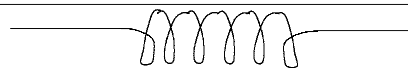
Magnetic Materials

- The inductor



$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t} \text{ (CGS)}$$

$$\iint \nabla \times E dS = -\frac{1}{c}\frac{\partial}{\partial t}\left(\iint BdS\right) = -\frac{1}{c}\frac{\partial \Phi_B}{\partial t}$$

$\Phi_B \equiv$ magnetic flux density

$$\iint \nabla \times E dS = \oint E \cdot d\ell \text{ (Green's Theorem)}$$

$$V = \int E \cdot d\ell = -\frac{1}{c}\frac{\partial \Phi_B}{\partial t} \text{ (explicit Faraday's Law)}$$

$$\Phi_B = LI \text{ (Q = CV)}$$

$$\frac{\partial \Phi_B}{\partial t} = L\frac{\partial I}{\partial t}$$

$$V_{EMF} = -\frac{\partial N\Phi_B}{\partial t} = -L\frac{\partial I}{\partial t}$$

$$V = L\frac{\partial I}{\partial t} \text{ (recall } I = C\frac{\partial V}{\partial t} \text{ for the capacitor)}$$

$$\text{Power} = VI = LI\frac{\partial I}{\partial t}$$

$$\text{Energy} = \int Power \cdot dt = \int LIdI = \frac{1}{2}LI^2 = \frac{1}{2}N\Phi_B I$$

$$\left(capacitor\ \frac{1}{2}CV^2\right)$$

The Inductor

$$\nabla \times B = \frac{4\pi}{c}J + \frac{1}{c}\frac{\partial E}{\partial t}$$

$$\iint \nabla \times BdS = \oint B \cdot d\ell = \frac{4\pi}{c}\iint J \cdot dS = \frac{4\pi}{c}I$$

$$B = \frac{4\pi}{c}In$$

$$N = n \cdot length = nl$$

$$L = \frac{N\phi_B}{I} = \frac{N(BA)}{I} = \frac{4\pi}{c}n^2 lA$$

Magnetic Permeability and Susceptibility

Insert magnetic material

Magnetic dipoles in material can line-up in magnetic field

$$B = H + 4\pi\chi H = H + 4\pi M$$

$$M = \chi H \quad \frac{\partial M}{\partial H} = \chi \quad \mu = 1 + 4\pi\chi$$

$$B = 4\pi M + 1 \quad B = \mu H$$

B magnetic induction
χ magnetic susceptibility
H magnetic field strength (applied field)
M magnetization

MKS:

$$B = \mu_0(H+M) = \mu_0 nI + \mu_0 M$$

$$\mu_r = B/\mu_0 H = 1 + (M/H) = 1 + \chi_m$$

Maxwell and Magnetic Materials

- Ampere's law $\oint H \cdot d\ell = I = 0$
- For a permanent magnet, there is no real current flow; if we use B, there is a need for a fictitious current (magnetization current)
- Magnetic material inserted inside inductor increases inductance

$$\Phi_B = BA \sim 4\pi MA = 4\pi\chi HA = 4\pi\chi\left(\frac{4\pi}{c}In\right)A$$

$$L = \frac{N\Phi_B}{I} = \frac{(4\pi)^2}{c}n^2 lA\chi$$

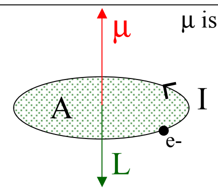
L increased by ~χ due to magnetic material

Material Type	χ
Paramagnetic	$+10^{-5}$-10^{-4}
Diamagnetic	-10^{-8}-10^{-5}
Ferromagnetic	$+10^{5}$

Microscopic Source of Magnetization

- No monopoles
- magnetic dipole comes from moving or spinning electrons

Orbital Angular Momentum



μ is the magnetic dipole moment

$$Energy = E = -\vec{\mu}\cdot\vec{H} = -|\mu||H|\cos\theta$$

What is μ? For θ=0, $E = -\mu H \approx -\Phi_B I$ since energy ~ LI^2 and for 1 loop $L = \frac{\Phi_B}{I}$

$$\Phi_B = \iint H \cdot dS \sim HA$$

$$\therefore \mu H = \Phi_B I = HAI \quad \text{and} \quad \boxed{\therefore \mu = IA}$$

$$I = -\frac{e}{c}\frac{\omega}{2\pi} \quad A = \pi r^2$$

$$\mu = -\frac{e}{2c}\omega r^2$$

Microscopic Source of Magnetization

- Classical mechanics gives orbital angular momentum as:

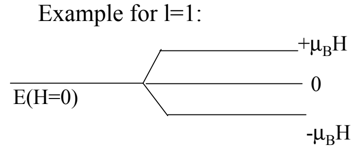
$$\vec{L} = \vec{r}\times\vec{p} = mr^2\omega$$

$$\mu_L = -\frac{e}{2mc}L_{QM} = -\frac{e\hbar}{2mc}L_Z = -\mu_B L_Z$$

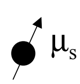
$$\left(\mu_B = \frac{e\hbar}{2mc}\right)$$

$$L_Z = m_\ell = -\ell,\ldots,0,\ldots\ell$$

Example for l=1:

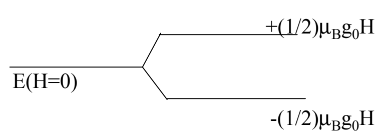


Spin Moment



$$\mu_s = -\frac{e}{mc}S \overset{Q.M.}{=} -g_0\frac{e\hbar}{2mc}S_z = -g_0\mu_B S_Z$$

$$S_Z = m_S = \pm\frac{1}{2} \quad g_0 = 2 \text{ for electron spin}$$



Exchange

E~-JS$_1$S$_2$

J negative, E~+S$_1$S$_2$--> Energy ↓ if ↓↑

J positive, E~-S$_1$S$_2$--> Energy ↓ if ↑↑

Fe, Ni, Co ---> J positive!

Other elements J is negative

Rule of Thumb:

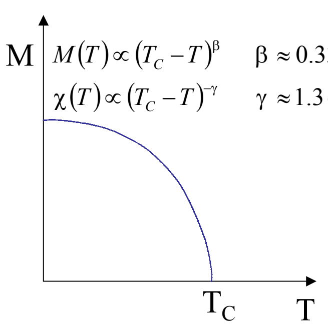
$$\frac{r}{2r_a} \equiv \frac{\text{interatomic distance}}{2(\text{atomic radius})} > 1.5$$

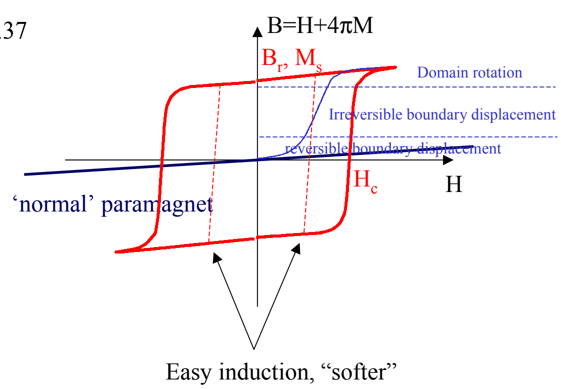
J is a function of distance!

Ferromagnetism

$M(T) \propto (T_C - T)^{\beta}$ $\beta \approx 0.33\text{-}0.37$

$\chi(T) \propto (T_C - T)^{-\gamma}$ $\gamma \approx 1.3\text{-}1.4$



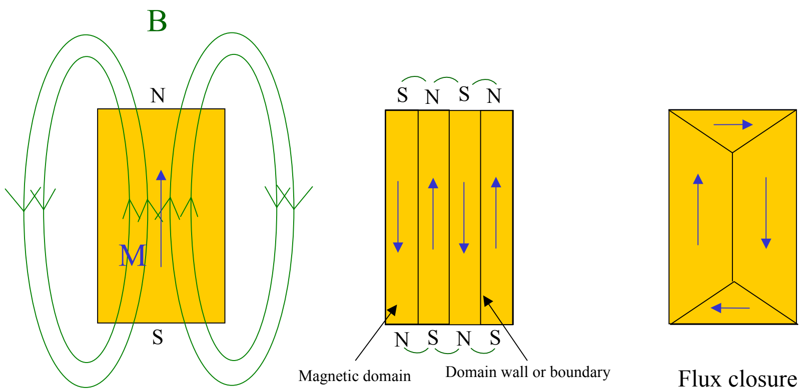


Magentic anisotropy

hardness of loop dependent on crystal direction

comes from spin interacting with bonding

Domains in Ferromagnetic Materials



Flux closure

Magnetic energy

$$=\frac{1}{8}\int B^2 dV$$

No external field