

Problem Set 4 Solutions

$$1. a) J_0 = e \left( \frac{D_e}{L_e} N_{ep} + \frac{D_h}{L_h} N_{hn} \right) \quad (\text{A/m}^2)$$

Need to find  $D_e, D_h, L_e, L_h$

$$\Rightarrow L_e = (D_e \tau_e)^{1/2}, \quad L_h = (D_h \tau_h)^{1/2}$$

Given find  $D_e, D_h$  using the Einstein relationship:

$$\frac{D}{\mu} = \frac{kT}{q}$$

$$D_e = \frac{kT}{q} \mu_e$$

$$= (0.026 \text{ V}) (801 \text{ cm}^2/\text{Vs}) (10^{-4} \text{ m}^2/\text{cm}^2)$$

$$= 2.08 \times 10^{-3} \text{ m}^2/\text{s}$$

$$D_h = \frac{kT}{q} \mu_h$$

$$= (0.026 \text{ V}) (400 \text{ cm}^2/\text{Vs}) (10^{-4} \text{ m}^2/\text{cm}^2)$$

$$= 1.04 \times 10^{-3} \text{ m}^2/\text{s}$$

$$L_e = (D_e \tau_e)^{1/2} = [(2.08 \times 10^{-3} \text{ m}^2/\text{s}) (0.1 \times 10^{-6} \text{ s})]^{1/2}$$

$$= 4.4 \times 10^{-3} \text{ m}$$

$$L_h = (D_h \tau_h)^{1/2} = [(1.04 \times 10^{-3} \text{ m}^2/\text{s}) (1 \times 10^{-6} \text{ s})]^{1/2}$$

$$= 3.2 \times 10^{-3} \text{ m}$$

$N_i$  for Silicon at R.T. is about  $10^{10} \text{ cm}^{-3}$

$$N_{ep} = \frac{N_i^2}{N_A} = \frac{10^{20} \text{ cm}^{-6}}{10^{17} \text{ cm}^{-3}} = 10^3 \text{ cm}^{-3} = 10^9 \text{ m}^{-3}$$

$$N_{hn} = \frac{N_i^2}{N_D} = \frac{10^{20} \text{ cm}^{-6}}{10^{15} \text{ cm}^{-3}} = 10^5 \text{ cm}^{-3} = 10^{11} \text{ m}^{-3}$$

$$\Rightarrow J_0 = e \left( \frac{D_e}{L_e} N_{ep} + \frac{D_h}{L_h} N_{hn} \right) = (1.6 \times 10^{-19} \text{ C}) \left[ \frac{2.08 \times 10^{-3} \text{ m}^2/\text{s}}{4.4 \times 10^{-3} \text{ m}} (10^9 \text{ m}^{-3}) + \frac{1.04 \times 10^{-3} \text{ m}^2/\text{s}}{3.2 \times 10^{-3} \text{ m}} (10^{11} \text{ m}^{-3}) \right]$$

$$= 5.87 \times 10^{-3} \text{ A/m}^2$$

$$I_0 = J_0 A = (5.87 \times 10^{-3} \text{ A/m}^2) (10^{-4} \text{ cm}^2) (10^{-4} \text{ m}^2/\text{cm}^2) = 5.87 \times 10^{-11} \text{ A}$$

$$I = I_0 (e^{\frac{eV}{kT}} - 1)$$

$$(i) I = (5.87 \cdot 10^{-15} \text{ A}) \left[ \exp\left(-\frac{50 \text{ V}}{0.026 \text{ V}}\right) - 1 \right] = \boxed{-5.87 \cdot 10^{-15} \text{ A}}$$

$$(ii) I = (5.87 \cdot 10^{-15} \text{ A}) \left[ \exp\left(-\frac{0.1 \text{ V}}{0.026 \text{ V}}\right) - 1 \right] = \boxed{-5.74 \cdot 10^{-15} \text{ A}}$$

$$(iii) I = (5.87 \cdot 10^{-15} \text{ A}) \left[ \exp\left(\frac{0.2 \text{ V}}{0.026 \text{ V}}\right) - 1 \right] = \boxed{1.29 \cdot 10^{-14} \text{ A}}$$

Assuming  $N$ 's and  $\tau$ 's don't change:

$$\frac{kT}{e} = \frac{(1.38 \cdot 10^{-23} \text{ J/K})(500 \text{ K})}{1.6 \cdot 10^{-19} \text{ Coul}} = 4.31 \cdot 10^{-2} \text{ V}$$

$$D_e = (4.31 \cdot 10^{-2} \text{ V}) (501 \text{ cm}^2/\text{Vs}) (10^{-4} \text{ m}^2/\text{cm}^2) = 3.46 \cdot 10^{-3} \text{ m}^2/\text{s}$$

$$D_h = (4.31 \cdot 10^{-2} \text{ V}) (477 \text{ cm}^2/\text{Vs}) (10^{-4} \text{ m}^2/\text{cm}^2) = 2.06 \cdot 10^{-3} \text{ m}^2/\text{s}$$

$$L_e = (D_e \tau_e)^{1/2} = [(3.46 \cdot 10^{-3} \text{ m}^2/\text{s})(0.1 \cdot 10^{-6} \text{ s})]^{1/2} = 1.86 \cdot 10^{-5} \text{ m}$$

$$L_h = (D_h \tau_h)^{1/2} = [(2.06 \cdot 10^{-3} \text{ m}^2/\text{s})(1 \cdot 10^{-6} \text{ s})]^{1/2} = 4.54 \cdot 10^{-5} \text{ m}$$

$$(N_i)_{500\text{K}} = (N_i)_{300\text{K}} \cdot \exp\left[-\left(\frac{E_g}{kT}\right)\left(\frac{1}{300} - \frac{1}{500}\right)\right]$$

$$(N_i)_{500\text{K}} = 10^{16} \text{ cm}^{-3} \cdot \exp\left[-\frac{1.1 \text{ eV}}{0.026 \text{ eV}} \left(\frac{1}{300} - \frac{1}{500}\right)\right]$$

$$(N_i)_{500\text{K}} = 2.46 \cdot 10^{27} \text{ cm}^{-3}$$

$$N_{ep} = \frac{N_i}{N_h} = \frac{2.46 \cdot 10^{27} \text{ cm}^{-3}}{10^{17} \text{ cm}^{-3}} = 2.46 \cdot 10^{10} \text{ cm}^{-3} = 2.46 \cdot 10^{16} \text{ m}^{-3}$$

$$N_{hn} = \frac{N_i}{N_0} = \frac{2.46 \cdot 10^{27} \text{ cm}^{-3}}{10^{15} \text{ cm}^{-3}} = 2.46 \cdot 10^{12} \text{ cm}^{-3} = 2.46 \cdot 10^{18} \text{ m}^{-3}$$

$$I_0 = J_0 A = (10^{-4} \text{ cm}^2)(10^{-4} \text{ m}^2/\text{cm}^2) (1.6 \cdot 10^{-19} \text{ Coul}) \left[ \frac{3.46 \cdot 10^{-3} \text{ m}^2/\text{s}}{1.86 \cdot 10^{-5} \text{ m}} (2.46 \cdot 10^{16} \text{ m}^{-3}) + \frac{2.06 \cdot 10^{-3} \text{ m}^2/\text{s}}{4.54 \cdot 10^{-5} \text{ m}} (2.46 \cdot 10^{18} \text{ m}^{-3}) \right]$$

$$= 1.86 \cdot 10^{-7} \text{ A}$$

$$(i) I = (1.86 \cdot 10^{-7} \text{ A}) \left[ \exp\left(-\frac{50 \text{ V}}{0.043 \text{ V}}\right) - 1 \right] = \boxed{-1.86 \cdot 10^{-7} \text{ A}}$$

$$(ii) I = (1.86 \cdot 10^{-7} \text{ A}) \left[ \exp\left(-\frac{0.1 \text{ V}}{0.043 \text{ V}}\right) - 1 \right] = \boxed{-1.68 \cdot 10^{-7} \text{ A}}$$

$$(iii) I = (1.86 \cdot 10^{-7} \text{ A}) \left[ \exp\left(\frac{0.1 \text{ V}}{0.043 \text{ V}}\right) - 1 \right] = \boxed{1.93 \cdot 10^{-7} \text{ A}}$$

$$2. a) I = A \cdot J_0 \left[ \exp\left(\frac{qV_b}{kT}\right) - 1 \right]$$

$$= A \cdot e \left( \frac{D_n}{L_n} N_{np} + \frac{D_p}{L_p} N_{pn} \right) \left[ \exp\left(\frac{qV_b}{kT}\right) - 1 \right]$$

$$N_{np} = \frac{N_A^2}{N_D}, \quad N_{pn} = \frac{N_D^2}{N_A}$$

$$N_i^2 = N_A N_D \exp\left(-\frac{E_g}{kT}\right)$$

$$N_D = C_1 T^{3/2}, \quad N_A = C_2 T^{3/2}$$

$$N_i^2 = C_3 T^3 \exp\left(-\frac{E_g}{kT}\right)$$

$$\Rightarrow I = A \cdot e \cdot \left( \frac{D_n}{L_n} \frac{1}{N_A} + \frac{D_p}{L_p} \frac{1}{N_D} \right) \cdot C_3 T^3 \exp\left(-\frac{E_g}{kT}\right) \cdot \left[ \exp\left(\frac{qV_b}{kT}\right) - 1 \right]$$

$\frac{D_n}{L_n}, \frac{D_p}{L_p}, N_A, N_D$  and  $A$  are all constant with respect to  $T$ , so

we can combine them into a constant =  $C$

$$I = C \cdot e \cdot T^3 \cdot \exp\left(-\frac{E_g}{kT}\right) \left[ \exp\left(\frac{qV_b}{kT}\right) - 1 \right]$$

using the given  $E_g$ ,

$$I = C \cdot e \cdot T^3 \cdot \exp\left[-\frac{1}{kT} \left( 1.17 - \frac{(4.73 \cdot 10^{-4}) T}{T+636} \right) eV \right] \left[ \exp\left(\frac{qV_b}{kT}\right) - 1 \right]$$

solving for  $V_b$

$$\frac{qV_b}{kT} = \ln \left\{ \frac{I}{C \cdot e \cdot T^3 \cdot \exp\left[-\frac{1}{kT} \left( 1.17 - \frac{4.73 \cdot 10^{-4} T}{T+636} \right) eV \right]} + 1 \right\}$$

$$V_b = \frac{kT}{q} \left( \ln \left\{ \frac{I}{C \cdot e \cdot T^3 \cdot \exp\left[-\frac{1}{kT} \left( 1.17 - \frac{4.73 \cdot 10^{-4} T}{T+636} \right) eV \right]} + 1 \right\} \right)$$

$$b) I_0 = 10^{-15} \text{ A}$$

$$I_0 = C \cdot e \cdot T^3 \cdot \exp\left[-\frac{1}{kT} \left(1.17 - \frac{4.73 \cdot 10^{-4} \text{ J}}{T + 636} \right) \text{ eV}\right]$$

$$T = 300 \text{ K}$$

$$10^{-15} \text{ A} = C \cdot (1.6 \cdot 10^{-19} \text{ coul}) \cdot (300 \text{ K})^3 \cdot \exp\left[-\frac{(8.6 \cdot 10^{-5} \text{ eV/K}) \cdot (300 \text{ K})}{1} \left(1.17 - \frac{(4.73 \cdot 10^{-4} \text{ J}) \cdot (300 \text{ K})}{936}\right) \text{ eV}\right]$$

$$C = 1.77 \cdot 10^{15} \text{ s}^{-1} \text{ K}^{-3}$$

$$I = 10^{-4} \text{ A}$$

plugging these into the equation from part (a)

$$V_A = \frac{(1.38 \cdot 10^{-23} \text{ J/K}) \cdot T}{(1.6 \cdot 10^{-19} \text{ coul})} \left( \ln \left\{ \frac{10^{-4} \text{ A}}{1.77 \cdot 10^{15} \text{ s}^{-1} \cdot (1.6 \cdot 10^{-19} \text{ coul})^3 \cdot \exp\left[-\frac{(8.6 \cdot 10^{-5} \text{ eV/K}) \cdot T}{1} \left(1.17 - \frac{(4.73 \cdot 10^{-4} \text{ J})}{T + 636}\right) \right]} \right\} \right)$$

$$V_A = (8.62 \cdot 10^{-5} \text{ eV/K}) \cdot T \cdot \left\{ \ln \left[ \frac{0.352}{T^3} \cdot \exp\left(-\frac{(8.6 \cdot 10^{-5} \text{ eV/K}) \cdot T}{1} \cdot \left\{ 1.17 - \frac{(4.73 \cdot 10^{-4} \text{ J})}{T + 636} \right\} \right) + 1 \right] \right\}$$

5 points on the curve are:

T	293 K	323 K	373 K	423 K	473 K
V <sub>A</sub>	0.670 V	0.607 V	0.500 V	0.391 V	0.279 V

This plot is, as the problem stated, linear over this temperature range and can be drawn by hand or by computer.

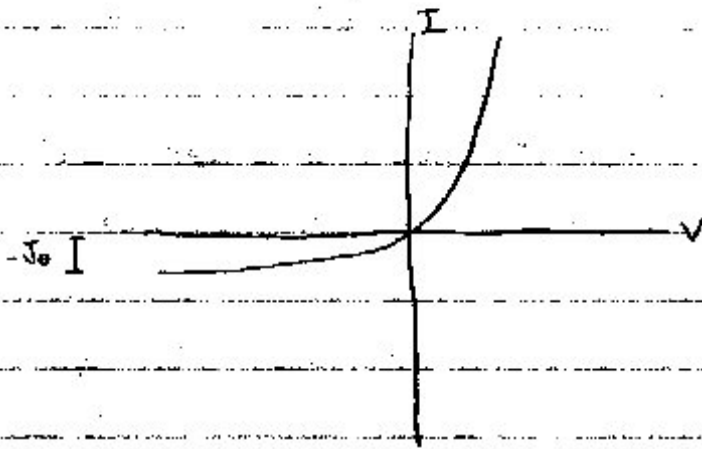
4) Taking the actual derivative of the equation above seems pretty tedious, so I think it makes more sense to determine a regression line to the five points found from the equation. Using these one obtains a linear regression with an  $r^2$  value of 0.9999 (pretty tight)!

$$V_A = 1.31 - 2.17 \cdot 10^{-3} \text{ eV/K} \cdot T$$

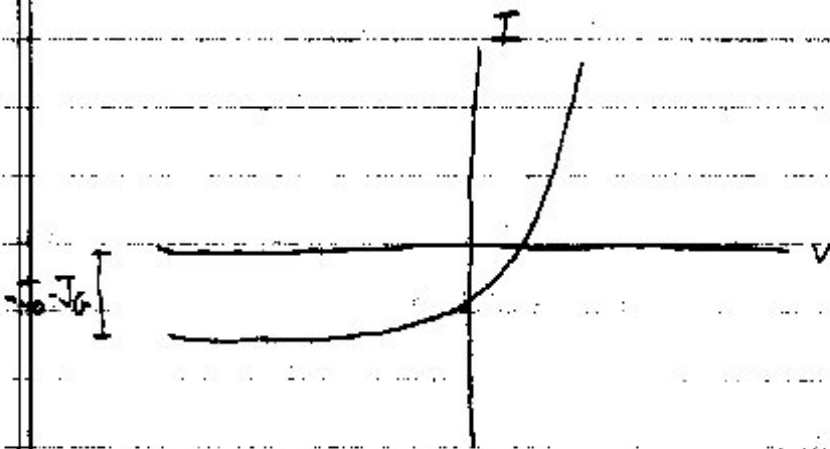
Therefore, the slope of the curve,  $\frac{dV_A}{dT} = -2.17 \text{ mV/K}$ .

- Q. 1) Reverse bias current is insensitive to applied bias.
- 2) As temperature increases, forward and reverse bias currents both increase.
- 3) As temperature increases, reverse bias current increases more with respect to forward bias current (i.e. the ability of the diode to rectify current is degraded).

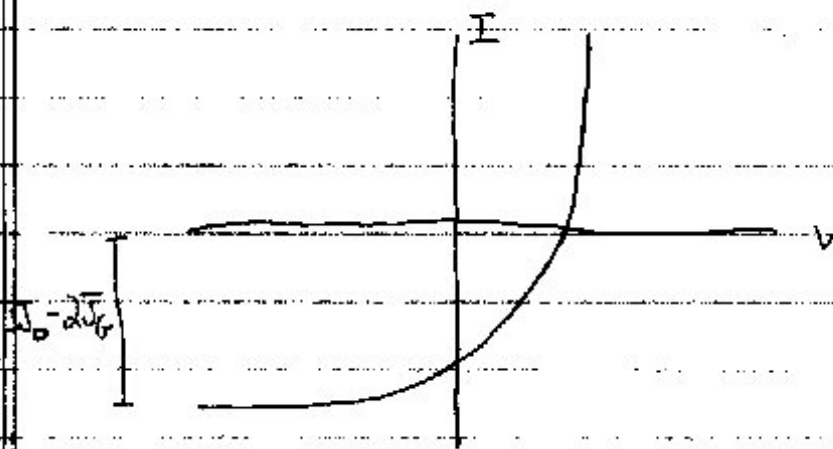
3. a) Under no illumination, simple diode



b) Under moderate illumination



c) Under illumination causing twice the generation rate



$$4. E_g(\text{ZnS}) = 3.6 \text{ eV}$$

$$E_g(\text{GaP}) = 2.3 \text{ eV}$$

ZnS has a bandgap of higher energy than visible light. This indicates that the visible photons present in sunlight will not be able to excite electrons across the bandgap and create a voltage. This would obviously make a pretty poor solar cell.

GaP has a bandgap energy within the visible spectrum. For this reason, many of the photons present in sunlight (specifically, the high-energy photons at the blue/indigo/violet part of the spectrum) will excite electrons over the bandgap and create a voltage. This semiconductor will work as a solar cell.

Just for interest, one cannot choose a semiconductor with a very small bandgap to use for a solar cell. Even though the semiconductor will absorb all of the photons and excite many electrons across the bandgap, there are problems with this idea. The energy one 'gets back' from each electron is the energy it loses when it recombines, i.e. the bandgap energy. Therefore, there is an optimum bandgap depending on the density of states in the valence and conduction bands.