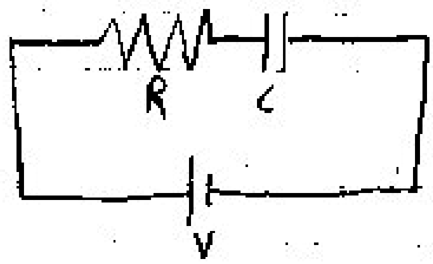
1. a)



→ at $t=0$, charge on the capacitor, $Q=0$

→ " " ", voltage across the capacitor, $V_C = \frac{Q}{C} = 0$

→ at all times $t>0$, $V = V_C + V_R$

→ at $t=0$, $V_R = V$ and so $I = \frac{V}{R}$

→ at $t=\infty$, $V_C = V$ and so $V_R = 0$ since the capacitor will then be fully charged

→ at $t=\infty$, charge on the capacitor, $Q = CV$

→ at $t=\infty$, no current can cross the charged capacitor, so $I=0$

→ at all times $I = I_R = I_C$

$$I = \frac{dQ}{dt} = \frac{d}{dt}(V_C \cdot C) = \frac{d}{dt}[C \cdot (V - V_R)]$$

→ voltage across the resistor is $V_R = I \cdot R$

$$I = \frac{d}{dt}(CV - CIR)$$

→ C, V, R are independent of time

$$I = -C \cdot R \frac{dI}{dt}$$

→ this is a differential equation with initial condition

$$I(t=0) = \frac{V}{R}$$

$$I \cdot dt = -C \cdot R \cdot dI$$

$$dt = -\frac{C \cdot R}{I}\, dI$$

$$\int dt = -\int \frac{CR}{I}\, dI$$

$$t = -CR \cdot \int \frac{dI}{I}$$

$$t = -CR \ln I + K$$ — the integration constant (the symbol C was already taken)

Using the initial condition,

$$0 = -C \cdot R \cdot \ln\left(\frac{V}{R}\right) + K$$

$$\Rightarrow K = C \cdot R \cdot \ln\left(\frac{V}{R}\right)$$

$$\therefore t = -C \cdot R \ln I + C \cdot R \cdot \ln\left(\frac{V}{R}\right)$$

$$\frac{t}{C \cdot R} = \ln\left(\frac{V}{R}\right) - \ln I = \ln\left(\frac{V}{RI}\right) = -\ln\left(\frac{RI}{V}\right)$$

$$\frac{R \cdot I}{V} = \exp\left(-\frac{t}{C \cdot R}\right)$$

$$\Rightarrow \boxed{I = \frac{V}{R} \exp\left(-\frac{t}{C \cdot R}\right)}$$

I think electrical engineers traditionally call $R \cdot C$ a time constant, τ so equivalently $I = \frac{V}{R} \exp\left(-\frac{t}{\tau}\right)$.

As a good check, this equation satisfies all of the conditions stated in the beginning.

b and c) It's easier to do these out of order:

$$V_R = I \cdot R = \left[\frac{V}{R} \cdot \exp\left(-\frac{t}{C \cdot R}\right)\right] R$$

$$\boxed{V_R = V \cdot \exp\left(-\frac{t}{C \cdot R}\right)}$$

$$V = V_C + V_R$$

$$V_C = V - V_R$$

$$V_C = V - V \exp\left(-\frac{t}{C R}\right)$$

$$\boxed{V_C = V\left[1 - \exp\left(-\frac{t}{C \cdot R}\right)\right]}$$

DATE

2\. a) $Q = CV$

$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$

$\varepsilon_r = 1$ for empty capacitor

$Q = \frac{\varepsilon_0 A}{d} V = \frac{(8.85\cdot10^{-12}\ F/m)(10\,cm^2)(10^{-4}\ m^2/cm^2)}{(3\cdot10^{-3}\ m)} \cdot 9V$

$Q = 2.66\cdot10^{-11}$ coul

b) ε_r now equals 25

$Q = \frac{\varepsilon_0 \varepsilon_r A}{d} \cdot V = \frac{25(8.85\cdot10^{-12}\ F/m)(10\ cm^2)(10^{-4}\ m^2/cm^2)}{(3\cdot10^{-3}\ m)} \cdot 9V$

$Q = 6.64\cdot10^{-10}$ coul , which is obviously 25× the charge originally

the charge on the surface of the dielectric, Q_s must be the difference between the Q found in parts (a) and (b).

$Q_s = 6.64\cdot10^{-10}$ coul $- 2.66\cdot10^{-11}$ coul

$Q_s = 6.38\cdot10^{-10}$ coul

total dipole moment per unit volume, P:

$P = \varepsilon_0 \cdot E \cdot \chi = \varepsilon_0 \cdot \frac{V}{d} \cdot (\varepsilon_r - 1) = (8.85\cdot10^{-12}\ F/m) \cdot \frac{9V}{3\cdot10^{-3}m} \cdot (24)$

$P = 6.38\cdot10^{-7}\ coul/m^2$

c) Now, instead of the voltage being constant in the $Q = CV$ equation the Q will be constant. So, as the capacitance increases by inserting the dielectric, instead of the charge increasing, the voltage will decrease.

The charge on the capacitor plates will be the same as found in part (a)

$$\boxed{Q = 2.66 \cdot 10^{-11}\ \text{coul}}$$

The voltage across the capacitor will be

$$V = \frac{Q}{C} = \frac{Q}{\left(\frac{\varepsilon_0 \varepsilon_r A}{d}\right)} = \frac{Q \cdot d}{\varepsilon_0 \varepsilon_r A} = \frac{(2.66 \cdot 10^{-11}\ \text{coul})(3 \cdot 10^{-3}\ \text{m})}{(8.85 \cdot 10^{-12}\ \text{F/m})(25)(10\ \text{cm}^2)(10^{-4}\ \text{m}^2/\text{cm}^2)}$$

$\boxed{V = 0.361\ \ V}$, which is obviously $\frac{1}{25}$ * the voltage originally

The total dipole moment per unit vol. is

$$P = \varepsilon_0 \cdot E \cdot \chi = \varepsilon_0 \frac{V}{d} \chi = (8.85 \cdot 10^{-12}\ \text{F/m}) \frac{0.361\ \text{V}}{3 \cdot 10^{-3}\ \text{m}} (24)$$

$$\boxed{P = 2.56 \cdot 10^{-8}\ \text{coul/m}^2}$$

The total dipole moment per unit vol. is equal to the charge on the dielectric surface per unit area, so we can use that to find the charge on the dielectric surface

$$P = \frac{Q_s}{A}$$

$$Q_s = P \cdot A = (2.56 \cdot 10^{-8}\ \text{coul/m}^2)(10\ \text{cm}^2)(10^{-4}\ \text{m}^2/\text{cm}^2)$$

$$\boxed{Q_s = 2.56 \cdot 10^{-11}\ \text{coul}}$$

3. The dielectric constant at high frequencies is due only to the electronic polarization. At low frequencies, there is contribution from both electronic and ionic polarization. Polarization is proportional to the susceptibility, so must use $(\varepsilon_r - 1)$ values

Electronic contribution $= \frac{0.9}{8.27} \cdot 100 = \boxed{10.9\%}$

Ionic contribution $= \frac{8.27 - 0.9}{8.27} \cdot 100 = \boxed{89.1\%}$

The larger ion with more electrons will contribute more to the electronic polarization. Looking at the periodic table, F is larger and has more electrons than Li, so F will contribute more.

4. $\varepsilon_r = 5.68$

$E = 1\ V/mm = 10^3\ V/m$

$P = \varepsilon_0 E(\varepsilon_r - 1) = (8.85 \cdot 10^{-12}\ F/m)(10^3\ V/m)(5.68-1)$

$\boxed{P = 4.14 \cdot 10^{-8}\ coul/m^2}$

$D = \varepsilon_0 \varepsilon_r \cdot E = (8.85 \cdot 10^{-12}\ F/m)(5.68)(10^3\ V/m)$

$\boxed{D = 5.03 \cdot 10^{-8}\ coul/m^2}$

$\chi = \varepsilon_r - 1 = 5.68 - 1$

$\boxed{\chi = 4.68}$