Due Date: Friday, November 14, 2003

1. A perennial concern in the design of "sin taxes," or environmental excise taxes more generally, is the tradeoff between distributional equity and efficiency improvement. The following problem illustrates some of these issues. A two-good economy is populated by two consumers, whose preferences are given by

$$U_1 = x_1 + \log z_1$$

$$U_2 = x_2 + \log z_2 - \beta \log z_1.$$

The parameter β satisfies $0 < \beta < 1$. Each consumer (i) chooses (x_i, z_i) subject to the budget constraint $2 = x_i + z_i$. A linear production technology allows one unit of good x to be costlessly transformed to good z, and vice versa. Person 1's consumption of good z reduces person 2's utility, but person 1 is not affected by person 2's consumption of z.

(a) Find the levels of x_1, x_2, z_1, and z_2 that will emerge from the competitive equilibrium. Contrast this equilibrium with the choices a social planner would make, if the social planner were maximizing a utilitarian social welfare function that placed equal weight on the utility of person 1 and person 2.

(b) Now assume that the utilitarian social planner can levy poll taxes on each individual, and can tax person-specific purchases of good z. The government has a balanced-budget requirement. What would the person-specific tax rates on purchases of good z be, and what would the lump sum taxes or subsidies be, if the social planner used these instruments to maximize an additive utilitarian social welfare function as in (a)?

(c) Now imagine that the social planner is constrained to levy a single tax on all consumption of z, without distinguishing consumption by the identity of the buyer. Further assume that lump sum taxes or subsidies must be identical for all individuals. In this case, what tax rate will the planner choose for good z? How does the resulting level of social welfare compare with the outcome in (b)?

2. Consider an individual who lives for two periods and has an additively separable utility function of the form

$$V(C_1, C_2) = \log C_1 + [1/(1+\delta)] \log C_2.$$

Assume that the individual receives an endowment of W_1 at the beginning of the first period, and then divides this endowment between consumption in the two periods. The pretax rate of return is r. The government has just announced that, for the first time, it will impose a capital tax at rate τ on capital income received in period two. The proceeds of this

tax will be paid as a lump-sum transfer to the next generation. Individuals alive today do not care about the next generation.

(a) Find the lifetime indirect utility function, $V(W_1, r(1-\tau))$, for an individual who is just beginning life. Using this expression, evaluate the change in initial endowment (W_1) that would be required to make the individual as well off with the capital tax as without it.

(b) What is $dC_1/d\tau$? What conclusions about the welfare cost of capital income taxation can you draw from this finding?

3. The government in Cloneland faces a long-standing constitutional constraint against levying lump-sum taxes. This is unfortunate, because everyone in Cloneland is identical. The government must raise revenue of R per person to pay tribute to a foreign colonial power that conquered Cloneland many centuries ago. The citizens of Cloneland consume two goods (x_1 and x_2) and they supply labor (L) to earn enough to cover their consumption purchases. The Clones live for only one period, and they all have a wage of unity (so each unit of labor supply earns income of one). The producer prices of the two consumption goods are also equal to unity. The government relies on commodity taxes on goods 1 and 2 at rates τ_1 and τ_2 to raise revenue. The Clones have identical utility functions given by:

$$U = (\alpha_1/(1-1/\varepsilon))*x_1^{1-1/\varepsilon} + (\alpha_2/(1-1/\eta))*x_2^{1-1/\eta} - L.$$

(a) Find the demands for the two consumption goods, and use these demands to write out the government's budget constraint. Then use these demands to obtain the indirect utility function for a representative Clone as a function of the two tax rates.

(b) Determine the relative values of the tax rates on the two goods. This requires finding an expression for τ_1/τ_2 or, what may be easier, $\{\tau_1/(1+\tau_1)\}/\{\tau_2/(1+\tau_2)\}$. You do not need to solve explicitly for each tax rate as a function of preferences and the required revenue level.

4. Now consider another individual with the same preferences as the person analyzed above:

$$V(C_1, C_2) = \log C_1 + [1/(1+\delta)] \log C_2.$$

This individual faces a wage income tax, so that the interest rate at which she can borrow and lend is simply r. Assume that her wage income in periods 1 and 2 is fixed at Y_1 and Y_2, and that her labor income tax rate in period 1 is τ_1 while that in period 2 is τ_2. Assume that $\tau_1 > \tau_2$, and that both taxes are linear. Further assume that the individual has access to a "tax avoidance technology" that permits wage income to be shifted from period 1 to period 2. If the individual chooses to shift A dollars from period 1 to period 2, where A is between 0 and Y_1, her taxable income in period 1 will be $Y_1 - A$ and that in period 2 will be $Y_2 + A$. Using the tax avoidance technology is costly; the cost of shifting A dollars is $\beta(A)$. This

cost can be viewed as the legal and administrative fees associated with tax avoidance, and it must be paid in period 1.

(a) Find the lifetime budget constraint for this individual, recognizing both the impact of tax avoidance on income net of taxes, and the cost of tax avoidance.

(b) Now obtain first order conditions for the optimal choice of A. Does the optimal level of A depend on the utility function? Explain why or why not.

(c) Consider the case in which $\beta(A) = \gamma A^2$, and assume that $r = 0$. Obtain a formula for A as a function of the tax rates in the two periods, and compute the elasticity of tax avoidance (A) with respect to $(1-\tau_1)$. Briefly describe the implications for the impact of tax changes on revenue.

5. The design of targeted saving incentive schemes is a subject of constant policy debate. Consider an economy in which consumers live for two periods, and have preferences given by:

$$U = \log C_1 + \log C_2.$$

Each consumer receives an endowment of 1 at the beginning of the first period of life. The pretax return available to consumers is 200%, so someone who saves $1 receives $3 at the beginning of the next period (the effective "period" is long!). The income tax rate is 50%. The government uses tax revenue to pay tribute to a foreign power.

(a) Find the optimal lifetime consumption profile of this consumer. Contrast this with the optimal choice if there were no income tax.

(b) Now assume that the government introduces a "retirement saving program" that allows each consumer to save up to .20 in the first period of life in a tax-free account. Sketch the consumer's budget constraint when the income tax rate is 50%, and there is no saving program, and again when the program is in place.

(c) Find the consumer's optimal lifetime consumption plan in the case with the retirement saving program. Explain how the introduction of this program affects first-period saving.

(d) When a new government takes power, it changes the structure of the saving incentive scheme to tax investment income on the first .50 of saving, but to exempt any saving in excess of 0.5 from taxation. Sketch the budget set associated with this policy, and find the consumer's optimal consumption profile. Can you explain the difference in the impact of this policy and that in (b)?

(e) In part (d), if the government raises the threshold for tax-exempt saving from .50 to .51, how would this policy change affect personal saving? What would it do to

national saving?