Lecture 11 - Carrier Flow (cont.)

September 28, 2001

Contents:

1. Majority-carrier type situations
2. Minority-carrier type situations

Reading assignment:

del Alamo, Ch. 5, §§5.5, 5.6 (5.6.1)
Key questions

• What characterizes *majority*-carrier type situations?
• What characterizes *minority*-carrier type situations?
Overview of simplified carrier flow formulations

- **General drift-diffusion situation** (Shockley's equations)
  - 1D approx.
  - **Quasi-neutral situation** (negligible volume charge)
  - **Space-charge situation** (field independent of n, p)
    - **Majority-carrier type situation** \((V \neq 0, n' = p' = 0)\)
    - **Minority-carrier type situation** \((V = 0, n' = p' \neq 0, LLI)\)
Simplified set of Shockley equations for 1D quasi-neutral situations

\[ p - n + N_D - N_A \simeq 0 \]

\[ J_e = -q n v_e^{drift} + q D_e \frac{\partial n}{\partial x} \]

\[ J_h = q p v_h^{drift} - q D_h \frac{\partial p}{\partial x} \]

\[ \frac{\partial n}{\partial t} = G_{ext} - U + \frac{1}{q} \frac{\partial J_e}{\partial x} \quad \text{or} \quad \frac{\partial p}{\partial t} = G_{ext} - U - \frac{1}{q} \frac{\partial J_h}{\partial x} \]

\[ \frac{\partial J_t}{\partial x} \simeq 0 \]

\[ J_t = J_e + J_h \]
1. Majority-carrier type situations

Voltage applied to extrinsic quasi-neutral semiconductor without upsetting the equilibrium carrier concentrations.

□ Remember what a battery does:

- Battery picks up electrons from positive terminal, increases their potential energy and puts them at the negative terminal.
- If provided with a path (resistor), electrons flow.
Characteristics of majority carrier-type situations:

- electric field imposed from outside
- electrons and holes drift
- electron and hole concentrations unperturbed from TE

Simplifications:

- neglect contribution of minority carriers
- neglect time derivatives of carrier concentrations

⇒ problem becomes completely quasi-static
Simplification of majority carrier current (n-type):

Must distinguish between internal field in TE ($E_o$) and total field outside equilibrium ($E$).

In equilibrium:

$$J_{eo} = -qn_0 v_e^{drift}(E_o) + qD_e \frac{dn_o}{dx} = 0$$

Out of equilibrium:

$$J_e \approx -qn_0 v_e^{drift}(E) + qD_e \frac{dn_o}{dx}$$

Hence:

$$J_e = -qn_0 [v_e^{drift}(E) - v_e^{drift}(E_o)]$$

For low fields,

$$J_e = qn_o \mu_e (E - E_o) = qn_o \mu_e E'$$
**Equation set for 1D majority-carrier type situations:**

<table>
<thead>
<tr>
<th>n-type</th>
<th>p-type</th>
</tr>
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<tbody>
<tr>
<td>( n \simeq n_0 \simeq N_D )</td>
<td>( p \simeq p_0 \simeq N_A )</td>
</tr>
<tr>
<td>( J_e = -q n_0 [v_{de}(E) - v_{de}(E_o)] )</td>
<td>( J_h = q p_0 [v_{dh}(E) - v_{dh}(E_o)] )</td>
</tr>
</tbody>
</table>

\[
\frac{dJ_e}{dx} \simeq 0, \quad \frac{dJ_h}{dx} \simeq 0, \quad \frac{dJ_t}{dx} \simeq 0
\]

\( J_t \simeq J_e \)

\( J_t \simeq J_h \)
Example 1: *Integrated Resistor* with uniform doping (n-type)

Uniform doping $\Rightarrow \mathcal{E}_o = 0$, then:

$$J_t = -qN_Dv_e^{drift}(\mathcal{E})$$

- If $\mathcal{E}$ not too high,

$$J_t \simeq qN_D\mu_e\mathcal{E}$$

I-V characteristics:

$$I = WtqN_D\mu_e\frac{V}{L}$$
• In general (low and high fields):

\[ I = W t q N_D \frac{v_{sat}}{1 + \frac{v_{sat}}{\mu_e} L/V} \]

which for high fields saturates to:

\[ I_{sat} = W t q N_D v_{sat} \]
2. Minority-carrier type situations

Situations characterized by:

- excess carriers over TE
- no external electric field applied (but small internal field generated by carrier injection: $\mathcal{E} = \mathcal{E}_o + \mathcal{E}'$)

Example: electron transport through base of npn BJT.

Two approximations:

1. $\mathcal{E}$ small $\Rightarrow |v^{drift}| \propto |\mathcal{E}|$

2. Low-level injection $\Rightarrow$ for n-type:
   - $n \simeq n_o$
   - $p \simeq p'$
   - $U \simeq \frac{p'}{\tau}$
   - negligible minority carrier drift due to $\mathcal{E}'$
     (but can’t say the same about majority carriers)
Shockley equations for 1D quasi-neutral situations

\[ \begin{align*}
    p - n + N_D - N_A & \simeq 0 \\
    J_e &= -qnv_e^{drift} + qD_e \frac{\partial n}{\partial x} \\
    J_h &= qpv_h^{drift} - qD_h \frac{\partial p}{\partial x} \\
    \frac{\partial n}{\partial t} &= G_{\text{ext}} - U + \frac{1}{q} \frac{\partial J_e}{\partial x} \quad \text{or} \quad \frac{\partial p}{\partial t} = G_{\text{ext}} - U - \frac{1}{q} \frac{\partial J_h}{\partial x} \\
    \frac{\partial J_t}{\partial x} &\simeq 0 \\
    J_t &= J_e + J_h
\end{align*} \]

\[  \Box \] Further simplifications for n-type minority-carrier-type situations

• Majority-carrier current equation:

\[ J_e \simeq q(n_o + n')\mu_e(\mathcal{E}_o + \mathcal{E}') + qD_e \left( \frac{\partial n_o}{\partial x} + \frac{\partial n'}{\partial x} \right) \]

but in TE:

\[ J_{eo} = qn_o\mu_e\mathcal{E}_o + qD_e \frac{\partial n_o}{\partial x} = 0 \]

Then:

\[ J_e \simeq qn_o\mu_e\mathcal{E}' + qn'\mu_e\mathcal{E}_o + qD_e \frac{\partial n'}{\partial x} \]
• Minority-carrier current equation:

\[ J_h \simeq q(p_o + p')\mu_h(\mathcal{E}_o + \mathcal{E}') - qD_h\left(\frac{\partial p_o}{\partial x} + \frac{\partial p'}{\partial x}\right) \]

In TE, \( J_{ho} = 0 \), and:

\[ J_h \simeq qp'\mu_h\mathcal{E}_o + qp'\mu_h\mathcal{E}' - qD_h\frac{\partial p'}{\partial x} \simeq qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x} \]

• Minority-carrier continuity equation:

\[ \frac{\partial p'}{\partial t} = G_{ext} - \frac{p'}{\tau} - \frac{1}{q} \frac{\partial J_h}{\partial x} \]

Now plug in \( J_h \) from above:

\[ D_h \frac{\partial^2 p'}{\partial x^2} - \mu_h\mathcal{E}_o\frac{\partial p'}{\partial x} - \frac{p'}{\tau} + G_{ext} = \frac{\partial p'}{\partial t} \]

One differential equation with one unknown: \( p' \).

If \( G_{ext} \) and BC’s are specified, problem can be solved.
### Shockley equations for 1D minority-carrier type situations

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<td>$p' \simeq n'$</td>
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<td>$J_e = qn_o \mu_e \mathcal{E}' + qn' \mu_e \mathcal{E}_o + qD_e \frac{\partial n'}{\partial x}$</td>
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<td>$D_h \frac{\partial^2 p'}{\partial x^2} - \mu_h \mathcal{E}<em>o \frac{\partial p'}{\partial x} - \frac{p'}{\tau} + G</em>{\text{ext}} = \frac{\partial p'}{\partial t}$</td>
<td>$D_e \frac{\partial^2 n'}{\partial x^2} + \mu_e \mathcal{E}<em>o \frac{\partial n'}{\partial x} - \frac{n'}{\tau} + G</em>{\text{ext}} = \frac{\partial n'}{\partial t}$</td>
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<tr>
<td>$\frac{\partial J_t}{\partial x} \simeq 0$</td>
<td>$J_t = J_e + J_h$</td>
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Example 1:

Diffusion and bulk recombination in a "long" bar

Uniform doping: $E_o = 0$; static conditions: $\frac{\partial}{\partial t} = 0$

Minority carrier profile:

Majority carrier profile? $n' = p'$ exactly?
Far away $J_t = 0 \Rightarrow J_t = 0$ everywhere.

$$J_t = J_e + J_h \simeq qn_o \mu_e \mathcal{E}' + q(D_e - D_h) \frac{dp'}{dx} = 0$$

If $D_e = D_h \Rightarrow$ diffusion term = 0
   $\Rightarrow$ drift term = 0
   $\Rightarrow \mathcal{E}' = 0$
   $\Rightarrow n' = p'$

But, typically $D_e > D_h \Rightarrow$ diffusion term < 0 (for $x > 0$)
   $\Rightarrow$ drift term > 0
   $\Rightarrow \mathcal{E}' > 0$ (for $x > 0$)
   $\Rightarrow$ and $\mathcal{E}' \propto D_e - D_h$
   $\Rightarrow n' \neq p'$ (but still $n' \simeq p'$)
   $\Rightarrow$ and $|n' - p'| \propto D_e - D_h$
The diagram illustrates the relationship between the carrier concentrations and the applied electric field in a semiconductor material. The top part of the diagram shows the absorption of light (hv) in a p-n junction, leading to an increase in electron (n') and hole (p') concentrations. The middle part of the diagram depicts the carrier concentration profiles for electron (n') and hole (p') as a function of distance (x), with the electron concentration dominating for p-n junctions.

The bottom part of the diagram focuses on the current density components, showing the drift (J_e) and diffusion (J_h) currents. The condition J_t = 0 everywhere ensures that the current density is conserved at any point along the x-axis.
Key conclusions

• *Majority carrier-type situations* characterized by application of external voltage without perturbing carrier concentrations.

• Majority-carrier type situations dominated by drift of majority carriers.

• Integrated resistor:
  
  – for low voltages, current proportional to voltage across
  
  – for high voltages, current saturates due to $v_{sat}$

• Minority-carrier type situations dominated by behavior of minority carriers: diffusion, recombination and drift.
Self-study

- Non-uniformly doped resistor
- Sheet resistance