Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061 Introduction to Power Systems

Problem Set 4 Solutions February 22, 2003

Problem 1: Real and reactive power at the sending and receiving ends (with sign convention *into* the line terminals are:

$$
\begin{aligned}
P_s &= -P_r = \frac{V_s V_r}{X_L}\sin\delta \\
Q_s &= V_s^2\left(\frac{1}{X_L}-\frac{1}{X_c}\right)-\frac{V_s V_r}{X_L}\cos\delta \\
Q_r &= V_r^2\left(\frac{1}{X_L}-\frac{1}{X_c}\right)-\frac{V_s V_r}{X_L}\cos\delta
\end{aligned}
$$

This is easily calculated and plotted: see the script which is appended to this solution. The *circle diagram* that results is shown in Figure 1. The maximum power that can be handled by this line is:

$$P_{\max} = \frac{13,800^2}{.03 * 377} \approx 16.84 \times 10^6$$

or just about 16.8 MW.

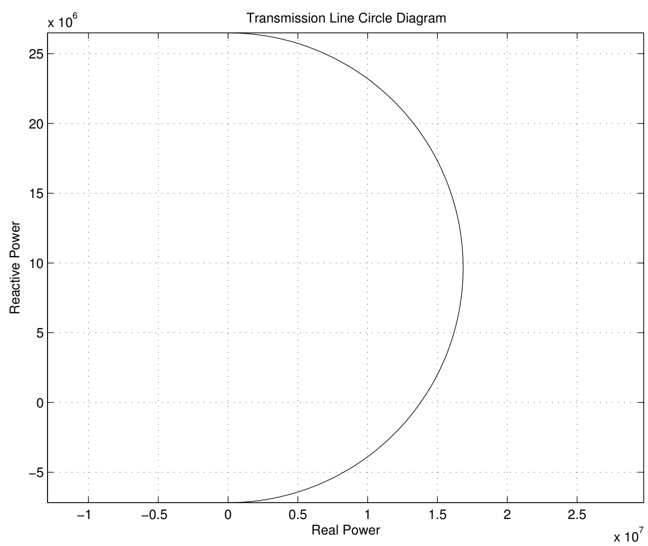

Figure 1: Transmission Line Circle Diagram

To compensate to unity power factor we first find the required phase angle for that power level:

$$\delta = \sin^{-1}\frac{10\times 10^6}{\frac{13,800^2}{.03\times 377}} \approx .6359\text{radians} \approx 36.4^\circ$$

Then if the sending end power factor is unity,

$$Q_s = V_s^2 \left(\frac{1}{X_L} - \frac{1}{X_c} \right) - \frac{V_s V_r}{X_L} \cos\delta = 0$$

or

$$\frac{1}{X_C} = \omega C = \frac{1 - \cos\delta}{X_L}$$

and C evaluates to $45.84 \mu F$.

Finally, for the 6.979 part of the problem, note that in some sense this whole thing can be parameterized by the angle δ. If the capacitance value affects voltage and the voltage in turn affects the phase angle required to push power through the line, we can look first at the back end of the problem, which is voltage:

If real power is

$$P = \frac{V_s V_r}{X_L} \sin\delta$$

we can get, first, the angle:

$$\delta = sin^{-1} \frac{X_L P}{V_s V_r}$$

Then observe that if the load has unity power factor:

$$Q_r = V_r^2 \left(\frac{1}{X_L} - \frac{1}{X_c} \right) - \frac{V_s V_r}{X_L} \cos\delta = 0$$

We may solve this for capacitance. (Does this sound familiar?)

$$\omega C = \frac{1}{X_C} = \frac{1}{X_L} \left(1 - \frac{V_s}{V_r} \cos\delta \right)$$

This is done in the second part of the script appended and the results are plotted in Figure 2 (actually, this is a reversed axis plot of C vs. V_r. We take some comfort in noting that this plot appears to agree with our earlier calculation of required capacitance when the sending and receiving end voltages have the same magnitude.

Problem 2: The three phase voltages are:

$$\begin{aligned} v_a &= \sqrt{2} \cdot 277 \cos(\omega t) \\ v_b &= \sqrt{2} \cdot 277 \cos(\omega t - \frac{2\pi}{3}) \\ v_c &= \sqrt{2} \cdot 277 \cos(\omega t + \frac{2\pi}{3}) \end{aligned}$$

and the center point of this source is grounded.

B We take this one out of order as it is the easiest. The voltages across each of the resistances is defined by the matching source, so that:

$$\begin{aligned} i_a &= \sqrt{2} \cdot \cos(\omega t) \\ i_b &= \sqrt{2} \cdot \cos(\omega t - \frac{2\pi}{3}) \\ i_c &= \sqrt{2} \cdot \cos(\omega t + \frac{2\pi}{3}) \end{aligned}$$

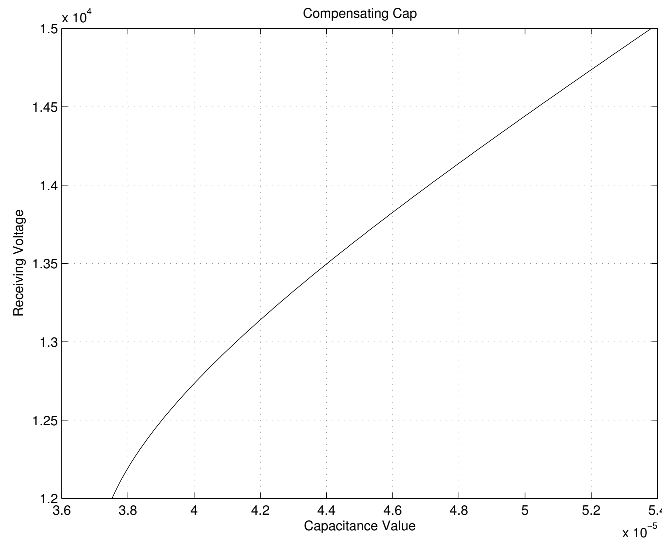

Figure 2: Receiving Voltage vs. Capacitance

A Noting that in part B, the sum of the three currents is zero, the neutral point at the junction of the three resistors can (and in fact will be) at zero potential and so the currents are exactly the same.

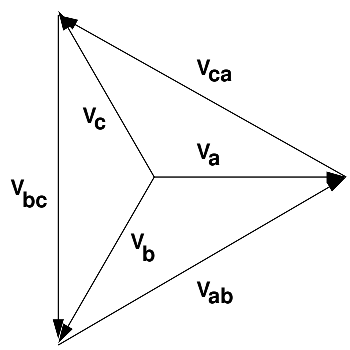

Figure 3: Three-Phase Voltages

C Refer to Figure 3. The voltage across the resistor is:

$$v_{ab} = v_a - v_b = \sqrt{2} \times 277 \left(\cos \omega t - \cos(\omega t - \frac{2\pi}{3}) = \sqrt{2} \times \sqrt{3} \times 277 \cos(\omega t + \frac{\pi}{6}) \right)$$

and of course $\sqrt{3} \times 277 \approx 480$. So

$$i_a = -i_b = \sqrt{2} \times \frac{480}{831} \left(\cos \omega t + \frac{\pi}{6} \right) \approx .816 \cos \left(\omega t + \frac{\pi}{6} \right)$$

D Since $3 \times 277 = 831$ this load is equivalent to that of Part A and so the currents are the same. If you want you can do this the hard way by following the recipe for Part E and adding the two resistor currents at each node.

E This one involves computing the two resistor voltages:

$$
\begin{aligned}
v_{ab} &= v_a - v_b = \sqrt{2}\times 277\left(\cos\omega t - \cos(\omega t - \frac{2\pi}{3})\right) = \sqrt{2}\times\sqrt{3}\times 277\cos(\omega t + \frac{\pi}{6}) \\
v_{ca} &= v_c - v_a = \sqrt{2}\times 277\left(\cos(\omega t + \frac{4\pi}{3}) - \cos\omega t\right) = \sqrt{2}\times\sqrt{3}\times 277\cos(\omega t + \frac{5\pi}{6})
\end{aligned}
$$

Noting that $\cos\omega t + \frac{5\pi}{6} = -\cos\omega t - \frac{\pi}{6}$, we may use the identity:

$$\cos(\omega t - \frac{\pi}{6}) + \cos(\omega t + \frac{\pi}{6}) = 2\cos\omega t\cos\frac{\pi}{6} = \sqrt{3}\cos\omega t$$

and, using the results obtained in Part C,

$$
\begin{aligned}
i_a &= \sqrt{2}\cos\omega t \\
i_b &= -\sqrt{2}\times .577\cos(\omega t + \frac{\pi}{6}) \\
i_c &= -\sqrt{2}\times .577\cos(\omega t - \frac{\pi}{6})
\end{aligned}
$$

F This is just like case B, except for phase C is not connected:

$$
\begin{aligned}
i_a &= \sqrt{2}\times\cos\omega t \\
i_b &= \sqrt{2}\times\cos\left(\omega t - \frac{2\pi}{3}\right) \\
i_c &= 0
\end{aligned}
$$

Problem 3: This one is best done graphically. Note that the current through the ground resistor is just the sum of the three phase currents. Shown in Figure 4 is the same figure that established the currents, but with this summation shown.

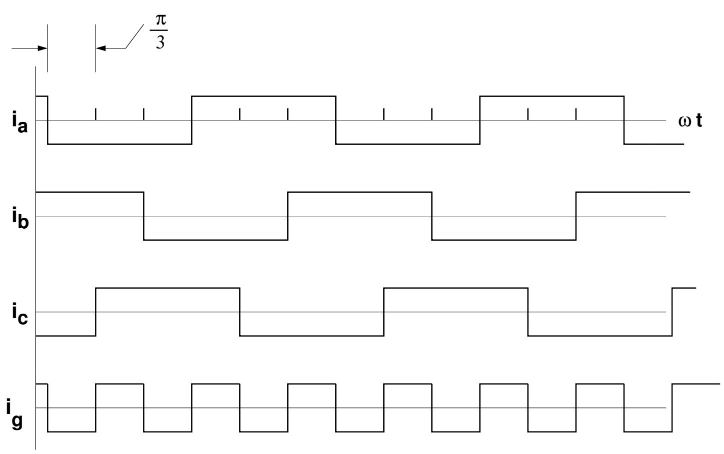

Figure 4: Currents

Now, the voltages in the individual resistors will be just the current sources times the one ohm resistance. The voltage across the ground resistance will, similarly, be just the bottom trace times the one ohm.

Problem 4: For 6.979 If we are to write expressions for the currents, they would be:

$$
\begin{aligned}
i_a &= \frac{v_a = v_n}{R} \\
i_b &= \frac{v_b = v_n}{R} \\
i_c &= \frac{v_c = v_n}{R}
\end{aligned}
$$

where v_n is the voltage of the 'star point' with respect to ground. Now we may note that, since the sum of the three currents must be zero the star point voltage must be the average of the three phase voltages:

$$v_n = \frac{v_a + v_b + v_c}{3}$$

which may be estimated graphically: see Figure 5

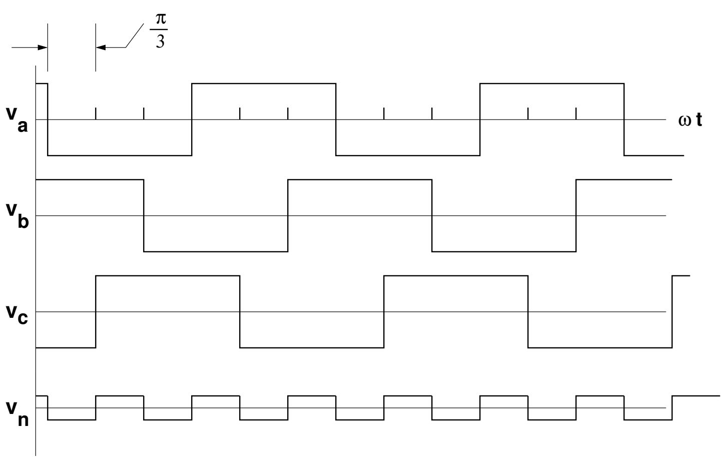

Figure 5: Voltage Source Waveforms

Now we may use this to find the voltage across each of the resistances, by graphically subtracting the neutral, or star point voltage from the phase voltage. This is shown for Phase A in Figure 6. Current in that lead will have the same shape. The same procedure follows for the other two phases: the answers are identical but shifted by 120 degrees.

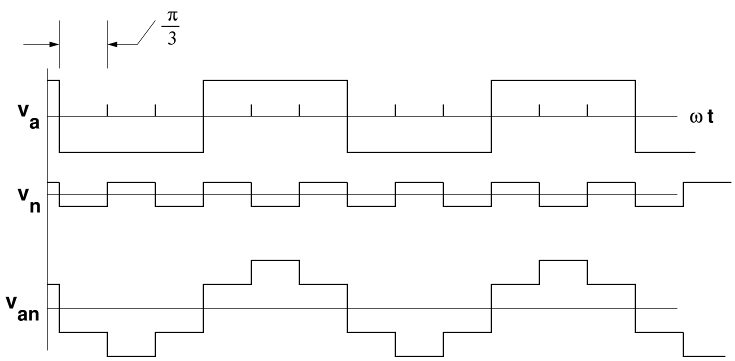

Figure 6: Reconstructed Phase A to Neutral Voltage

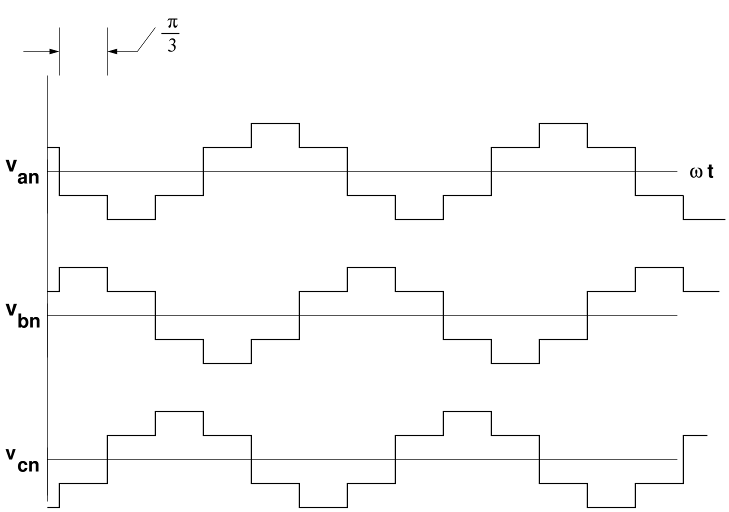

Figure 7: Three Phase Voltages with Neutral Open

Appendix: MATLAB script for Problem 1:

```
% PS 4, Problem 1
V = 13800;
L = .03;
C = 100e-6;
om = 377;
Xl = om*L;
Xc = 1/(om*C);
delt = 0:pi/100:pi;
Ps = (V^2/Xl) .* sin(delt);
Qs = V^2*(1/Xl-1/Xc) - (V^2/Xl) .* cos(delt);

figure(1)
plot(Ps, Qs)
axis equal
grid on
title('Transmission Line Circle Diagram')
ylabel('Reactive Power');
xlabel('Real Power');
Pmax = max(Ps)

% last part
Pr = 10e6;
Vr = 12000:10:15000;          % try this range
d = asin((Xl*Pr) ./ (V .* Vr));  % angle required to make power
Cr = (1/(om*Xl)) .* (1 - (V ./ Vr) .* cos(d));
figure(2)
plot(Cr, Vr)
title('Compensating Cap');
ylabel('Receiving Voltage')
xlabel('Capacitance Value')
grid on
```