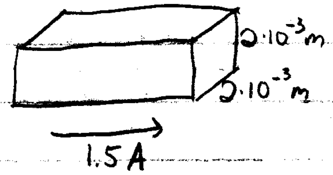
Problem Set 1 Solutions

1.



$\rho = 2.69\cdot10^{-6}\ \Omega\cdot cm = 2.69\cdot10^{-8}\ \Omega\cdot m$

at. Vol. = $10\ cc/mol = 1\cdot10^{-5}\ m^3/mol$

$Z = 3$ electrons/atom

a) $R_H = -\frac{1}{ne}$

n = # of electrons / unit vol.

$= 3\left(\frac{e^-}{atom}\right)\cdot 6\cdot10^{23}\left(\frac{atom}{mol}\right)\cdot\frac{1}{1\cdot10^{-5}}\left(\frac{mol}{m^3}\right) = 1.8\cdot10^{29}\ e^-/m^3$

$R_H = -\frac{1}{1.8\cdot10^{29}\cdot1.6\cdot10^{-19}}\ m^3/coul$

$= \boxed{-3.47\cdot10^{-11}\ m^3/coul}$

b) $\sigma = \frac{1}{\rho} = ne\mu$

$\Rightarrow \mu = \frac{1}{ne\cdot\rho} = -\frac{R_H}{\rho}$

$= -\frac{-3.47\cdot10^{-11}\ m^3/coul}{2.69\cdot10^{-8}\ \Omega\cdot m}$

$= \boxed{1.29\cdot10^{-3}\ coul\cdot s/Kg}$

c) $V_D = -\frac{J}{e\cdot n} = J\cdot R_H$

$J = \frac{I}{A} = \frac{1.5A}{(2\cdot10^{-3}m)^2} = 3.75\cdot10^{5}\ A/m^2$

$V_D = 3.75\cdot10^{5}\ A/m^2 \cdot -3.47\cdot10^{-11}\ m^3/coul$

$= \boxed{-1.30\cdot10^{-5}\ m/s}$ ← negative because we implied current flow as +X direction, so electrons are flowing in −X direction!

d) $\mu = \frac{e\tau}{m}$

$\Rightarrow \tau = \frac{\mu m}{e}$

$= \frac{1.29\cdot10^{-3}\ Coul\cdot s/Kg\cdot 9.11\cdot10^{-31}\ Kg}{1.6\cdot10^{-19}\ coul}$

$= \boxed{7.34\cdot10^{-15}\ s}$

e) $E = J \cdot \rho$

$= (3.75 \cdot 10^{5}\ A/m^2)(2.69 \cdot 10^{-8}\ \Omega \cdot m)$

$= \boxed{1.01 \cdot 10^{-2}\ V/m}$

f) $P/\ell = I \cdot \frac{V}{\ell} = I \cdot E$

$= (1.5\ A)(1.01 \cdot 10^{-2}\ V/m)$

$= 1.52 \cdot 10^{-2}\ W/m$

$= \boxed{1.52 \cdot 10^{-4}\ W/cm}$

g) $E_H = -R_H \cdot J_x \cdot B_z$

$= (-3.47 \cdot 10^{-11}\ m^3/coul)(3.75 \cdot 10^{5}\ A/m^2)(1\ T)$

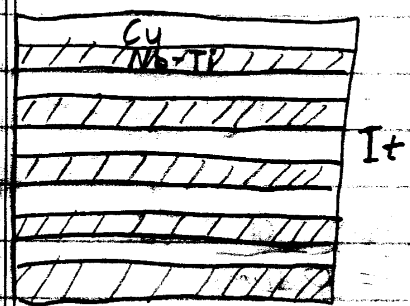
$= +1.30 \cdot 10^{-5}\ V/m$

$V_H = E_H \cdot \ell = (+1.30 \cdot 10^{-5}\ V/m)(2 \cdot 10^{-3}\ m)$

$= \boxed{+2.60 \cdot 10^{-8}\ V}$

DATE

2.

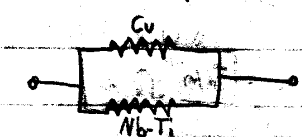


$\rho_{Cu} = 1.69\cdot10^{-6}\,\Omega\cdot cm = 1.69\cdot10^{-8}\,\Omega\cdot m$

$\rho_{Nb-Ti} = 7\cdot10^{-5}\,\Omega\cdot cm = 7\cdot10^{-7}\,\Omega\cdot m$

a) parallel to lamellae:

resistors in parallel

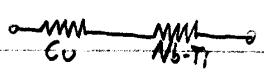


$$\rho_{TOT} = \frac{1}{\frac{1}{2\rho_{Cu}}+\frac{1}{2\rho_{Nb-Ti}}} = 2\left(\frac{1}{1.69\cdot10^{-8}\,\Omega\cdot m}+\frac{1}{7\cdot10^{-7}\,\Omega\cdot m}\right)^{-1}$$
$$= \boxed{3.30\cdot10^{-8}\,\Omega\cdot m}$$

$$J = \frac{E}{\rho}$$
$$J_{TOT} = \frac{E}{\rho_{TOT}}, \quad J_{Cu} = \frac{E}{2\cdot\rho_{Cu}}$$
$$\frac{I_{Cu}}{I_{TOT}} = \frac{J_{Cu}}{J_{TOT}} = \frac{\rho_{TOT}}{2\cdot\rho_{Cu}} = \frac{3.30\cdot10^{-8}\,\Omega\cdot m}{3.38\cdot10^{-8}\,\Omega\cdot m} = \boxed{0.98}$$

$$\boxed{E_{Cu} = E_{Nb-Ti}}$$

b) perpendicular to lamellae: resistors in series



$$\rho_{TOT} = \tfrac{1}{2}(\rho_{Cu}+\rho_{Nb-Ti}) = \tfrac{1}{2}(1.69\cdot10^{-8}\,\Omega\cdot m + 7\cdot10^{-7}\,\Omega\cdot m) = \boxed{3.58\cdot10^{-7}\,\Omega\cdot m}$$
$$J = \frac{E_{Cu}}{\rho_{Cu}} = \frac{E_{Nb-Ti}}{\rho_{Nb-Ti}}$$
$$\Rightarrow \frac{E_{Nb-Ti}}{E_{Cu}} = \frac{\rho_{Nb-Ti}}{\rho_{Cu}} = \frac{7\cdot10^{-7}\,\Omega\cdot m}{1.69\cdot10^{-8}\,\Omega\cdot m} = \boxed{41.4}$$

c) $\rho_{Cu} = 10^{-8}\,\Omega\cdot cm = 10^{-10}\,\Omega\cdot m$

$\rho_{Nb\text{-}Ti} = 0$

parallel:

$$\rho_{TOT} = \left(\frac{1}{\rho_{Cu}} + \frac{1}{\rho_{Nb\text{-}Ti}}\right)^{-1} = \left(\frac{1}{10^{-10}\,\Omega\cdot m} + \frac{1}{0}\right)^{-1} = \boxed{0\,\Omega\cdot m}$$

$$\frac{I_{Cu}}{I_{TOT}} = \frac{\rho_{TOT}}{\rho_{Cu}} = \frac{0}{10^{-10}\,\Omega\cdot m} = \boxed{0}$$

$$\boxed{E_{Cu} = E_{Nb\text{-}Ti} = 0}$$

perpendicular:

$$\rho_{TOT} = \rho_{Cu} + \rho_{Nb\text{-}Ti} = \frac{1}{2}\cdot 10^{-10}\,\Omega\cdot m + 0\,\Omega\cdot m = \boxed{5\cdot 10^{-11}\,\Omega\cdot m}$$

$$\frac{E_{Nb\text{-}Ti}}{E_{Cu}} = \frac{\rho_{Nb\text{-}Ti}}{\rho_{Cu}} = \frac{0}{10^{-10}\,\Omega\cdot m} = \boxed{0}$$

3 From red light!

$$\frac{I}{I_0} = \frac{1}{10} = \exp\left(-\frac{2z}{\delta}\right) = \exp\left(-\frac{2(10^{-8}\,m)}{\delta}\right)$$

$$\Rightarrow \delta = 8.69 \cdot 10^{-9}\,m$$

$$\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$$

$$\mu\cdot\sigma = \frac{2}{\omega\cdot\delta^2}$$

$$= \frac{2\lambda}{2\pi c\cdot\delta^2}$$

$$= \frac{2(700\cdot10^{-9}\,m)}{2\pi(3\cdot10^8\,m/s)(8.69\cdot10^{-9}\,m)^2} = 9.84\ S/m^2$$

$\mu\sigma$ is a material const., so we can use this for the blue light

$$\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$$

$$= \left(\frac{2\lambda}{2\pi c\mu\sigma}\right)^{1/2}$$

$$= \left(\frac{400\cdot10^{-9}\,m}{\pi(3\cdot10^8\,m/s)\cdot 9.84}\right)^{1/2}$$

$$= 6.57\cdot10^{-9}\,m$$

$$\frac{I_{blue}}{I_0} = \exp\left(-\frac{2z}{\delta}\right)$$

$$= \exp\left(-\frac{2(10^{-8}\,m)}{6.57\cdot10^{-9}\,m}\right)$$

$$= \boxed{4.76\cdot10^{-2}}$$

i.e. light has decreased to 4.8% of original intensity or has decreased by 95.2%

After 20 nm,

Red light: $$\frac{I_{red}}{I_0} = \exp\left(-\frac{2z}{\delta}\right)$$

$$= \exp\left(-\frac{2(2\cdot10^{-8}\,m)}{8.69\cdot10^{-9}\,m}\right)$$

$$= \boxed{1.00\cdot10^{-2}}$$

light has decreased by 99%

Blue light: $$\frac{I_{blue}}{I_0} = \exp\left(-\frac{2z}{\delta}\right)$$

$$= \exp\left(-\frac{2(2\cdot10^{-8}\,m)}{6.57\cdot10^{-9}\,m}\right)$$

$$= \boxed{2.27\cdot10^{-3}}$$

light has decreased by 99.8%