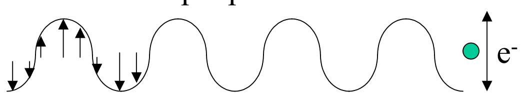
Response of Free e- to AC Electric Fields

- Microscopic picture



$$E_Z = E_O e^{-i\omega t}$$

B=0 in conductor,

and $\vec{F}(\vec{E}) >> \vec{F}(\vec{B})$

$$\frac{dp(t)}{dt} = -\frac{p(t)}{\tau} - eE_0 e^{-i\omega t}$$

try $\quad p(t) = p_0 e^{-i\omega t}$

$$-i\omega p_0 = -\frac{p_0}{\tau} - eE_0$$

$$p_0 = \frac{eE_0}{i\omega - \frac{1}{\tau}}$$

ω>>1/τ, p out of phase with E

$$p_0 = \frac{eE_0}{i\omega} \qquad \omega \to \infty, p \to 0$$

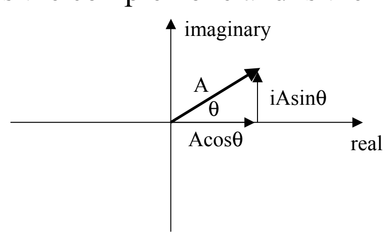
ω<<1/τ, p in phase with E

$$p_0 = eE_0\tau$$

Complex Representation of Waves

sin(kx-ωt), cos(kx-ωt), and $e^{-i(kx-\omega t)}$ are all waves

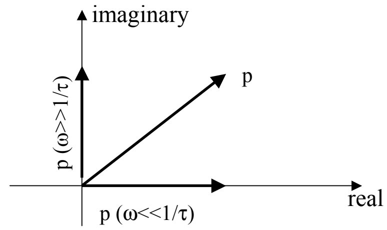
$e^{-i(kx-\omega t)}$ is the complex one and is the most general



$e^{i\theta} = \cos\theta + i\sin\theta$

Response of e- to AC Electric Fields

- Momentum represented in the complex plane



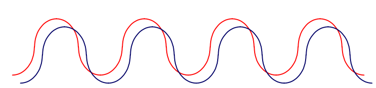
Instead of a complex momentum, we can go back to macroscopic and create a complex J and σ

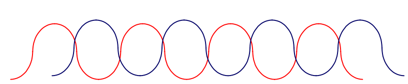
$$J(t) = J_0 e^{-i\omega\tau} \qquad J_0 = -nev = \frac{-nep_0}{m} = \frac{ne^2}{m(\frac{1}{\tau} - i\omega)} E_0$$

$$\sigma = \frac{\sigma_0}{1 - i\omega\tau}, \sigma_0 = \frac{ne^2\tau}{m}$$

Response of e- to AC Electric Fields

- Low frequency (ω<<1/τ)
  - electron has many collisions before direction change
  - Ohm's Law: J follows E, σ real
- High frequency (ω>>1/τ)
  - electron has nearly 1 collision or less when direction is changed
  - J imaginary and 90 degrees out of phase with E, σ is imaginary





Qualitatively:

ωτ<<1, electrons in phase, re-irradiate, $E_i=E_r+E_t$, *reflection*

ωτ>>1, electrons out of phase, electrons too slow, less interaction, *transmission* $\varepsilon=\varepsilon_r\varepsilon_0$ $\varepsilon_r=1$

$$\tau \approx 10^{-14}\sec, \nu\lambda = c, \nu = \frac{3x10^{10}\,cm/\sec}{5000x10^{-8}\,cm} \approx 10^{14}\,Hz$$

E-fields with frequencies greater than visible light frequency expected to be beyond influence of free electrons

Response of Light to Interaction with Material

- Need Maxwell's equations
  - from experiments: Gauss, Faraday, Ampere's laws
  - second term in Ampere's is from the unification
  - electromagnetic waves!

SI Units (MKS)

$$\nabla \bullet \vec{D} = \rho$$
$$\nabla \bullet \vec{B} = 0$$
$$\nabla x\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla x\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$
$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$
$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu \vec{H}$$
$$\mu = \mu_r \mu_0 ; \varepsilon = \varepsilon_r \varepsilon_0$$

Gaussian Units (CGS)

$$\nabla \bullet \vec{D} = 4\pi\rho$$
$$\nabla \bullet \vec{B} = 0$$
$$\nabla x\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$
$$\nabla x\vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$$
$$\vec{D} = \vec{E} + 4\pi\vec{P}$$
$$\vec{B} = \vec{H} + 4\pi\vec{M}$$

Waves in Materials

- Non-magnetic material, $\mu = \mu_0$
- Polarization non-existent or swamped by free electrons, P=0

$$\nabla x\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla x\vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$
$$\nabla x(\nabla x\vec{E}) = -\frac{\partial \nabla x\vec{B}}{\partial t}$$
$$-\nabla^2 E = -\frac{\partial}{\partial t}[\mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}]$$
$$\nabla^2 E = \mu_0 \sigma \frac{\partial E}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

For a typical wave,

$$E = E_0 e^{i(k \bullet r - \varpi t)} = E_0 e^{ik \bullet r} e^{-i\varpi t} = E(r) e^{-i\varpi t}$$
$$\nabla^2 E(r) = -i\varpi \mu_0 \sigma E(r) - \mu_0 \varepsilon_0 \omega^2 E(r)$$
$$\boxed{\nabla^2 E(r) = -\frac{\omega^2}{c^2}\varepsilon(\omega) E(r)}$$ Wave Equation
$$\varepsilon(\omega) = 1 + \frac{i\sigma}{\varepsilon_0 \omega}$$
$$E(r) = E_0 e^{ik \bullet r}$$
$$k^2 = \frac{\omega^2}{c^2}\varepsilon(\omega)$$
$$v = \frac{\omega}{k} = \frac{c}{\sqrt{\varepsilon(\omega)}}$$

Waves in Materials

- Waves slow down in materials (depends on $\varepsilon(\omega)$)
- Wavelength decreases (depends on $\varepsilon(\omega)$)
- Frequency dependence in $\varepsilon(\omega)$

$$\varepsilon(\omega) = 1 + \frac{i\sigma}{\varepsilon_0 \omega} = 1 + \frac{i\sigma_0}{\varepsilon_0 \omega (1 - i\omega\tau)}$$

$$\varepsilon(\omega) = 1 + \frac{i\omega_p^2 \tau}{\omega - i\omega^2 \tau}$$

$$\omega_p^2 = \frac{ne^2}{\varepsilon_0 m} \quad \text{Plasma Frequency}$$

For $\omega\tau >>> 1$, $\varepsilon(\omega)$ goes to 1

For an excellent conductor (σ_0 large), ignore 1, look at case for $\omega\tau << 1$

$$\varepsilon(\omega) \approx \frac{i\omega_p^2 \tau}{\omega - i\omega^2 \tau} \approx \frac{i\omega_p^2 \tau}{\omega}$$

Waves in Materials

$$k = \frac{\omega}{c}\sqrt{\varepsilon(\omega)} = \frac{\omega}{c}\sqrt{i}\sqrt{\frac{\sigma_0}{\omega \varepsilon_0}}$$

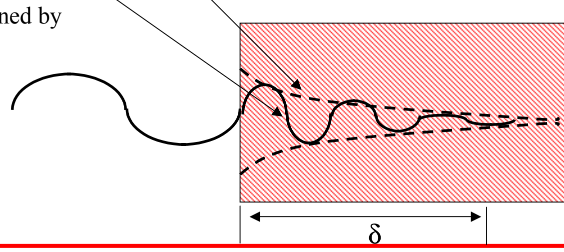
$$k = \frac{\omega}{c}\left(\frac{1+i}{\sqrt{2}}\right)\sqrt{\frac{\sigma_0}{\omega\varepsilon_0}} = \left(\sqrt{\frac{\sigma_0 \omega}{2\varepsilon_0 c^2}} + i\sqrt{\frac{\sigma_0 \omega}{2\varepsilon_0 c^2}}\right)$$

For a wave $E = E_0 e^{i(kz - \omega t)}$ Let $k = k_{real} + k_{imaginary} = k_r + ik_i$

$$E = E_0 e^{i[k_r z - \omega t]} e^{-|k_i| z}$$

The skin depth can be defined by

$$\delta = \frac{1}{|k_i|} = \sqrt{\frac{2\varepsilon_o c^2}{\sigma \omega_o}} = \sqrt{\frac{2}{\sigma_o \mu_o \omega}}$$



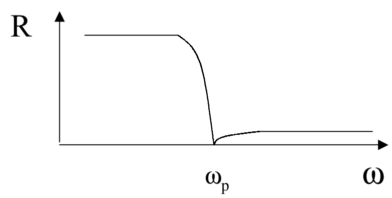
Waves in Materials

For a material with any σ_0, look at case for $\omega\tau \gg 1$

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$\omega<\omega_p$, ε is negative, $k=k_i$, wave reflected

$\omega>\omega_p$, ε is positive, $k=k_r$, wave propagates



Success and Failure of Free e- Picture

- Success
  - Metal conductivity
  - Hall effect valence=1
  - Skin Depth
  - Wiedmann-Franz law
- Examples of Failure
  - Insulators, Semiconductors
  - Hall effect valence>1
  - Thermoelectric effect
  - Colors of metals

K/σ=thermal conduct./electrical conduct.~CT

$$K = \frac{1}{3} c_v v_{therm}^2 \tau$$

$$c_v = \left(\frac{\partial E}{\partial T}\right)_v = \frac{3}{2} n k_b ; v_{therm}^2 = \frac{3k_b T}{m}$$

$$K = \frac{1}{3}\left(\frac{3}{2} n k_b\right)\left(\frac{3k_b T}{m}\right)\tau = \frac{3}{2}\frac{n k_b^2 T \tau}{m}$$

$$\sigma = \frac{ne^2\tau}{m}$$

Therefore : $\frac{K}{\sigma} = \frac{3}{2}\left(\frac{k_b}{e}\right)^2 T$

~C!

Luck: $c_{vreal}=c_{vclass}/100$; $v_{real}^2=v_{class}^2*10$
0

Wiedmann-Franz 'Success'

EXPERIMENTAL THERMAL CONDUCTIVITIES AND LORENZ NUMBERS OF SELECTED METALS

	273 K		373 K	
ELEMENT	κ (watt/cm-K)	$\kappa/\sigma T$ (watt-ohm/K^2)	κ (watt/cm-K)	$\kappa/\sigma T$ (watt-ohm/K^2)
Li	0.71	2.22×10^{-8}	0.73	2.43×10^{-8}
Na	1.38	2.12		
K	1.0	2.23		
Rb	0.6	2.42		
Cu	3.85	2.20	3.82	2.29
Ag	4.18	2.31	4.17	2.38
Au	3.1	2.32	3.1	2.36
Be	2.3	2.36	1.7	2.42
Mg	1.5	2.14	1.5	2.25
Nb	0.52	2.90	0.54	2.78
Fe	0.80	2.61	0.73	2.88
Zn	1.13	2.28	1.1	2.30
Cd	1.0	2.49	1.0	
Al	2.38	2.14	2.30	2.19
In	0.88	2.58	0.80	2.60
Tl	0.5	2.75	0.45	2.75
Sn	0.64	2.48	0.60	2.54
Pb	0.38	2.64	0.35	2.53
Bi	0.09	3.53	0.08	3.35
Sb	0.18	2.57	0.17	2.69

Thermoelectric Effect

Exposed Failure when c_v and v^2 are not both in property

$$E = Q\nabla T$$

Thermopower Q is $Q = -\frac{c_v}{3ne} = \frac{-\frac{3}{2}nk_b}{3ne} = -\frac{nk_b}{2e}$

Thermopower is about 100 times too large!

Waves in Vacuum

- $J, \rho = 0$
- $\mu = \mu_0; \varepsilon = \varepsilon_0$

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$ Wave Equation

For typical wave:

$$E = E_0 e^{ik\cdot r - \omega t} \qquad k = 2\pi/\lambda ; \omega = 2\pi\nu$$

$$k^2 = \mu_0 \varepsilon_0 \omega^2$$

$$\Rightarrow \omega/k = \nu\lambda = (\mu_0 \varepsilon_0)^{-1/2}$$

For constant phase: (kx-ωt)

$$v_{phase} = \omega/k = c \rightarrow c = (\mu_0 \varepsilon_0)^{-1/2}$$

Example:
Violet light ($\nu = 7.5 \times 10^{14}$ Hz)
$\lambda = c/\nu = 400$ nm
$k = 2\pi/\lambda = 1.57 \times 10^7$ m^{-1}
$\omega = 2\pi\nu = 4.71 \times 10^{15}$ s^{-1}

After Livingston

Waves in Materials; Skin Depth

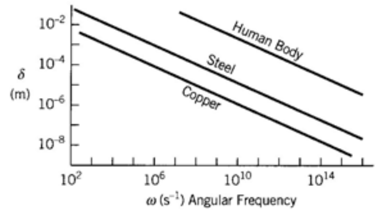
$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} + \mu\sigma \frac{\partial E}{\partial t} \quad ; \mu \cong \mu_0$$

$$k^2 = \omega^2 \mu\varepsilon + i\omega\mu\sigma$$

$$k \approx (i\omega\mu\sigma)^{1/2} \qquad \text{Conductive materials}$$

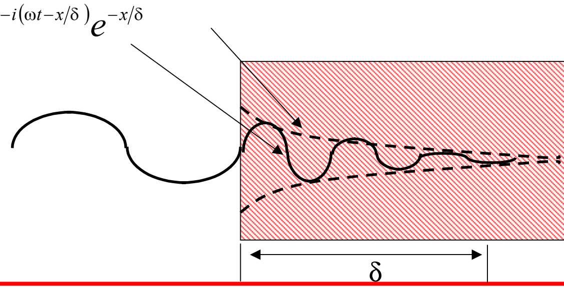
$$= \pm \frac{1+i}{\sqrt{2}} (\omega\mu\sigma)^{1/2} = \pm \frac{1+i}{\delta}$$

$$\Rightarrow E = E_0 e^{i(kx-\omega t)} = E_0 e^{-i(\omega t - x/\delta)} e^{-x/\delta}$$



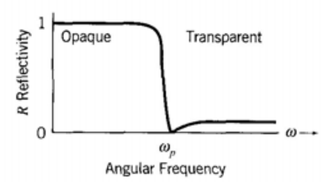
The skin depth is defined by

$$\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$$



Plasma Frequency

Remember: $k^2 = \omega^2 \mu\varepsilon + i\omega\mu\sigma$



where $\sigma = \dfrac{\sigma_0}{1 - i\omega\tau} \approx \dfrac{-\sigma_0}{i\omega\tau} \; (\omega\tau >> 1)$

then $k^2 \cong \omega^2 \mu\varepsilon - \dfrac{\mu\sigma_0}{\tau} = \omega^2 \mu\varepsilon \left(1 - \dfrac{\omega_p^2}{\omega^2}\right)$

where $\omega_p \cong \left(\dfrac{ne^2}{m\varepsilon}\right)^{1/2} \Rightarrow$ Plasma Frequency

$\Rightarrow$ For $\omega > \omega_p$; k is real number no attenuation!
$\omega < \omega_p$; k contains imaginary component, wave reflected

$\Rightarrow$ Criteria for transparent electrode?

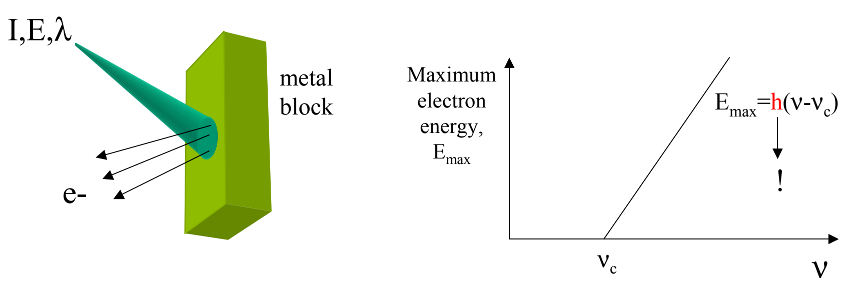
(Example: $n = 5.8 \times 10^{27}/m^3$; $\omega_p = 4.3 \times 10^{15} s^{-1}$)

Wave-particle Duality: Electrons are not *just* particles

- Compton, Planck, Einstein
  - light (xrays) can be 'particle-like'
- DeBroglie
  - matter can act like it has a 'wave-nature'
- Schrodinger, Born
  - Unification of wave-particle duality, Schrodinger Equation

Light is always quantized: Photoelectric effect (Einstein)

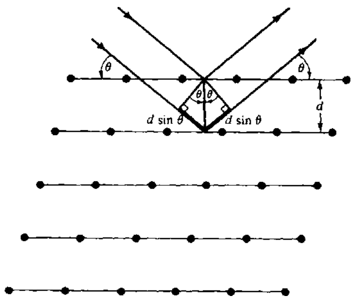
- Photoelectric effect shows that E=hν even outside the box



For light with $\nu<\nu_c$, no matter what the intensity, no e-

DeBroglie: Matter is Wave

- His PhD thesis!
- λ=h/p also for matter
- To verify, need very light matter (p small) so λ is large enough
- Need small periodic structure on scale of λ to see if wave is there (diffraction)
- Solution:electron diffraction from a crystal



$N\lambda=2d\sin\theta$

For small θ, $\theta\sim\lambda/d$, so λ must be on order of d in order to measure easily

Unification: Wave-particle Duality

Ψ must be able to represent everything from a particle to a wave (the two extremes)

wave ⟷ particle

$$\Psi = Ae^{i(kx-\omega t)}$$

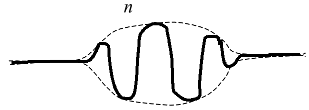
$$\Psi = \sum_n a_n e^{i(k_n x - \omega_n t)}$$

k and p known exactly

$n=\infty$ to create a delta function in ψ^2

generalized

$$\Psi = \sum_n a_n e^{i(k_n x - \omega_n t)}$$



Quantum Mechanics - Wave Equation

Classical Hamiltonian

$$\frac{p^2}{2m} + V(x, y, z) = E$$

QM Operators

$$p = \left(\frac{\hbar}{i}\right)\Delta \qquad E = -\left(\frac{\hbar}{i}\right)\frac{\partial}{\partial t}$$

$$-\frac{\hbar^2}{2m}\Delta^2\psi + V(x, y, z)\psi = -\frac{-\hbar}{i}\frac{\partial\psi}{\partial t}$$

1. ψ and $\Delta\,\psi$ must be finite, continuous and single valued.
2. $\psi\psi^*$ real with $\psi\psi^* dV$ = probability of finding particle in volume dV.
3. Average or expectation value of variable

$$\langle\alpha\rangle = \int_v \psi^*\alpha_{op}\psi\, dV$$

Time and Spatial Dependence of ψ

Assume ψ (x,y,z,t) separable into ψ(x,y,z) and ϕ(t)

Applying separation of variables:

$$\frac{-\hbar^2}{2m}\frac{\nabla^2\psi}{\psi} + V = -\frac{\hbar}{i}\frac{1}{\phi}\frac{\partial\phi}{\partial t} = \varepsilon \quad = \text{constant}$$

Time-Dependent Equation:

$$\phi(t) = Ae^{-i(\varepsilon/\hbar)t} = Ae^{-i\omega t}$$

$$\Rightarrow \boxed{\varepsilon = \hbar\omega}$$

Time-Independent Equation:

$$\nabla^2\psi + \frac{2m}{\hbar^2}(\varepsilon - V)\psi = 0$$

Solutions: ψ_n -eigenfunctions; ε_n -eigenvalues

Free Particle

• One dimensional $V = O$

$$\frac{d^2\psi}{dx^2} = -\frac{2m\varepsilon}{\hbar^2}\psi = -k^2\psi$$

$$\psi = Ae^{ikx}$$

$$\boxed{\varepsilon = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}}$$

$$\boxed{p = \hbar k}$$ Crystal Momentum

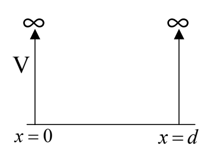
$$\psi(x,t) = Ae^{i(kx-\omega t)}$$

• Momentum

$$\langle p_x \rangle = \int \psi^* \frac{\hbar}{i}\frac{\partial \psi}{\partial x} dx = \hbar k$$

Particle in Box

$$\psi = Ae^{ikx} + Be^{-ikx}; \; k^2 = \frac{2m\varepsilon}{\hbar^2}$$



Boundary Conditions:

- $\psi(0) = \psi(d) = 0$
- $\psi(0) = A + B = 0$ $\quad \Rightarrow A = -B$
- $\psi(d) = A(e^{ikd} - e^{-ikd}) = 0$ $\quad \Rightarrow 2ASinkd = 0$

$$\boxed{k = \frac{n\pi}{d} \qquad n = 1,2,3\ldots}$$

$$\boxed{\psi = \sqrt{2/d}\,\sin\left(\frac{n\pi x}{d}\right) \qquad n = 1,2,3\ldots}$$

Particle in Box

$$\psi_n(x,y,z) = A\sin k_1 x \cdot \sin k_2 y \cdot \sin k_3 z$$

$$k^2 = k_1^2 + k_2^2 + k_3^2 \qquad k_i = {n_i\pi}/{d}\,; \qquad n_i = 1,2,3\ldots$$

$$\varepsilon_n = \frac{\hbar^2}{8md^2}(n_1^2 + n_2^2 + n_3^2) = \frac{\hbar^2 k^2}{2m}$$

n = Quantum numbers

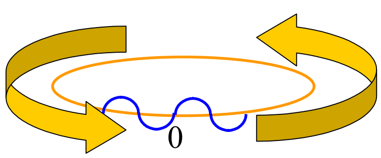
• Ground state E ; $n_1 = n_2 = n_3 = 1$ not zero!

• Degeneracy

First excited state 112, 211, 121

Consequence of Electrons as Waves on Free Electron Model

Traveling wave picture



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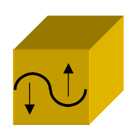
Standing wave picture



0 L

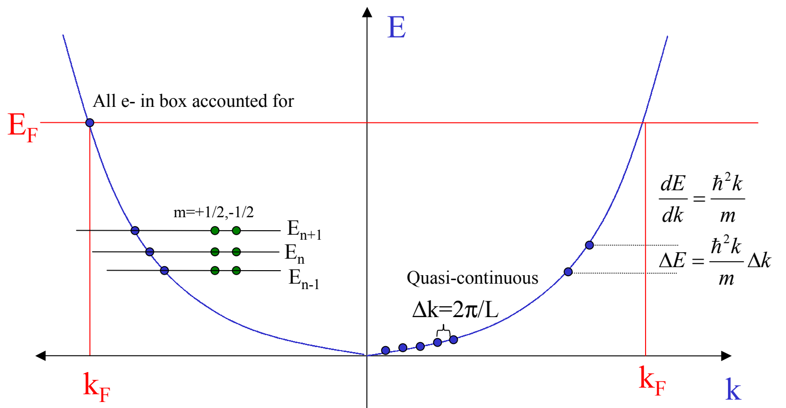
$$\Psi(x) = \Psi(x+L)$$
$$e^{ikx} = e^{ik(x+L)}$$
$$e^{ikx} = 1$$
$$k = \frac{2\pi n}{L}$$

Just having a boundary condition means that k and E are quasi-continuous, i.e. for large L, they appear continuous but are discrete

Representation of E,k for 1-D Material

- states
- electrons

$$E = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}$$



Total number of electrons=N=2*2k_F*L/2π