Lecture 27 - The "Long" Metal-Oxide-Semiconductor Field-Effect Transistor (cont.)

November 6, 2002

Contents:

1. Current-voltage characteristics of ideal MOSFET (cont.)

Reading assignment:

del Alamo, Ch. 9, §9.2

Announcements:

To comply with MIT rules regarding evening quizzes, there is no recitation tomorrow (as planned).
Key questions

• The potential of the inversion layer increases along the channel. This should change the \textit{local} threshold voltage. Does this affect the I-V characteristics of the MOSFET?

• What happens to MOSFET I-V characteristics if we apply a bias to the body with respect to the source?
1. Current-voltage characteristics of ideal MOSFET (cont.)

Introduce three significant refinements to model:

- Body effect (impact of $y$-dependence of $V_{th}$)
- Back bias (impact of $V_{BS}$)
- Channel length modulation (impact of $V_{DS} > V_{DSsat}$) [next time]
**Body effect**

In a MOSFET biased in linear or saturation regimes, channel voltage $V(y)$ depends on position:

- $\Rightarrow$ voltage difference between channel and body $V(y)$
- $\Rightarrow V_{th}(y)$ (increases along $y$)

Dependence of $V_{th}(y)$ further debiases transistor:

- $\Rightarrow I_D$ lower than ideal
- $\Rightarrow V_{DS_{sat}}$ lower than ideal
Voltage dependence of $V_{th}$:

$$V_{th}(V) = V_{tho} + \gamma(\sqrt{\phi_{sth} + V} - \sqrt{\phi_{sth}})$$

$V_{tho}$ is $V_{th}$ for $V_{SB} = 0$.

Charge control relation becomes:

$$Q_i = -C_{ox}(V_{GS} - V_{th}) = -C_{ox}[V_{GS} - V - V_{tho} - \gamma(\sqrt{\phi_{sth} + V} - \sqrt{\phi_{sth}})]$$

Insert into current equation:

$$J_e = \mu_e Q_i \frac{dV}{dy} = -\mu_e C_{ox}[V_{GS} - V - V_{tho} - \gamma(\sqrt{\phi_{sth} + V} - \sqrt{\phi_{sth}})] \frac{dV}{dy}$$

Integrate from $y = 0$ to $y = L \Rightarrow$ MOSFET current in linear regime:

$$I_D = \frac{W}{L} \mu_e C_{ox} \{ (V_{GS} - V_{tho} + \gamma \sqrt{\phi_{sth}} - \frac{1}{2} V_{DS}) V_{DS} - \frac{2}{3} \gamma [(\phi_{sth} + V_{DS})^{3/2} - (\phi_{sth})^{3/2}] \}$$

Note new terms multiplied by $\gamma \Rightarrow$ if $\gamma \to 0$, body effect $\to 0$. 
To get $V_{DS_{sat}}$, look at $Q_i$ at $y = L$:

$$Q_i(y = L) = -C_{ox}[V_{GS} - V_{DS_{sat}} - V_{tho} - \gamma(\sqrt{\phi_{sth}} + V_{DS_{sat}} - \sqrt{\phi_{sth}})] = 0$$

Solve for $V_{DS_{sat}}$:

$$V_{DS_{sat}} = V_{GS} - V_{tho} + \gamma\sqrt{\phi_{sth}} - \frac{\gamma^2}{2} \left[ \sqrt{1 + \frac{4}{\gamma^2}(V_{GS} - V_{FB})} - 1 \right]$$

MOSFET saturated current: plug $V_{Dssat}$ into current equation in linear regime:

$$I_{Dsat} = \frac{W}{L} \mu e C_{ox} \left\{ (V_{GS} - V_{tho} + \gamma\sqrt{\phi_{sth}} - \frac{1}{2}V_{DS_{sat}})V_{DS_{sat}} \right. \left. - \frac{2}{3}\gamma[\left( \phi_{sth} + V_{DS_{sat}} \right)^{3/2} - (\phi_{sth})^{3/2}] \right\}$$
Three noticeable features:

- for all values of $V_{GS}$ and $V_{DS}$, body effect reduces $I_D$
- for given $V_{GS}$, body effect reduces $V_{DS_{sat}}$
- body effect goes away as transistor is turned off
Key observations for model simplification:

- $V_{DS_{sat}}$ dependence on $V_{GS}$ remains roughly linear:

- $I_{D_{sat}}$ dependence on $V_{GS}$ remains roughly quadratic:
Linearize dependence of $V_{th}$ on $V$ ($V \ll \phi_{sth}$):

$$V_{th}(V) = V_{tho} + \gamma(\sqrt{\phi_{sth}} + V - \sqrt{\phi_{sth}}) \simeq V_{tho} + \frac{\gamma}{2\sqrt{\phi_{sth}}} V$$

Solve again differential equation to get MOSFET current in linear regime:

$$I_D \simeq \frac{W}{L} \mu_e C_{ox}(V_{GS} - V_{tho} - \frac{m}{2} V_{DS}) V_{DS}$$

with:

$$m = 1 + \frac{\gamma}{2\sqrt{\phi_{sth}}} > 1$$

$V_{DS_{sat}}$ becomes:

$$V_{DS_{sat}} \simeq \frac{1}{m}(V_{GS} - V_{tho})$$

Current in saturation regime:

$$I_{Ds} \simeq \frac{W}{2mL} \mu_e C_{ox}(V_{GS} - V_{tho})^2$$
$m$ is body-effect coefficient ($m > 1$):

$$m = 1 + \frac{\gamma}{2\sqrt{\phi_{sth}}}$$

$m$ has same dependences as $\gamma$:

- $x_{ox} \downarrow \Rightarrow \gamma \downarrow \Rightarrow m \downarrow$ (less severe body effect)
- $N_A \uparrow \Rightarrow \gamma \uparrow \Rightarrow m \uparrow$ (more severe body effect)

$m$ and $\gamma$ represent relative electrostatic influence of gate and body on inversion layer; if $\gamma = 0 \rightarrow m = 1$ (negligible impact of body).

In circuit CAD, $m$ used as fitting parameter. Typically $m \sim 1.1 - 1.4$. 
☐ Back bias

If bias applied to body with respect to source \((V_{SB} > 0)\):

\[\Rightarrow V_{th} \text{ shifts positive}\]

\[\Rightarrow \text{for constant } V_{GS} \text{ and } V_{DS}, \quad I_D \text{ reduced}\]

Model in absence of body effect \(\Rightarrow\) just replace \(V_{th}\) in first order model by:

\[
V_{th}(V_{SB}) = V_{tho} + \gamma(\sqrt{\phi_{sth}} + V_{SB} - \sqrt{\phi_{sth}})
\]
\begin{align*}
\text{\textbf{Lecture 27-13}}
\end{align*}
**I-V characteristics of n-channel MOSFET** \((L = 1.5 \, \mu m)\)

- Output characteristics \((V_{GS} = 0 - 3 \, V, \Delta V_{GS} = 0.5 \, V)\):
Transfer characteristics ($V_{DS} = 4 \text{ V}$):
Output characteristics vs. back bias ($V_{SB} = 0, \ 2 \text{ V}$):
Transfer characteristics vs. back bias ($V_{DS} = 4 \, V$, $V_{SB} = 0 - 2 \, V$, $\Delta V_{SB} = 0.5 \, V$):
Backgate output characteristics ($V_{SB} = 0 - 3 \, V$ in $0.5 \, V$ increments, $V_{GS} = 1.5 \, V$):
Key conclusions

• "Body effect" arises from spatial dependence of $V_{th}$: local gate overdrive reduced.

• Main consequences of body effect:
  - $I_D$ lower than ideal,
  - $V_{DS_{sat}}$ lower than ideal.

• Simple formulation of body effect is fairly accurate:

$$I_{D_{sat}} \approx \frac{W}{2mL} \mu C_{ox} (V_{GS} - V_{tho})^2$$

with $m \geq 1$.

• $m$ captures relative electrostatic influence of gate and body (want $m \rightarrow 1$).

• Application of back bias shifts $V_{th}$ positive and reduces $I_D$. 