0000 + 0.1521i	0.0002 - 0.0001i
0.0002 - 0.0001i	-0.0002 - 0.0001i	-0.0000 + 0.0110i

Currents

I =

0.0000 - 2.0302i
0.0049 + 0.0028i
0.0339 - 0.0196i

>> abs(I(2))

ans =

0.0057

Note that we have computed a zero sequence current because, again due to sloth, we did not set all of the zero sequence admittances to zero as would be the case with the delta transformer connections. However, that omission does not affect the negative sequence current, which is about 0.5%.

Problem 4: The best way of working this one is to copy the example script from the course web page and modify it to look like the problem that is posed. Shown in Figure 3 is the problem repeated with the buses and lines numbered. (Apologies as the figure becomes kind of busy).

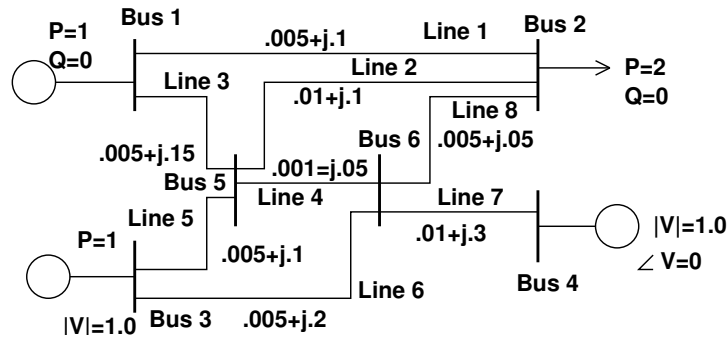


Figure 3: Load Flow Problem with Elements Numbered

One important thing to notice here is that it is necessary to find the bus-admittance network. A little reverse engineering of the script would indicate that the use of what is called the *node-incidence* matrix makes this a lot easier. The node-incidence matrix describes the geometry of the network. It has as many rows as there are buses and as many columns as there are lines. For each line there is an entry for the two buses to which the line is connected. (Those entries are ± 1 , and it doesn't matter which end gets the positive sign and which gets the negative sign but there must be one of each). A line connected to only one bus would appear to be shorted at the other end (That is one way of producing a fixed impedance to ground). The node incidence matrix may be used to calculate voltages across the lines in terms of bus voltage:

$$\underline{V}_{\text{line}} = \underline{N}^T \underline{V}_{\text{bus}}$$

Or to compute the bus currents in terms of line currents:

$$\underline{I}_{\text{bus}} = \underline{I}_{\text{line}} \underline{N}$$

and of course the bus admittance network is then calculated from the line admittance network by:

$$\underline{Y}_{\text{bus}} = \underline{N} \underline{Y}_{\text{line}} \underline{N}^T$$

The script which does all of the work is appended and the answer is:

>> p4
 Here is the line admittance matrix:

Y =

Columns 1 through 5

0.4988 - 9.9751i	0	0	0	0
0	0.9901 - 9.9010i	0	0	0
0	0	0.2220 - 6.6593i	0	0
0	0	0	0.3998 - 19.9920i	0
0	0	0	0	0.4988 - 9.9751i
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

Columns 6 through 8

0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0.1249 - 4.9969i	0	0
0	0.1110 - 3.3296i	0
0	0	1.9802 - 19.8020i

And here is the bus admittance matrix

YN =

Columns 1 through 5

0.7207 - 16.6343i	-0.4988 + 9.9751i	0	0	-0.2220 + 6.6593i
-0.4988 + 9.9751i	3.4691 - 39.6780i	0	0	-0.9901 + 9.9010i
0	0	0.6237 - 14.9719i	0	-0.4988 + 9.9751i
0	0	0	0.1110 - 3.3296i	0
-0.2220 + 6.6593i	-0.9901 + 9.9010i	-0.4988 + 9.9751i	0	2.1107 - 46.5273i
0	-1.9802 + 19.8020i	-0.1249 + 4.9969i	-0.1110 + 3.3296i	-0.3998 + 19.9920i

Column 6

0
-1.9802 + 19.8020i
-0.1249 + 4.9969i
-0.1110 + 3.3296i
-0.3998 + 19.9920i
2.6159 - 48.1205i

Here are the voltages

V =

1.0703 + 0.0367i
1.0658 - 0.0381i
1.0981 + 0.0651i
1.0000
1.0754 + 0.0087i
1.0696 - 0.0055i

Real Power

PI =

1.0000
-2.0000
1.0000
0.0106
0.0000
0.0000

Reactive Power

QI =

-0.0000
-0.0000
0.4245
-0.2324
-0.0000
0.0000

Line Voltages are

Vline =

0.0045 + 0.0748i
0.0096 + 0.0468i
-0.0050 + 0.0280i
0.0058 + 0.0142i
0.0227 + 0.0564i
0.0284 + 0.0706i
0.0696 - 0.0055i
0.0038 + 0.0326i

Line Currents are

Iline =

0.7481 - 0.0079i
0.4729 - 0.0485i
0.1851 + 0.0398i
0.2862 - 0.1098i
0.5740 - 0.1980i
0.3564 - 0.1333i
-0.0106 - 0.2324i
0.6531 - 0.0107i

Scripts

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% Problem 6.1

N = 4160/600; % Transformer voltage ratio
a = exp(j*2*pi/3); % our good old friend and shorthand
T = (1/3) .* [1 a a^2;1 a^2 a;1 1 1]; % symmetrical components transformation
TI = inv(T); % and its inverse
Vs = [600/sqrt(3) 0 0]; % source is positive sequence only
rf = exp(j*pi/6); % 30 degree rotation across transformer
rb = exp(-j*pi/6); % negative sequence rotates the other way
fprintf('Part 1\n');
Yph = [0 0 0;0 1/20 0;0 0 1/20]; % phases B and C solidly to ground
Ys = T*Yph*TI; % symmetrical component admittance
Is = Ys*Vs'; % times voltage is current
Isl = (1/N).* [rb*Is(1); rf*Is(2); 0]; % on the left
Iph = TI*Isl; % and here is phase current
Iphl = TI*Isl; % phase current on the left
fprintf('Symmetrical Component Currents = %g e^-j%g, %g e^-j%g, %g e^-j%g\n',abs(Is(1)), angle(Is(1)),...
abs(Is(2)), angle(Is(2)),abs(Is(3)), angle(Is(3)))
fprintf('Load Phase Currents = %g e^-j%g, %g e^-j%g, %g e^-j%g\n',abs(Iph(1)), angle(Iph(1)),...
abs(Iph(2)), angle(Iph(2)),abs(Iph(3)), angle(Iph(3)))
fprintf('Ground Current = %g e^-j%g\n',abs(Iph(1)+Iph(2)+Iph(3)),angle(Iph(1)+Iph(2)+Iph(3)));
fprintf('Source Phase Currents = %g e^-j%g, %g e^-j%g, %g e^-j%g\n',abs(Iphl(1)), angle(Iphl(1)),...
abs(Iphl(2)), angle(Iphl(2)),abs(Iphl(3)), angle(Iphl(3)))
fprintf('Check: Sum of Phase Currents = %g\n', Iphl(1)+Iphl(2)+Iphl(3));
fprintf('Part 2\n');
Yph = [0 0 0;0 1/40 -1/40;0 -1/40 1/40]; % phases B and C solidly together
Ys = T*Yph*TI; % symmetrical component admittance
Is = Ys*Vs'; % times voltage is current
Isl = (1/N).* [rb*Is(1); rf*Is(2); 0]; % on the left
Iph = TI*Isl; % and here is phase current
Iphl = TI*Isl; % phase current on the left
fprintf('Symmetrical Component Currents = %g e^-j%g, %g e^-j%g, %g e^-j%g\n',abs(Is(1)), angle(Is(1)),...
abs(Is(2)), angle(Is(2)),abs(Is(3)), angle(Is(3)))
fprintf('Load Phase Currents = %g e^-j%g, %g e^-j%g, %g e^-j%g\n',abs(Iph(1)), angle(Iph(1)),...
abs(Iph(2)), angle(Iph(2)),abs(Iph(3)), angle(Iph(3)))
fprintf('Ground Current = %g e^-j%g\n',abs(Iph(1)+Iph(2)+Iph(3)),angle(Iph(1)+Iph(2)+Iph(3)));
fprintf('Source Phase Currents = %g e^-j%g, %g e^-j%g, %g e^-j%g\n',abs(Iphl(1)), angle(Iphl(1)),...
abs(Iphl(2)), angle(Iphl(2)),abs(Iphl(3)), angle(Iphl(3)))
fprintf('Check: Sum of Phase Currents = %g\n', Iphl(1)+Iphl(2)+Iphl(3));
fprintf('Part 3\n');
Yph = [0 0 0;0 1/24 -1/120;0 -1/120 1/24]; % phases B and C to ground through Rg
Ys = T*Yph*TI; % symmetrical component admittance
Is = Ys*Vs'; % times voltage is current
Isl = (1/N).* [rb*Is(1); rf*Is(2); 0]; % on the left
Iph = TI*Isl; % and here is phase current
Iphl = TI*Isl; % phase current on the left
fprintf('Symmetrical Component Currents = %g e^-j%g, %g e^-j%g, %g e^-j%g\n',abs(Is(1)), angle(Is(1)),...
abs(Is(2)), angle(Is(2)),abs(Is(3)), angle(Is(3)))
fprintf('Load Phase Currents = %g e^-j%g, %g e^-j%g, %g e^-j%g\n',abs(Iph(1)), angle(Iph(1)),...
abs(Iph(2)), angle(Iph(2)),abs(Iph(3)), angle(Iph(3)))
fprintf('Ground Current = %g e^-j%g\n',abs(Iph(1)+Iph(2)+Iph(3)),angle(Iph(1)+Iph(2)+Iph(3)));
fprintf('Source Phase Currents = %g e^-j%g, %g e^-j%g, %g e^-j%g\n',abs(Iphl(1)), angle(Iphl(1)),...
abs(Iphl(2)), angle(Iphl(2)),abs(Iphl(3)), angle(Iphl(3)))
fprintf('Check: Sum of Phase Currents = %g\n', Iphl(1)+Iphl(2)+Iphl(3));
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% Problem 6.2
a = exp(j*2*pi/3);
T = (1/3) .* [1 a a^2;1 a^2 a;1 1 1];
TI = inv(T);
Zl = j*377 .* [.016 .01 .008;.01 .016 .01; .008 .01 .016];
zl = Zl ./ 1190;
zls = T*zl*TI;

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z1 = j*.02+j*.005+j*.025+j*.1;
ze = [z1 0 0;0 z1 0;0 0 0];
zt = z1s+ze;
yt = inv(zt);
x = .0266+.12595;
delt = asin(2*x);
v = 2*sin(delt/2);
V = [v; 0; 0];
I = yt*V;
fprintf('Problem 6.3\n')
fprintf('Transmission Line Impedance, per-unit\n')
z1
fprintf('Symmetrical Component Impedances\n')
z1s
fprintf('Total Impedances\n');
zt
fprintf('Currents\n')
I
-----
% Simple-Minded Load Flow Example
% First, impedances
Z1=.005+j*.1;
Z2=.01+j*.1;
Z3=.005+j*.15;
Z4=.001+j*.05;
Z5=.005+j*.1;
Z6=.005+j*.2;
Z7=.01+j*.3;
Z8=.005+j*.05;
%
% This is the node-incidence Matrix
NI=[1 0 1 0 0 0 0 0;
    -1 -1 0 0 0 0 0 -1;
    0 0 0 0 1 1 0 0;
    0 0 0 0 0 0 -1 0;
    0 1 -1 1 -1 0 0 0;
    0 0 0 -1 0 -1 1 1];
% This is the vector of "known" voltage magnitudes
VNM = [0 0 1.1 1 0 0]';
% And the vector of known voltage angles
VNA = [0 0 0 0 0 0]';
% and this is the "key" to which are actually known
KNM = [0 0 1 1 0 0]';
KNA = [0 0 0 1 0 0]';
% and which are to be manipulated by the system
KUM = 1 - KNM;
KUA = 1 - KNA;
% Here are the known loads (positive is INTO network
% Use zeros for unknowns
P=[1 -2 1 0 0 0]';
Q=[0 0 0 0 0 0]';
% and here are the corresponding vectors to indicate
% which elements should be checked in error checking
PC = [1 1 1 0 1 1]';
QC = [1 1 0 0 1 1]';
Check = KNM + KNA + PC + QC;
% Unknown P and Q vectors
PU = 1 - PC;
QU = 1 - QC;
fprintf('Here is the line admittance matrix:\n');
Y=[1/Z1 0 0 0 0 0 0 0;
    0 1/Z2 0 0 0 0 0 0;
    0 0 1/Z3 0 0 0 0 0;
    0 0 0 1/Z4 0 0 0 0;

```

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    0 0 0 0 1/Z5 0 0 0;
    0 0 0 0 0 1/Z6 0 0;
    0 0 0 0 0 0 1/Z7 0;
    0 0 0 0 0 0 0 1/Z8]
% Construct Node-Admittance Matrix
fprintf('And here is the bus admittance matrix\n')
YN=NI*Y*NI'
% Now: here are some starting voltage magnitudes and angles
VM = [1 1 1 1 1 1]';
VA = [.0965 .146 .00713 .0261 0 0]';
% Here starts a loop
Error = 1;
Tol=1e-16;

N = length(VNM);
% Construct a candidate voltage from what we have so far
VMAG = VNM .* KNM + VM .* KUM;
VANG = VNA .* KNA + VA .* KUA;
V = VMAG .* exp(j .* VANG);

% and calculate power to start
I = (YN*V);
PI = real(V .* conj(I));
QI = imag(V .* conj(I));
%pause
while(Error>Tol);
    for i=1:N,% Run through all of the buses
        % What we do depends on what bus!
        if (KUM(i) == 1) & (KUA(i) == 1),% don't know voltage magnitude or angle
            pvc= (P(i)-j*Q(i))/conj(V(i));
            for n=1:N,
                if n ~=i, pvc = pvc - (YN(i,n) * V(n)); end
            end
            V(i) = pvc/YN(i,i);

        elseif (KUM(i) == 0) & (KUA(i) == 1), % know magnitude but not angle
            % first must generate an estimate for Q
            Qn = imag(V(i) * conj(YN(i,:)*V));
            pvc= (P(i)-j*Qn)/conj(V(i));
            for n=1:N,
                if n ~=i, pvc = pvc - (YN(i,n) * V(n)); end
            end
            pv=pvc/YN(i,i);
            V(i) = VNM(i) * exp(j*angle(pv));
        end % probably should have more cases
    end % one shot through voltage list: check error

% Now calculate currents indicated by this voltage expression
I = (YN*V);
% For error checking purposes, compute indicated power
PI = real(V .* conj(I));
QI = imag(V .* conj(I));
% Now we find out how close we are to desired conditions
PERR = (P-PI) .* PC;
QERR = (Q-QI) .* QC;

Error = sum(abs(PERR) .^2 + abs(QERR) .^2);
end
fprintf('Here are the voltages\n')
V
fprintf('Real Power\n')
PI
fprintf('Reactive Power\n')
QI

```

```
fprintf('Line Voltages are\n')
Vline = NI'*V
fprintf('Line Currents are\n')
Iline = Y*Vline
```