Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.061 Introduction to Power Systems

Problem Set 5 Solutions

March 2, 2003

Problem 1: The current source will launch two pulses along the line, one going to the right as v_+ and one to the left as v_-. These will have associated currents and as the point of injection,

$$\begin{aligned} v(t) &= && v_+ + v_- \\ i(t) &= I_0(t) = && i_+ - i_- = \frac{v_+}{Z_0} + \frac{V_-}{Z_0} \end{aligned}$$

From symmetry we can see that the two pulses that get launched must be of the same magnitude, so that:

$$v_+ = v_- = \frac{Z_0}{2} I_0 = 1000A \times 125\Omega = 125,000V$$

Time of flight is $T = \frac{3\times10^5 \text{m}}{3\times10^8 \text{m/s}} = 1\text{mS} = 1000\mu S$.

At the right-hand end, when the forward going pulse reaches the short, a second, reverse direction pulse is generated, and there because of the short, $v_+ + v_- = 0$. The current in the pulse is doubled, so that at peak it has value of 1000 A. It takes $2 \times T = 20\mu S$ for this pulse to get to the left hand end. Since the left-hand end is terminated in impedance $R = Z_0$, both pulses appear across the resistor but no reflections are generated. The results are shown in graphical form in Figure 1

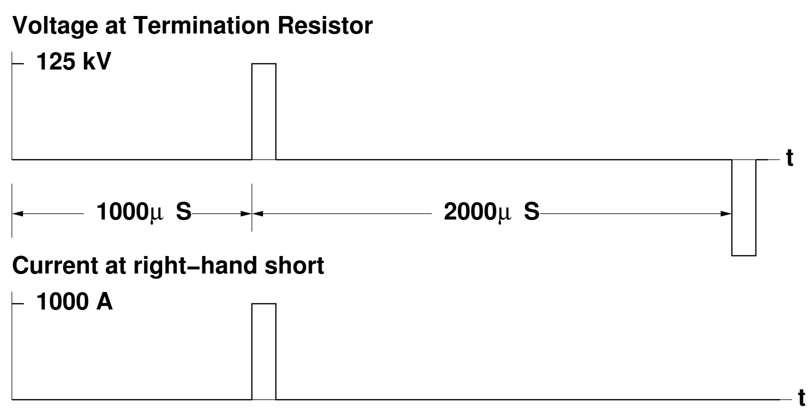

Figure 1: Transmission Line Example

Problem 2: Once corrected, the notes give us the pertinent formula: the ratio of receiving end to sending end voltage is:

$$\frac{V_r}{V_s} = \frac{\frac{R}{Z_0}}{\frac{R}{Z_0}\cos(k\ell) + j\sin(k\ell)}$$

Since the wavelength is $\lambda = \frac{3\times 10^8}{60} = 5 \times 10^6\text{m} = 5000\text{km}$, Here,

$$k\ell = 2\pi\frac{600}{5000}$$

To find out what happens at the sending end, note that, at $x = 0$, voltage and current are:

$$\begin{aligned} V_s &= V_+ + V_- \\ I_s &= \frac{1}{Z_0}(V_+ - V_-) \end{aligned}$$

Since the ratio of reflected to incident wave voltage is:

$$\frac{V_-}{V_+} = \frac{\frac{R}{Z_0} - 1}{\frac{R}{Z_0} + 1} e^{-2jk\ell}$$

then, with a little algebraic pain we may conclude that:

$$\begin{aligned} V_+ &= \frac{V_s\left(\frac{R}{Z_0} + 1\right)}{\left(\frac{R}{Z_0} + 1\right) + \left(\frac{R}{Z_0} - 1\right)e^{-j2l\ell}} \\ V_- &= \frac{V_s\left(\frac{R}{Z_0} - 1\right)e^{-j2k\ell}}{\left(\frac{R}{Z_0} + 1\right) + \left(\frac{R}{Z_0} - 1\right)e^{-j2l\ell}} \end{aligned}$$

Then, with a little more algebra the sending end current can be found to be:

$$I_s = \frac{V_s}{Z_0}\frac{j\frac{R}{Z_0}\sin k\ell + \cos k\ell}{\frac{R}{Z_0}\cos k\ell + j\sin k\ell}$$

This leads to the expression for sending end complex (real + reactive) power as:

$$P_s + jQ_s = \frac{|V_s|^2}{Z_0}\left\{\frac{-j\frac{R}{Z_0}\sin k\ell + \cos k\ell}{\frac{R}{Z_0}\cos k\ell - j\sin k\ell}\right\}$$

The calculations from here are carried out in MATLAB. Here is the session. More results show up later

```
>> p2
R/ZO     Vr      Ps     Qs
    Open       685900.6              0     9.39e+08
    1.2500     548391.2     9.62e+08    -2.16e+08
    1.0000     500000.0        1e+09    -8.69e-08
    0.8333     455263.5     9.95e+08     1.82e+08
```

For the final two parts of this problem, note that the derivation done in the notes can be generalized to a termination with a complex impedance. In this case the complex amplitudes of receiving end and sending end voltages are related by:

$$\underline{V}_r = \underline{V}_s \frac{\frac{Z_R}{Z_0}}{\frac{Z_R}{Z_0}\cos(k\ell) + j\sin(k\ell)}$$

For the first part, we note that the load is drawing surge impedance load when $Z_R = Z_0$. As a practical matter we can use load *admittance* $G_R = 1/Z_R$ as a parameter, calculating both load power and voltage as a function of that parameter and doing a cross-plot. The result is shown in Figure 2

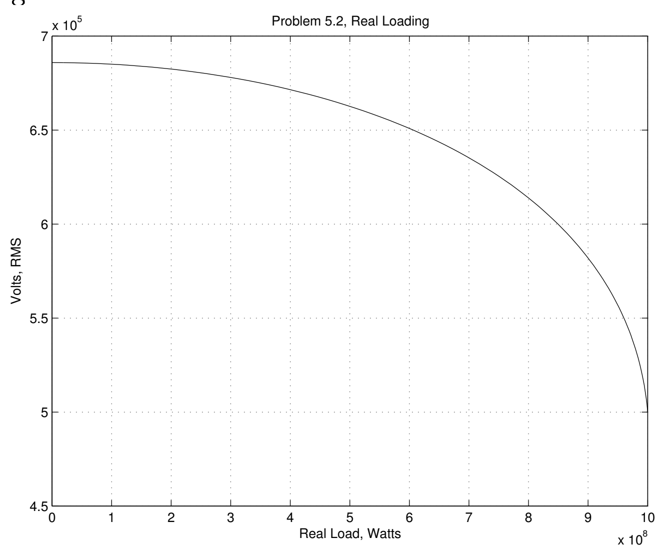

Figure 2: Voltage vs. power

Now for the second part, we may approximate things by assuming a real part of the terminating admittance suitable for the power levels involved and then note that reactive power *supplied by the termination* is:

$$Q_s = |V_R|^2 Im\{Y_s\}$$

A cross-plot of receiving end voltage against Q_s is shown in Figure 3.

The script that implements these calculations is in the appendix.

Problem 3: Voltage across the resistor is:

$$\frac{600}{\sqrt{3}} \approx 346v$$

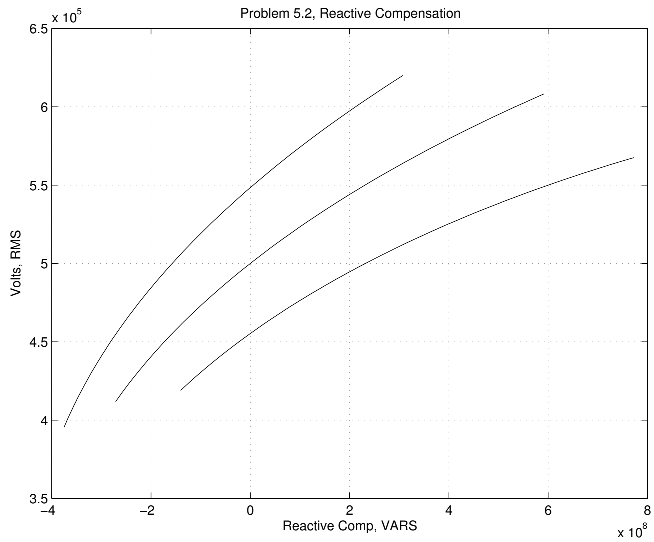

Figure 3: Voltage vs. reactive injection

So that power is $P = 100 \times 346 \approx 34.6kW$.

The turns ratio of the transformer is $\frac{4200}{346} \approx 12.1$.

Current on the primary side of the transformer is in phases A and C and is:

$$i_A = -i_C = 8.25Ae^{-j\frac{\pi}{6}}$$

Since the primary side voltage is $\frac{4200}{\sqrt{3}} \approx 2425v$,

$$\begin{aligned} P_a &= P_c = & 2425 \times 8.25 \times \cos\frac{\pi}{6} \approx 17.3kW \\ Q_a &= -Q_c = & 2425 \times 8.25 \times \sin\frac{\pi}{6} \approx 10\text{kVAR} \end{aligned}$$

Since there is no current in Phase B, its real and reactive powers are both zero. Current on the primary side is shown in Figure 4.

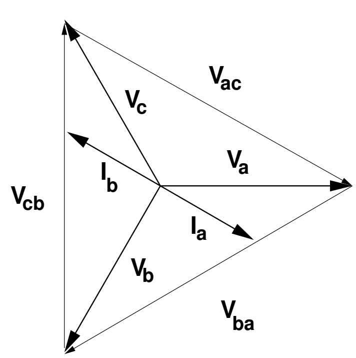

Figure 4: Currents for Problem 3

Problem 4: For 6.979: This one is actually even easier, at least to start. Since the voltage is 600 V and current is 100 A, power is $P = 600 \times 100 = 60kW$.

The turns ratio is $N = \frac{2425}{600} \approx 4.04$.

Note that since the primary is ungrounded, $i_A + i_B + i_C = 0$. And since there is no connection to the B-C corner, $i_C = i_B$, leaving us with $i_C = i_B = -\frac{i_A}{2}$. Then:

$$i_A = \frac{2}{3} \times \frac{1}{4.04} \times 100 \approx 16.5A$$

This is shown in Figure 5.

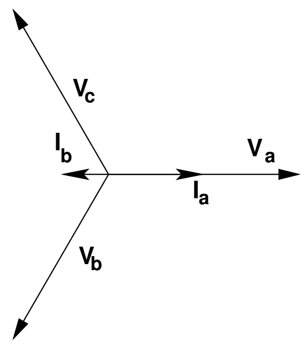

Figure 5: Phasor Diagram for Problem 4

Then:

$$\begin{aligned}
P_A &= 16.5 \times 2425 \approx 40kW \\
Q_A &= 0 \\
P_B = P_C &= 8.25 \times 2425 \times \cos\frac{\pi}{3} \approx 10kW \\
Q_B = -Q_c &= 8.25 \times 2425 \times \sin\frac{\pi}{3} \approx 17.3kVAR
\end{aligned}$$

Finally, nothing happens when the ground is restored. To see this, view the equivalent delta side of the transformer as is shown in Figure 5. Since the combination of Phase B and Phase C voltage is equal to the Phase A voltage, the thevenin equivalent, including 'leakage' impedance, has the same back voltage but twice the series impedance. That combination will source half the current of Phase A, so there is really no difference between the grounded and ungrounded situations here.

Scripts

```
% Problem Set 5, Problem 2
C=3e8;            % speed of light
L=600e3;          % line length
Z0 = 250;         % characteristic impedance
Vs = 500e3;       % line voltage
```

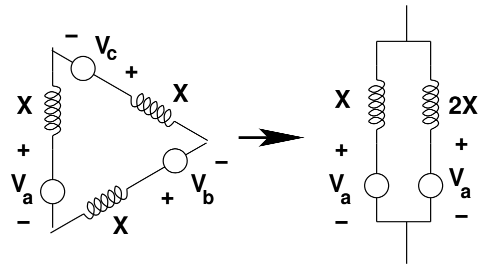

Figure 6: Delta Equivalent

```
f = 60;
lambda = C/f;
k = 2*pi/lambda;
% part 0: easy part
Vro = Vs /cos(k*L);
Qso = (Vs^2/Z0) * tan(k*L);
rr = [1/.8 1 1/1.2];
Vrr= Vs .* rr ./ (rr .* cos(k*L) + j*sin(k*L));
VAs = (Vs^2/Z0) .* (-rr .* j*sin(k*L) + cos(k*L)) ./ (rr .* cos(k*L) -j*sin(k*L));
fprintf('R/Z0    Vr    Ps    Qs\n');
    fprintf('   Open     %10.1f  %10.3g  %10.3g\n',abs(Vro), 0, Qso);
for i = 1:length(rr)
    fprintf('%10.4f  %10.1f  %10.3g  %10.3g\n',rr(i),abs(Vrr(i)),real(VAs(i)),imag(VAs(i)))
end


% part 1: Real loading only
rf = 1;          % fudge factor
Gr = 0:.01:rf;
Gi = 0;
Vr = 1 ./ (cos(k*L) + j*sin(k*L) .* (Gr + j * Gi));
V_r = Vs .* abs(Vr);
P_r = V_r .^2 .* (Gr/Z0);

figure(1)
plot(P_r, V_r)
title('Problem 5.2, Real Loading');
ylabel('Volts, RMS');
xlabel('Real Load, Watts')
grid on

%part 2: Reactive Loading
```

```
Gr = 0.8;
Gi = -.6:.01:.2;
Vr = 1 ./ (cos(k*L) + j*sin(k*L) .* (Gr + j .* Gi));
V_r1 = Vs .* abs(Vr);
Q_r1 = V_r1 .^2 .* (Gi/Z0);
Gr = 1.0;
Gi = -.4:.01:.4;
Vr = 1 ./ (cos(k*L) + j*sin(k*L) .* (Gr + j .* Gi));
V_r2 = Vs .* abs(Vr);
Q_r2 = V_r2 .^2 .* (Gi/Z0);
Gr = 1.2;
Gi = -.2:.01:.6;
Vr = 1 ./ (cos(k*L) + j*sin(k*L) .* (Gr + j .* Gi));
V_r3 = Vs .* abs(Vr);
Q_r3 = V_r3 .^2 .* (Gi/Z0);

figure(2)
plot(Q_r1, V_r1, Q_r2, V_r2, Q_r3, V_r3)
title('Problem 5.2, Reactive Compensation');
ylabel('Volts, RMS');
xlabel('Reactive Comp, VARS')
grid on
```