# Open vs. Closed Apple Music Distribution Platform 


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#### Abstract

In this thesis, based on the example case study of the Apple iTunes-iPods platform technology, two simple models are analyzed to gain a better understanding of open vs. closed business models as management and market strategies for multi-sided platforms. First, a simple model of a firm with a two-sided platform serving two distinct types of customers is evaluated, assuming network effects as the only intrinsic benefits to joining such a platform. Three different cases of market structure are investigated: (i) monopoly, (ii) open duopoly (iii) closed duopoly. Using game theory, comparative results of prices, profits, consumer surplus and social welfare among the three regimes are presented. The second model focuses on the effects of competition and compatibility between a profit-maximizing closed platform and an open, freely accessible platform. Given certain conditions, it is shown that compatibility can in fact be a profitable strategy for closed platforms while improving social welfare at the same time.


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## Chapter 1

## Introduction

### 1.1 Introduction to Multi-sided Platforms

Many diverse industries are populated by businesses that operate multi-sided platforms. These businesses serve several distinct groups of customers who need each other in some way, and the core business of the multi-sided platform is to provide a common (real or virtual) meeting place and to facilitate interactions between members of the two or more distinct customer groups. Multi-sided platforms are common in old-economy industries such as those based on advertising-supported media and new-economy industries such as those based on software platforms, search engines, Internet service providers and web portals. Digital media platforms, for example, often mediate four distinct markets: users, developers, hardware makers, and content providers. They play an important role throughout the economy by minimizing transactions costs between entities that can benefit from getting together.

These multi-sided markets do present certain unique practical problems. Not surprisingly, the complexity primarily arises from the presence of many unique, but interdependent, classes of customers. In a traditional market, the analysis centers around the responses of a single set of customers to changes in supply (either price or output) and the responses of the vendors to changes in demand. In a multi-sided market the analysis becomes multi-dimensional. The analysis needs to account for (1) the responses of multiple sets of customers to the vendors, (2) the vendors' responses to multiple sets of customers, and (3) the responses of one class of customers to changes in the others' behavior and vice versa. The dialogue over multiple-sided markets has been fueled in part by a growing scholarship that has increased understanding of these markets, combined with a number of significant empirical case studies that involved two-sided markets. Some classic examples include Yellow Pages [37], credit cards [3], newspapers [1], video games [9], etc.

The interdependency of the multiple customer groups also impacts the analysis of the likelihood and success of new entry in multiple-sided markets. First, because multiple sides of the market are needed for the product or service to function (i.e., the platform provider must get both sides of the market on board), new entrants face a form of the chicken-and-egg problem. This problem is probably fairly easy to overcome in some multiple-sided markets, but quite difficult in others. For example, the owner of an attractive new nightclub may find it relatively easy to get the necessary critical mass of both men and women customers. In contrast, a new payment network likely would find it considerably more difficult to obtain the required critical mass of both issuers and merchants.

The difficulty of entry is further increased in some multiple-sided markets because of the presence of indirect network effects (i.e., the value of the product or service to one class of customers often increases directly with the level of usage by the other customer class). Thus, not only must the new entrant simultaneously convince both sets of customers to purchase its product, but it must also overcome the challenge that for many customers the value of purchasing the product or service from the established provider is likely to be significantly greater than from purchasing from the start-up. Obtaining the information needed to analyze these issues is often complex. For example, what critical mass of both sides of the market does a new entrant need to compete effectively? Does conduct by incumbents designed to get both sides of the market on board (e.g., a payment network signing bonuses to issuers) increase the difficulty of entry, and potentially constitute unlawful exclusionary conduct with disparity between prices charged to one side vs. the other? Answering these types of questions is difficult, but it can be done through careful focus on the multiple-sided nature of the market.

For the purposes of this thesis, it is helpful to clarify some terminology that is used in the economics literature and which sometimes causes confusion. Rochet and Tirole [34] used the term multi-sided markets to refer to situations in which businesses were catering to more than one interdependent group of customers through an intermediary product or service. From this point onwards, multisided markets is used synonymously with multi-sided platform as in much of the economics literature. Multi-sided platforms often compete with ordinary single-sided firms and sometimes compete on one side with multi-sided platforms that serve a different second side.

### 1.2 To Platform or not to Platform

Before going onto the next section of open vs. closed platforms, a brief discussion of why many businesses today are following the platform model as opposed to traditional merchant or supply chain model of manufacturer-wholesaler-retailer is highlighted [19]. In order to stay competitive in fast-paced and demanding high technology markets, it is no longer enough for companies to develop
one product at a time. Increasingly, good product development means good platform development. A product platform strategy is the foundation of product strategy, especially in high-technology companies such as Apple that have multiple products (iPhones, iPods, iTouch, etc) connected to a common technology (iTunes). By sharing common components and processes across a platform of products, companies can develop differentiated products efficiently, increase the flexibility and responsiveness of their processes, and take market share away from competitors that develop only one product at a time [35].

In general, a platform is the lowest common denominator of relevant technology in a set of products or a product line [28]. Keeping in mind that these common components or technologies are not necessarily complete in the sense that they could be sold to a customer, a platform can be thought alternately as a vehicle for planning, decision making, and strategic thinking. The nature of product platforms varies widely across industries and product applications. For the purpose of this thesis, aligning with Apple's e-commerce business model, the discussion will concentrate on product platforms in terms of application software products, such as software, hardware and computer-based information services. A platform for application software products comprises the architecture, input/output interfaces, and application functionality. For Apple, the interfaces (iTunes) between subsystems (iPod, iTouch, iPhone) can easily be more important than the subsystems themselves; controlling the interface's design and evolution can lead to long lived systems and is one element of market domination as proven repeatedly by Apple; according to wikipedia, iPod commands 70 percent of the MP3 player market, 4 billion songs have been purchased from iTunes, iPhone is reshaping the entire wireless industry. Even the underdog Mac operating system has begun to nibble into Windows' once-unassailable dominance; for 2007, its share of the US market topped 6 percent, more than double its portion in 2003. Further qualitative analysis of Apple iTunes platform will be considered in the next chapter.

### 1.3 Open vs. Closed Platforms

In recent years fueled by the new capabilities enabled by rapidly growing information and communications technologies, the benefits of being open, i.e. enabling compatibility, modularity, accessibility and integrability, are becoming more apparent and are likely to increase in the near future. These benefits challenge some conventional wisdom about management of businesses, market strategies and the incentives needed to stimulate profitability in an ever changing environment of multi-sided networks. In this thesis, for tractability, two-sided platforms are analyzed from the perspective of different business strategies that platform providers can utilize: open vs. proprietary or closed. The simple models of two-sided platforms can be extended to include platforms with more than two sides.

The openness phenomenon generates extremely interesting theoretical puzzles for economic analysis, which can be formulated as follows: Why do platforms businesses want to open up their networks if no one pays them to do it? How do you coordinate with different sides of platform users in providing access and maintaining applications in the absence of a significant hierarchical structure based on the ownership of assets? Why is it that open business models are becoming so widespread in a world dominated traditionally by proprietary-imposed standards? Contrary to the closed model of Apple iTunes, consider the social networking Facebook platform and its applications developers. What system of incentives regulates the behavior of how hundreds of individuals continually writing applications which are then made available free of charge to the users? The spontaneous, decentralized functioning of the open platform represents challenge to many current notions of coordination, which is crucial for the transformation of an invention into economically advantageous and marketable innovations. How is it possible to align the incentives of several different individuals without resorting to property rights and related contracts? How is it possible to specify everyone's tasks without resorting to a formal hierarchical organization? Regarding the conditions for the diffusion of open platforms, how is it possible to increase the diffusion of new technology without a well-established standard and thus increasing returns from the adoption of such technology?

From an economic point of view, the production of information involves high fixed costs and negligible marginal costs: the majority of this cost is concentrated in establishing the platform and accumulating the number of users on each side. Producing applications through the platform has relatively low costs. This is due to the presence of network externalities, i.e, the demand side economies of scale the utility that a user derives from consumption of the good increases with the number of other agents consuming the good [14]. Externalities can be direct, indirect or deriving from complementary services. Network externalities deeply influence the diffusion process that, in such a case, corresponds to the problem of the emergence of a domineering standard. If a platform manages to gain a significant market share, a virtuous cycle is set in motion such that consumers will have even more incentive to use it, there will be an increase in the supply of complementary products (applications, maintenance) and that particular piece of software will start to dominate the market and become the standard as in the case of Facebook becoming the most popular social networking platform.

### 1.3.1 Waning Dominance of Closed Platforms

Given all the apparent advantages of being open [22, 26, 27], it would seem that having an open platform or value chain would be preferable, at least, from a consumer perspective. Traditionally closed platforms have been seen as enormously profitable from the producers' standpoint, especially given sufficient market share to develop a dominant (or even better, monopolistic) position. Pro-
prietary or closed platforms are great for producers once they have achieved scale precisely because they can lock consumers into their network platform and make switching difficult. Building scale at the beginning can be very tricky. One can imagine how most consumers would generally prefer open to closed platforms as long as the content available is similar in quantity and quality.

Increasingly, closed networks are starting to lose their grip on the consumers with the business transactions moving onto the digital realm. In the last decade (or so) an enormous open network (the Internet) has emerged and is quickly supplanting closed networks as a rival (and increasingly superior) platform. As an example, if one were to look at the main broad media distribution platform categories today, the dominance of closed platforms can still be witnessed but one can also observe the seeds of a very different market emerging:

- Text: Books, magazines, and newspapers are still popular, but blogs have provided consumers with a simple method to publish and promote their own written works by taking advantage of the inherent bilateral communications of the Web, drawing consumer attention away from more traditional closed platform sources.
- Audio: Terrestrial and satellite radio continue to draw listeners (although the overall market for radio is probably not growing, but rather shifting) and labels still sell millions of records, but open platform technologies continue their relentless assault on legacy distribution. While companies such as Apple have taken a half step (by layering a closed network atop an open infrastructure), consumers still turn to myriad P2P network platforms to share and download audio files.
- Video: Broadcast, cable, satellite and DVDs still rule with an iron fist. However, increased broadband penetration has begun to reveal the potential of open video distribution platforms to a much larger audience. While video may not expand at the same rate as open text and audio networks (due to continuing bandwidth constraints), technology platforms such as Pod casts and online movies are becoming ever more popular.

While some economists argue that closed platform business models will continue to operate for many years $[30,25]$, others $[29,18,17]$ suggest that the market will be dominated by hybrids that attempt to layer closed network architectures atop open networks. In many realms, if firms cannot figure out how to make profits on openness, they may not survive. The most likely equilibrium scenario is probably an oscillation between open and closed model as consumers and profit seeking firms vie for the economic profits, resembling the double helix [16]. Perhaps given the overwhelming advantage consumers experience within an open distribution platform, this transition of closed to open or to hybrid is as inevitable as the transition from analog to digital. What promises to be the biggest
challenge of this transition, for the producers, will be the abrupt transition in economic models, since open platforms will not operate in the same way as closed platforms. As the landscape of consumer demand has changed with time and attention comprising the primary currency in today's environment, maximizing the utility of these consumer assets will be the key to success.

### 1.3.2 Effect of Time-Attention Economy on Open Platforms

Without a solid platform, any multi-sided framework (no matter how beneficial to consumers) is ultimately crippled. The reason for this is that these multi-sided systems require agents from all different sides in order to work. If one consumer segment cannot be compensated for his or her efforts in some fashion, the platform will have a hard time expanding beyond a niche community. By definition, all groups have to participate maximally to achieve the richest possible experience although there may be some exceptions. However bad open platforms seem to producers, they have some significant advantages over closed platforms. These advantages can drive new business models that, while different from existing models, can be quite profitable. Two primary advantages are (1) open networks offer bi-directional communication: communication between producer and consumer is direct, simple, and immediate; (2) open networks encourage diversity and specialization: low cost of production encourages consumer participation and enables focus on niche subject matter. If utilized effectively, these advantages can help producers derive maximum benefit from open platforms through an alternate form of currency.

If open platform models begin to dominate and closed platform revenue streams begin to evaporate, producers and managers will have to make changes to their business strategies for multi-sided markets. In media industries, cheap productions will more easily find the advertising partners they need for hyper distribution; costly productions will find themselves competing against so many cheap productions that they'll find it progressively harder to justify their costs in the face of ever-smaller ratings. The mass audiences of the future may less often number in the millions. The micro audiences of hyper distribution will range from hundreds to hundreds of thousands, but in that long tail of television productions there is a vast appetite for an incredible variety of programs. This is no longer an era of mass media and mass audiences: the dinosaurs of media are about to give way to the mammals. Producers must learn to master the economics of time and attention. They must focus less effort on building walls and more on building community; less focus on the lecture and more focus on the conversation. But some old media dinosaurs will make the leap from closed to open. In doing so, they will join an emerging global community of millions of producers, advertisers, and consumers, each striving to contribute something valuable to the conversation. The result will be a platform value chain defined by a staggering degree of diversity and openness, but also rich in the possibility of profit.

Even as producers focus on the virtues of open platforms, in many cases money still needs to be spent to produce and maintain the platform. Without economic incentive, content richness and diversity ultimately suffers. But in an open platform business model, charging consumers cash for content erects barriers to consumption that can dramatically reduce demand and limit reach. As open networks become more mature the problem will only get worse. In the Economy of Bartered Time and Attention, you might not be able to charge the consumer directly for access or usage, but you are nonetheless receiving something from the consumer that is very valuable (albeit less liquid than cash). Time and attention are the only variables which remain scarce in an open platform, and they are about the most valuable things you can request of a consumer.

While time and attention is not itself liquid, there is already a group of people who specialize in converting time and attention into real dollars. They are called advertisers. While many advertisers have struggled recently (many still focus on traditional push marketing), open platforms provide the tools necessary to increase advertising efficiency by orders of magnitude. Bi-directional communication provides advertisers with the potential to communicate directly with consumers. Niche audiences enable advertisers to focus messaging on very tight demographic communities (even individuals). Inexpensive production and distribution enable effective reach, even if audiences are fragmented. Contextual advertising is still in its infancy; however, improved software algorithms and direct consumer interaction will continue to drive greater efficiencies in the conversion of time and attention into cash. Ultimately, advertising and sponsorship receipts should meet the funding requirements of almost any production. But while advertising and sponsorships will generate a bulk of the revenue, producers will still have other ways in which they can monetize content assets in an open platform such as soliciting voluntary payments for access or downloads, content filtering, device integration, third party software developers, maintenance services, to name a few. This is where the current work presented in this thesis in multi-sided platform businesses intersects with open vs. closed platforms.

The next chapter provides a case study of Apple iTunes platform and its dominance of online music distribution industry. In order to further understand the models presented in chapter 4 and 5 , chapter 3 showcases traditional economic principles of two-sided platforms, the simplest form of multi-sided platforms and briefly reviews existing significant theoretical and empirical literature on basic issues in two-sided markets using some simple linear pricing models based on pioneering work by Rochet and Tirole [33]. Monopoly, first-best and second-best prices when a single platform charges either usage fees or membership fees are carefully examined. Chapter 4 and 5 details the two models of open vs. closed platforms, motivated by the case study in chapter 2. Chapter 6 concludes with directions for future research.

## Chapter 2

## Multi-sided Platform Case Study: Apple iTunes Online Media Distribution Platform

Consider the Apple music distribution platform through iTunes-iPod interface which is clearly a multi-sided platform, the two sides primarily being music producers and consumers. Along the value chain upstream to downstream, there are artists, publishers, marketers/promoters, online music sales to consumers of music. Each entity can be thought of as a node in the supply chain and it is interesting to observe how open or closed each node is in relation to the next link or to the external consumers. To provide a visual example of open vs. closed in this scenario, imagine the content (pod casts, songs, TV shows, music videos, movies, etc) available through iTunes website as closed to non-MP3 music formats as seen in Figure 2.1.

An interesting question about Apples solution, which is clearly a closed one (their service, their device, their player), is not whether being open or closed is the correct business model for Apple, but whether openness really matters. Philosophically, there's something democratic about the notion of openness: open government, open communications, open systems. But openness in a technology platform has an Achilles' heel as well. Offering compatibility means accepting all adopters no matter what their expertise or the quality of their products. Windows indeed supports far more developers and devices than Apple, but suffers attendant reliability and security woes partly as a result. Amazingly, Apple managed to resolve these issues after many other companies had established the market, essentially by getting the music production companies or labels to buy into its closed network architecture. Apple has enormous incentive to keep the network closed in order to keep people using iTunes to find tracks (preferably on a Mac) and iPods to play them. The fact that


Figure 2-1: Value Chain of Apple iTunes Music Distribution Platform
the iTunes-iPod pairing has turned into a phenomenal success means that Apple has little gain to make iTunes compatible with Windows players. Apple wants to sell more iPods, and the best way to do that is to keep customers in the Apple universe. By keeping its music business under one roof, Apple makes tracking and restricting unauthorized uses of iTunes music far more manageable.

While the rest of its Silicon Valley cohorts have embraced the values of openness and interoperability, Apple continues to tie its proprietary software to its proprietary hardware, maintaining its status as one of the few companies to remain vertically integrated and closed. And yet many consumers don't seem to mind Apple's closed model. The fact that iPod hardware and the iTunes software are inextricably linked may be the reason why they work so well together. And now, PC-based iPod users, impressed with the experience, have started converting to Macs, further investing themselves in the Apple ecosystem. Some Apple competitors have tried to emulate its tactics. Microsoft's MP3 strategy used to be like its mobile strategy, licensing its software to (almost) all newcomers. This is no longer the case; the operating system for Microsoft's Zune player is designed uniquely for the device, mimicking the iPod's vertical integration. Amazon's Kindle e-reader provides seamless access to a proprietary selection of downloadable books, much as the iTunes Music Store provides direct access to an Apple-curated storefront. And the Nintendo Wii, the Sony PlayStation 3, and the Xbox360 each offer users access to self-contained online marketplaces for downloading games and special features.

Apple's successful product platform strategy seems to be the biggest determinant of its success in high-technology industry. Apple clearly understands the underlying elements of its iTunes platform. It's all too easy to oversimplify a product platform and fail to really understand its subsystems. A
product platform consists of a number of subsystems, each with different characteristics. Since the subsystems underlying a product platform define the potential and limits of its performance, it's essential for companies to understand how these subsystems fit together, how they may change over time, and how they differ from competitors' platforms. Apple represents one industry standard for the future of the Internet: three tiered systems that blend hardware, installed software and proprietary web applications. As consumers increasingly access the Web using scaled-down appliances like mobile phones and Kindle readers, they will demand applications that are tailored to work with those devices. True, such systems could theoretically be open, with any developer allowed to throw its own applications and services into the mix. But for now, Apple is the benchmark for how to build a three-tiered system. A few years ago, when hardware and software makers were focused on winning business clients, price and interoperability were more important than the user experience. But now that end users make up the most profitable market segment, usability and design have become priorities. Customers expect a reliable and intuitive experience just like they do with any other consumer product. The Apple platform provides the ultimate proprietary solution.

The Apple platform's defining technology is clearly distinguishable from other platform subsystems. In any product platform, there often exists one subsystem above all others that best defines the nature of that platform. This critical subsystem defines the unique characteristic of all products developed from that platform. Most importantly, this defining subsystems or defining technology is also the key to understanding a product platform. The inability to understand this defining technology will lead to the eventual failure of a platform strategy; AOL, Friendster, Compuserve, just to name a few examples. Apple's approach to platform development and innovation is another important success factor. In general, a robust platform and a well-planned strategy can lead to longlived platforms, market dominance through interfaces becoming industry standard as mentioned in the previous chapter, market advantage through more timely new product introduction and upgrades, a richer product family covering a broader scope of a particular market, and barriers to entry for other competitors that lack an equivalent product line.

In addition, Apple's platform strategy focuses management of key decisions at the right time. Because there are fewer platforms than products, developing a product control strategy at the platform level simplifies the product strategy process and enables senior management in a company to concentrate on the most critical platform-level decisions, instead of diluting attention across numerous products. Apple regularly releases updates for iPhones which penalizes the users who make unauthorized changes to the software on their phones, including unlocking techniques, voiding their warranties or deleting all third-party software applications downloaded through the update. Apple's platform strategy enables products to be deployed rapidly and consistently which implies greater ability to tailor products to the needs of different market segments or customers. Apple is well-
positioned to make more applications available through its current Web-hosted approach through its Safari web browser. The Apple platform significantly enhances responsiveness of firms, as the product variants can be developed quickly once time has been invested in building and developing the platform. Apple is always churning out updated iTunes versions as well as multiple generations of iPods.

Apple is resistant to an open standard for music distribution. Its platform approach encourages a longer-term view of a proprietary product strategy. Since it takes a product platform much longer to progress through a life cycle than the individual products derived from that platform, Apple uses a platform approach to help them create a longer-term opportunity and roadmap for their strategy to stay dominant in the changing industry landscape while Apple's competitors have gone the open and interoperable route: Android, Google's operating system for mobile phones, is designed to work on any participating handset, Amazon.com began selling DRM-free songs that can be played on any MP3 player in 2007, even Microsoft has begun to embrace the open movement toward Web-based applications, software that runs on any platform. Is Apple best served by keeping its products closed to competitor's complementary products in the long run? For example, should Apple design the iPod to play only songs purchased from iTunes or should the online iTunes store sell files that can be played only on the iPod? Might Apple be better off by accepting an open standard? How are these dynamics affected by the relative attractiveness of the firms products vis-a-vis competitors products? Under what conditions is being open better than being closed? These are the very core questions that rooted this thesis.

There are some who argue that Apple's closed loop model will eventually limit its users to too few songs when the next link in the value chain, the music companies, decides to close their connection by withdrawing or refusing to license. If they do, it could stem more from opposing Apple's dollar-per-song price point than from the accessibility of its technology. In any case, system openness will play a big part with the breadth or success of Apple's offering. On the other hand, one can easily imagine that the iPod's success was clearly in large part driven by its adoption of two open modular architectures: the open MP3 format and commodity hard-drive technology. iPod took off because it gave people on-the-go access to all the music they had already ripped. Even the vaunted tight connectivity between iTunes and iPod is built on the open USB standard. iPod-iTunes has only a small dose of proprietary architecture in its digital rights management - everything else is essentially open and modular. In fact it's arguable that Apple's core competency of late has been its ability to integrate open standards in a user-centered manner with great fit and finish, with the minimum necessary proprietary secret sauce. This is the main motivation behind the mathematical models presented in later chapters: How does openness or closedness of each link in the value chain affect the viability of the platform?

## Chapter 3

## Review on Multi-Sided Platforms: Literature and Models

The objective of this chapter is to present some simplified and tractable models to illustrate key principles that drive the economic analysis of multi-sided markets to help better understand mathematical constructs presented in later chapters. Examples and mathematical results shown are based on the simplest scenario of two-sided markets. Concentration will be on the case of a single platform with monopoly. Socially optimal prices are also examined. Competition between platforms is a more complex issue, and the models presented in later chapters address the case of competition between platforms. Linear forms for demand are assumed to make the analyses tractable and to give closed-form solutions that can be interpreted easily.

### 3.1 Pricing principles for a single platform

Most of the content of this chapter is based on simple versions of the models by Rochet \& Tirole [34, 32] and Armstrong [2]. The main focus is for established platforms, motivated by the case of the Apple iTunes-iPod Platform mentioned in the previous chapter. A related problem is how to solve the so-called chicken and egg problem [4] and get the platform off the ground in the first place, since neither side of the market will be willing to use the platform unless participants on the other side do. This problem is only briefly discussed in the context of membership pricing. In the next section pricing by a single platform is discussed. Usage pricing and membership pricing are considered separately. In each case monopoly and socially optimal prices are examined and some factors driving prices are explained citing the similarities and differences. The chapter concludes with a brief section of some implications for analysis of competition and regulation in two-sided markets.

Many articles in the existing literature have examined the economics of price determination in multisided platform markets. A key finding is that optimal prices for the multiple customer groups must align or balance the demand among these groups and indeed the emergence of a pricing structure as well as a pricing level is the defining characteristic of such industries [12]. Optimal prices are not proportional to marginal costs as is the case with the familiar Lerner conditions or its multi-product variants. Indeed, it is possible that the optimal price for one side will be less than the marginal cost for that side. The assignment of costs to one side or another may not be well defined either. When it is necessary to get both sides together for a platform product to exist that is for either customer to have anything to purchase one may not be able to say that one side or another 'caused' a cost [31]. Platform businesses may tend to skew prices towards one side or another depending upon the magnitude of the indirect network externalities resulting from that side. If side $A$ generates a much greater degree of externalities for side $B$ than side $B$ does for side $A$, side $A$ may tend to get a lower price. Most literature is based on quite rarefied assumptions and has thus far focused on static pricing issues.

Evans [13] tries to put put some order in this fast growing field, by providing general introduction, overview and discussion of lessons to be drawn and Roson [36] surveys on general theoretical and definitional frameworks. From the supply point of view, useful distinctions can be introduced, on the basis of the price, or non-price, instruments available for the platforms. Much of the literature has considered two classes of price instruments: membership and usage charges. Membership fees have been considered mainly in the context of market intermediation, and in all cases where transactions are not perfectly observed, or are costly to monitor. Usage fees have been considered mainly in association with credit cards and payment systems, for example in terms of merchant fees [3].

The use of one or the other price instrument, however, is sometimes a purely conventional choice. From the point of view of an agent on one side, who has expectations about the number of interactions carried out in equilibrium, there is always some equivalence between the two types of price (as far as price changes can be compensated, to keep expected utility constant). The distinction between the two prices makes sense only if the choice of joining a platform is logically separated from the subsequent choice of making a certain interaction on the same platform. This is the approach taken, for example, by Caillaud and Julien [4], where the realization of a transaction on an intermediation platform is probabilistic. In this setting, there can be side-payments between buyers and sellers, so that only the total transaction cost matter (not the price structure). Higher transaction prices reduce the probability of realization of a transaction on the platform.

The crucial feature of demand in two-sided markets is that demand on each side depends to some extent on what happens on the other side of the market. The models used in this section will be
special cases of the canonical demand model proposed by Rochet \& Tirole [34]. In their model, with a single platform the payoff to a user on side $i=A, B$ of the market from using the platform is:

$$
\begin{equation*}
U_{i}=\left(b_{i}-p_{i}\right) N_{j}+B_{i}-r_{i} \tag{3.1}
\end{equation*}
$$

where $b_{i}$ is the user's benefit per interaction with members on the other side of the market, $p_{i}$ is the price the platform charges per interaction, $N_{j}$ is the number of platform members on the other side of the market, $B_{i}$ is the user's fixed benefit (or cost) of joining the platform that does not depend on interactions with the other side, and $r_{i}$ is the platforms membership fee.

In this model, users on each side of the market may be differentiated in terms of their values of $b_{i}$ and $B_{i}$. The platform also has four prices available to it: usage fees and membership fees on both sides. Analyzing platform behavior in the context of this general model is somewhat complicated, so some further simplifying assumptions are needed about prices or the users' benefits, to derive basic results about prices.

### 3.2 Pure usage pricing

First consider the case where platforms only charge for usage, i.e. $r_{i}=0$ on both sides. Suppose also that there are no fixed benefits or costs of joining the platform, so $B_{i}=0$ on both sides. Then the payoff of a consumer on side $i=A, B$ of the market from joining the platform is $U_{i}=\left(b_{i}-p_{i}\right) N_{j}$. There are two ways to justify the assumption that fixed benefits are zero $\left(B_{i}=0\right)$. One way is that with no membership fees, users are able to choose whether to use the platform on a case-by-case basis with respect to each interaction that they make with a user on the other side of the market. The other way is to assume that membership of the platform is already fixed. This is probably the most simple type of two-sided market model that one can analyze. For illustration, let's also assume that the per-interaction benefits are uniformly distributed on both sides of the market. In particular, $b_{i}$ is drawn from a uniform distribution on $\left[0, \alpha_{i}\right]$. A consumer on side $i$ will use the platform if $b_{i} \geq p_{i}$ and $N_{j}>0$. Normalizing the total numbers of consumers on both sides of the market to 1 , the demand for interactions generated by side $i$ is therefore

$$
\begin{equation*}
N_{i}=\frac{\alpha_{i}-p_{i}}{\alpha_{i}} \tag{3.2}
\end{equation*}
$$

In this model, $\alpha_{i}$ represents the maximum willingness to pay of users on side $i$ per interaction with a user on the other side. An increase in $\alpha_{i}$ raises the willingness to pay per interaction of all users on side $i$, but leaves the total potential market size fixed at 1 (by assumption). Since only usage is charged, the demand for interactions on each side of the market is independent of the behavior
of users on the other side of the market. This can be seen from the demand function for side $i$ given in (3.2). For simplicity, assume that the total number of interactions that takes place on the platform is the product of the demands on the two sides, $N_{A} N_{B}$. In a more complex model one might allow for congestion, that is, the total number of interactions is some function of $N_{A} N_{B}$ that exhibits diminishing returns at some point. Assume also that the costs of operating the platform are proportional to the number of interactions. If $c$ is the constant marginal and average cost per interaction then the total cost is $c N_{A} N_{B}$. It is assumed that at least some interactions on the platform generate positive social value, so $\alpha_{A}+\alpha_{B}>c$.

### 3.2.1 Monopoly prices

Under these assumptions, the profit of a monopoly platform is

$$
\pi=\left(p_{A}+p_{B}-c\right) N_{A} N_{B}
$$

or

$$
\pi=\left(p_{A}+p_{B}-c\right)\left(\frac{\alpha_{A}-p_{A}}{\alpha_{A}}\right)\left(\frac{\alpha_{B}-p_{B}}{\alpha_{B}}\right)
$$

A monopoly will choose $p_{A}$ and $p_{B}$ to maximize profit. The first-order conditions are

$$
\frac{\partial \pi}{\partial p_{i}}=\frac{-\left(p_{i}+p_{j}-c\right)\left(\alpha_{j}-p_{j}\right)+\left(\alpha_{i}-p_{i}\right)\left(\alpha_{j}-p_{j}\right)}{\alpha_{A} \alpha_{B}}=0
$$

$i \neq j=A, B$. Solving for $p_{i}$ gives

$$
\begin{equation*}
p_{i}=\frac{1}{2}\left(\alpha_{i}+c-p_{j}\right) \tag{3.3}
\end{equation*}
$$

This shows that the profit-maximizing price charged on side $i$ is increasing in willingness to pay on side $i$ and in the per-interaction cost, as would be expected, but is also clecreasing in the price charged on the other side of the market. This can be explained as follows. Look at the profit function $\pi=\left(p_{A}+p_{B}-c\right) N_{A} N_{B}$. If $p_{B}$ is fixed, say, then $N_{B}$ is fixed and so is irrelevant for the choice of $p_{A}$. But $p_{B}$ also affects the marginal revenues from increasing $p_{A}$, since the platform earns $p_{A}+p_{B}$ per interaction. Higher $p_{B}$ means higher marginal revenue from an additional interaction. Thus the platform has an incentive to cut $p_{A}$ and increase $N_{A}$, to boost usage.

This analysis shows the first crucial fact about pricing in two-sided markets: the prices charged to the two sides are linked. Since the volume of usage of the platform is determined by the products of the demands on both sides and per-interaction revenue is the sum of both prices, the two prices must be set in concert. Demand conditions on both sides of the market affect both prices. Figure 3.1 illustrates this latter point. Starting from a symmetric situation where $\alpha_{A}=\alpha_{B}=\alpha$, the figure
shows what happens to optimal prices when $\alpha_{A}$ increases while $\alpha_{B}$ remains constant: the optimal price charged to $A$ side users increases, while the optimal price on the $B$ side decreases. This is because the total revenue that can be extracted from usage charges depends on the demands on both sides. When $\alpha_{A}$ increases, $A$ side users are willing to pay higher usage fees per interaction to the platform, thus the optimal price charged to them increases, as shown by (3.3). In addition, the higher price on the $A$ side means the platform can raise profits further by decreasing prices on the $B$ side, which will boost the number of interactions on its platform and increase the revenue that can be extracted from usage pricing.


Figure 3-1: Starting from a symmetric situation where $\alpha_{A}=\alpha_{B}=\alpha$, effect on profit maximizing usage prices of an increase in $\alpha_{B}$ to $\alpha^{\prime}$

Solving (3.3) for $p_{A}$ and $p_{B}$ at a profit maximum we get monopoly usage prices:

$$
\begin{equation*}
p_{i}^{M}=\frac{1}{3}\left(c+2 \alpha_{i}-\alpha_{j}\right) \tag{3.4}
\end{equation*}
$$

for $i \neq j=A, B$. In general, the side with the higher willingness to pay should be charged a relatively higher price. This reflects not only the higher willingness to pay, but the fact that revenues can be further boosted by reducing prices on the lower willingness to pay side to increase the volume of interactions on the platform. In other words, the side that generates relatively more benefits for the other side should be charged a lower price. This feature of two-sided pricing is termed the skewness principle [34]: a factor that supports higher pricing on one side tends to support lower pricing on the other side. Thus it is generally expected of platforms to charge asymmetric prices on the two sides of the market, not only because the two groups of consumers have different preferences but also because of the way the platforms profits depend on the interaction between the two sides.

Another way to characterize prices in two-sided markets is in terms of their level and structure. The level is the total price charged to the two sides, and the structure is the split of this total across the
two sides. In the model of this section define $L=p_{A}+p_{B}$ and $s=\frac{p_{A}}{L}$. Therefore, $p_{A}=s L$ and $p_{B}=(1-s) L$, i.e, a fraction $s$ of the price level is paid by the $A$ side and $1-s$ is paid by the $B$ side. In these terms the monopoly profit function under our liner demand assumptions can be rewritten as

$$
\pi=(L-c)\left(\frac{\alpha_{A}-s L}{\alpha_{A}}\right)\left(\frac{\alpha_{B}-(1-s) L}{\alpha_{B}}\right)
$$

One can think of the platform choosing $s$ and $L$ to maximize its profits. This is equivalent to maximizing with respect to $p_{A}$ and $p_{B}$ since given $s$ and $L, p_{A}$ and $p_{B}$ are uniquely determined.

In this formulation, the first-order condition for the optimal choice of price structure is

$$
\frac{\partial \pi}{\partial s}=-\frac{L}{\alpha_{A}}(L-c)\left(\frac{\alpha_{B}-(1-s) L}{\alpha_{B}}\right)+\frac{L}{\alpha_{B}}(L-c)\left(\frac{\alpha_{A}-s L}{\alpha_{A}}\right)=0
$$

Simplification results in

$$
\begin{equation*}
\frac{L}{\alpha_{A}}\left(\frac{\alpha_{B}-(1-s) L}{\alpha_{B}}\right)=\frac{L}{\alpha_{B}}\left(\frac{\alpha_{A}-s L}{\alpha_{A}}\right) . \tag{3.5}
\end{equation*}
$$

This shows the role of the price structure in usage pricing. For a given price level, the per-interaction profit, $L-c$ is given. Thus given $L$ the platform chooses $s$ to maximize the volume of interactions on the platform, $N_{A} N_{B}$. Equation (3.5) shows that $s$ should be chosen so that the marginal effect of an increase in $s$ in terms of the resulting reduction in $N_{A}$ on the volume of usage of the platform should equal the effect of the increase in $N_{B}$ on the volume of usage. Furthermore, solving (3.5) for $s$ gives

$$
\begin{equation*}
s=\frac{1}{2}+\frac{\left(\alpha_{A}-\alpha_{B}\right)}{2 L} . \tag{3.6}
\end{equation*}
$$

If demands on the two sides of the market are symmetric $\left(\alpha_{A}=\alpha_{B}\right)$ then clearly the optimal price structure is for each side to pay an equal share of the price level, regardless of what the level is. Otherwise, if $\alpha_{A}>\alpha_{B}$ then $s>\frac{1}{2}$, so the $A$ side pays a relatively higher price, for the reasons explained above. In this particular model, the condition for the profit-maximizing choice of $L$ for a given $s$ is algebraically complex and does not give any useful insights. However, consider more general demand functions $q_{A}(s L)$ and $q_{B}((1-s) L)$. Then the platforms profit in terms of $L$ and $s$ is $\pi=(L-c) q_{A}(s L) q_{B}((1-s) L)$. The first-order condition for choice of $L$ is

$$
\frac{\partial \pi}{\partial L}=q_{A} q_{B}+(L-c)\left(s q_{A}^{\prime} q_{B}+(1-s) q_{A} q_{B}^{\prime}\right)=0
$$

Rearranging, the markup of the price level over cost is obtained as

$$
\begin{equation*}
\frac{L-c}{L}=-\frac{q_{A} q_{B}}{s L q_{A}^{\prime} q_{B}+(1-s) L q_{A} q_{B}^{\prime}} \tag{3.7}
\end{equation*}
$$

Demand elasticities on the two sides of the market are

$$
\begin{aligned}
\epsilon_{A} & =-\frac{s L q_{A}^{\prime}}{q_{A}} \\
\epsilon_{B} & =-\frac{(1-s) L q_{B}^{\prime}}{q_{B}}
\end{aligned}
$$

Rewriting (3.7) gives

$$
\begin{equation*}
\frac{L-c}{L}=\frac{1}{\epsilon_{A}+\epsilon_{B}} \tag{3.8}
\end{equation*}
$$

Thus, as with a normal monopoly, the markup of the price level over cost is equal to the inverse demand elasticity, but with a two-sided platform the total demand elasticity is the sum of the elasticities on the two sides of the market.

Returning to the linear model, maximizing profit with respect to $s$ and $L$ should give the same results as maximizing with respect to $p_{A}$ and $p_{B}$ directly. Indeed, it is straightforward to verify that the monopoly $s$ and $L$ satisfy $L^{M}=p_{A}^{M}+p_{B}^{M}$ and $s^{M}=\frac{p_{A}^{M}}{\left(p_{A}^{M}+p_{B}^{M}\right)}$, where $p_{A}^{M}$ and $p_{B}^{M}$ are the monopoly prices given by (3.4). For later reference, these are

$$
\begin{aligned}
L^{M} & =\frac{1}{3}\left(\alpha_{A}+\alpha_{B}+2 c\right) \\
s^{M} & =\frac{1}{2}+\frac{3\left(\alpha_{A}-\alpha_{B}\right)}{2\left(2 c+\alpha_{A}+\alpha_{B}\right)}
\end{aligned}
$$

In summary, the factors that affect monopoly usage prices can be thought of in two ways. First, the price charged to each side reflects a markup over cost, plus an adjustment so that the side which creates more benefits for users on the other side pays a relatively lower price. Alternatively, the overall level of prices is determined by a standard Lerner markup over marginal cost, where the markup is proportional to the inverse of the sum of the elasticities across the two sides. Given this price level, the split of it across the two sides is determined by maximizing the total volume of usage of the platform.

### 3.2.2 First-Best Prices

As with normal markets, monopoly pricing in two-sided markets involves a distortion due to market power that causes a deadweight loss. From (3.8) a monopoly will set the price level greater than marginal cost, which clearly cannot be socially optimal. In fact, the first-best (unconstrained welfare maximizing) prices require that the price level be less than the per interaction marginal cost. To
see this, in the linear model, the gross surplus created on side $i$ of the market is

$$
\begin{align*}
V_{i} & =N_{j} \int_{p_{i}}^{a_{i}} \frac{b_{i}}{\alpha_{i}} d b_{i} \\
& =\frac{\alpha_{j}-p_{j}}{\alpha_{j}} \times \frac{\left(\alpha_{i}-p_{i}\right)\left(\alpha_{i}+p_{i}\right)}{2 \alpha_{i}} \tag{3.9}
\end{align*}
$$

Thus total welfare across both sides of the market is $W=V_{A}+V_{B}-c N_{A} N_{B}$, or

$$
\begin{equation*}
W=\frac{\left(\alpha_{A}+\alpha_{B}+p_{A}+p_{B}-2 c\right)\left(\alpha_{A}-p_{A}\right)\left(\alpha_{B}-p_{B}\right)}{2 \alpha_{A} \alpha_{B}} \tag{3.10}
\end{equation*}
$$

Then the first-best price on side $i$ satisfies

$$
\frac{\partial W}{\partial p_{i}}=\frac{-\left(\alpha_{i}+\alpha_{j}+p_{i}+p_{j}-2 c\right)\left(\alpha_{j}-p_{j}\right)+\left(\alpha_{i}-p_{i}\right)\left(\alpha_{j}-p_{j}\right)}{2 \alpha_{i} \alpha_{j}}=0
$$

which gives

$$
\begin{equation*}
p_{i}=c-\frac{1}{2}\left(\alpha_{j}+p_{j}\right) . \tag{3.11}
\end{equation*}
$$

Comparing (3.11) with the monopolist's choice of price given by (3.3) reveals some similarities and differences. In both cases $p_{i}$ is increasing in $c$, although the monopolist does not pass through all of the cost increase into prices. In both cases $p_{i}$ is also decreasing in $p_{j}$, but the intuitive reasons for this are different. With monopoly higher $p_{j}$ means higher marginal revenue on side $i$ for any given $p_{i}$, so the profit-maximizing response is to reduce $p_{i}$. From a total welfare point of view, higher $p_{j}$ reduces demand on side $j$ meaning that the average per-interaction valuation of consumers on side $B$ who do use the platform at the higher price increases. Thus given this welfare can be increased by reducing the price on $i$ and stimulating demand on that side.

However, the striking difference is that the monopoly price on side $i$ is increasing in $\alpha_{i}$ and independent of $\alpha_{j}$, while the first-best price is decreasing in $\alpha_{j}$ and independent of $\alpha_{i}$. To explain this, imagine for example that $p_{B}=0$. Then $N_{B}=1$ (maximum possible demand on the $B$ side) and a monopolist would set $p_{A}=\frac{1}{2}\left(c+\alpha_{A}\right)$ while the first-best price is $p_{A}=c-\frac{1}{2} \alpha_{B}$. What accounts for this difference? For the monopolist, when $p_{B}=0$, profit comes entirely from the $A$ side of the market, thus the optimal choice of $p_{A}$ depends only on $\alpha_{A}$. For welfare, when $p_{B}=0$, the welfare generated on the $B$ side of the market still matters, and the welfare generated on the $B$ side depends on $p_{A}$, since $p_{A}$ determines $N_{A}$, and that determines how many interactions each consumer on the $B$ side makes. If the welfare on the $B$ side did not matter, $p_{A}=c$ would generate maximum welfare on the $A$ side. However, higher $p_{A}$ reduces welfare on the $B$ side, since it reduces the number of interactions for each customer on the $B$ side. The importance of this effect depends on the welfare generated per interaction on the $B$ side, which is measured by $\alpha_{B}$. Thus the first-best price on the
$A$ side is decreasing in $\alpha_{B}$.

Solving the first-order conditions for the first-best prices gives

$$
\begin{equation*}
p_{i}^{F B}=\frac{1}{3}\left(2 c+\alpha_{i}-2 \alpha_{j}\right) \tag{3.12}
\end{equation*}
$$

As with monopoly, first-best prices on both sides depend on costs and also demand conditions on both sides of the market. As mentioned before, the first-best prices imply a per-transaction subsidy. To see this, note that the first-best price level is $L^{F B}=\frac{1}{3}\left(4 c-\alpha_{A}-\alpha_{B}\right)$. Thus $c-L^{F B}=\frac{1}{3}\left(\alpha_{A}+\alpha B-c\right)$ which is positive under the assumption that $\alpha_{A}+\alpha_{B}>c$. Thus the sum of the first-best prices is lower than the marginal per-interaction cost. To see why, consider starting from $L=c$. Now imagine, for example, cutting the price on $A$ side of the market slightly, so that $L<c$. In a normal market this would reduce welfare, since the gross benefits from the price reduction would be less than the cost of producing the extra output. This will be true on the $A$ side of the market, but in a two-sided market, benefits are also generated on the $B$ side. Taking this into account, reducing $p_{A}$ slightly in this way will increase total welfare across both sides of the market. Therefore, the first-best price level involves pricing below marginal cost. The first-best price structure also differs from that chosen by a monopoly.

Under monopoly we have

$$
s^{M}=\frac{1}{2}+\frac{3\left(\alpha_{A}-\alpha_{B}\right)}{2\left(2 c+\alpha_{A}+\alpha_{B}\right)}
$$

while from (3.12) the following is obtained:

$$
s^{F B}=\frac{1}{2}+\frac{3\left(\alpha_{A}-\alpha_{B}\right)}{2\left(4 c-\alpha_{A}+\alpha_{B}\right)}
$$

These are identical with $s^{M}=s^{F B}=\frac{1}{2}$ only in the special case that consumer preferences are identical on the two sides of the market $\left(\alpha_{A}=\alpha_{B}\right)$. The difference comes because the monopoly platform does not fully account for the effects of prices on welfare generated on both sides of the market. In addition, the more different are consumer preferences on the two sides, the greater the divergence of the monopoly price structure from the first-best structure. For example, normalize $\alpha_{A}=1$ and suppose $c=0$, then

$$
s^{M}-s^{F B}=\frac{3\left(1-\alpha_{B}\right)}{1+\alpha_{B}}
$$

which is larger (in absolute value) the further $\alpha_{B}$ is from 1, i.e, the greater the difference in willingness to pay across the two sides.

### 3.2.3 Second-best Prices

As explained above, first best usage prices involve a subsidy since the price level is below the perinteraction marginal cost. Thus the revenue that a platform generates under the first-best prices will not cover its total costs, and it will require a subsidy to achieve these prices. A more feasible solution is the second-best prices that satisfy the break-even constraint $p_{A}^{S B}+p_{B}^{S B}=c$ or $L^{S B}=c$. Second-best prices are thus chosen to maximize gross welfare benefits across both markets, given by (3.10), subject to this break-even constraint.

Substituting $p_{B}=c-p_{A}$ into (3.10) gives

$$
W=\frac{\left(\alpha_{A}+\alpha_{B}-c\right)\left(\alpha_{A}-p_{A}\right)\left(\alpha_{B}-c+p_{A}\right)}{2 \alpha_{A} \alpha_{B}}
$$

Then the first-order condition for $p_{A}$ is

$$
\frac{\partial W}{\partial p_{A}}=\frac{-\left(\alpha_{A}+\alpha_{B}-c\right)\left(\alpha_{B}-c+p_{A}\right)+\left(\alpha_{A}+\alpha_{B}-c\right)\left(\alpha_{A}-p_{A}\right)}{2 \alpha_{A} \alpha_{B}}=0
$$

which gives second-best prices $p_{i}^{S B}=\frac{1}{2}\left(c+\alpha_{i}-\alpha_{j}\right)$.

As usual, these prices depend on demand conditions on both sides of the market, as well as the per-interaction marginal cost. Given the price level $L=c$, the second-best price structure is

$$
s^{S B}=\frac{1}{2}+\frac{\alpha_{A}-\alpha_{B}}{2 c} .
$$

Again, the side with the higher valuation of interactions pays a relatively higher price, and the price on each side is decreasing in the per-interaction value created on the other side. Recall from (3.6) that the monopoly price structure for a given price level is $s=\frac{1}{2}+\frac{\left(\alpha_{A}-\alpha_{B}\right)}{2 L}$. Thus if $L=c$, the monopoly will implement the second-best price structure. However, this is not a general result and depends on the linear demand functions assumed in this example. Intuitively, given that $L=c$, suppose that $s=\frac{1}{2}$ but $\alpha_{A}>\alpha_{B}$. Then welfare can be increased while maintaining the cost-recovery constraint by increasing $p_{A}$ while decreasing $p_{B}$ by the same amount .i.e, by increasing $s$. When $\alpha_{A}>\alpha_{B}$, consumers on the $A$ side value interactions more highly, so it is optimal to cut the price on the $B$ side to boost usage of the platform and create more value on the $A$ side. However, to maintain cost recovery this requires simultaneously increasing the price on the $A$ side. Since $\alpha_{A}$ is high, the revenue generated from the $A$ side is high, and while the required price increase reduces demand on the $A$ side, it does not completely offset the additional welfare generated from reducing $p_{B}$.

### 3.2.4 Usage Pricing Comparison

Let's briefly compare the monopoly, first-best and second-best usage prices in terms of the price levels and structures generated. First, as in any market, market power distorts the price level, so the monopoly price level exceeds both the first- and second-best levels. In contrast, the first-best price level involves a per-interaction subsidy, due to the fact that a marginal decrease in price on either side of the market generates benefits on both sides of the market. Obviously, the second-best price level is in the middle, equal to the per-interaction marginal cost.

In the linear example earlier it turns out that the monopoly and first-best price levels are equal distance from the second-best price level:

$$
L^{M}-L^{S B}=L^{S B}-L^{F B}=\frac{1}{3}\left(\alpha_{A}+\alpha_{B}-c\right)
$$

Thus the greater are $\alpha_{A}$ and $\alpha_{B}$, the greater the spread of the monopoly and first-best price levels around the second-best price level. Intuitively, higher $\alpha_{A}$ or $\alpha_{B}$ means greater cross-platform benefits from reducing the price on either side of the market, so the first-best price level is lower. On the other hand, higher $\alpha_{A}$ or $\alpha_{B}$ means that a monopoly platform can exploit stronger willingness to pay on at least one side of the market, so monopoly prices are higher.

The price structure also differs across the three cases, unless consumer preferences are the same on both sides of the market. Comparing the three price structures,

$$
\begin{aligned}
s^{M} & =\frac{1}{2}+\frac{3\left(\alpha_{A}-\alpha_{B}\right)}{2\left(2 c+\alpha_{A}+\alpha_{B}\right)} \\
s^{S B} & =\frac{1}{2}+\frac{\alpha_{A}-\alpha_{B}}{2 c} \\
s^{F B} & =\frac{1}{2}+\frac{3\left(\alpha_{A}-\alpha_{B}\right)}{2\left(4 c-\alpha_{A}-\alpha_{B}\right)}
\end{aligned}
$$

Since it must be assumed $4 c>\alpha_{A}+\alpha_{B}$ to ensure the first-best price level is positive, all three price structures have the feature that the $A$ side of the market pays a relatively higher price when $\alpha_{A}>\alpha_{B}$.However, the mechanism underlying the determination of the price structure is somewhat different across the cases. To illustrate, start from a situation of symmetric demand, where $\alpha_{A}=\alpha_{B}$. Then the optimal price structure is $s=\frac{1}{2}$ in all three cases. Now suppose we increase $\alpha_{A}$ slightly. Under monopoly, willingness to pay on the $A$ side is higher, which supports a higher price on that side, and revenues can be further boosted by cutting the price on the $B$ side to stimulate greater usage of the platform.

With second-best pricing, greater value per interaction on the $A$ side means greater cross-platform benefits from increasing usage on the $B$ side, and welfare can be increased by cutting price on the $B$
side to increase the volume of usage of the platform, but this must be accompanied by an increase in price on the $A$ side to maintain cost recover. With first-best pricing, an increase in $\alpha_{A}$ also means that usage of the platform on the $B$ side creates greater externalities on the $A$ side, thus the $B$ side price should be reduced to subsidize usage on that side and take account of this externality. This reduction in price on the $B$ side should be accompanied by an increase in price on the $A$ side because when the $B$ side price reduces, the new consumers who use the platform have relatively lower valuations of an interaction compared to the existing $B$ type consumers. This reduces the marginal value of an $A$ side consumer on the platform from a welfare point of view, and it follows that the $A$ side price should increase.

### 3.3 Pure membership pricing

Next consider the case where a single platform charges only membership fees. Unlike with usage fees this can cause analytical difficulties because the network nature of demand can result in multiple equilibria. Another difference with usage pricing is that the structure of membership fees will not generally be neutralized by payments between users on the two sides of the market. This is because membership fees are fixed and sunk when users on the two sides interact using the platform. Thus if the platform facilitates a perfectly competitive market, for example, membership fees will affect the level of demand and supply in the market and thus the market quantity. The market price will not neutralize the structure of membership fees because when users transact the membership fees are already sunk, and will not affect the market outcome.

### 3.3.1 Network Effects and Expectations

To illustrate the expectations problem, in the general model (3.1), suppose that the platform only charges membership fees ( $p_{i}=0$ on both sides), and all benefits come from usage ( $B_{i}=0$ on both sides). Then the payoff of a consumer on side $i$ of the market from joining the platform is $U_{i}=b_{i} N_{j}-r_{i}$. These assumptions could reflect the platforms inability to monitor interactions on its platform and impose fees on them. Again assume that $b_{i}$ is uniformly distributed on $\left[0, \alpha_{i}\right]$ and the number of potential users on both sides is normalized to 1 . A user on side $i$ will join the platform if $b_{i}=r_{i} / N_{j}$ and so demand on side $i$ is given by

$$
\begin{equation*}
N_{i}=\frac{\alpha_{i}-\frac{r_{i}}{N_{j}}}{\alpha_{i}} \tag{3.13}
\end{equation*}
$$

As before, $\alpha_{i}$ represents the maximum willingness to pay per interaction on side $i$. In this model, demands on the two sides of the market are given by the solutions to (3.13). For any given membership fees, provided that fees are not too high, these equations have two solutions for each of $N_{A}$ and
$N_{B}$. Unlike with usage fees, under membership pricing a consumers decision to use the platform depends on the actions of consumers on the other side of the market. Consider consumers on each side basing their membership decision on their expectation of what consumers on the other side will do, the model generates multiple equilibria where pessimistic expectations lead to low demand and optimistic expectations lead to high demand, for the same prices. Multiple equilibria are a common characteristic of demand in models with network externalities. In two-sided markets with membership pricing, these externalities flow across the platform, and generate multiple equilibria on both sides of the market.

Figure 3.2 illustrates for the asymmetric case of $r_{A}=\frac{2}{25}$ and $r_{B}=\frac{8}{25}$ and shows the effect of a decrease in $r_{B}$ to $\frac{4}{25}$ while keeping $r_{U}$ constant. The value that users in each group receive from joining the platform increases with the number of users in the other group that are expected to join. If expectations are high, this increases the expected utility from joining. High expectations thus reinforce high demand, and low expectations reinforce low demand. In this simple linear utility setup this leads to two demand configurations for any given prices, provided that the prices are not too high.


Figure 3-2: Solutions to the equations in (3.13) when $\alpha_{A}=\alpha_{B}=1$

Of the two demand levels for given prices, only the larger solution is stable in the sense that demands on both sides of the market would converge to this solution if perturbed slightly away from it for given prices. A better interpretation of the lower solution is the critical mass that the platform needs to achieve on both sides of the market. Once demands exceeding this critical mass have been achieved, the demands will converge to the stable upper solution. In fact, with a two-sided platform, it is only necessary to achieve critical mass on one side of the market. For example, once a sufficient number of consumers on side $A$ of the market have joined a platform, it will induce more consumers on side $B$ to join, even if the initial membership on side $B$ was low. Thus provided that critical mass is achieved on one side of the market, the expectations problem can be solved on both sides. This suggests that a platform with limited resources seeking to establish itself would be best to target one side of the market and try to achieve a high membership level on that side. One way do to this is to charge low or even zero prices on one side of the market. Doing so would make that side more likely to join the platform even if the consumers had very pessimistic expectations about the success of the platform. Knowing this, members on the other side will be more willing to join for a positive membership fee.

Without further detailed analysis it is difficult to predict which side of the market should be targeted with low prices to solve the expectations problem. In terms of the expectations problem it makes sense to charge lower prices on the side of the market with lowest willingness to pay, since the revenues foregone will be less than if lower prices were provided to the side with relatively high willingness to pay. In the simple model above, each consumer on one side of the market that joins the platform is equally valuable to any given consumer on the other side. In reality, some important consumers might raise the value to consumers on the other side disproportionately. Thus it may be necessary only to attract a relatively small number of such consumers on one side to overcome the expectations problem and it may be desirable for the platform to use more complex price discrimination strategies such as offering membership discounts to users who are highly valuable to members on the other side.

The possibility for low or high levels of demand at the same membership prices can also be characterized as a coordination problem between the two sides of the market. Agents on both sides would prefer the high demand outcome as it gives them more gross benefits at the same prices. However, when agents on the two sides make membership decisions independently, they may not be able to coordinate on achieving this equilibrium. This suggests another way for platforms to overcome the expectations problem is to target local groups of consumers that have some relationship outside the platform. Such groups of agents may have the incentive and ability to coordinate their membership decisions, thus reducing the need for the platform to establish optimistic expectations.

### 3.3.2 Simplified Membership Pricing

The expectations problem and the resulting multiple equilibria makes analysis of membership pricing technically complex. Even in the simple linear models assumed here, the solutions to (3.13) that define demands for given membership fees are algebraically messy. To abstract from the expectations problem while still providing some insight into membership pricing, let's consider an extra simplification in this section. This simplification assumes away the expectations problem while still keeping the two-sided nature of demand.

The way to simplify is to use the model of usage benefits and membership prices from above, but assume that the platform can solve the expectations problem somehow and can coordinate demand on the two sides of the market to achieve the demand level that it desires, i.e, assume that the platform maximizes its profits by choice of the network sizes $N_{A}$ and $N_{B}$ directly, while the membership fees that it receives are determined by whatever is consistent with these network sizes, that is, from the inverse demand functions. In other words, assume that the platform chooses quantities rather than prices. In this case, rearranging (3.13) gives the membership fee received on side $i$ of the market (inverse demand on side $i$ ) as $r_{i}=a_{i} N_{j}\left(1-N_{i}\right)$. When analyzing single-platform prices, Rochet \& Tirole $[33,34]$ assume that the platform can set the effective per interaction fee faced by consumers on each side of the market. In the simple model presented before, this means that the platform can set $\frac{r_{j}}{N_{j}}$, i.e, the membership fee paid on side $i$ divided by the number of interactions that consumers on that side make on the platform. If $P_{i}=\frac{r_{j}}{N_{j}}$ is assumed to be the effective per-interaction price then the payoff to a consumer on side $i$ from joining the platform is $U_{i}=\left(b_{i}-P_{i}\right) N_{j}$. Then a consumer on side $i$ will join if $b_{i}=P_{i}$. Thus choosing $P_{i}$ is equivalent to choosing $N_{i}$ directly, and demand on side $i$ is

$$
\begin{equation*}
N_{i}=\frac{\alpha_{i}-P_{i}}{\alpha_{i}} \tag{3.14}
\end{equation*}
$$

So with a single platform, choosing the effective per-interaction fees $P_{A}$ and $P_{B}$ or choosing the quantities $N_{A}$ and $N_{B}$ directly are equivalent and will produce exactly the same results. In terms of gaining intuitive understanding of membership pricing, it turns out to be better to use the [34] approach of choosing per-interaction fees. With membership pricing, the platforms profit is

$$
\pi=N_{A} r_{A}+N_{B} r_{B}-c N_{A} N_{B}
$$

Since $r_{i}=N_{j} P_{i}$, rewrite this as $p=\left(P_{A}+P_{B}-c\right) N_{A} N_{B}$, where $N_{A}$ and $N_{B}$ are given by (3.14). Thus in this version of the model platform pricing with membership fees is equivalent to pricing with usage fees, if the platform can set per-interaction fees (equivalently quantities) directly.

This observation implies that all of the insights from the section about usage pricing carry over
to the determination of per-interaction fees in membership pricing. The only additional step is to convert the per-interaction fees into membership fees. Since $r_{i}=N_{j} P_{i}$, for given per-interaction fees $P_{A}$ and $P_{B}$, from (3.14) the equivalent membership fees are

$$
r_{i}=\alpha_{j}-P_{j} \alpha_{j} \times P_{i}
$$

That is, multiply the per-interaction fee on side $i$ by the demand on the other side of the market to get the equivalent membership fee for side $i$. Given this, it is straightforward to calculate the monopoly, second-best and first-best membership prices in this model. With monopoly, from (3.4) the profitmaximizing per-interaction fees are $P_{i}^{M}=\frac{1}{3}\left(c+2 \alpha_{i}-\alpha_{j}\right)$. This gives demand of $N_{i}=\frac{\alpha_{i}+\alpha_{j}-c}{3 \alpha_{i}}$ and so the monopoly membership fees are

$$
\begin{equation*}
r_{i}^{M}=\frac{\left(\alpha_{i}+\alpha_{j}-c\right)\left(c+2 \alpha_{i}-\alpha_{j}\right)}{9 \alpha_{j}} . \tag{3.15}
\end{equation*}
$$

From this, it can be easily deduced that compared to usage pricing, membership pricing requires an additional consideration. With membership pricing, the determination of the effective perinteraction fees $P_{A}$ and $P_{B}$ (or equivalently the network sizes $N_{A}$ and $N_{B}$ ) is the same as determination of usage fees with pure usage pricing. The per-interaction fee on each side is increasing in willingness to pay on that side, but decreasing in willingness to pay on the other side, for the same reasons that usage fees are. However, for a given per-interaction fee on side $i$, the membership fee that generates it must take account of the number of interactions, since platform members on side $i$ will make $N_{j}$ interactions on the platform and $r_{i}=N_{j} P_{i}$.

Thus unlike usage pricing, a change in $\alpha_{j}$ has two opposing effects on the membership fee charged on side $i$. Higher $\alpha_{j}$ reduces the optimal per-interaction fee on side $i$, while at the same time higher $\alpha_{j}$ increases membership of the platform on side $j$ at given prices, which increases the number of interactions made by a member on side $i$. This latter effect requires an increase in the membership fee to achieve a given per-interaction fee. Therefore, these two effects work in opposite directions, and $r_{i}^{M}$ may be increasing or decreasing in $\alpha_{j}$ depending on which effect is stronger. Nevertheless, the structure of membership fees follows a similar pattern to usage fees. Defining the monopoly structure as $s^{M}=\frac{r_{A}^{M}}{r_{A}^{M}+r_{B}^{M}}$, from (3.15) it can be derived that

$$
s^{M}=\frac{1}{2}+\left(\alpha_{A}-\alpha_{B}\right)\left(2\left(\alpha_{A}+\alpha_{B}\right)+c\right) 2\left(\left(\alpha_{A}-\alpha_{B}\right)^{2}+\alpha_{A}^{2}+\alpha_{B}^{2}+\left(\alpha_{A}+\alpha_{B}\right) c\right)
$$

As with usage pricing, the side of the market that creates more benefits for members on the other side who pay a relatively lower price. Similar calculations allow derivation of the first- and secondbest membership fees from the respective usage prices. The important conclusion is that the basic
principle of linkage between the two sides of the market, and distortion of the price level and structure by market power carry over to the case of membership pricing. The only extra complication is the adjustment of per-interaction fees to equivalent membership fees as discussed above.

### 3.4 Summary and implications for analysis of competition

The discussion and simple models that are examined have revealed a number of principles relevant to pricing in two-sided markets. The most fundamental principle is that prices on the two sides of the market are linked. Prices on one side cannot be set without consideration for the effects on both sides. Thus the pricing problem is better thought of as a joint problem where prices on the two sides are chosen together. It is important to note that this principle applies regardless of whether the objective is profit maximization or welfare maximization, and whether usage or membership is charged.

As seen in this chapter, the optimal prices on the two sides depend on costs as well as demand conditions on both sides. In particular, the price set on one side should generally be lower if users on that side create greater benefits for users on the other side, everything else equal. In addition, even if there are costs that can be identified as being caused by one side of the market, it does not follow that these costs should be passed through entirely to consumers on that side. Doing so will result in relatively high prices on that side, which will also reduce benefits generated on the other side of the market. Depending on consumer preferences across the two sides, this may not be optimal.

These insights have a few important implications for the analysis of competitive behavior and regulation in two-sided markets which is explored in the next two chapters. First, price regulation in a two-sided market is more complex than usual. In the simplest textbook case, a regulator need only set price equal to marginal cost to maximize welfare. Thus the regulator only needs information about production costs. Even in the simplest two-sided market model this is no longer true. Second-best prices involve the price level across the two sides of the market equaling marginal cost, but the split of this price level across consumers on the two sides still matters for welfare. Passing through all of the cost to one side or the other, or an equal sharing of costs across both sides are unlikely to result in welfare maximization. Thus a regulator in a two-sided market always needs information about demands as well as costs.

Second, the linkage in prices across the two sides means that it never makes sense to analyze the price on one side in isolation. This is important from a competition policy point of view. High prices on one side of the market do not necessarily imply the existence of market power. Instead, market power will be reflected in the level of prices across both sides. Similarly, very low prices on one side do not imply predatory pricing. These may simply be optimal pricing structures in response to
consumer preferences across both sides. As evidenced by the simple models in this chapter, it is also possible for first- or second-best pricing structures to be skewed. In terms of regulation, linkage also implies that regulation of the price on only one side of the market will cause the price charged on the other side to change. This may have unintended and detrimental consequences for the objective of the regulator. In simple cases, when platforms charge only usage fees or only membership fees, the prices charged to the two sides are uniquely determined by the price level and structure. Thus firms and regulators in two-sided markets should focus on these two variables rather than thinking about the prices charged to the two sides independently. In most of the discussion above the concepts of the level and structure of prices as an alternative representation of prices in two-sided markets are emphasized to form a basic understanding and formal introduction of the existing pricing models of two-sided markets. In the following two chapters, these pricing models are used as building blocks to analyze competition between platforms, particularly regarding the issue of profitability and welfare benefits of open versus closed platform business models.

## Chapter 4

## Simple Model of Open vs. Closed Platform

In this chapter a firm with a two-sided platform serving two distinct types of consumer is modeled assuming network effects as the only intrinsic benefits to joining such a platform. Imagine that Apple is the firm and iTunes is the platform serving music download purchasers and the record companies who own the rights for the songs. Again, the scenario can be extended to include more than two sides of the iTunes platform, namely advertisers and other content providers. Given that the utility of each type of consumer is increasing in the number of the other type accessible through the firm's platform, three possible cases of market structure are investigated: (i) monopoly, (ii) open duopoly (ii) closed duopoly.

The model presented is closely related to several papers that analyze competition between firms with two-sided markets such as analysis of Bertrand competition between firms in a matching industry [4], who focus on the ability of a firm to dominate such a market and the pricing strategies used to do so. In contrast, this chapter considers a different type of competition, namely, Cournot competition as in [21] and analyzes welfare issues and pricing strategies of imperfect competitors. Detailed analysis of the model relies heavily on material previously introduced in chapter 3. The model's approach is similar to [24], which uses the set up of competing platforms to analyze the Internet as a two-sided network and look at interconnection and access pricing between Internet backbone providers, in that it focuses on elasticity issues and parallels with traditional Ramsey pricing. Finally, this paper should be distinguished from the literature on systems competition. In the systems literature, firms produce different products (for example, computer CPUs and monitors), but they are consumed together as a system by the same consumer, not by two different types of consumer [15].

### 4.1 Model Setup and Assumptions

Assume there are two distinct populations of consumers, Type $A$ and Type $B$. Consumers of both types are differentiated according to their valuation of getting access to the other type through the networked platform service provider. Suppose Type $A$ consumer gets total utility $u_{A}$ from accessing Type $B$ consumers, where for simplicity of argument $u_{A}$ is uniformly distributed on $[0,1]$. Similarly, Type $B$ consumer gets total utility $\theta u_{B}$ from finding Type $A$ consumers, where $\theta>0$ and $u_{B}$ is again uniformly distributed on $[0,1]$. The parameter $\theta$ measures how much on average Type $B$ consumers value gaining access to Type $A$ consumers relative to how much Type $A$ consumers value Type B. Without loss of generality, assume $\theta \leq 1$.

Suppose the firm has $n_{A} \in[0,1]$ Type $A$ and $n_{B} \in[0,1]$ Type $B$ consumers connected to its twosided network. In terms of interpreting the basic model, if the firm is a platform provider, $n_{B}$ becomes the expected number of transactions that Type $A$ consumers makes using the platform, given that every consumer of both types can make one "transaction" with every consumer of the other type. Total expected utilities are then $n_{B} u_{A}$ and $n_{A} \theta u_{B}$. In the case of the firm being a matching service provider such as employment and dating agencies or real estate companies, assume that there exists a unique match for each consumer of the same type. A Type $A$ consumer gets total utility $u_{A}$ from being matched, and zero otherwise. The same utility outcome is true for Type $B$ consumers. Valuations of each consumer and that of their perfect match are assumed to be independent. Thus, Type $A$ consumer meets his or her unique match with probability $n_{B}$ and attains total expected utility $n_{B} u_{A}$ from joining the matching service. Similarly, Type $B$ consumer's total expected utility equals $n_{A} \theta u_{B}$.

For further analysis, it is also needed that the firms commit to their output levels prior to consumers forming expectations about the equilibrium network sizes (Cournot competition). For simplicity, firms produce at zero marginal cost and face small fixed cost associated with having a strictly positive network size of either type of consumer. It is shown in a later section that this particular assumption eliminates some 'undesirable' equilibrium outcomes in the duopoly case. With the lack of costs, the first best outcome is to set $n_{A}=n_{B}=1$, which gives us total welfare $W_{\text {best }}=\frac{1}{2}(1+\theta)$. There can be no interoperability or complementarity between platforms or services. This issue is explored in the subsequent chapter in this thesis. Let's also allow for the possibility that the market can be flooded with either type of consumers and that the network effects are multiplicative with quantity effects. This is reasonable since through the two-sided network structure, the firm can cross-subsidize between the two consumer types and therefore may aim to flood one side of the market. In addition, consumers are heterogeneous according to their levels of network externalities.

### 4.2 Pricing Schemes

Two different pricing schemes that a firm can typically implement are examined. One possibility is to charge a fixed subscription fee to use the service irrespective of whether a unique match is found in the matching service situation or how many transactions the consumer makes through the platform. Let $s_{A}$ and $s_{B}$ be the subscription charges to Type $A$ and Type $B$. Alternatively, the firm can charge a success fee contingent of a consumer finding a unique match or a per transaction fee. Label $t_{A}$ and $t_{B}$ as the transaction fees charged to Type $A$ and Type $B$ consumers respectively. In the following sections, it is shown that the two pricing regimes are equivalent from the firm's point of view since they produce the same profit functions. Hence, in this simplified setup, equilibrium quantities and profits are guaranteed to be identical whichever pricing scheme is used.

### 4.3 Monopoly

This is the case where there is only one firm in the market providing the platform or matching service.

- Subscription Fees: The inverse demand functions faced by the monopolist $\operatorname{ares}_{A}\left(n_{A}, n_{B}\right)=$ $n_{B}\left(1-n_{A}\right)$ for Type $A$ consumers and $s_{B}\left(n_{A}, n_{B}\right)=\theta n_{A}\left(1-n_{B}\right)$ for Type $B$ consumers. The monopolist's profit function is $\pi\left(n_{A}, n_{B}\right)=n_{A} s_{A}\left(n_{A}, n_{B}\right)+n_{B} s_{B}\left(n_{A}, n_{B}\right)$.
- Transaction Fees: The inverse demand functions faced by the monopolist are $t_{A}\left(n_{A}\right)=1-n_{A}$ for Type $A$ consumers and $t_{B}\left(n_{B}\right)=\theta\left(1-n_{B}\right)$ for Type $B$ consumers. The monopolist's profit function is $\pi\left(n_{A}, n_{B}\right)=n_{A} n_{B}\left[t_{A}\left(n_{A}\right)+t_{B}\left(n_{B}\right)\right]$.

Substituting the appropriate inverse demand functions into the two profit functions above gives the payoff function:

$$
\begin{equation*}
\left(n_{A}, n_{B}\right)=n_{A} n_{B}\left[\left(1-n_{A}\right)+\theta\left(1-n_{B}\right)\right] \tag{4.1}
\end{equation*}
$$

The monopolist chooses $n_{A}$ and $n_{B}$ to maximize (3.1) subject to $n_{A}, n_{B} \in[0,1]$.
Proposition 4.1. For $\theta \in\left(\frac{1}{2}, 1\right]$, the solution to the monopolist's optimization problem is interior with $n_{A}=\frac{1}{3}(1+\theta)$ and $n_{B}=\left(\frac{1}{3 \theta}\right)(1+\theta)$. If $\theta \in\left(0, \frac{1}{2}\right]$, the monopolist sets $n_{A}=\frac{1}{2}$ and $n_{B}=1$.

Proof: The monopolist chooses $n_{A}$ and $n_{B}$ to maximize (4.1) subject to $n_{A}, n_{B} \in[0,1]$. Let $\Gamma=n_{A} n_{B}\left[\left(1-n_{A}\right)+\theta\left(1-n_{B}\right)\right]-\lambda\left(n_{A}-1\right)-\mu\left(n_{B}-1\right)$. The first order conditions are:

$$
\begin{aligned}
& \frac{\partial \Gamma}{\partial n_{A}}=n_{B}\left(1-n_{A}\right)-n_{A} n_{B}+n_{B} \theta\left(1-n_{B}\right)-\lambda=0 \\
& \frac{\partial \Gamma}{\partial n_{B}}=n_{A}\left(1-n_{A}\right)-\theta n_{A} n_{B}+n_{A} \theta\left(1-n_{B}\right)-\mu=0
\end{aligned}
$$

The complimentary slackness conditions are:

$$
\begin{aligned}
& \lambda\left(1-n_{A}\right)=0 \\
& \mu\left(1-n_{B}\right)=0
\end{aligned}
$$

In addition, the solution should also satisfy $\lambda \geq 0, \mu \geq 0, n_{A} \in[0,1]$ and $n_{B} \in[0,1]$.
If neither constraint is binding, the first order conditions yield $n_{A}=\frac{1}{3}(1+\theta)$ and $n_{B}=\frac{1+\theta}{3 \theta}$. Given that $\theta>0$, for this solution to be valid, it must be that $n_{A}<1$ which implies $\theta<2$ and $n_{B}<1$ resulting in $\theta>\frac{1}{2}$. There are also three different solutions to the first order conditions in which neither constraint binds with at least $n_{A}$ or $n_{B}$ is zero. Such solutions do not maximize the monopolist's profit. Therefore, the solution is interior for $\theta \in\left(\frac{1}{2}, 2\right)$. If only Type $A$ constraint binds, the solution is $n_{A}=1, n_{B}=\frac{1}{2}, \lambda=\frac{1}{2}\left(\frac{\theta}{2}-1\right)$ and $\mu=0$. Here $\lambda \geq 0$ implies $\theta \geq 2$. If Type $B$ is the only constraint that is binding, the solution is $n_{A}=\frac{1}{2}, n_{B}=1, \lambda=0$ and $\mu=\frac{1}{2}\left(\frac{1}{2}-\theta\right)$. In this case $\mu \geq 0$ implies $\theta \leq \frac{1}{2}$. If both constraints for both types are binding, $\lambda=-1$ and $\mu=-\theta$; thus the solution does not exist.

Proposition (4.1) states that if the two types of consumers are not too different in terms of their valuations of being matched, the solution to the monopolist's problem is interior with neither market being completely covered. Given that $\theta \in[0,1]$, in an interior solution the monopolist will always set $n_{A}<n_{B}$, i.e. the monopolist will restrict the quantity of relatively high valuation type. If Type $A$ on average value being matched more than twice as much as Type $B$, the monopolist maximizes profit by flooding the market of Type $B$ consumers. For the interior solution case, the monopolist charges transaction fees of $t_{A}=\frac{1}{3}(2-\theta)$ and $t_{B}=\frac{1}{3}(2 \theta-1)$ or subscription fees of $s_{A}=\left(\frac{1}{9 \theta}\right)(1+\theta)(2-\theta)$ and $s_{B}=\frac{1}{9}(1+\theta)(2 \theta-1)$. If Type $B$ market is flooded, the consumers of this type pay a zero price while Type $A$ consumers pay a strictly positive price, under both pricing schemes. The conclusion is that the consumer type with relatively higher valuation (Type $A$ by assumption in the model) always pays a relatively higher price than the other type.

The explanation why the monopolist should set $n_{A}<n_{B}$ while charging a higher price to Type $A$ is different depending on the type of pricing scheme the firm is implementing. Consider the subscription fees; if the prices to the two types of consumer are equal, certainly more Type $A$ will join. However, Type $B$ consumers exert a relatively greater positive externality on Type $A$ consumers. Then the firm's profits can be increased by charging a relatively higher price to Type $A$ consumers. In the case of transaction fees, Type $B$ consumers' demand is more elastic than their counterparts, thus any given percentage change in $n_{B}$ induces a smaller percentage change in the transaction fee paid by Type $B$ consumers compared to the same percentage change in $n_{A}$ on the transaction fee paid by Type $A$ consumers. The monopolist should therefore choose a relatively larger Type $B$ consumers
network size in order to maximize profits, which are the product of the total number of transactions and the total fee for each transaction. In terms on consumer surplus either under a matching service or platform, $C S_{A}=\frac{1}{2} n_{A}^{2} n_{B}$ and $C S_{B}=\frac{1}{2} \theta n_{A} n_{B}^{2}$.

### 4.4 Duopoly

Consider a market with two competing platforms. Both firms serve both types of consumers such that the case where one platform has monopoly control over one type of consumer while the other platform has monopoly control over the other type of consumer is not permitted. Two duopolistic scenarios in which the platforms' networks are either separated or interconnected.

### 4.4.1 Closed Duopoly

Under closed duopoly, the two platforms have separate networks, meaning consumers who join one platform will not be matched with consumers of the other platform or consumers cannot make crossplatform transactions. This is generally true for platforms providing matching services where the networks consist of proprietary databases. Let $n_{A}^{i} \in[0,1]$ and $n_{B}^{i} \in[0,1]$ be the size of platform $i$ 's Type $A$ and Type $B$ networks, for $i=1,2$. Assume $n_{A}^{1}+n_{A}^{2} \leq 1$ and $n_{B}^{1}+n_{B}^{2} \leq 1$,i.e. a single consumer cannot purchase from both platforms.

- Subscription Fees: Platform $i$ charges subscription fees $s_{A}^{i}$ to Type $A$ and $s_{B}^{i}$ to Type $B$ consumer. For both platforms to have positive market shares and for the market to clear, the quality adjusted prices of the two platforms must be equal. Therefore, $n_{A}^{1}+n_{A}^{2}=1-s_{A}^{i} n_{B}^{i}$ for $i=1,2$. The inverse demand function of Type $A$ faced by platform $i$ is $s_{A}^{i}\left(n_{A}^{i}, n_{A}^{i}, n_{B}^{i}\right)=$ $n_{B}^{i}\left(1-n_{A}^{i}-n_{A}^{j}\right)$. Similarly, the inverse demand function of Type $B$ consumers faced by platform $i$ is $s_{B}^{i}\left(n_{A}^{i}, n_{B}^{i}, n_{B}^{j}\right)=\theta n_{A}^{i}\left(1-n_{B}^{i}-n_{B}^{j}\right)$. Platform $i$ 's profit function is $\pi_{i}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}, n_{B}^{j}\right)=$ $n_{A}^{i} s_{A}^{i}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}\right)+n_{B}^{i} s_{B}^{i}\left(n_{A}^{i}, n_{B}^{i}, n_{B}^{j}\right)$.
- Transaction Fees: Platform $i$ charges success or per transaction fees of $t_{A}^{i}$ to Type $A$ and $t_{B}^{i}$ to Type $B$ consumers. The inverse demand function of Type $A$ consumers faced by platform $i$ is $t_{A}^{i}\left(n_{A}^{i}, n_{A}^{j}\right)=1-n_{A}^{i}-n_{B}^{i}$ and of Type $B$ consumers is $t_{B}^{i}\left(n_{B}^{i}, n_{B}^{j}=\theta\left(1-n_{B}^{i}-n_{B}^{j}\right)\right.$. Platform $i$ 's profit function is $\pi_{i}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}, n_{B}^{j}\right)=n_{A}^{i} n_{B}^{i}\left[t_{A}^{i}\left(n_{A}^{i}, n_{A}^{j}\right)+t_{B}^{i}\left(n_{B}^{i}, n_{B}^{j}\right)\right]$.

Substituting the appropriate inverse demand functions as above to obtain the following:

$$
\begin{equation*}
\pi_{i}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}, n_{B}^{j}\right)=n_{A}^{i}, n_{B}^{i}\left[\left(1-n_{A}^{i}-n_{A}^{j}+\theta\left(1-n_{B}^{i}, n_{B}^{j}\right)\right]\right. \tag{4.2}
\end{equation*}
$$

The platform chooses its network sizes $n_{A}^{i}$ and $n_{B}^{i}$ simultaneously to maximize (4.2), subject to $n_{A}^{i} \in\left[0,1-n_{A}^{j}\right]$ and $n_{B}^{i} \in\left[0,1-n_{B}^{j}\right]$. As in the monopoly case, subscription fees and transaction
fees are equivalent from the perspective of the platform.
Proposition 4.2. Under duopoly with closed networks, for $\theta \in\left(\frac{2}{3}, 1\right]$, there exists a unique Nash equilibrium with $n_{A}^{1}=n_{A}^{2}=\frac{1}{5}(1+\theta)$ and $n_{B}^{1}=n_{B}^{2}=\frac{1}{5 \theta}(1+\theta)$. For $\theta \in\left(0, \frac{2}{3}\right]$ the equilibrium takes the form $n_{A}^{1}=n_{A}^{2}=\frac{1}{3}, n_{B}^{1}=y$ and $n_{B}^{2}=1-y$, where $y \in\left[\max \left(0,1-\frac{1}{3 \theta}\right), \min \left(1, \frac{1}{3 \theta}\right)\right]$. For $\theta \in\left(0, \frac{1}{2}\right]$, there are also equilibria in which $n_{A}^{i}=\frac{1}{2}, n_{B}^{i}=1$, and $n_{B}^{j}=0$ for $i \neq j=1,2$.

Proof: Platform $i$ chooses $n_{A}^{i}$ and $n_{B}^{i}$ to maximize (3.2), subject to $n_{A}^{i} \in\left[0,1-n_{A}^{j}\right]$ and $n_{B}^{i} \in$ $\left[0,1-n_{B}^{j}\right]$ for $i=1,2$. Let $\Gamma_{i}=n_{A}^{i} n_{B}^{i}\left[\left(1-n_{A}^{i}-n_{A}^{j}\right)+\theta\left(1-n_{B}^{i}+n_{B}^{j}\right)\right]-\lambda_{i}\left(n_{A}^{i}+n_{A}^{j}-1\right)-\mu_{i}\left(n_{B}^{i} n_{B}^{j}-1\right)$. The first order conditions for platform $i$ are $\frac{\partial \Gamma_{i}}{\partial n_{A}^{i}}=n_{B}^{i}\left(1-n_{A}^{i}-n_{A}^{j}\right)-n_{A}^{i} n_{B}^{i}+n_{B}^{i} \theta\left(1-n_{B}^{i}-n_{B}^{j}\right)-\lambda_{i}=0$ and $\frac{\partial \Gamma_{i}}{\partial n_{B}^{i}}=n_{A}^{i}\left(1-n_{A}^{i}-n_{A}^{j}\right)-\theta n_{A}^{i} n_{B}^{i}+n_{A}^{i} \theta\left(1-n_{B}^{i}-n_{B}^{j}\right)-\mu_{i}=0$. The complementary slackness conditions are $\lambda_{i}\left(1-n_{A}^{i}-n_{A}^{j}\right)=0$ and $\mu_{i}\left(1-n_{B}^{i}-n_{B}^{j}\right)=0$. An equilibrium must satisfy these conditions as well as $\lambda_{i} \geq 0, \mu_{i} \geq 0, n_{A}^{i} \geq 0, n_{B}^{i} \geq 0, n_{A}^{1}+n_{A}^{2} \leq 1$ and $n_{B}^{1}+n_{B}^{2} \leq 1$.

If neither constraint binds, the first order conditions yield $n_{A}^{1}=n_{A}^{2}=\frac{1}{5}(1+\theta)$ and $n_{B}^{1}=n_{B}^{2}=$ $\frac{1}{5 \theta}(1+\theta)$. Seven other different solutions are excluded to the first order and complementary slackness conditions in which neither constraint is binding and at least one of the platform sets $n_{A}^{i}$ or $n_{B}^{i}$ or both equal zero. Such solutions cannot be an equilibrium because if a platform sets either $n_{A}^{i}$ or $n_{B}^{i}$ equal to zero, it makes zero profit, however if neither constraint is binding, it can increase $n_{A}^{i}$ or $n_{B}^{i}$ slightly so as to make positive profit, while keeping both constraints slack. In order for Type Aconstraint to be non-binding, it must be that $n_{A}^{1}+n_{A}^{2} \leq 1$ which implies $\theta<\frac{2}{3}$, and for Type $B$ constraint to be non-binding, $n_{B}^{1}+n_{B}^{2} \leq 1$ which implies $\theta>\frac{2}{3}$. Thus, there is a symmetric interior equilibrium for $\theta \in\left(\frac{2}{3}, \frac{3}{2}\right)$.

If only Type $A$ constraint is binding, the first order and complementary slackness conditions yield $n_{A}^{1}=x, n_{A}^{2}=1-x, n_{B}^{1}=n_{B}^{2}=\frac{1}{3}, \lambda_{1}=\frac{1}{3}(x-1)+\frac{\theta}{9}$ and $\lambda_{2}=\frac{1}{3}\left(\frac{\theta}{3}-x\right) . \lambda_{1} \geq 0$ implies $x \geq 1-\frac{\theta}{3}$ and $\lambda_{2} \geq 0$ implies $x \leq \frac{\theta}{3}$. For there to exist some $x$ such that both of these conditions are satisfied, it must have $1-\frac{\theta}{3} \leq \frac{\theta}{3}$, which implies $\theta \geq \frac{2}{3}$. When the Type $A$ constraint is binding, the first order and complementary slackness conditions also yield $n_{A}^{i}=1, n_{A}^{j}=0, n_{B}^{i}=\frac{1}{2}(1-z), n_{B}^{j}=z$, $\lambda_{i}=\frac{1}{4}(1-z)(\theta(1-z)-2)$ and $\lambda_{j}=\frac{\theta}{2}(1-z) z$. Given that platform $i$ sets $n_{A}^{i}=1$, platform $j$ must choose $n_{A}^{j}=0$ and thus gets zero profit whatever $n_{B}^{j}$ it chooses. The presence of a small fixed cost means that the platform $j$ will set $n_{B}^{j}=0$ and thus $n_{B}^{i}=\frac{1}{2}$. For this to be a solution, $\lambda_{i} \geq 0$ implies $\theta \geq 2$ and $\lambda_{j} \leq 0$ is satisfied for any $\theta>0$ and $z \in[0,1]$.

Using a similar argument, if only Type $B$ constraint is binding, the first order and complementary slackness conditions yield equilibria of $n_{A}^{1}=n_{A}^{2}=\frac{1}{3}, n_{B}^{1}=y, n_{B}^{2}=1-y$ where $y \in\left[1-\frac{1}{3 \theta}, \frac{1}{3 \theta}\right]$ for $\theta \in\left(\frac{1}{3}, \frac{2}{3}\right]$ and $y \in(0,1)$ for $\theta \in(0, f r a c 13]$. For $\theta \in\left(0, \frac{1}{2}\right]$ there are also equilibria in which $n_{A}^{i}=\frac{1}{2}, n_{A}^{j}=0, n_{B}^{i}=1$ and $n_{B}^{j}=0$. Finally, there are no equilibria in which both constraints are
binding. If both constraints bind, $n_{A}^{1}=x, n_{A}^{2}=1-x, n_{B}^{1}=y$ and $n_{B}^{2}=1-y$ for $x, y \in[0,1]$. This yields $\lambda_{1}=-x y, \lambda_{2}=-(1-x)(1-y), \mu_{1}=-\theta x y$ and $\mu_{2}=-\theta(1-x)(1-y)$. At least one of these multipliers is negative for any $x, y \in(0,1)$. Equilibria in which $x=1$ and $y=0$ or $x=0$ and $y=1$ can be ruled out by the small fixed cost.

The proposition (4.2) states that the Nash equilibrium under closed duopoly is a unique with neither side of the market flooded only when the two types of consumers are sufficiently similar $\left(\theta>\frac{2}{3}\right)$. When $\theta \leq \frac{2}{3}$, the low valuation (Type $B$ ) market is flooded in equilibrium. If one platform floods the Type $B$ market, the other platform makes zero profit whatever it does, and with the assumption of small fixed cost will maximize profit by setting both its network sizes to zero. This outcome is a highlight to the strong network effects that can arise in matching and platform service providers. With either subscription or transaction fees, consumer surplus of Type $A$ equals $C S_{A}=\frac{1}{2}\left(n_{A}^{1}+\right.$ $\left.n_{A}^{2}\right)^{2} \frac{n_{B}^{1}+n_{B}^{2}}{2}$ and consumer surplus of Type $B$ consumers is given by $C S_{B}=\frac{\theta}{2}\left(n_{A}^{1}+n_{A}^{2}\right)^{2}\left(\frac{n_{A}^{1}+n_{A}^{2}}{2}\right)$.

### 4.4.2 Open Duopoly

Now suppose that the two platforms form an open network. In the business of matching services, this means that the platforms share their databases of at least one type of consumer. The analysis does not depend on whether the platforms share information about Type $A$ or Type $B$ or both types of consumers. In the case of platform provider, an open network means that the platforms are interconnected, such that consumers of both types using either platform can transparently make transactions with consumers using the other platform.

- Subscription Fees: The inverse Type $A$ demand function faced by both platforms is

$$
s_{A}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}, n_{B}^{j}\right)=\left(n_{B}^{i}+n_{B}^{j}\right)\left(1-n_{A}^{i}, n_{A}^{j}\right)
$$

and the inverse demand faced by Type B of both platforms is

$$
s_{B}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}, n_{B}^{j}\right)=\theta\left(n_{A}^{i}+n_{A}^{j}\right)\left(1-n_{B}^{i}, n_{B}^{j}\right)
$$

The platform $i$ 's profit function is

$$
\pi_{i}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}, n_{B}^{j}\right)=n_{A}^{i} s_{A}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}, n_{B}^{j}\right)+n_{B}^{i} s_{B}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}, n_{B}^{j}\right)
$$

- Transaction Fees: With compatibility/interoperability, it is arbitrarily assumed that if a Type $A$ consumer of platform 1 is matched with or transacts with a Type $B$ consumer of platform 2, for example, then platform 1 charges the Type $A$ and platform 2 charges the Type $B$. The inverse demand functions faced by both platforms are $t_{A}\left(n_{A}^{i}, n_{A}^{j}\right)=\left(1-n_{A}^{i}-n_{A}^{j}\right)$ from

Type $A$ consumers and $t_{B}\left(n_{B}^{i}, n_{B}^{j}\right)=\theta\left(1-n_{B}^{i}-n_{B}^{j}\right)$ from Type $B$ consumers. Platform $i$ 's profit function is $\pi_{i}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}, n_{B}^{j}\right)=n_{A}^{i} n_{B}^{i}\left[t_{A}\left(n_{A}^{i}, n_{A}^{j}\right)+t_{B}\left(n_{B}^{i}, n_{B}^{j}\right)\right]+n_{A}^{i} n_{B}^{j} s_{A}\left(n_{A}^{i}, n_{A}^{j}\right)+$ $n_{A}^{i} n_{B}^{j} s_{B}\left(n_{B}^{i}, n_{B}^{j}\right)$

Substituting the appropriate inverse demand functions to obtain:

$$
\begin{equation*}
\pi_{i}\left(n_{A}^{i}, n_{A}^{j}, n_{B}^{i}, n_{B}^{j}\right)=n_{A}^{i}\left(n_{B}^{i}+n_{B}^{j}\right)\left(1-n_{A}^{i}-n_{A}^{j}\right)+n_{B}^{i} \theta\left(n_{A}^{i}+n_{A}^{j}\right)\left(1-n_{B}^{i}-n_{B}^{j}\right) \tag{4.3}
\end{equation*}
$$

Again platforms choose $n_{A}^{i}$ and $n_{B}^{i}$ simultaneously to maximize (3.3), subject to $n_{A}^{i} \in\left[0,1-n_{A}^{i}\right]$ and $n_{B}^{i} \in\left[0,1-n_{B}^{j}\right]$.

Proposition 4.3. Under open duopoly, Nash equilibrium is unique and interior for $\theta \in\left(\frac{1}{3}, 1\right]$, with $n_{A}^{1}=n_{A}^{2}=\frac{5+\theta}{16}$ and $n_{B}^{1}=n_{B}^{2}=\frac{5 \theta+1}{16 \theta}$. If $\theta \in\left(0, \frac{1}{3}\right]$, the equilibrium is characterized by $n_{A}^{1}=n_{A}^{2}=\frac{1}{3}, n_{B}^{1}=y, n_{B}^{2}=1-y$, where $y \in\left[\max \left(0,1-\frac{1}{16}\right), \min \left(1, \frac{1}{6 \theta}\right)\right]$.

Proof: Platform $i$ chooses $n_{A}^{i}$ and $n_{B}^{i}$ maximize (4.3), subject to $n_{A}^{i} \in\left[0,1-n_{A}^{i}\right]$ and $n_{B}^{i} \in\left[0,1-n_{B}^{i}\right]$ for $i=1,2$. Let $\Gamma_{i}=n_{A}^{i}\left(n_{A}^{i} n_{B}^{j}\right)\left(1-n_{A}^{i}-n_{A}^{j}\right)+n_{B}^{i} \theta\left(n_{A}^{i}+n_{A}^{j}\right)\left(1-n_{B}^{i}-n_{B}^{j}\right)-\lambda_{i}\left(n_{A}^{i}+n_{A}^{j}-1\right)-\mu_{i}\left(n_{B}^{i}+\right.$ $\left.n_{B}^{j}-1\right)$. The first order conditions for platform $i$ are $\frac{\partial \Gamma_{i}}{\partial n_{A}^{i}}=\left(n_{B}^{i}+n_{B}^{j}\right)\left(1-n_{A}^{i}-n_{A}^{j}\right)-n_{A}^{i}\left(n_{B}^{i}+n_{B}^{j}\right)+$ $\theta n_{B}^{i}\left(1-n_{A}^{i}-n_{A}^{j}\right)-\lambda_{i}=0$ and $\frac{\partial \Gamma_{i}}{\partial n_{B}^{i}}=n_{A}^{i}\left(1-n_{A}^{i}-n_{A}^{j}\right)+\theta\left(n_{A}^{i}+n_{A}^{j}\right)\left(1-n_{B}^{i}-n_{B}^{j}\right)+n_{B}^{i} \theta\left(n_{A}^{i}+n_{A}^{j}\right)-\mu_{i}=$ 0 . The complementary slackness conditions are $\lambda_{i}\left(1-n_{A}^{i}-n_{A}^{j}\right)=0$ and $\mu_{i}\left(1-n_{B}^{i}-n_{B}^{j}\right)=0$.
An equilibrium must satisfy these conditions as well as $\lambda_{i} \geq 1, \mu_{i} \geq 0, n_{A}^{i} \geq 0, n_{B}^{i} \geq 0, n_{A}^{1}+n_{A}^{2} \leq 1$ and $n_{B}^{1}+n_{B}^{2} \leq 1$. If neither constraint binds, the first order conditions yield $n_{A}^{1}=n_{A}^{2}=\frac{1}{16}(5+\theta)$ and $n_{B}^{1}=n_{B}^{2}=\frac{1}{16 \theta}(1+5 \theta)$. This is valid for $n_{A}^{1}+n_{A}^{2}<1$ which implies $\theta<3$ and $n_{B}^{1}+n_{B}^{2}<1$ which implies $\theta>\frac{1}{3}$. Thus the equilibrium is interior for $\theta \in\left(\frac{1}{3}, 3\right)$. If only Type $A$ constraint is binding, the first order and complimentary slackness conditions yield $n_{A}^{1}=x, n_{A}^{2}=1-x, n_{B}^{1}=n_{B}^{2}=\frac{1}{3}$, $\lambda_{1}=\frac{2}{3}(x-1)+\frac{\theta}{9}$ and $\lambda_{2}=\frac{\theta}{9}-\frac{2 x}{3} . \lambda_{1} \geq 0$ implies $x \geq 1-\frac{\theta}{6}$ and $\lambda_{2} \geq 0$ implies $x \leq \frac{\theta}{6}$.

For there to exist some $x \in[0,1]$ such that both of these conditions are satisfied, it must be that $\theta \leq 3$. If Type $B$ constraint is binding, the first order and complementary slackness conditions yield $n_{A}^{1}=n_{A}^{2}=\frac{1}{3}, n_{B}^{1}=y, n_{B}^{2}=1-y, \mu_{1}=\frac{1}{9}-\frac{2 \theta y}{3}$ and $\mu_{2}=\frac{1}{9}+\frac{2 \theta}{3}(y-1) . \mu_{1} \geq 0$ implies $y \leq \frac{1}{6 \theta}$ and $\mu_{2} \geq 0$ implies $y \geq 1-\frac{1}{6 \theta}$, thus $\theta \leq \frac{1}{3}$. Finally, if both constraints are binding, $n_{A}^{1}=x$, $n_{A}^{2}=1-x, n_{B}^{1}=y$ and $n_{B}^{2}=1-y$ for $x, y \in[0,1]$. This yields $\lambda_{1}=-x, \lambda_{2}=x-1, \mu_{1}=-\theta y$ and $\mu_{2}=\theta(y-1)$. At least one of these multipliers is negative for any $x, y \in(0,1)$ and thus there is no equilibrium in which both constraints are binding.

As under closed duopoly, the proposition says that the Nash equilibrium is unique and involves neither market being flooded only if the two types of consumers are sufficiently similar ( $\theta>\frac{1}{3}$ ) although a lower degree of similarity is required compared to the closed duopoly. If ( $\theta \leq \frac{1}{3}$ ), the

Type $B$ market is flooded in equilibrium and the distribution of of Type $B$ consumers between the two platforms is indeterminate, within a given range. The monopoly equilibrium cannot be replicated with open networks, unlike with closed networks. The reason for this is that the integration of the networks means that even if one platform corners the Type $B$ market, the other platform can still charge a positive price in Type $A$ side market, and thus will set a non-zero network size in that market. Consumer surplus of Type $A$ is given by $C S_{A}=\frac{1}{2}\left(n_{A}^{1}+n_{A}^{2}\right)^{2}\left(n_{B}^{1}+n_{B}^{2}\right)$ and that of Type $B$ consumers is $C S_{B}=\frac{1}{2} \theta\left(n_{A}^{1}+n_{A}^{2}\right)\left(n_{B}^{1}+n_{B}^{2}\right)^{2}$. Comparing all the consumer surpluses between open and closed platforms reveals that for any given network sizes, consumers are twice as well off under open duopoly than the closed platform.

### 4.5 Comparison between Regimes

In this section, comparisons are made between the equilibrium results of the different regimes considered above. Figure 4.1 shows that equilibrium subscription fees charged by a platform under the three different competitive regimes as a function of $\theta$. In the case of closed duopoly, from the second proposition, if the Type $B$ side market is flooded in equilibrium, there may be a continuum of equilibria. The dashed lines on the graph for this case show the minimum and maximum equilibrium prices that may be charged by a platform. All prices between the lines are also equilibria.

It is clear that the monopolist always charges the highest joining fee to Type $A$ consumers, while the Type $A$ price under open duopoly is always at least as high as the maximum equilibrium price under closed duopoly. On the other hand, the price charged to Type $B$ consumers is highest under open duopoly for intermediate values of $\theta$ and under monopoly for high values of $\theta$. The price charged to Type $B$ consumers under closed duopoly is always lowest.


Figure 4-1: Equilibrium Subscription Fees Charged to Consumers Type $A$ (left) and Type $B$ (right)

The reason that prices are higher under open duopoly compared to closed duopoly is that with compatibility, if one platform expands its output in one side of the market, it benefits both platforms in terms of the higher prices that can be charged to consumers in the other market. On the other hand, with closed duopoly, the platform can capture the benefits of its own expansion in one
market. Therefore, compatibility of platforms or sharing consumer data reduces the incentives of both platforms to increase output and results in higher equilibrium prices. From Figure 4.5 it can also observed that, for intermediate values of $\theta$, where Type $B$ market is not flooded, higher than monopoly prices are profit maximizing under open duopoly. This is again due to the inability of the platforms to capture all the benefits of setting a high $n_{B}$ under open duopoly.

Similarly Figure 4.2 represents the equilibrium success or transaction fees as a function of $\theta$. Although there is still a continuum of equilibria for some values of $\theta$ where the Type $B$ market is flooded in the closed duopoly case, this does not produce a range of equilibrium prices to either Type $A$ or $B$ consumers under a transaction fee regime. This is because if the Type $B$ market is flooded, both platforms will receive a zero price in that market, while Type $A$ price that a platform receives does not depend on the quantity of Type $B$ consumers that are accessible on its network, unlike the case of subscription fees. From Figure 4.2, as with the subscription fees, it can be seen that the Type $A$ consumers price charged by the monopolist is the highest, while the Type $B$ price is highest under open duopoly for intermediate values of $\theta$ and under monopoly for high values of $\theta$. However, the lowest price charged to Type $A$ consumers is not always under closed duopoly; the price is lower under open duopoly for intermediate values of $\theta$.


Figure 4-2: Equilibrium Transaction Fees Charged to Consumers Type $A$ (left) and Type $B$ (right)

Figure 4.3 shows total equilibrium consumer surplus of both types of consumers as a function of $\theta$, for either subscription or transaction fees. The range of equilibria that are possible under closed duopoly all produce the same level of consumer surplus, i.e. total consumer surplus is independent of the split of Type $B$ consumers between the platforms. For all values of $\theta$, equilibrium consumer surplus of both types is highest under open duopoly, followed by monopoly and then by closed duopoly. From Figure 4.1 and Figure 4.2, prices may rise or fall when going from closed to open duopoly, depending on the value of $\theta$. However, in this model, equilibrium consumer surplus of both types always rises following a regime change. This occurs because the network benefits in the model are assumed to be very large and outweigh any increase in the fees that consumers must pay.

Figure 4.4 illustrates the equilibrium profit of one platform as a function of $\theta$. In the model,


Figure 4-3: Equilibrium Consumer Surplus of Type $A$ (left) and Type $B$ (right) Consumers
a platform's profits are identical whether it charges subscription or transaction fees. Profits are clearly the highest under monopoly, while profits under open duopoly are always at least as great as the highest profit level under closed duopoly. This is a reflection of weakened competition under open duopoly compared to closed duopoly. The results here indicate that platforms in matching and platform industries may seek to form open networks as a means of increasing their profits.


Figure 4-4: Equilibrium Profit of One Platform under Either Pricing Schemes

Figure 4.5 summarizes the total equilibrium welfare, the sum of consumer surplus of both types, and the profit of the platform as a function of $\theta$. Social welfare seems to be highest under open duopoly, followed by monopoly and by closed duopoly. This coincides with the standard literature on one-sided networks in which compatibility ii socially optimal. Furthermore, in the absence of any additional cost of forming open networks, the platforms' incentives to do so are in line with the social incentives, reflecting a classic result from the literature on one-sided networks. If, however, there is a fixed cost associated with achieving compatibility, there may be cases in which open networks are
socially desirable but would not be chosen by the platforms. This arises because the difference in profits between open and closed duopoly is smaller than the difference in welfare, due to the fact that consumers also tend to prefer open duopoly.


Figure 4-5: Total Equilibrium Welfare

If not precluded by antitrust law, the platforms could further increase their joint profits over open duopoly by merging to become a monopolist, which is not socially optimal, although it is an improvement on closed duopoly. On the other hand, these results also suggest that if compatibility is not possible due to technical or other reasons, a monopoly provider of matching or platform services may be preferable to closed duopoly since the monopolist is better able to internalize the spillovers from one type of consumer to the other, and the gain from consumers to the larger network more than outweighs the negative effect from the reduction in quantities.

### 4.6 Conclusion

This chapter analyzes the behavior of platforms and the welfare implications of different competitive regimes in two-sided network markets. A simple model is developed which encompassed both subscription fee and success or transaction fee pricing schemes. For both type of business models, the Nash equilibria under monopoly, and open and closed duopoly are derived. With regard to the equilibrium prices, a common theme is that if the two types of consumers are sufficiently different in their valuation of being matched or valuation of a transaction, then the market of the low valuation type will be flooded in equilibrium, and that type will pay a zero price. The difference in valuations between the two types necessary for this to happen depends on the competitive valuation
market, with closed duopolists being most likely to flood the low valuation market, followed by the monopolist, followed by open duopolists.

The welfare implications of the different competitive regimes are also investigated. Due to the assumption of strong network effects and lack of intrinsic benefits inherent in the type of market considered, the increase in consumer surplus from network benefits when going from a more competitive regime to a less competitive one always outweighs any restriction in quantities, such that total welfare increases. Thus, of the three regimes, open duopoly is socially preferred to monopoly, which is in turn preferred to closed duopoly. On the other hand, while platforms face incentives to form open networks as opposed to closed ones, they face stronger incentives to merge and become a monopolist. Therefore, a policy implication is that the formation of open two-sided networks should be encouraged, while mergers should be discouraged. However, if open networks are infeasible, a monopoly provider is socially desirable to closed duopoly.

## Chapter 5

## Effects of Compatibility on Open vs. Closed Platforms

In the case of Apple, the platform provider can charge access fees to users, music production companies and their artists, or both, or even choose to subsidize one side of the market, thereby using its pricing strategies to eliminate the issues associated with what would otherwise be a coordination game. What would happen if Apple allows other plug and play devices to connect to iTunes? How would it impact the dominance of Apple's closed platform? How would Apple's competitors of other closed and open platforms respond? The model presented in this chapter aims to highlight these key questions.

Unlike the model in chapter 4, this chapter's results do not rely on demand elasticities. Instead, prices and purchase decisions are modeled as functions of the degree of differentiation between the two platforms, using a Hotelling model. Church et al. [6] develop a two-sided model of hardware and software markets with free entry, and show that the market outcome need not maximize social welfare, since it can lead to under adoption. In 2006, Choi [5] and Wright et al. [10] used a Hotelling structure to model the two types of users in models of competing platforms and the effects of compatibility. However, their model does not accommodate transactions between the two types of users, which is a crucial feature of multi-sided markets. They find that compatibility is always welfareincreasing relative to the case when users multi-home (adopt both platforms simultaneously), but firms may have an excessive incentive to become compatible if users were single-homing previously in the case of one-sided markets. When markets are two-sided but there is no product differentiation, incentive to choose compatibility as long as consumers cannot multi-home. Otherwise, their incentive is insufficient. This chapter's findings are in line with their result, in that the proprietary platform has an insufficient incentive to choose compatibility in some cases. However, in the model
presented here, firms never have an excessive incentive to choose compatibility.

The model is also based on Economides' [11] theoretical analysis of a comparison of a closed software platform versus an open one. While approaching the issue as a study of two-sided platforms consisting of an operating system and applications, Economides et al. [11] find that although variety of available applications is greater in an open platform, the closed platform dominates the open platform in terms of market share. Katz \& Shapiro [21] examine the effects of compatibility on one-sided networks, and deduce that it improves social welfare as long as firm profits increase under compatibility, and that firms may fail to achieve it in some cases even when it is socially optimal. This paper confirms the stronger result for the specific case of two competing two-sided platforms, where one is open and the other closed, since it is always socially optimal to impose compatibility even when it reduces the platform's profits. Hagiu [19] finds that the closed platforms may be more efficient, as they can internalize adoption externalities and induce greater, sometimes even excessive, application variety. He [18] also shows that competition between an open and closed platform may also be socially undesirable if it prevents platforms from sufficiently internalizing network externalities and direct competitive, or business-stealing effects.

The video games industry is another example where the issue of compatibility between platforms is paramount. Sony and Microsoft have been known to sell their Playstation and XBox video game consoles to users at a price below cost [7], while charging game developers high fees to develop games for the respective consoles. On the other hand, Microsoft charges users a relatively high fee to use its Windows operating system, while charging application developers a low to zero fee to use its Application Programming Interface (API) to develop Windows applications. Linux, being an open platform developed for the most part by volunteers, does not charge access fees to users or developers; however, application developers such as Red Hat and Novell sell Enterprise editions of Linux that bundle the platform with a suite of applications [8]. The purpose of this chapter is to analyze the implications of compatibility between a closed platform and the open platform that it competes with, and its implications on strategies, pricing, profits and welfare.

### 5.1 Model for Incompatible Platforms

There are two platforms, one closed and one open, denoted throughout as $C$ and $O$ respectively. The closed platform is developed by a strategic profit-maximizing firm, while the open platform is freely accessible setting its user and developer prices equal to zero. The closed platform competes with the open platform in a modified Hotelling setup forming a single-agent game. Consumers who wants to download music or other types of platform users (watching music videos, listening to pod casts, etc), are uniformly distributed along a 0,1 interval as in Figure 5.1. The parameter $t$,
where $t>0$, represents the users' taste for a particular platform, while $x$ represents the consumer's location. Assume the market is covered, so that all consumers purchase at least one platform. This can be thought of a consumer owning an iPod versus owning some other plug and play device. Users are not strategic agents but their values for each application available for the chosen platform are independent and uniformly distributed on the unit interval. Users must also purchase applications to use on the platform. In the case of Apple iTunes platform, applications could mean any type of downloadable digital media such as songs, videos, games, etc.


Developers:


Figure 5-1: Division of Users and Developers between Two Incompatible Platforms

Assume that application developers are strategic, profit-maximizing firms, and differentiated by their fixed costs, which are uniformly distributed along a separate, unrelated 0,1 interval as in Figure 5.1. For a multi-sided platform like Apple, developers can be individual artists, record companies, music promoters, etc. Each developer produces one type of application and there is a continuum of developers. The parameter $s$, where $s>0$, represents their preference for a particular platform and $y$ represents their location. This captures the degree of differentiation between platforms from a developer's point of view ( $s$ ) and the developer's degree of investment in, or preference for, coding for a particular platform ( $y$ ). For example, certain TV shows are exclusively available to download through iTunes. sy is a developer's fixed cost of developing an application for the closed platform, and $s(1-y)$ is a developer's fixed cost of developing an application for the open platform. Developers choose one platform and do not multi-home. However, their location is exogenously determined. It is also assumed that all developers enter the market. This is because the welfare analysis focuses on the case where the equilibrium developer price charged by the closed platform equals zero, and this case is not compatible with less than full entry of developers under most parameter value ranges (See Appendix A for proof).

The market proceeds in the following way, with the closed platform and all developers as strategic agents:

1. First, the closed platform sets user prices, denoted as $p_{u}^{C}$ and developer access fees, denoted as $p_{d}^{C}$. The prices for the open platform, $p_{u}^{o}$ and $p_{d}^{o}$ are zero by definition.
2. Next, developers choose which platform to develop for. Let $n_{C}^{d}$ denote the number of developers choosing to work with the closed platform and $n_{o}^{d}$ the number of developers choosing to work with the open platform, given $p_{u}^{C}$ (since the equilibrium proportion of users on each platform is a known function of $p_{u}^{C}$ ) and $p_{d}^{C}$.
3. After choosing a platform, developers set application prices, given $p_{u}^{C}$ and $p_{d}^{C}$. For developer $j$, this is denoted by $p_{C j}^{d}$ for developers on the closed platform and $p_{O j}^{d}$ for developers on the open platform.
4. Lastly, users decide which platform to adopt. Let $n_{C}^{u}$ denote the number, or proportion of users who choose the closed platform, and $n_{O}^{u}$ be the number of users choosing the open platform. Users also simultaneously purchase applications to use on the platform, choosing from among the available applications on the chosen platform and buying exactly one unit of every selected application.

Each user's valuations for each application are independent. The probability that any user on platform $k$ buys application $j$ with a price $p_{k j}^{d}$ is therefore the probability that their value for the application is greater than $p_{k j}^{d}$, which is $\left(1-p_{k j}^{d}\right)$. Each developer then faces the following decision problem: the expected number of applications that developer $j$ can sell is $\int_{0}^{n_{C}^{u}}\left(1-p_{C j}^{d}\right) d j$ for a developer on the closed platform and $\int_{n_{C}^{u}}^{1}\left(1-p_{O j}^{d}\right) d j$ for a developer on the open platform, since developers can only sell to users on their own platform. As a result, each application also faces the same demand curve: $\left(q_{j} \mid p_{u}^{C}, p_{d}^{C}\right)=\left(1-p_{C}^{d}\right) n_{C}^{u}$ for firm $j$ on the closed platform and $\left(q_{j} \mid p_{u}^{C}, p_{d}^{C}\right)=\left(1-p_{O j}^{d}\right) n_{O}^{u}$ for firm $j$ on the open platform.

Assuming developers only face fixed costs and have a zero marginal cost, $\left(p_{d}^{C} \mid p_{u}^{C}, p_{d}^{C}\right)=\frac{1}{2}$ as the equilibrium price for all applications on the closed platform and $\left(p_{O}^{d} \mid p_{u}^{C}, p_{d}^{C}\right)=\frac{1}{2}$ as the equilibrium price for all applications on the open platform. Figure 5.2 shows the demand curve faced by any single closed application developer. At a price $p_{d}^{C}$ as shown, it can expect to sell one application each to $\frac{n_{C}^{u}}{2}$ users. Since each user's valuation for any given application is independent and does not depend on the prices or valuations for the other applications, the equilibrium price for any one application does not depend on the prices charged by the other applications. Therefore, no developer has an incentive to deviate from a price of $\frac{1}{2}$, since by doing so they would neither increase their own profit, nor lower the profits of the other developers, and setting any other price is a strictly dominated strategy.


Figure 5-2: Developers' Demand Curve for Applications on Closed Platform

Since all application prices are the same, the expected mass of applications any user on the closed platform purchases is $\int_{0}^{n_{C}^{d}}\left(1-p_{C}^{d}\right) d z$ and $\int_{n_{C}^{d}}^{1}\left(1-p_{O}^{d}\right) d z$ for any user on the open platform. Each user therefore has an application demand: $q_{i}=\left(1-p_{C}^{d}\right) n_{C}^{d}$ for user $i$ on the closed platform as shown in Figure 5.3, and $q_{i}=\left(1-p_{O}^{d}\right) n_{O}^{d}$ for user $i$ on the open platform. However, the demand curve in Figure 5.3 is not typical, since each application is different from the others. At the same time, it denotes the total mass of applications purchased by any single user, which, at an equilibrium application price of $\frac{1}{2}$, equals $\frac{n_{c}^{z}}{2}$. User utility is a function of the base utility from using a platform, $V$, the price paid for the platform, the user's location (i.e. preference for the chosen platform), and the net consumer surplus from consumption of the applications, which is the willingness to pay for each application less its price, integrated over the quantity of applications purchased. This is increasing in the number of available applications (or application variety) and decreasing in their price.


Figure 5-3: Users' Demand Curve for Applications on Closed Platform

User utility is therefore:

$$
U_{i}^{C}=V-p_{u}^{C}-t x+\int_{0}^{\left(1-p_{C}^{d}\right) n_{C}^{d}}\left(1-\frac{z}{n_{C}^{d}}-p_{C}^{d}\right) d z
$$

for a user who adopts the closed platform and

$$
U_{i}^{O}=V-t(1-x)+\int_{0}^{\left(1-p_{O}^{d}\right) n_{O}^{d}}\left(1-\frac{z}{n_{O}^{d}}-p_{O}^{d}\right) d z
$$

for a user who adopts the open platform. The utility from using a platform, $V$, is assumed to be high enough that in equilibrium all consumers purchase at least one platform. At this point it is assumed that users cannot multi-home, or purchase both applications. Since $p_{C}^{d}=p^{d} O=\frac{1}{2}$ in equilibrium, substitute that price into user utility functions to obtain the following two expressions:

$$
\begin{aligned}
& U_{i}^{C}=V+\frac{\left(1-p_{C}^{d}\right)^{2} n_{C}^{d}}{2}-p_{u}^{C}-t x \\
& U_{i}^{O}=V+\frac{\left(1-p_{O}^{d}\right)^{2} n_{O}^{d}}{2}-t(1-x)
\end{aligned}
$$

Consumer surplus, and therefore user utility, is increasing in the proportion of developers on the chosen platform. This constitutes the indirect network effect that makes the operating system a two-sided platform. By locating the indifferent consumer, $x=n_{C}^{u}=\frac{1}{2}+\frac{n_{C}^{d}-n_{o}^{d}}{16 t}-\frac{p_{c}^{C}}{2 t}$ and since $n_{O}^{u}=1-n_{O}^{u}$ by definition, $n_{O}^{u}=\frac{1}{2}-\frac{n_{C}^{d}-n_{O}^{d}}{16 t}+\frac{p_{u}^{c}}{2 t}$. In the limit, as $t \rightarrow \infty, n_{C}^{u} \rightarrow \frac{1}{2}$. If $t \rightarrow 0$, then $n_{C}^{u} \rightarrow 0$ (solving through $n_{C}^{d}-n_{O}^{d}$ ).

Developer profits, for a developer $j$ who chooses to develop for the closed platform, are represented by:

$$
\pi_{C j}^{d}=p_{C j}^{d}\left(1-p_{C j}^{d}\right) n_{C}^{u}-p_{d}^{C}-s y
$$

where $s y$ is the fixed cost of developing the application, $y$ represents a developer's location with regards to the preference for developing for a particular platform, and $s$ is a parameter indicating the level of differentiation between the platforms from a developer's perspective. This can be understood as the investment a developer makes in learning to code for, and becoming comfortable with, a particular platform. Similarly, profits for a developer $j$ who chooses the open platform are:

$$
\pi_{O_{j}}^{d}=p_{O j}^{d}\left(1-p_{O j}^{d}\right) n_{O}^{u}-s(1-y)
$$

Since all developer prices must equal $\frac{1}{2}$ in equilibrium, substitute this and the values obtained for
$n_{p}^{u}$ and $n_{o}^{u}$ back into the profit functions to obtain:

$$
\begin{aligned}
& \pi_{p}^{d}=\frac{1}{8}+\frac{n_{C}^{d}-n_{O}^{d}}{64 t}-\frac{p_{u}^{C}}{8 t}-p_{d}^{C}-s y \\
& \pi_{O}^{d}=\frac{1}{8}-\frac{n_{C}^{d}-n_{O}^{d}}{64 t}-\frac{p_{u}^{C}}{8 t}-s(1-y)
\end{aligned}
$$

To calculate the proportion of application developers entering the market on each platform, the indifferent developer is found to obtain that:

$$
\begin{gathered}
y=n_{C}^{d}=\frac{1}{2}-\frac{4 p_{u}^{C}}{32 s t-1}-\frac{16 t p_{d}^{C}}{32 s t-1} \\
n_{O}^{d}=\frac{1}{2}+\frac{4 p_{u}^{C}}{32 s t-1}+\frac{16 t p_{d}^{C}}{32 s t-1}
\end{gathered}
$$

Platform profits can be represented as

$$
\begin{equation*}
\Pi_{C}=p_{u}^{C} n_{C}^{u}+p_{d}^{C} n_{C}^{d}-F \tag{5.1}
\end{equation*}
$$

for the closed platform, where $F$ is the platform's fixed cost. Substituting for $n_{C}^{d}$ and $n_{O}^{d}$,

$$
\Pi_{C}=p_{u}^{C}\left[\frac{1}{2}-\frac{p_{u}^{C}}{2 t(32 s t-1)}-\frac{2 p_{d}^{C}}{32 s t-1}-\frac{p_{u}^{C}}{2 t}\right]+p_{d}^{C}\left[\frac{1}{2}-\frac{4 p_{u}^{C}}{32 s t-1}-\frac{16 t p_{d}^{C}}{32 s t-1}\right]-F
$$

This results in the following:

$$
\begin{aligned}
& p_{u}^{p *}=\frac{32 s t-1}{64 t}-\frac{3 p_{d}^{C}}{16 s} \\
& p_{d}^{p *}=\frac{32 s t-1}{64 s}-\frac{3 p_{d}^{C}}{16 t}
\end{aligned}
$$

Substitution gives:

$$
\begin{aligned}
& p_{p}^{C *}=\frac{(32 s t-1)(16 t-3)}{4(256 s t-9)} \\
& p_{C *}^{d}=\frac{(32 s t-1)(16 s-3)}{4(256 s t-9)}
\end{aligned}
$$

Clearly the sum of $p_{u}^{C}$ and $p_{d}^{C}$ is a constant for fixed values of $s$ and $t$. As the platform raises one price it must lower the other; however for given values of $t$ and $s$ there is a unique equilibrium pair of prices. The closed platform's market share, $n_{C}^{u}$, is increasing in $t$ as long as $s>\frac{3}{16}$. Since the closed platform dominates the market as switching costs (represented by levels of differentiation, in
this model) increase.

Currently, Microsoft sets its developer price at or close to zero. In this chapter's basic framework, a value of $s=\frac{3}{16}$ rationalizes this price as long as $t>\frac{3}{16}$. A low level of platform differentiation with respect to developers and a high level of differentiation with respect to users justifies a zero developer price (since the developers need to be brought on board) and a positive user price (since there are enough users with a strong preference for the closed platform even when it has a positive price). Negative prices, or subsidies to either side are also possible if the level of differentiation for that side is especially low.

### 5.2 Model for Compatible Platforms

Compatibility is defined as giving users the ability to use applications made for any platform, having installed either platform. Platforms can achieve this at some additional fixed cost $F^{\prime}$. Each user now can purchase any of the applications from 0 to 1 as in Figure 5.2, not just from 0 to $n_{C}^{d}$ for closed platform users and $n_{O}^{d}$ to 1 for open platform users, as was the case when platforms were incompatible. Developers now have a potential market of all users from 0 to 1 as in Figure 5.3. Each developer on platform $k$ can now expect to sell to $\int_{0}^{1}\left(1-p_{k j}^{d}\right) d j$ users. The demand curve faced by any developer, regardless of the platform, is now $q_{j}=\left(1-p_{k}^{d}\right)$ for developer $j$ on platform $k$. User demand curve for applications does not depend on the platform chosen by the application developer and is $q_{i}=\left(1-p_{k}^{d}\right)$ for user $i$. The equilibrium application price does not change, since the developers still have a zero marginal cost and positive fixed cost indexed by their location (preference for a platform), and thus $p_{C}^{d}=p_{O}^{d}=\frac{1}{2}$.

User utility is now:

$$
U_{i}^{C}=V-p_{u}^{C}-t x+\int_{0}^{\left(1-p_{C}^{d}\right)}\left(1-z-p_{C}^{d}\right) d z
$$

for a user who adopts the closed platform and

$$
U_{i}^{O}=V-t(1-x)+\int_{0}^{\left(1-p_{O}^{d}\right)}\left(1-z-p_{O}^{d}\right) d z
$$

for a user who adopts the open platform. As in the previous mode, assume $V$, the utility for using a platform, to be high enough that in equilibrium all consumers purchase at least one platform. Since $p_{C}^{d}=p_{O}^{d}=\frac{1}{2}$ in equilibrium, again substitute that price into user utility functions to obtain:

$$
\begin{aligned}
U_{i}^{C} & =V+\frac{1}{8}-p_{u}^{C}-t x \\
U_{i}^{O} & =V+\frac{1}{8}-t(1-x)
\end{aligned}
$$

By locating the indifferent consumer, $x=n_{C}^{u}=\frac{1}{2}-\frac{p_{u}^{C}}{2 t}$ and $n_{O}^{u}=\frac{1}{2}+\frac{p_{u}^{C}}{2 t}$. Since the demand for any given developer's application is $1-\frac{1}{2}=\frac{1}{2}$, developer profits for closed and open platforms respectively are:

$$
\begin{aligned}
\pi_{C j}^{d} & =\frac{p_{C j}^{d}}{2}-p_{d}^{C}-s y=\frac{1}{4}-p_{d}^{C}-s y \\
\pi_{O j}^{d} & =\frac{1}{4}-s(1-y)
\end{aligned}
$$

As before, $s$ represents the fixed cost and the location parameter $y$ accounts for a developer's taste for a particular platform. Note that since platforms are now compatible, developers can potentially sell to all users by choosing any platform. So if $p_{d}^{C}>s$, no developers will choose to develop for the closed platform, since even the developers located at $y \rightarrow 0$ will find it more profitable to switch to the open platform; as the platforms are compatible the demand for their application will be unchanged. The proportion of developers choosing each platform can be written as:

$$
\begin{aligned}
n_{C}^{d} & =\frac{1}{2}-\frac{p_{d}^{C}}{2 s} \\
n_{O}^{d} & =\frac{1}{2}+\frac{p_{d}^{C}}{2 s}
\end{aligned}
$$

The platform's profit can now be written as

$$
\begin{equation*}
\Pi_{C}=p_{u}^{C}\left(\frac{1}{2}-\frac{p_{u}^{C}}{2 t}\right)+p_{d}^{C}\left(\frac{1}{2}-\frac{p_{d}^{C}}{2 s}\right) \tag{5.2}
\end{equation*}
$$

The platform's prices are now: $p_{u}^{C}=\frac{t}{2}$ and $p_{d}^{C}=\frac{s}{2}$. Platform profit equals $\frac{s+t}{8}$.

Comparing (5.1) and (5.2), it is clear that under some cases if the assumptions hold with positive $s$ and $t$, compatibility can increase profits for the closed platform. Let's examine situations where social welfare is improved by imposing compatibility even if the closed platform's profits are lowered. Since this framework leads to the conclusion that Microsoft's zero price for developers when platforms are incompatible requires $c=\frac{3}{16}$, the developer and platform profits and consumer welfare are compared for that particular case. When the platforms are incompatible, the closed platform makes a profit of:

$$
\Pi_{C}=\frac{32 s t-1}{4(256 s t-9)}\left[(16 t-3) n^{u} C+(16 c-3) n^{d} C\right]
$$

This is greater than $\frac{s+t}{8}$ when $s=\frac{3}{16}$ if $t>0.263$. However, if $t<0.263$, then the closed platform is better off choosing to be compatible with the open platform. So as the differentiation with respect to users falls, compatibility becomes a better option.

Total profits for the developers on the closed platform when platforms are incompatible can be
written as:

$$
\int_{0}^{n_{C}^{d}}\left(\frac{1}{8}+\frac{2 n_{C}^{d}-1}{64 t}-\frac{p_{u}^{C}}{8 t}-p_{d}^{C}-s y\right) d y
$$

When the platforms are compatible, developers on the closed platform have a total profit of $\frac{4-3 s}{32}$, which equals 0.1074 when $s=\frac{3}{16}$. In this case, total profits for developers on the closed platform are always higher when the platforms are compatible. This results from the higher demand for their application, even though the price the developers pay the platform is higher when they are compatible.

Total profits for developers on the open platform are always higher in the case of compatibility, since demand for their product increases and they face no change in the platform price they pay; it is always equal to zero. Consumer welfare for all users under incompatibility is:

$$
\int_{0}^{n_{C}^{u}}\left(V-p_{u}^{C}+\frac{n_{C}^{d}}{8}-t x\right) d x+\int_{n_{C}^{u}}^{1}\left(V+\frac{1-n_{C}^{d}}{8}-t(1-x)\right) d x
$$

Consumer welfare for all users when platforms are compatible can be expressed as $\frac{16 \mathrm{~V}+2-7 t}{16}$. When $s=\frac{3}{16}$, consumer welfare is always greater when the platforms are compatible.

### 5.3 Welfare Analysis

Under incompatibility, summing up closed platform profits, developer profits for open and closed developers, and consumer welfare, and find that total social welfare is:

$$
\begin{gathered}
\frac{32 s t-1}{4(256 t-9)}\left[(16 t-3)\left(\frac{1}{2}+\frac{15-16(s+4 t)}{2(256 s t-9)}\right)+(16 s-3)\left(\frac{1}{2}+\frac{15-16(t+4 s)}{2(256 s t-9)}\right)\right] \\
+\int_{0}^{n_{C}^{d}}\left(\frac{1}{8}+\frac{2 n_{C}^{d}-1}{64 t}-\frac{p_{u}^{C}}{8 t}-p_{d}^{C}-s y\right) d y+\int_{n_{C}^{d}}^{1}\left(\frac{1}{8}-\frac{2 n_{C}^{d}-1}{64 t}+\frac{p_{u}^{C}}{8 t}-s(1-y)\right) d y \\
\quad+\int_{0}^{n_{C}^{u}}\left(V-p_{u}^{C}+\frac{n_{C}^{d}}{8}-t x\right) d x+\int_{n_{C}^{u}}^{1}\left(V+\frac{\left(1-n_{C}^{d}\right)}{8}-t(1-x)\right) d x
\end{gathered}
$$

When $s=\frac{3}{16}$, the above expression equals $\frac{21 t+3 V+1}{144}+\frac{17 t-5}{192 t}$. When platforms are compatible and $s=\frac{3}{16}$, total welfare equals $\frac{256 V+81-80 t}{256}$ which is always greater than total welfare when platforms are incompatible and $s=\frac{3}{16}$, assuming the platform has the same fixed cost in both cases. However, the open platform will only choose compatibility of its own accord when $t<0.263$, so if there is a comparatively high level of differentiation between platforms with regards to consumer preferences, the closed platform has no incentive to choose to be compatible with the open platform. Within this framework, then, if $s=\frac{3}{16}$ so that $p_{d}^{p}=0$, it is welfare- improving to impose compatibility on the platforms as a matter of policy. This is an even stronger result than that of [21], since in this
case social welfare rises under compatibility even if the closed platform's profits are lower, which is when $t>0.263$.

Compatibility between platforms can be a sustainable equilibrium, and as the level of differentiation between the platforms falls from the user's perspective, a closed platform is better off choosing to be compatible with the open platform. In this light, Microsoft's recent interoperability initiative can potentially be explained as a result of efforts within the Linux community to make the Linux operating system user interface more conventionally user-friendly, dare one say more Windows-like, i.e. effectively lowering $t$. Perhaps Apple should follow suit by opening up iTunes. With closed platforms, the firm can run it as a service or sell it as a product. It can also leverage both and generate sales through support and add-ons, as well. With open platform models, the firm can still decide to go for service or product but the question becomes why would somebody pay for it when they can download and run the application for free? The only business model left is that of support and add-ons. As proven by the enormous success of Facebook's open platform with its set of application programming interfaces and services that allow outside developers to inject new features and content into users' experience, ultimately, the add-on applications may just be enough to build a solid business. As has been demonstrated theoretically by this chapter, this would make compatibility the better option in terms of profits as well as social welfare. In addition, in this particular case it is also always welfare-improving to impose compatibility on the closed platform.

## Chapter 6

## Extensions and Future Directions

While this thesis focuses on open and closed models of one specific multi-sided platform: Apple iTunes-iPod platform, other multi-sided platform businesses are becoming an increasingly important part of the economy. They range from relatively small emerging companies like eBay, Yahoo!, and Palm, to relatively large and mature companies like American Express. These markets also have had a large impact on the recent information technology boom, and undoubtedly, they will continue to be important, as Internet-based commerce expands its scope to include both new and old economy firms. In addition to Internet commerce, other increasingly important industries, such as credit cards, operating systems, shopping centers, and mass media, are all governed by the economics of multi-sided platforms. Multi-sided platforms have business models that are not yet well understood and engage in highly complex business strategies. While the majority of the literature explored the asymmetric pricing problems between the multiple sides of the platforms, and other broad range of issues such as multi-homing, dynamic evolution of open vs. closed platform business models, value chain of platform businesses and unbiased empirical studies of different platform strategies need to be investigated in more depth.

The simplified model in chapter 4 can be extended into multi-sided networks where there are more than two types of consumers. A reasonable extension is to look at alternate pricing schemes such as the case where both subscription and transaction fees are required to access a platform. One issue not addressed here is the possibility for consumers to use more than one platform, or to multihome. A tractable analysis of multi-homing in the context of multi-sided networks would be a useful addition to the literature. Another possibility for future research is to incorporate negative network effects whereby consumers' utility is decreasing in the number of their own type attached to the same network. For example, males may prefer to join a dating agency with fewer other males (for a given number of females) because it would presumably raise their chances of meeting a better match. However, such an analysis would require a more detailed model of the matching technology
used by such platforms.

The emergence of open platform model as a new business paradigm does not necessarily mean the end of the closed models but the possibility of a new equilibrium in which the two paradigms compete. For example, large IT operators such as IBM have moved towards an explicit legitimization of the openness movement and the development solutions that are compatible with the Microsoft standard and with Open Source Software. It seems rather difficult to foresee the final outcomes of this dynamic in terms of market share of the two paradigms. One direction of future research is to make an attempt to present a simulation model of technological competition between closed and open platforms that may explain long term behavior of both paradigms given the presence of network externalities. Thus a relevant question to consider is whether an open technology platform could defeat a proprietary opponent when the former was not supported by some kind of psychological feeling favoring the open-source alternative against the dominant proprietary standard. Not all dominant closed standards create a strong antagonistic response. It may also be noteworthy to see how adoption of open vs. closed platform depends on users preference distribution or nonhomogeneous preferences. Simulation experiments can be carried out with uniform, bimodal and normal distribution of adopters preference patterns. One could explore whether differentiation could be a good strategy for open technology when it is subject to strong network effects. It is intuitively easier for such a new technology to become a standard when adopters mainly belong to two subpopulations which prefer either technology, i.e. when the new technology is able to differentiate from the existing proprietary standard than when adopters simply have normally distributed heterogeneous preferences.

Consider technological hybridization as another possible strategy for closed platform providers. By technological hybridization, it means either technology can adopt one or more feature of the other. This phenomenon has recently been emphasized by economic historian Kirsch [23] during both the early and recent histories of the automobile. Early on, the success of combustion engine vehicles was consistently related to the fact that they had implemented an electric starter, taken from their electric vehicle competitor, which allowed for a much easier start and therefore proved extremely important for many users, and specially for women. Now, there is once again consistent discussions about new 'hybrid' vehicles which would possess both a combustion engine and an electric engine (or a fuel cell). A new hybrid variant might therefore be on its way, with the idea of combining the environmental interest of electric vehicle and the autonomy and power of combustion engine. A working model could be formulated to simulate a multi-staged dynamic game where platforms take on strategic decisions on operating open vs. closed network to maximize their short and long term profits and the impact of such decisions on competition, public policy, regulation and industry landscape as they switch their business models from closed to open or vice verse.

Another key unanswered question is to examine that possible dual existence of open and closed platforms. Hybridization is a key strategy for producers who face technological competition. It is not only that they try to make their technology better by copying characteristics from other technologies, but hybridization also plays an important role in influencing demand by modifying preferences within different subpopulations of potential adopters who would prefer either technology ab initio, and who might make the switch if the other technology includes several features of their favorite. Modifying the underlying preferences of potential adopters can prove an extremely valuable strategy, specially for a dominant standard to resist technological invaders, as it influences the nonlinear dynamics of diffusion and competition processes. A relevant question here would be the sensitivity of users who prefer open technology to hybridization strategies. Recent example from Microsoft has proven that it might sometimes not be the case and should also render proprietary technology close enough to open technology to convince users for which the openness of sources is a key condition of their preferences. One could wonder whether it does not purely mean turning closed platform into open platform, and if the ongoing invention of less public license will retain enough characteristics of open business model.

Can open platform defeat closed platform? Under what conditions? The answer may depend on the efficiency of its organization. Although the basics of this organization are determined by the openness of sources, it relies of course on the community of developers and on its internal organization. In a way, one could interpret the fact that many such communities are not associated with ancillary business firms as a further development of their 'internal' organization. The model points out that the organizational level is here crucial, since it will crucially determine the outcome of the competition between open and closed technology. In what extent then are open technology communities able to organize themselves, to 'self-organize', and is it sufficient to grant them with enough competitive power? This is still an open question, which further studies should try to address: this is an important question also if one was to consider public support to open platforms, as is already the case in some countries, as a means to correct market failures monopolies and closed de facto proprietary standards due to network effects, at least in software markets. Public intervention cannot simply consider existing open-source communities and give them a hint: economists have to suggest ways to help these communities improve their organizational structure.

Apart from this, the competitiveness of open technology also depends on its compatibility or interoperability with existing proprietary solutions, and on the distribution of adopter preferences. This can be either a crucial strength or a crucial weakness for open platforms. To take but one important example, think about the very different destiny of Linux on the server market and on the global PC market: it has won considerable market shares in the first, while it is still stagnant in the second. A simple straightforward explanation is that the lack of an appropriate graphic user interface (GUI,
or desktop) for Linux which is not needed for servers since they are maintained by skilled users geeks, once again who even prefer the older 'command line', while normal users need an efficient GUI. In future research framework, it may mean that the distribution of adopter preferences in the server market allowed for the diffusion of Linux, whereas it did not in the global market. A possible conjecture in this last respect might be that there were not enough early adopters, that the left mode of a bimodal distribution was not 'big' enough.

An even more interesting point comes when significant effort is now being done is accounted for to develop such a GUI for Linux, i.e. this time to hybridize open with closed technology. Perhaps asymmetric hybridization by an open-technology platform provider has no discernible effect on the outcome of its competition against a dominant proprietary standard. This precisely means that developing a GUI for Linux will probably not be sufficient to change its destiny on the PC market. To put it differently, and although these issues are of course still blurred, it might only be part of a potential solution, which should also include a further improvement of Linux compatibility with Windows, and a further increase in the efficiency of its development. It is then an open question whether such an increase in organizational efficiency is simply feasible, since it certainly depends not only on the number and the personality of kernel developers, but also on the involvement of ancillary business firms which would not only provide dedicated services, as is generally the case with open platform businesses, but which would also act as quasi-editors. But the question then is about how they would then earn money since they could not sell free open technology. Linux will perhaps never replace Windows in the offices: perhaps efforts in this direction are even misguided, all the more so as they are diverting efforts from other projects which could win their own open vs. closed technological competition.

Further empirical case studies could be carried out to validate the theoretical work, since platforms are central to many key industries including computer games, information technology, many Internetbased industries, media, mobile telephony and other telecommunications industries, and payment systems. Future research in the economics of open platforms could also focus on business models and competition strategies in different industry, and on appropriate governance structures for open source projects. In this regard, one question considers the positioning of closed and open products. For example, regarding software, users vary in their technical sophistication and requirements. Casual observation suggests that open source software is largely aimed at sophisticated users, while closed source software is often more 'user-friendly'. To some extent this can be explained by the fact that open source programmers seek recognition from their peers, who are sophisticated users. However, it could also be thought of as a product differentiation strategy by the closed source firm(s). The question is then what the optimal degree and determinants of user-friendliness of a closed source program are when faced with open source competition. Second, as noted above, a possible reaction
of closed source firms to the open phenomenon is to try to emulate some of the open incentives within the firm. In-depth case studies of modern closed source software development (if possible) would shed light on the extent to which firms are doing this, and the methods that they use. Theoretical work could also give some insight as to the effects of this mimicking behavior on the open community, and whether it is likely to be successful for the closed source firms.

For network dependent platform industries, standards adoption is a key prerequisite for attracting complementary assets. Producer firms that hope to profit from their standards success must trade off control of the standard against the imperative for adoption. During the systems era of computing, mainframe producers maximized their control by offering vertically integrated standards architectures. In the PC era, IBM unintentionally surrendered control to two key suppliers in its haste to launch the IBM PC and maximize its adoption. Microsoft and Intel in turn sought pervasive adoption of their technologies by appropriating only a single layer of the standards architecture and publishing a subset of the interfaces to other layers. In reaction to these proprietary strategies, the open source movement developed software that relinquishes control in favor of adoption. Such free software has played an important role in Internet infrastructure, and its adherents argue that it will supplant such proprietary standards in the network era. Vertically integrated proprietary standards architectures were the norm for the first three decades of the postwar computer industry. Each computer maker developed most if not all of its technology internally, and sold that technology only as part of an integrated computer system.

This strategy was challenged by two different approaches. One was the fragmentation of proprietary standards in the PC industry between different suppliers, which led firms like Microsoft and Intel to seek industry-wide dominance for their proprietary component of the overall system architecture, marking the PC era. The second was a movement by users and second-tier producers to create industry-wide open systems, in which the standard was not owned by a single firm. The explosive adoption of the Linux operating system in the late nineties was a response to these earlier approaches. Linux was the most commercially successful example of a new wave of open source software, in which the software and even source code are freely distributed to use and modify. At the same time, its adherents emphasized its advantages in contrast to the proprietary PC standards, particularly software standards controlled by Microsoft. For future investigation, the dynamic evolution of standards competition strategies in the computer industry from the proprietary integrated systems to both horizontal specialization and open systems should be examined.

Finally, open standards have direct implications on public policy concerning competition as standards allow for interoperability and interconnection. Much of the success of the information age can be attributed to the use of standards, whether it is for transporting information (TCP/IP, FTP) or representing information (HTML, XML, JPEG). When everyone uses the same standard, powerful
network effects increase the utility of a technology for everyone. With the growth and power of standards, a noticeable shift in how standards are produced and used has occurred. In the past, standards were created through two means: de facto and de jure. De facto standards are those created by vendors. These proprietary standards are not publicly distributed and often require a licensing fee for others to use them. A simple example of a de facto standard is the Microsoft Word document format. De jure standards, on the other hand, are those developed by formal standardsetting organizations, such as the International Organization for Standardization (ISO). This simple dichotomy does not capture how standards are currently developed.

Firms, individuals, and governments have all recognized the value of standards and developed a new organizational form, consortia, to develop standards collectively. There are significant variations in the design and processes of consortia that affect the development of standards. However, the standards can no longer be categorized simply as de facto or de jure. Instead, a variety of new approaches to creating and distributing standards has emerged. Policymakers need a robust empirical analysis so that they might understand the impact of emerging standards. Without such evidence, it would be ill advised to blindly put into place preferential policies that favor open platforms. Research must be done to assess how standards develop and provide core strategies to improve the process to explore the appropriate strategies to adopt in order to become the standard whether the business is using the open or closed model.

## Appendix A

## Proof

Relaxing the assumption that all developers enter the market, let's assume that, among the developers on the 0,1 interval, $0<y<1$ developers work on the closed platform, and $0<1-\theta<1$ developers work on the open platform as shown in the figure below. $y$ may or may not be equal to $\theta$ and by definition $\theta>y$. The demand functions for applications do not change, so $p_{C}^{d}=p_{O}^{d}=\frac{1}{2}$. When platforms are incompatible,

$$
\begin{aligned}
& n_{C}^{u}=\frac{1}{2}+\frac{n_{C}^{d}-n_{O}^{d}}{16 t}-\frac{p_{u}^{C}}{2 t} \\
& n_{O}^{u}=\frac{1}{2}-\frac{n_{C}^{d}-n_{O}^{d}}{16 t}+\frac{p_{u}^{C}}{2 t}
\end{aligned}
$$

since it is still assumed all users enter the market.


Figure A-1: Division of Developers between Two Incompatible Platforms, assuming Free Entry

Solving for developer profits,

$$
\begin{aligned}
\pi_{p}^{d} & =\frac{1}{8}+\frac{n_{C}^{d}-n_{O}^{d}}{64 t}-\frac{p_{u}^{C}}{8 t}-p_{d}^{C}-s y \\
\pi_{O}^{d} & =\frac{1}{8}-\frac{n_{C}^{d}-n_{O}^{d}}{64 t}-s(1-\theta)
\end{aligned}
$$

Developers continue to enter the market until the marginal developer's profit is zero. Therefore:

$$
y=n_{C}^{d}=\frac{1}{8 s}+\frac{n_{C}^{d}-n_{O}^{d}}{64 s t}-\frac{p_{u}^{C}}{8 s t}-\frac{p_{d}^{C}}{s}
$$

and

$$
1-\theta=n_{O}^{d}=\frac{1}{8 s}+\frac{n_{C}^{d}-n_{O}^{d}}{64 s t}-\frac{p_{u}^{C}}{8 s t}
$$

After some algebra, $\theta-y>0$ if $p_{d}^{C}>\frac{1}{4}-s$. In addition, solving through for $n_{C}^{d}$ and $n_{O}^{d}$ gives:

$$
\begin{gathered}
n_{C}^{d}=\frac{4 s}{\left(2048 s^{2} t^{2}-1\right)}-\frac{(32 s t+1) p_{u}^{C}}{8\left(2048 s^{2} t^{2}-1\right)}-\frac{p_{d}^{C}}{128 t\left(2048 s^{2} t^{2}-1\right)} \\
n_{O}^{d}=\frac{1}{8 t(64 s t+1)}+\frac{p_{u}^{C}}{8(64 s t+1)}-\frac{1}{64 s t+1}\left(\frac{4 s}{2048 s^{2} t^{2}-1}-\frac{(32 s t+1) p_{u}^{C}}{8\left(2048 s^{2} t^{2}-1\right)}-\frac{p_{D}^{C}}{128 t\left(2048 s^{2} t^{2}-1\right)}\right)
\end{gathered}
$$

The closed platform's profit is derived as:

$$
\Pi_{C}=p_{d}^{C}\left(\frac{512 s t-16 t(32 s t+1) p_{u}^{C}-p_{d}^{C}}{128 t\left(2048 s^{2} t^{2}-1\right)}\right)+p_{u}^{C}\left(\frac{1}{2}-\frac{p_{u}^{C}}{2 t(32 s t-1)}-\frac{2 p_{d}^{C}}{t(32 s t-1)}-\frac{p_{u}^{C}}{2 t}\right)
$$

Solving for $p_{d}^{C}$ and $p_{O}^{C}$ :

$$
\begin{aligned}
p_{d}^{C} & =256 t-\frac{p_{u}^{C}}{32 s t-1}\left(8 t\left(1024 s^{2} t^{2}-1\right)-128\left(2048 s^{2} t^{2}-1\right)\right) \\
p_{u}^{C} & =\frac{32 s t-1}{64 s}+p_{d}^{C}\left(\frac{t\left(1024 s^{2} t^{2}-1\right)-16\left(2048 s^{2} t^{2}-1\right)}{245 s t\left(2048 s^{2} t^{2}-1\right)}\right)
\end{aligned}
$$

If $p_{d}^{C}=0$ in equilibrium, this implies that $256 s t=\frac{8 t\left(1024 s^{2} t^{2}-1\right)-128\left(2048 s^{2} t^{2}-1\right)}{64 s}$ and thus $2048 s^{2} t=$ $t\left(1024 s^{2} t^{2}-1\right)-16\left(2048 s^{2} t^{2}-1\right)$. Since $s$ and $t$ are both positive, $2048 s^{2} t$ must be greater than zero. However, $t\left(1024 s^{2} t^{2}-1\right)-16\left(2048 s^{2} t^{2}-1\right)$ cannot be greater than zero as long as $t \leq 16$. Such extreme product differentiation from the point of view of users is unrealistic in the current setup, and therefore, for the purposes of the model, it is assumed that all developers enter the market.

## Bibliography

[1] Argentesi, E. and Filistrucchi, L. "Estimating market power in a two-sided market: The case of newspapers," Journal of Applied Econometrics, Vol. 22, 2007, pp. 1247-1266.
[2] Armstrong, M. "Competition in Two-Sided Markets," mimeo, University College, London (2004).
[3] Baxter, W. F. "Bank Interchange of Transactional Paper- Legal and Economic Perspectives," Journal of Law and Economics, Vol. 26, 1983, pp. 541-588.
[4] Caillaud, B. and Jullien, B. "Chicken \& Egg- Competition among Intermediation Service Providers," RAND Journal of Economics, Vol. 24, 2003, pp. 309-328.
[5] Choi, J. P. "Tying in two-sided markets with multi-homing," NET Institute (2006).
[6] Church, J., Gandal, N. and Krause, D. P. "Indirect Network Effects and Adoption Externalities," Foerder Institute for Economic Research Working Paper (2002).
[7] "Is Sony eating hundreds of dollars on each PS3?" available at http://news.com.com/Is+Sony+eating+hundreds+of $\quad+$ dollars+on+each + PS3/2100-1043 36136204.html (2006).
[8] "Linux to work with windows" available at http://money.cnn.com/2006/11/02/technology/microsoft (2006).
[9] Corts, K. S. and Lederman, M. "Software Exclusivity and the Scope of Indirect Network Effects in the U.S. Home Video Game Market," NET Institute (2007).
[10] Doganoglu, T. and Wright, J. "Multi-homing and compatibility," International Journal of Industrial Organization Vol. 24, 2003, pp. 45-67.
[11] Economides, N. and Katsamakas, E. "Two-Sided Competition of Proprietary vs. Open Source Technology Platforms and the Implications for the Software Industry," Management Science, Vol. 52, 2006, pp. 1057-1071.
[12] Eisenmann, T.E., Parker, G. G. and Van Alstyne, M. "Strategies for Two-Sided Markets," Harvard Business Review (2006).
[13] Evans, D. "The Antitrust Economics of Multi-Sided Platform Markets," Yale Journal on Regulation, Vol. 20, 2003, pp. 325-382.
[14] Farrell, J. and Saloner, G. "Standardization, Compatibility and Innovation," Rand Journal of Economics, Vol. 16, 1985, pp. 70-83.
[15] Farrell, J., Monroe, H. and Saloner, G. "The Vertical Organization of Industry- Systems Competition vs. Component Competition," Journal of Economics 8 Management Strategy, Vol. 7, 1998, pp. 143-182.
[16] Fine, C. H. "Clockspeed: Winning Industry Control in the Age of Temporary Advantage," Perseus Books, Reading, MA (1998).
[17] Hagiu, A. and Lee, R. "Exclusivity and control," HBS Working Paper(2007).
[18] Hagiu, A. "Proprietary vs. open two-sided platforms and social effciency," HBS Working Paper 07-095 (2007).
[19] Hagiu, A. "Merchant or two-sided platform?" Review of Network Economics, Vol. 6, 2007, pp. 115-133.
[20] Kaiser, U. and Wright, J. "Price structure in two-sided markets- Evidence from the magazine industry," International Journal of Industrial Organization, Vol. 24, 2006, pp. 1-28.
[21] Katz, M. L. and Shapiro, C. "Network Externalities, Competition, and Compatibility," American Economic Review, Vol. 75, 1985, pp. 424-440.
[22] Kende, M. "Profitability under an open versus a closed system," Journal of Economics and Management Strategy, Vol. 7, 1998, pp. 307-326.
[23] Kirsch, D.A. "Technological hybrids and the automobile system: historical considerations and future directions," mimeo, UCLA (1997).
[24] Laffont, J. J., Marcus, S., Rey, P. and Tirole, J. "Internet interconnection and the off-net-cost pricing principle," Mimeo, Institut dEconomie Industrielle (2001).
[25] Lee, R. "Vertical integration and exclusivity in platform and two-sided markets," NET Institute Working Paper 07-39 (2007).
[26] Lerner, J. and Tirole, J. "The Open Source Movement: Key Research Questions," European Economic Review, Vol. 45, 2001, pp. 819-826.
[27] Lerner, J. and Tirole, J. "Some Simple Economics of Open Source," Journal of Industrial Economics, Vol. 50, 2002, pp. 197-234.
[28] McGrath,M. E. "Product Strategy for High Technology Companies," McGraw-Hill New York (2001).
[29] Mustonen, M. "Copyleft The Economics of Linux and Other Open Source Software," Information Economics and Policy, Vol. 15, 2003, pp. 99-121.
[30] Nocke, V., Peitz, M. and Stahl, C. "Platform Ownership in Two-Sided Markets," mimeo, University of Pennsylvania and University of Mannheim (2004).
[31] Parker, G. G. and Van Alstyne, M. "Two-Sided Network Effects- A Theory of Information Product Design," Management Science, Vol. 51, 2005, pp. 1494-1504.
[32] Rochet, J. and Tirole, J. "Platform Competition in Two-Sided Markets," Journal of European Economic Association, Vol.1, 2003, pp. 990-1029.
[33] Rochet, J. and Tirole, J. "Two-Sided Markets: An Overview," mimeo, IDEI University of Toulouse (2004).
[34] Rochet, J. and Tirole, J. "Two-sided markets: A progress report," RAND Journal of Economics, Vol. 37, 2006, pp. 645-667.
[35] Robertson, D. and Ulrich, K. "Planning for Product Platforms," Sloan Management Review, Vol. 39, 1998, pp. 19-31.
[36] Roson, R. "Two-Sided Markets: A Tentative Survey," Review of Network Economics, Vol. 4, 2005, pp. 142-160.
[37] Rysmans, M. "Competition Between Networks: A Study of the Market for Yellow Pages," Review of Economic Studies, Vol. 71, 2004, pp. 483-512.
[38] Valloppillil V. "Open Source Software, a (new?) development methodology," Microsoft Internal Report (1989).

