3 Phase Systems

3 sources \((V_a, V_b, V_c)\) separated by 120° in phase \((\frac{2\pi}{3}\text{ rad})\)

\[V_a = V_s \sin(\omega t)\]
\[V_b = V_s \sin(\omega t + \frac{2\pi}{3})\]
\[V_c = V_s \sin(\omega t - \frac{2\pi}{3})\]

Represented as phasors:

\[V_x(t) = \Re \{ V_x e^{j\omega t} \} \]

Connection of 3\(\phi\) sources

- Most common: \(Y\)
- \(\Delta\) Connected (no neutral)

Why 3\(\phi\)?

1. Constant Power Sourcing

\[P(t) = \frac{V^2(t)}{R} \sin^2(\omega t) = \frac{V_s^2}{2R} \left[ 1 - \cos(2\omega t) \right]\]
Power Electronics Notes - D. Perreault

1. At unity power factor, instantaneous power fluctuates between zero and twice average at double the line frequency (no const. power out). Makes sense: can't get power when voltage is zero. This is bad for supplying power to machines, rectifiers, etc. which would like to draw constant power, must buffer the fluctuations (via inertia, capacitors, etc.)

3. Solves thus: \[ P_{tor} = \frac{V_s^2}{2R} \left[ \sin^2(\omega t) + \sin^2(\omega t + \frac{2\pi}{3}) + \sin^2(\omega t - \frac{2\pi}{3}) \right] \]

\[ P_{tor} = \frac{V_s^2}{2R} \left[ 3 + \cos(2\omega t) + \cos(2\omega t - \frac{2\pi}{3}) + \cos(2\omega t + \frac{4\pi}{3}) \right] \]

\[ P_{tor} = \frac{3V_s^2}{2R} \]

\[ \therefore \text{3} \phi \text{ sets can deliver constant total output power without fluctuations!} \]

\[ \text{(note: 2} \phi \text{ sets can as well, since } \sin^2(\omega t) + \cos^2(\omega t) = 1) \]

2. Neutral wire return not needed: unlike 2 \phi power, one can deliver 3 \phi power w/o the need for a neutral return. So 3 wires are needed for both 2 \phi + 3 \phi, but can deliver more power for same amount of wire, cutting in 3 \phi... important for cost of utility lines.

3. 3 \phi systems allow cancellation of all triplen harmonics (harmonics that are multiples of 3). How?

If I take a waveform (not necessarily sinusoidal)

\[ f(t) = \sum a_n \sin(n\omega t + \phi_n) \text{ and shift it by } \pm \frac{T}{3} \left( \pm \frac{\pi}{3} \text{ radians of fundamental } = 120^\circ \right) \]
The \( \pm \) shifted waveforms will have fundamentals that differ by \( 120^\circ = \frac{2\pi}{3} \) radians. 3n harmonics will be shifted by \( 3n \times 120^\circ = n \times 360^\circ \). Thus, if we take the difference of the shifted waveforms, (e.g. \( l-n \)), the 3n harmonics will drop out!

Thus, no even harmonics, 3n's gone, 5th, 7th, 11th, 13th lowest harmonics left, \( \rightarrow \) great!

Line-line voltages can be vector constructed from line-neutral:

\[
V_{ab} = V_a - V_b = \sqrt{3} V_s \sin (\omega t - \frac{\pi}{6})
\]

- 120V l-n
- 208V l-l (rms)

\( \rightarrow \) So l-l magnitude is scaled by \( \sqrt{3} \)

- l-l phase is shifted by \( \frac{\pi}{2} \) (30°) from l-n

Phase shift can be useful for converters fed from \( \Delta/\Delta, \Delta/Y \) transformers,

e.g. 12-pulse rectifiers
Given a 3D set of voltages, we can create a set with any phase relation we desire.

\[ \begin{align*}
V_a(t) &= \text{Re}\left\{V_0 e^{j\frac{2\pi}{3}} e^{j\omega t}\right\} \\
V_b(t) &= \text{Re}\left\{V_0 e^{j\frac{\pi}{3}} e^{j\omega t}\right\} \\
V_c(t) &= \text{Re}\left\{V_0 e^{j\pi} e^{j\omega t}\right\}
\end{align*} \]

In rectangular coordinates, we could represent

\[ \vec{V}_a = \begin{bmatrix} \cos\left(-\frac{\pi}{3}\right) \\ \sin\left(-\frac{\pi}{3}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} V_0 \]

\[ \vec{V}_b = \begin{bmatrix} \cos\left(\frac{\pi}{3}\right) \\ \sin\left(\frac{\pi}{3}\right) \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} V_0 \]

If we wanted to synthesize a phase \( \vec{V}_d \) with \( \alpha = 0 \), \( \| \| = V_0 \)

we could do this by summing parts of \( \vec{V}_a, \vec{V}_b \)

\[ \vec{V}_d = \alpha \vec{V}_a + \beta \vec{V}_b \]

\[ \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} \\ -1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \implies \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \]

We could similarly synthesize any other angle to other phases.

There are multiple ways to do this, since \( \vec{V}_a, \vec{V}_b, \vec{V}_c \)

are a linearly dependent set.
3 Phase Rectification

\( \frac{1}{2} \) wave rectifier

\[ V_d \]

\( V_d \) is the diode "or" of the 3 voltages

If we connect things negatively, we get other halves

Connecting both together, we get the full-bridge rectifier:

\[ V_d \text{ is "most positive of } \{V_a, V_b, V_c\} \text{ - most negative of } \{V_a, V_b, V_c\} \]

This is the same as the largest of \( \{V_{ab}, V_{ac}, V_{bc}, V_{ba}, V_{cb}, V_{ca}\} \)

This "full-bridge" connection operates on line-to-line voltages!
The full-bridge rectifier can be drawn as follows:

Note: The bridge diodes are numbered in the order in which they conduct over the cycle. (e.g., D1, D2, D3, D4, D5, D6, D7, D8, D9, ...)

We can calculate power factor (for phase A, for example)

\[ V_{a_{rms}} = \frac{V_s}{\sqrt{2}} \]
\[ I_{a_{rms}} = \sqrt{\frac{2}{3}} I_d \]
\[ \langle P \rangle = \frac{1}{\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} V_s I_d \sin(\phi) d\phi = \frac{\sqrt{3}}{\pi} V_s I_d \]
\[ K_p = \frac{\langle P \rangle}{V_{\text{rms}} I_{\text{rms}}} = \frac{\sqrt{3}}{\pi} V_{5} I_{d} \cdot \left(\frac{\sqrt{3}}{V_{5} I_{d}}\right) = \frac{3}{\pi} \approx 0.96 \]

(Compare to 0.91 for a 1-phase bridge)

Also, 1-phase bridge \(\rightarrow\) 4 diodes
3-phase bridge \(\rightarrow\) only 6 diodes

Ripple voltage is at 6 \(f_{\text{line}}\) not 2 \(f_{\text{line}}\) \(\Rightarrow\) easier to filter

\[ V_{d1,1P} \rightarrow V_{d1,3P} \]

Single phase rectifier

3-phase rectifier

The ripple voltage magnitude is also smaller \(\Rightarrow\) easier to filter
Higher-Order Rectifiers

Suppose we use two phase-shifted transformer sets on series-stacked six-pulse bridges.

\[ \frac{V_{d1}}{V_{d2}} \] 

The Y/Y, Δ/Δ transformer sets generate equal voltage magnitudes, with a 30° phase difference between their 3φ outputs.

Since all nodes are isolated, constant current in the bridges \( \rightarrow \) the two six-pulse bridges act independently.

Since input waveforms shifted by 30° \((\frac{T}{12})\) and output ripple is at 6x input frequency \((T_{out} = \frac{T}{6})\),

Output ripple voltages are shifted by \(T_{out}/2\) \((30° fund, 150° c.w.)\)

We have a 12-pulse rectifier:

- Smaller ripple magnitude \( \rightarrow \) easier output filter
- Higher ripple frequency
- Net input current + power factor also improves
consider parallel case (direct connection)

again,

V_d is 12 pulse

12-pulse

\[ I_{d_{rms}} = \sqrt{\frac{2^{12}}{2} \cdot \frac{1}{2\pi} \cdot I_d^2} = \frac{I_d}{\sqrt{2}} \]

6-pulse

\[ I_{d_{rms}} = \sqrt{\frac{2^6}{2} \cdot \frac{1}{2\pi} \cdot I_d^2} = \frac{I_d}{\sqrt{3}} \]

So 12 pulse, I_{rms} \downarrow by \sqrt{2}, but twice as many devices

Each device carries the full current for \( \frac{1}{2} \) the time!

(Damage loss in devices, transformers, thus depend on RMS!)
What we would like is to get the 12-pulse waveform with 2 bridges acting independently so each carries half the current.

\[
\text{Ideal:} \quad \text{Forces the current to split evenly.}
\]
\[
\text{Forces the voltage at the output to be the average of 2 input voltages.}
\]

What happens? \(\rightarrow\) Each bridge operates independently.

\[
I_{d1,\text{rms}} = \sqrt{\frac{4\pi}{6} \cdot \frac{1}{2\pi} \left(\frac{I_d}{2}\right)^2} = \frac{I_d}{2\sqrt{3}}
\]

So we have 2x as many devices as a 6-pulse rectifier which carry half the rms current.

The voltage is the average of 2 shifted 6-pulse waveforms giving harmonic cancellation in the output voltage and the input current (as w/ series connection).

This connection is often used for high-current systems (MIT tokamak, Railroad converters, etc.)

Can extend to 13, 24, 36 pulse etc.