Lecture 22 - The Si surface and the Metal-Oxide-Semiconductor Structure (cont.)

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1. Ideal MOS structure in thermal equilibrium (cont.)
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Reading assignment:

del Alamo, Ch. 8, §§8.2 (8.2.3, 8.2.4), 8.3 (8.3.1)
Key questions

• What happens if a voltage is applied to the metal with respect to the semiconductor in a MOS structure?

• How much voltage needs to be applied to bring the MOS structure to the onset of inversion?

• How does the inversion charge evolve with voltage?
1. Ideal MOS structure in thermal equilibrium

- **Inversion**

\[ Q_s = Q_d + Q_i \]
Do *sheet-charge approximation*: inversion layer much thinner than any other vertical dimensions of the problem.

Two consequences:

1. $\phi_s$ depends on $Q_d$ but is independent of $Q_i$ ($\phi$ does not change while crossing a sheet of charge):

   $$Q_d = -qN_Ax_d \simeq -\sqrt{2\epsilon_s qN_A\phi_s}$$

   [same relationship as in depletion]

2. $\phi_s$ in inversion rather insensitive to actual value of $W_M$:

   $$\phi_s \simeq \phi_{sth}$$

   $\phi_{sth}$ is *surface potential at threshold*
Rough estimate of $\phi_{sth}$: value that brings surface right at edge of inversion, that is, $n_o(0) = N_A$:

$$n_o(x = 0)|_{\text{threshold}} = \frac{n_i^2}{N_A} \exp \frac{q\phi_{sth}}{kT} = N_A$$

Then:

$$\phi_{sth} = 2kT \frac{q}{q} \ln \frac{N_A}{n_i} \equiv 2\phi_f$$

\[E_c\quad E_i\quad E_F\quad E_v\]
Within the *sheet-charge approximation*, electrostatic problem is easy to solve:

- In inversion, $\phi_s$ independent of $W_M \Rightarrow x_d$ independent of $W_M$:

  \[ x_d \approx x_{dmax} = \sqrt{\frac{2\epsilon_s\phi_{sth}}{qN_A}} \]

  Total depletion region charge:

  \[ Q_d \approx Q_{dmax} = -qN_Ax_{dmax} = -\sqrt{2\epsilon_s qN_A\phi_{sth}} \]

- From fundamental electrostatics relation in inversion:

  \[ \phi_{bi} = \phi_s - \frac{Q_s}{C_{ox}} = \phi_{sth} - \frac{Q_i + Q_{dmax}}{C_{ox}} \]

  Derive expression for $Q_i$ in inversion:

  \[ Q_i = -C_{ox}(\phi_{bi} - \phi_{sth}) - Q_{dmax} \]
\section*{Accumulation}

\[
Q_s = Q_a
\]
Sheet-charge approximation again:

\[ \phi_s \approx 0 \]

Use fundamental electrostatics equation again:

\[ Q_a \approx -C_{ox} \phi_{bi} \]

□ Flatband

If \( \phi_{bi} = 0 \) \( \Rightarrow Q_s = 0, E_s = 0, E_{ox} = 0, \phi_s = 0 \)

Perfect charge neutrality everywhere.
2. Ideal MOS structure outside equilibrium

- Apply voltage $V$ to metal with respect to semiconductor:

- $V < V_{FB}$: accumulation
- $V = V_{FB}$: flatband
- $V_{FB} < V < 0$: depletion
- $V = 0$: equilibrium
- $0 < V < V_{th}$: depletion
- $V = V_{th}$: threshold
- $V > V_{th}$: inversion
• MOS structure can swing all the way from accumulation to inversion.

• semiconductor remains in quasi-equilibrium (no carrier flow, no carrier injection)

\[ E_{fe} = E_{fh} = E_F \]

• electrostatics identical to TE but potential difference across structure changed:

\[ \phi_{bi} \rightarrow \phi_{bi} + V \]
Total potential build-up across structure:

\[ \phi_{bi} + V = \phi_s - \frac{Q_s}{C_{ox}} \]

Several important results:

- **Flatband voltage**: \( \phi_s = 0, Q_s = 0 \):
  \[ V_{FB} = -\phi_{bi} \]

- **Threshold voltage** for inversion: \( Q_i = 0, Q_d = Q_{dmax}, \phi_s = \phi_{sth} \):
  \[ V_{th} = -\phi_{bi} + \phi_{sth} - \frac{Q_{dmax}}{C_{ox}} = V_{FB} + \phi_{sth} + \gamma \sqrt{\phi_{sth}} \]
\[ V_{th} = -\phi_{bi} + \phi_{sth} - \frac{Q_{dmax}}{C_{ox}} = V_{FB} + \phi_{sth} + \gamma \sqrt{\phi_{sth}} \]

\[ V_t \] plays key role in MOSFET operation.

Key dependencies: \( N_A \uparrow \rightarrow V_t \uparrow \)

\[ x_{ox} \uparrow \rightarrow V_t \uparrow \]

- In inversion \((V > V_{th})\):

\[ Q_i = -C_{ox}(V - V_{th}) \quad \text{for } V \geq V_{th} \]

Once reached inversion, the inversion charge increases \textit{linearly} with the applied voltage in excess of \( V_{th} \).
Poisson-Boltzmann formulation

In uncompensated uniformly-doped p-type semiconductor:

\[ \rho = q(p - n - N_A) \]

Poisson equation:

\[ \frac{d^2 \phi}{dx^2} = -\frac{q}{\epsilon_s}(p - n - N_A) \]

Quasi-equilibrium prevails for electrons and holes:

\[ n = n_{oB} \exp \frac{q\phi}{kT} \approx \frac{n_i^2}{N_A} \exp \frac{q\phi}{kT} \]
\[ p = p_{oB} \exp -\frac{q\phi}{kT} \approx N_A \exp -\frac{q\phi}{kT} \]

Charge neutrality in bulk:

\[ p_{oB} - n_{oB} - N_A = 0 \]

All together - Poisson-Boltzmann equation:

\[ \frac{d^2 \phi}{dx^2} = -\frac{qN_A}{\epsilon_s} \left[ \exp -\frac{q\phi}{kT} - 1 \right] - \frac{n_i^2}{N_A^2} \left( \exp \frac{q\phi}{kT} - 1 \right) \]

Double integration of this equation leads to complete solution.
First integration tricky [see notes]:

\[ \frac{d\phi}{dx} = -\sqrt{\frac{2kT N_A}{\varepsilon_s}} F(\phi) \]

with:

\[ F(\phi) = \frac{\phi}{|\phi|} \left[ \left( \exp\left(-\frac{q\phi}{kT}\right) + \frac{q\phi}{kT} - 1 \right) + \frac{n_i^2}{N_A^2} \left( \exp\left(\frac{q\phi}{kT}\right) - \frac{q\phi}{kT} - 1 \right) \right]^{1/2} \]

Check signs:

- if \( \phi > 0 \), \( \frac{\phi}{|\phi|} = +1 \), and \( \frac{d\phi}{dx} < 0 \) (depletion and inversion)

- if \( \phi < 0 \), \( \frac{\phi}{|\phi|} = -1 \), \( \frac{d\phi}{dx} > 0 \) (accumulation)
Second integration from surface \((x = 0, \phi = \phi_s)\) towards bulk:

\[
\int_{\phi_s}^{\phi} \frac{d\phi}{F(\phi)} = -\sqrt{\frac{2kTN_A}{\epsilon_s}} x
\]

Complete solution, but..., in general, not analytical.
Even without second integration, we have interesting results:

- **Electric field:**
  \[
  \mathcal{E} = -\frac{d\phi}{dx} = \sqrt{\frac{2kTN_A}{\epsilon_s}} F(\phi)
  \]

- **Electric field at surface:**
  \[
  \mathcal{E}_s = \sqrt{\frac{2kTN_A}{\epsilon_s}} F(\phi_s)
  \]

- **Charge in semiconductor:**
  \[
  Q_s = -\epsilon_s \mathcal{E}_s = -\sqrt{2\epsilon_s kTN_A} F(\phi_s)
  \]

- **Relation between \( V \) and \( \phi_s \):**
  \[
  V = -\phi_{bi} + \phi_s + \frac{\sqrt{2\epsilon_s kTN_A}}{C_{ox}} F(\phi_s) = V_{FB} + \phi_s + \sqrt{\frac{kT}{q}} \gamma F(\phi_s)
  \]
Key conclusions

• In inversion, $\phi_s \simeq \phi_{sth}$, roughly independent of metal.

• In accumulation, $\phi_s \simeq 0$, roughly independent of metal.

• At Si/SiO$_2$ interface, semiconductor can swing all the way from accumulation to inversion by the application of a voltage.

• To first order, surface potential in inversion and accumulation does not change with $V$.

• **Flatband voltage**: voltage that produces flatband:

\[ V_{FB} = -\phi_{bi} \]

• **Threshold voltage**: voltage beyond which an inversion layer of electrons is formed:

\[ V_{th} = V_{FB} + 2\phi_f + \gamma \sqrt{2\phi_f} \]

• Beyond threshold, absolute inversion charge increases linearly with voltage:

\[ Q_i = -C_{ox}(V - V_{th}) \]
Self study

• Mathematics of Poisson-Boltzmann formulation