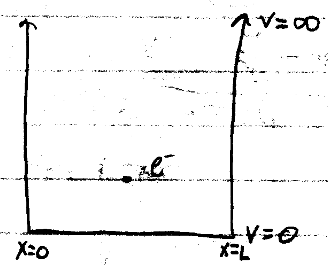
1.



a) need 3 lowest ΔE values from ground state, i.e.

$$\Delta E_{1\to2},\ \Delta E_{1\to3},\ \Delta E_{1\to4}$$

in 1-d infinite potential well, energy levels are given by

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{n^2 h^2}{8mL^2} \qquad [\text{Eq. } 8.25]$$

$$\Delta E_{1\to2} = \frac{h^2}{8mL^2}(2^2-1^2) = \frac{(6.63\cdot10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11\cdot10^{-31}\ \text{Kg})(10^{-9}\ \text{m})^2}\cdot(3)$$

$$= (6.02\cdot10^{-20}\ \text{J})\cdot3 = \boxed{1.81\cdot10^{-19}\ \text{J} = 1.13\ \text{eV}}$$

$$\Delta E_{1\to3} = (6.02\cdot10^{-20}\ \text{J})\cdot(3^2-1^2) = \boxed{4.82\cdot10^{-19}\ \text{J} = 3.01\ \text{eV}}$$

$$\Delta E_{1\to4} = (6.02\cdot10^{-20}\ \text{J})\cdot(4^2-1^2) = \boxed{9.04\cdot10^{-19}\ \text{J} = 5.64\ \text{eV}}$$

oops! need wavelength. Using $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$

$\lambda_{1\to2} = \boxed{1.10\cdot10^{-6}\ \text{m}}$

$\lambda_{1\to3} = \boxed{4.12\cdot10^{-7}\ \text{m}}$

$\lambda_{1\to4} = \boxed{2.20\cdot10^{-7}\ \text{m}}$

b) $$\Delta E_{1\to2} = \frac{h^2}{8mL^2}(2^2-1^2) = \frac{3\,(6.63\cdot10^{-34}\ \text{J}\cdot\text{s})^2}{8\,(9.11\cdot10^{-31}\ \text{Kg})\,L^2} = \frac{1.81\cdot10^{-37}\ \text{J}\cdot\text{m}^2}{L^2}$$

at $T = 300\text{K}$, $k_B T = (1.38\cdot10^{-23}\ \text{J/K})(300\ \text{K}) = 4.14\cdot10^{-21}\ \text{J}$

$$\Rightarrow \frac{1.81\cdot10^{-37}\ \text{J}\cdot\text{m}^2}{L^2} = 4.14\cdot10^{-21}\ \text{J}$$

$$L^2 = 4.37\cdot10^{-17}\ \text{m}^2$$

$$\boxed{L = 6.61\cdot10^{-9}\ \text{m} = 6.61\ \text{nm}}$$

c) Want $\Delta E_{1\to n} = K_B T$ for $L = 1$ cm, $T = 300$ K

$$\Delta E = \frac{h^2}{8mL^2}(n^2 - 1^2) = \frac{(6.63\cdot 10^{-34}\ J\cdot s)^2}{8(9.11\cdot 10^{-31}\ kg)(10^{-2}m)^2}(n^2-1)$$

$$= 6.02\cdot 10^{-34}\cdot (n^2-1)\ J$$

$$K_B T = 4.14\cdot 10^{-21}\ J = \Delta E$$

$$6.02\cdot 10^{-34}(n^2-1) = 4.14\cdot 10^{-21}$$

$$n^2 - 1 = 6.87\cdot 10^{12}$$

$$\boxed{n = 2.62\cdot 10^6}$$

2. This was a tricky question... parts b and c threw me for a loop for a long time...

a) $E = \frac{1}{2} m V^2$

$\therefore V = \sqrt{\frac{2E}{m}}$

$V_F = \sqrt{\frac{2 E_F}{m}}$

$= \sqrt{\frac{2(2.8\,eV)(1.6 \cdot 10^{-19}\,J/eV)}{9.11 \cdot 10^{-31}\,Kg}}$

$= \boxed{9.92 \cdot 10^5\ m/s}$

$E = \frac{\hbar^2 K^2}{2m}$

$\therefore K = \sqrt{\frac{2mE}{\hbar^2}}$

$K_F = \sqrt{\frac{2m E_F}{\hbar^2}}$

$= \sqrt{\frac{2(9.11 \cdot 10^{-31}\,Kg)(2.8\,eV)(1.6 \cdot 10^{-19}\,J/eV)}{(1.05 \cdot 10^{-34}\,J \cdot s)^2}}$

$= \boxed{8.61 \cdot 10^9\ m^{-1}}$

b) Average Velocity is zero, because the free electrons are moving with random orientation! However, they have a speed related to the Kinetic energy.

$E = \frac{1}{2} mV^2$

$\therefore V = \sqrt{\frac{2m}{E}}$

average speed is found analytically by

$$V_{avg} = \frac{\int V \cdot Z(E) \cdot F(E)\, dE}{\int Z(E) \cdot F(E)\, dE}$$

$Z(E)$ for a 3-D metal is given by $C E^{1/2}$, where C is a constant not dependent upon energy.

$$F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{KT}\right)}$$

So the average speed is

$$V_{avg} = \frac{\int_0^\infty \sqrt{\frac{2E}{m}} \cdot C E^{1/2} \cdot \frac{1}{1+\exp\left(\frac{E-E_F}{KT}\right)} \, dE}{\int_0^\infty C E^{1/2} \frac{1}{1+\exp\left(\frac{E-E_F}{KT}\right)} \, dE}$$

$$= \frac{C\sqrt{\frac{2}{m}} \int_0^\infty \frac{E}{1+\exp\left(\frac{E-E_F}{KT}\right)} \, dE}{C \int_0^\infty \frac{E^{1/2}}{1+\exp\left(\frac{E-E_F}{KT}\right)} \, dE}$$

These are two pretty hard integrals! We'll have to make an assumption;

Even at room temp., $E_F \gg K_B T$, so almost all of the electrons will have energies less than the Fermi energy. In other words, the vast majority of electrons will have the same speeds as at $T = 0$ K. Therefore, taking $T = 0$ K and thus $E < E_F$ for all electrons does not introduce much error.

$$V_{avg} = \sqrt{\frac{2}{m}} \cdot \frac{\int_0^{E_F} \frac{E}{1+\exp(-\infty)} \, dE}{\int_0^{E_F} \frac{E^{1/2}}{1+\exp(-\infty)} \, dE}$$

$$= \sqrt{\frac{2}{m}} \frac{\int_0^{E_F} E \, dE}{\int_0^{E_F} E^{1/2} \, dE} = \sqrt{\frac{2}{m}} \frac{\frac{1}{2} E^2 \Big]_0^{E_F}}{\frac{2}{3} E^{3/2} \Big]_0^{E_F}}$$

$$= \sqrt{\frac{2}{m}} \cdot \frac{3}{4} E_F^{1/2} = \frac{3}{4}\sqrt{\frac{2E_F}{m}} = \frac{3}{4}\sqrt{\frac{2(2.8\,\text{eV})(1.6\cdot 10^{-19}\,\text{J/eV})}{9.11\cdot 10^{-31}\,\text{Kg}}}$$

$$= \boxed{7.44 \cdot 10^5 \text{ m/s}}$$

A much tougher problem than I think the writer intended....

2. c) The root mean square speed is the square root of the average "velocity-squared", i.e.

$$V_{rms} = \sqrt{\langle V^2 \rangle}$$

We could find the average "velocity-squared" using a similar method as part b, replacing V with V^2 in the equation

$$\langle V^2 \rangle = \frac{\int_0^\infty V^2 \cdot Z(E) \cdot F(E) dE}{\int_0^\infty Z(E) \cdot F(E) dE}$$

However from the energy-velocity relation,

$$E = \frac{1}{2} m V^2,$$

we can see that the average "velocity-squared" is related to the average energy:

$$\langle E \rangle = \frac{1}{2} m \langle V^2 \rangle$$

This is useful because we know the average energy (it is worked out in the book).

$$\langle E \rangle = \frac{3}{5} E_F = \frac{3}{5}(2.8eV)(1.6 \cdot 10^{-19}\ J/eV) = \boxed{2.7 \cdot 10^{-19}\ J}$$

The analysis assumed $T = 0\,K$, but as we saw in part (b) the error should be small in using this assumption. So:

$$\langle V^2 \rangle = \frac{2\langle E \rangle}{m} = \frac{6 E_F}{5m}$$

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{6 E_F}{5m}} = \sqrt{\frac{6(2.8eV)(1.6 \cdot 10^{-19}\ J/eV)}{5(9.11 \cdot 10^{-31}\ Kg)}}$$

$$V_{rms} = \boxed{7.69 \cdot 10^5\ m/s}$$

3. a) $F(E) = \dfrac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$

at $T = 300\,K$, $k_BT = 4.14\cdot 10^{-21}\,J = 2.59\cdot 10^{-2}\,eV$

$$F(E) = \frac{1}{1+\exp\left(\frac{E-3eV}{2.59\cdot 10^{-2}\,eV}\right)}$$

$$F(2.9eV) = \frac{1}{1+\exp\left(-\frac{0.1}{2.59\cdot 10^{-2}}\right)} = 0.979$$

$F(3.1\,eV) = 0.021$ [you can determine this from the F(E) equation or realize that F(E) has a symmetry about E_F, such that $F(3.1eV) = 1 - F(2.9eV)$]

$F(3.0eV) = 0.5$

density of filled states is $Z(E)\cdot F(E)$

$Z(E)$ for a 3-D metal is

$$Z(E) = CE^{1/2}$$

$$\frac{Z(2.9\,eV)\cdot F(2.9eV)}{Z(3.0eV)\cdot F(3.0eV)} = \frac{(2.9)^{1/2}(0.979)}{(3.0)^{1/2}(0.5)} = \boxed{1.93}$$

$$\frac{Z(3.1\,eV)\cdot F(3.1eV)}{Z(3.0eV)\cdot F(3.0eV)} = \frac{(3.1)^{1/2}(0.021)}{(3.0)^{1/2}(0.5)} = \boxed{0.043}$$

b) $F(2.95\,eV) = 0.873$

$F(3.05\,eV) = 0.127$

$$\frac{Z(2.95\,eV)\,F(2.95eV)}{Z(3.0\,eV)\cdot F(3.0eV)} = \frac{(2.95)^{1/2}(0.873)}{(3.0)^{1/2}\,(0.5)} = \boxed{1.73}$$

$$\frac{Z(3.05\,eV)\cdot F(3.05eV)}{Z(3.0eV)\cdot F(3.0eV)} = \frac{(3.05\,eV)^{1/2}(0.127)}{(3.0)^{1/2}\,(0.5)} = \boxed{0.26}$$