Problem Set 6 Solutions

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1. This is very similar to problem 3 in problem set #5.

$\varepsilon_r = 3.85$ (low freq.)

$n = 1.46$

$\varepsilon_r = n^2 = 2.13$ (high freq.)

Polarization is proportional to susceptibility, $\chi = (\varepsilon_r - 1)$

There is electronic contribution at all frequencies, but ionic at only low frequencies.

a) Ionic contribution at low freq.:

$$\frac{\chi_{low} - \chi_{high}}{\chi_{low}} = \frac{2.85 - 1.13}{2.85} = \boxed{60.4\%}$$

b) At high freq., only electronic contributions.

$\boxed{0\%}$

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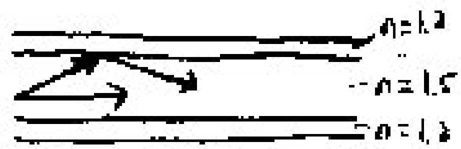

a) $\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right) = \sin^{-1}\left(\frac{1.2}{1.5}\right)$

$\theta_c = 53.1^\circ$

If light is uniformly distributed over all angles, the amount transmitted by total internal reflection is

$$\frac{90-\theta_c}{90} = \frac{90-53.1}{90} = \boxed{40.9\%}$$

b) Maximum difference in path length will occur between light travelling straight down the center and light bouncing between the two ends at the critical angle.

Light beam A will travel a distance of 10 diameters.

Light beam B will travel a distance given by:

$r_B = r_A \cdot \frac{1}{\sin(\theta_c)}$

$r_B = (10 \text{ diameters}) \cdot \frac{1}{\sin(53.1)}$

$r_B = 12.5$ diameters

The maximum difference in path length is $r_B - r_A = \boxed{25 \text{ diameters}}$

3. Light intensity dropping to 0.1% corresponds to a dB of

$$10 \cdot \log_{10} \frac{I_0}{0.1\% I_0} = 10 \cdot \log_{10} \frac{1}{0.001} = 10 \cdot \log_{10} 10^3 = 30 \text{ dB}$$

So, every time the dB decreases by 30, must add an amplifier

a) 100 dB/Km lost

$$\frac{30 \text{ dB/amp}}{100 \text{ dB/Km}} = 0.3 \text{ Km/amp}$$

$$\frac{1200 \text{ Km}}{0.3 \text{ Km/amp}} = \boxed{4000 \text{ amplifiers}}$$

b) 10 dB/Km

$$\frac{1200 \text{ Km}}{\left(\frac{30 \text{ dB/amp}}{10 \text{ dB/Km}}\right)} = \boxed{400 \text{ amplifiers}}$$

c) 1 dB/Km:

(notice the pattern?)

$\boxed{40 \text{ amplifiers}}$

d) 0.1 dB/Km

$\boxed{4 \text{ amplifiers}}$

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4. I think the main thing to realize from this problem is that ionic polarizability has a frequency dependence related to

$$\omega = \sqrt{\frac{K}{m}}$$

where m is the mass of the ions present (K is a spring constant). By reducing the mass of the ions, the absorption we get in the near-infrared will be pushed to higher frequencies. This would create absorption in the visible spectrum which is presumably where a lot of our signal is being carried. Since we want to get rid of absorption in our signal range (as we saw in problem 3, absorption = more amplifiers = more \$), we should probably put in some heavier ions and push the absorption to very low frequencies.