

LIBRARY
OF THE
MASSACHUSETTS INSTITUTE
OF TECHNOLOGY

WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

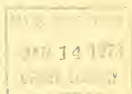
Alternative Markup Decision Rules and
Their Profit Implications

612-72
September, 1972

Philippe A. Naert**

MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139





Alternative Markup Decision Rules and
Their Profit Implications

612-72
September, 1972

Philippe A. Naert*+

* Assistant Professor of Management Science, Alfred P. Sloan School of Management, Massachusetts Institute of Technology (M.I.T.)

+ The author is indebted to Alain Bultez for helpful suggestions and comments.

HD28
.M414
no. 612-72

RECEIVED
JAN 18 1973
M. I. T. LIBRARIES

Abstract

In studies of a company's marketing mix, markup decisions have not received much attention. A meaningful analysis of such decisions requires knowledge of the channel of distribution through which a product flows before reaching the final consumer, and of the distribution of power among various vertical levels in the channel. In this paper we will examine profit implications of alternative markup decision rules. The general ideas will be illustrated with examples.

I. Introduction

Markup has been defined as the difference between price of a product to a consumer (retail price) and cost of the product to the retailer (factory price). Markup is then measured in monetary units. Alternatively, markup is often expressed as a percentage either of cost, or of sales price. In this paper several of these definitions will be used. Which one applies will be clear from the context.

Most studies of markup decisions have centered on the retail level with little or no consideration given to other parts of the vertical market structure, in particular the firms from whom the retailers buy.¹

On the other hand, studies of marketing programs for producers show little interest in the economic impact of intermediaries through whom their products are sold, and markup is thus in general not given much attention in marketing decision models. In this paper markup decisions will be examined from both the producer's and the retailers' point of view. We will consider a market consisting of a monopolistic producer, who sells to independent intermediaries, who in turn sell to final consumers.

In Section II we will study the case where markup decisions are made by retailers. We will review the commonly believed rationale behind constant percentage markup pricing, which is based on its equivalence with marginalist pricing under the assumptions of constancy of marginal cost and of price elasticity.² We will provide a more general rationale by showing that constant percentage markup pricing may lead to higher retailer profits. We will also derive optimality conditions for the producer, and analyze the effect on these of ignoring the existence of intermediaries.

In Section III we will examine the situation where the producer makes markup decisions.

II. Markup Decisions Are Made By The Retailers

Let us first review how constant percentage markup pricing has traditionally been rationalized.

The individual retailer, i say, faces a demand curve

$$q_i = f_i(p),$$

where q_i is quantity sold by retailer i , and p is retail price. With c_{Ri} equal to average variable cost per unit, the i^{th} retailer's profit, π_{Ri} , is ³

$$\pi_{Ri} = (p - c_{Ri}) q_i,$$

which is maximum for ⁴

$$\frac{d\pi_{Ri}}{dp} = q_i + p \frac{dq_i}{dp} - q_i \frac{dc_{Ri}}{dq_i} \frac{dq_i}{dp} - c_{Ri} \frac{dq_i}{dp} = 0$$

Dividing through by dq_i/dp , and rearranging terms gives

$$c_{Ri} + q_i \frac{dc_{Ri}}{dq_i} = p \left(1 + \frac{1}{\eta_{pi}} \right), \quad (1)$$

where η_{pi} = price elasticity = $\frac{dq_i/q_i}{dp/p}$.

Assume now that (i) average cost is constant, and (ii) price elasticity is constant. Average cost constant implies $dc_{Ri}/dq_i = 0$. With markup, m_i , defined as price minus average cost, we have at optimality

$$m_i = p - c_{Ri} = (-1/\eta_{pi})p. \quad (2)$$

Given a constant price elasticity, markup is then a constant percentage of price. The less elastic the demand the larger the constant will be.

Traditionally one then argues that constant percentage markup pricing makes economic sense because it is equivalent to marginalist pricing if average cost and price elasticity are constant [6] [7]. There is evidence that for some kinds of retailers, supermarkets for example, constancy of unit cost over fairly wide quantity ranges is typical [7]. In making the constant elasticity assumption, the key phrase may be "in the relevant range," that is, although elasticity may not be constant over the whole demand curve, assuming constancy over a certain price range may well be a reasonable approximation.

What if these assumptions are isolated? A meaningful investigation of the problem will require taking into account the market relationship between producer and retailers.

The following market structure will be considered: A monopolistic producer who sells to a group of independent retailers, who in turn sell to the final consumers. The producer's decision variable is p_R , price charged to the retailers. Other decision variables such as advertising and promotion will not be considered here, because the main points of this section can be made without these.

We will assume that all retailers face the same demand curve, in which case price elasticity $\eta_{pi} = \eta_p$ for all i , and price elasticity for the industry is also η_p . We will also assume that the only variable cost incurred by retailers is the price they pay to the producer, p_R (hereafter called factory price). That is retailers' average cost c_R is equal to p_R .

Retailers buy from the producer at price p_R , to which a given markup policy (specific policies to be considered later) is applied that transforms it into the retail price p .⁵ Consumer demand q , that

is, the demand faced by all retailers combined, will be a function of price charged by the retailers.⁶

$$q = q(p)$$

The producer's profit function is then

$$\pi_M = p_R q(p) - c(q) q(p)$$

The profit maximizing producer will set p_R such as to make $d\pi_M/dp_R$ equal zero.

$$\frac{d\pi_M}{dp_R} = q + p_R \frac{dq}{dp} \frac{dp}{dp_R} - c \frac{dq}{dp} \frac{dp}{dp_R} - q \frac{dc}{dq} \frac{dq}{dp} \frac{dp}{dp_R} = 0$$

Dividing every term by $\frac{dq}{dp} \frac{dp}{dp_R}$ gives

$$\frac{q}{\frac{dq}{dp} \frac{dp}{dp_R}} + p_R - c - q \frac{dc}{dq} = 0$$

Letting $MC = \text{marginal cost} = c + q \frac{dc}{dq}$, we can write

$$MC = p_R \left(1 + \frac{1}{\frac{p_R}{q} \frac{dq}{dp} \frac{dp}{dp_R}} \right) \quad (3)$$

with $w_M = (p_R - MC)/p_R = \text{percentage of gross margin}$, and

$$\eta_{p_R} = \frac{p_R}{q} \frac{dq}{dp} \frac{dp}{dp_R}, \quad \text{equation (3)}$$

reduces to

$$w_M = -1/\eta_{p_R} \rho_p = -1/\eta_p \rho_p \quad (4)$$

where $\rho_p = \frac{dp}{dp_R} \frac{p}{p} = \text{elasticity of retail price with respect to}$

factory price. An alternative way of writing equation (4) is

$$p_R = MC \left(\frac{\eta_p \rho_p}{1 + \eta_p \rho_p} \right) \quad (5)$$

Writing η_{p_R} as $\eta_p \rho_p$ allows us to incorporate markup policy, as expressed by ρ_p explicitly.

Let us now turn to the retail level. Two markup policies will be considered: Simple marginalist pricing and constant markup pricing.

1. Simple Marginalist Pricing

With simple marginalist pricing, the profit function for the retailers is

$$\pi_R = (p - p_R) q(p) \quad (6)$$

The retailers select p such as to maximize profit. That is,

$$\frac{d\pi_R}{dp} = (p - p_R) \frac{dq}{dp} + q = 0, \quad (7)$$

which reduces to

$$p = \left(\frac{\eta_p}{1 + \eta_p} \right) p_R. \quad (8)$$

Markup m is then

$$m = \left(\frac{-1}{1 + \eta_p} \right) p_R = \left(\frac{-1}{\eta_p} \right) p \quad (9)$$

If η_p is constant, it follows from (9) that markup is a constant percentage of price. Note that no assumptions have to be made with regard to the producer's marginal cost. It remains to be seen whether the constant itself will be the same if we derive optimality conditions when retailers use a constant percentage markup rule. Before doing this we may observe that with η_p constant, $dp/dp_R = \text{constant} = \eta_p / (1 + \eta_p)$, and ρ_p is equal to one. Factory price is then equal to

$$p_R = MC \left(\frac{\eta_p}{1 + \eta_p} \right)$$

and

$$p = MC \left(\frac{\eta_p}{1+\eta_p} \right)$$

2. Constant Percentage Markup Pricing

Suppose now instead that a constant percentage markup policy is adopted. Multiplying factory price by a constant b then determines retail price.

$$p = bp_R \quad (10)$$

The problem for the retailers is to find that value of b that maximizes profit. Retailers' profit will now be π'_R

$$\begin{aligned} \pi'_R &= (p - p_R) q(p) \\ \pi'_R &= (b-1) p_R q(p) \end{aligned} \quad (11)$$

The first order condition for maximization is

$$\frac{d\pi'_R}{db} = p_R q + (b-1)q \frac{dp_R}{db} + (b-1)p_R \frac{dq}{dp} (p_R + b \frac{dp_R}{db}) = 0 \quad (12)$$

With $\rho_b = \frac{dp_R}{db} \frac{b}{p_R}$, equation (12) reduces to

$$b = \frac{\eta_p + \rho_b (1+\eta_p)}{(1+\eta_p) (1+\rho_b)} \quad (13)$$

We know that ρ_b is negative. But it should also be greater than -1 since for ρ_b equal to minus one there is a discontinuity in b .

Equation (5) gives the optimal value for p_R . The markup rule used by the retailers intervenes through the retail price - factory price elasticity. With a constant percent markup rule $dp/dp_R = b$ and ρ_p equals 1. Equation (5) reduces to

$$p_R = MC \eta_p / (1+\eta_p), \quad (14)$$

Suppose now that η_p is constant. If in addition MC , the producer's marginal cost, is constant, factory price will not depend on b . With p_R independent of b , ρ_b is zero and

$$b = \eta_p / (1+\eta_p),$$

in which case we obtain the same result as under simple marginalist retail pricing. Below we will discuss some of the implications of the results obtained so far. These implications will be illustrated with a series of examples.

3. Constant Elasticity Case

We will study the constant price elasticity case with an example.

Example 1:

Consider the following demand curve

$$q = kp^\alpha$$

Price elasticity η_p is constant and equal to α . First assume that marginal cost is constant and equal to c .

i) MC = c

With marginalist pricing we have

$$\eta_p / (1 + \eta_p) = \alpha / (1 + \alpha) = \text{constant, and from (8)}$$

$$P = \frac{\alpha}{1 + \alpha} P_R$$

Since $\alpha / (1 + \alpha)$ is constant, ρ_p is equal to one and from (5)

$$P_R = c\alpha / (1 + \alpha)$$

With constant percentage markup pricing we start with the equation for p_R (14)

$$p_R = c\alpha / (1 + \alpha)$$

Since c is a constant, p_R' is not a function of b , and hence ρ_b is zero. From (13) we then find,

$$b = \alpha / (1 + \alpha),$$

and single marginalist pricing and optimal constant percentage markup pricing lead to the same results.

ii) MC = c + dq

Simple marginalist pricing again leads to

$$P = \frac{\alpha}{1 + \alpha} P_R,$$

with p_R the solution to

$$p_R = (c+dq) \alpha / (1+\alpha).$$

With constant percentage markup pricing equation (14) is

$$p_R = (c + dq) \alpha / (1+\alpha).$$

q as a function of p_R depends on b , and hence $d p_R / db$ is not zero. The average revenue curve for the producer is

$$q = k b^\alpha p_R^\alpha$$

Optimal factory price p_R is then the solution to

$$c + d k b^\alpha p_R^\alpha - p_R \frac{(1+\alpha)}{\alpha} = 0.$$

Optimal p_R will be smaller for larger values of b . ρ_b is then negative and

$$b = \frac{\alpha + \rho_b (1+\alpha)}{(1+\alpha) (1+\rho_b)} > \frac{\alpha}{1+\alpha}.$$

This shows that when price elasticity is constant but the producer's marginal cost is not constant, simple marginalist pricing and constant percentage markup pricing produce different results. Both lead to constant retail price-factory price ratios, but the constants themselves are different.

4. Intermediaries Assumed Away

The existence of intermediaries is often assumed away (implicitly) in economics and also in applications of marginal analysis in marketing. Yet somehow intermediaries still enter the picture since it is usually made clear that percentage of gross margin is based on factory price and not on retail price. However, the right hand side of equation (4) is written $-1/\eta_p$ rather than $-1/\eta_{p_R}$.⁷ How does this affect the optimal value for p_R ?

First of all let us examine under what conditions η_p and η_{p_R} are equal. From equation (4) we know that this will be so if $\rho_p = 1$, that is for

$$p = bp_R$$

where b is a constant. From what we have seen before, b is constant by definition in the case of constant percentage markup pricing. With η_p constant (an assumption often made in marketing applications), simple marginalist pricing also entails $\rho_p = 1$. Furthermore in many of these studies marginal cost is assumed constant (see for example [3]), so that simple marginalist pricing or constant percentage pricing by retailers will lead to the same ratio p/p_R .

Suppose now that η_p is not constant, and that retailers use simple marginalist pricing. With η_p not constant, ρ_p is not equal to one and η_p is not equal to η_{p_R} . Ignoring the existence of intermediaries means that the producer uses $w_M = -1/\eta_p$ as optimality condition rather than $w_N = -1/\eta_p \rho_p$. An example will be used to study the impact on the producer's profit performance.

Example 2:

Let AR_R be the demand curve faced by the retailers (see Figure 1). The quantity demanded by the retailers is given by the intersection of factory price (retailers' marginal cost) and the retailers' marginal revenue curve MR_R . So in making their ordering decisions the retailers move along their marginal revenue curve, which therefore becomes the producer's average revenue curve, i.e. $MR_R = AR_M$. The producer maximizes his profit by equating marginal cost MC_M to marginal revenue MR_M . Referring again to Figure 1, the producer will quote a price P_R , the retailers will buy q^* units and will sell at a price p . The producer and retailers will make a profit of $ABCD$ and $ABC'D'$ respectively.

The retailers would make a larger profit if they could sell more than q^* , since markup increases in proportion to q . If the retailers are powerful in comparison to the producer they may be able to force the producer to sell a quantity larger than q^* . The producer on the other hand may try to prevent such pressure by applying resale price maintenance (fair trade pricing). Where resale price maintenance is legal the producer can put a lower bound on retail price, for example, OD' , the price corresponding to q^* . With a minimum price of OD' , retailers would not benefit by charging any price other than OD' , assuming of course that the minimum price can be properly enforced. Whether or not enforcement is possible will depend to a large extent on the relative power of the producer and the retailers [8].

The situation presented in Figure 1 will now be studied in more detail. Average revenue curve for the retailers is:

$$p = k - aq, \quad (15)$$

where k and a are constants.⁸ Total revenue for the retailers is

$$TR_R = pq = kq - aq^2.$$

Marginal revenue MR_R is then

$$MR_R = dTR_R/dq = k - 2aq,$$

and since $MR_R = AR_M = p_R$

$$p_R = k - 2aq \quad (16)$$

Solving (15) and (16) for q , the following relationship between p and p_R is obtained

$$p = (p_R + k)/2.$$

Markup m is then

$$m = (k - p_R)/2$$

At optimality we should have $w_M = -1/\eta_p^0 p$. In this example

$$\eta_p = -(k - aq)/aq,$$

$$\text{and } \rho_p = \frac{dp}{d p_R} \cdot \frac{p_R}{p} = \frac{1}{2} \frac{k-2aq}{k-aq} = \frac{1}{2} \frac{1 + \eta_p}{\eta_p}.$$

We then have $\eta_{p_R} = \frac{1}{2} \eta_p + \frac{1}{2}$, and the optimality condition becomes

$$w_M = -1 / \left(\frac{1}{2} \eta_p + \frac{1}{2} \right) = -1 / \left(\eta_p + \frac{k}{2qa} \right)$$

By ignoring the existence of intermediaries and using $w_M = -1/\eta_p$ as optimality condition, we will obtain a price p_R which, in fact, lies below its true optimal value. To clarify this somewhat it can be shown that using η_p rather than η_{p_R} is equivalent to the producer considering MR'_M as his marginal revenue curve (see Figure 1). This assumed marginal revenue (hereafter called quasi marginal revenue) is

$$MR'_M = p_R \left(1 + \frac{1}{\eta_p} \right), \text{ or}$$

$$MR'_M = (k-2aq) \left(1 + \frac{1}{-(k-aq)/aq} \right),$$

which reduces to

$$MR'_M = k-3aq + (aq)^2/(k-aq).$$

The correct marginal revenue curve is

$$MR_M = d TR_M/dq = k-4aq.$$

The quasi-optimal solution for the producer is obtained by equating MR'_M and MC_M . The producer's price will be p'_R , he will sell q^0 units, and retail price will be p' . The prices p'_R and p' fall below p_R and p respectively. Observe that the producer makes less profit ($A'B'C''D''$ versus an optimal value of $ABCD$), whereas the retailers' profit increases (from $ABC'D'$ to $A'B'C''D''$).

5. Constant Percentage Markup Pricing Versus Simple Marginalist

Pricing

At this point we want to examine why retailers might prefer to use constant percentage markup pricing over simple marginalist pricing in cases where they do not give equivalent results. Let π_R^* be the maximum possible profit under marginalist pricing, and $\pi_R^{*'} under constant percentage markup pricing.$

$$\pi_R^* = (p^* - p_R^*) q(p^*)$$

Using equation (5) and (8), π_R^* becomes

$$\pi_R^* = \frac{-MC^* \eta_{p^*} \rho_{p^*} q(p^*)}{(1+\eta_{p^*}) (1+\eta_{p^*} \rho_{p^*})} , \quad (15)$$

$$p^* = \frac{\eta_{p^*}^2 \rho_{p^*} MC^*}{(1+\eta_{p^*}) (1+\eta_{p^*} \rho_{p^*})} \quad (16)$$

Similarly

$$\pi_R^{*'} = (b^*-1) p_R^{*'} q(p^{*'})$$

Using equations (13) and (14), $\pi_R^{*'}$ becomes

$$\pi_R^{*'} = \frac{-MC^{*'} \eta_{p^{*'}} q(p^{*'})}{(1+\eta_{p^{*'}})^2 (1+\rho_{b^*})} , \quad (17)$$

where

$$p^{*' } = \frac{[\eta_{p^{*'}}^2 + \rho_{b^*} \eta_{p^{*'}} (1+\eta_{p^{*'}})] MC^{*' }}{(1+\eta_{p^{*'}})^2 (1+\rho_{b^*})} . \quad (18)$$

Retailers will prefer constant percentage markup pricing to marginalist pricing if $\pi_R^{*' } > \pi_R^*$. A direct comparison of $\pi_R^{*'}$ and π_R^* is difficult since p^* and $p^{*'}$ will in general not be equal. Example three will illustrate a case where $\pi_R^{*'}$ is larger than π_R^* .

The reader may be somewhat puzzled at this point to find that a pricing rule other than a marginalist one leads to maximum profit. There

is no need to worry, however. We can look at the constant percentage markup pricing as a marginalist rule where the retailer takes into account how the producer will react to his pricing decision.⁹

With $\pi_R = (p-p_R)q(p)$, and taking into account the producer's reaction, the first order optimality condition becomes,

$$\frac{d\pi_R}{dp} = (p-p_R) \frac{dq}{dp} + q(1 - \frac{dp_R}{dp}) = 0 \quad (19)$$

Dividing each term by dq/dp and letting $\rho_{P_R, P} = \frac{p}{P_R} \frac{dp_R}{dp}$, (19) simplifies to

$$(p-p_R) + \frac{p}{\eta_p} - P_R \frac{\rho_{P_R, P}}{\eta_p} = 0.$$

Solving for p , we find

$$p = \left(\frac{\eta_p + \rho_{P_R, P}}{1 + \eta_p} \right) P_R, \quad (20)$$

as compared to $p = \left(\frac{\eta_p}{1 + \eta_p} \right) P_R$, in the simple marginalist case.

It remains to be shown that the coefficient of P_R in (20) is equal to the value of b obtained in (13). That is, we must show that

$$\frac{\eta_p + \rho_{P_R, P}}{1 + \eta_p} = \frac{\eta_p + \rho_b (1 + \eta_p)}{(1 + \eta_p) (1 + \rho_b)},$$

or that

$$\rho_{P_R, P} = \frac{\rho_b}{1 + \rho_b}. \quad (21)$$

The total differential of p is

$$dp = bdp_R + p_R db. \quad (22)$$

Using (22) and $p/p_R = b$, we can write

$$\rho_{p_R, p} = \frac{bdp_R}{bdp_R + p_R db}.$$

Dividing numerator and denominator by db , it is readily seen that (21) holds.

Example 3:

Consider the same final demand curve as in Example 2,

$$p = k - aq.$$

With constant percentage markup pricing $p = bp_R$, and the producer's demand curve is

$$p_R = (k - aq)/b$$

The producer's marginal cost curve is

$$MC_M = c + dq,$$

and average cost

$$AC_M = c + (d/2)q.$$

The range of relevant values for b is

$$1 \leq b \leq k/c,$$

since for b less than one markup is negative, and for b larger than k/c the demand curve faced by the producer lies below his average cost curve.

Total revenue TR'_M is

$$TR'_M = (kq - aq^2)/b,$$

and corresponding marginal revenue

$$MR'_M = (k - 2aq)/b$$

For a given value of b , producer profit π'_M will be maximum when

$$q' = (k - bc)/(2a + bd) \quad (23)$$

Maximum profit is then

$$\begin{aligned}\pi'_M &= (p'_R - AC'_M) q' \\ &= \left\{ \frac{k}{b} - \frac{a(k-bc)}{b(2a+bd)} - \left[c + \frac{d(k-bc)}{2(2a+bd)} \right] \right\} \left(\frac{k-bc}{2a+bd} \right) \\ \pi'_M &= \frac{(k-bc)^2}{2b(2a+bd)}\end{aligned}\quad (24)$$

When $b = b^*$, the optimal value for b , π'_M will be equal π'^*_M . Retailer

profit is $\pi'_R = (b-1) p'_R q'$

$$\begin{aligned}&= (b-1) \left(\frac{k}{b} - \frac{a(k-bc)}{b(2a+bd)} \right) \left(\frac{k-bc}{2a+bd} \right) \\ \pi'_R &= \frac{(b-1)(ak+kbd+abc)(k-bc)}{b(2a+bd)^2}\end{aligned}\quad (25)$$

To obtain the optimal value of b we set the derivative of π'_R with respect to b equal to zero, and solve for a value of b between 1 and k/c . Alternatively we can find b^* directly by solving equation (13). With

$$\eta_p = -(ak + kbd + abc)/a(k-bc)$$

$$1 + \rho_b = ab(2ac+dk)/(2a+bd)(ak+kbd+abc)$$

Substituting these two expressions into equation (13) we find

$$\begin{aligned}[-3\{cd^2k+ac^2d + 4acdk + 4a^2c^2 + d^2k^2\} + 2b^2\{k^2d^2 \\ + a^2c^2 + acdk\} + 3b\{adk^2\} + 2a^2k^2]/b^2(2a+bd)^4 = 0\end{aligned}\quad (26)$$

Here we are only interested in a root which lies between 1 and k/c , and which negative second derivative. A numerical example will be studied below.

Let us now compare the constant percentage markup case with simple marginalist pricing. With the final demand curve equal to $p = k-aq$, the marginal revenue curve for the retailers is $MR_R = k-2aq$, which in turn becomes the demand curve faced by the producer, that is,

$p_R = k-2aq$. The producer's marginal revenue curve is $MR_M = k-4aq$. The optimal value of q is obtained from equating MC_M and MR_M , that is

$$k-4aq = c+dq$$

$$\text{and} \quad q^* = (k-c)/(4a+d) \quad (27)$$

Maximum profit for the producer is

$$\begin{aligned} \pi_M^* &= (p_B^* - AC^*) q^* \\ &= \left\{ \frac{k-2a(k-c)}{4a+d} - \left[c + \frac{d}{2} \frac{(k-c)}{(4a+d)} \right] \right\} \frac{(k-c)}{4a+d} \\ \pi_M^* &= \frac{(k-c)^2}{2(4a+d)} \end{aligned} \quad (28)$$

Corresponding profit for the retailers is then

$$\begin{aligned} \pi_R^* &= (p^* - p_R^*) q^* \\ &= \left\{ k - \frac{a(k-c)}{4a+d} - \left[k - \frac{2a(k-c)}{4a+d} \right] \right\} \frac{(k-c)}{4a+d} \\ \pi_R^* &= \frac{a(k-c)^2}{(4a+d)^2} \end{aligned} \quad (29)$$

Figure 2 shows a comparison of the simple marginalist pricing policy and the constant percentage markup policy for a given value of b .

At this point it may be instructive to consider a numerical example. Let $k=1,100$, $a=1$, $c=100$, and $d=1$. With simple marginalist pricing we have $\pi_M^* = \$100,000$ and $\pi_R^* = \$40,000$. Total profit in the market π_T^* , is then $\$140,000$.

Figure 3 shows π_M^* , π_R^* , π_T^* and π_M' , π_R' and π_T' for various values of b . The first order derivative of retailers profit with respect to b reduces to

$$(-181b^3 + 266b^2 + 363b + 242)/b^2(2+b)^4 = 0$$

The derivative is positive for the lower bound $b = 1$, and is negative for $b/c = 11$. The derivative is zero for only one value of b in the interval

$1 < b < 11$. This is the optimum value $b^* \approx 2.490$, or a markup of 149 percent over cost. With this markup the retailers' profit amounts to \$103,261 versus \$40,000 under marginalist pricing. Clearly then, by choosing the optimum constant percentage markup retailers can substantially increase their profit as compared to the simple marginalist solution. On the other hand, the producer is worse off than under simple marginalist pricing. He now makes \$32,375 rather than \$100,000. So the producer will probably not sit idle, but will try to prevent retailers from charging a markup of 149 percent over cost. How can he achieve this? Well, first of all let us consider an extreme case. The producer could decide to sell through wholly owned intermediaries. In that case the producer has the power to decide on both price and quantity. The final demand curve becomes the producer's demand curve and his marginal revenue curve becomes MR_R in which case quantity produced becomes q^+ (see Figure 2a). Technically, this is equivalent to retailers adding a zero percentage markup. The producer then makes a profit of \$166,667, and the retailers make no profit.¹⁰ In order to realize this profit, however, the producer has to gain full control over the intermediaries.¹¹ In many cases, achieving control over intermediaries may not be worth the cost or may not even be possible. For example, if the product is one normally sold in foodstores where thousands of other products are sold it may simply be impossible for the producer to try to gain full control.

What other possible courses of action does the producer have? Looking back at Figure 3, we see that constant percentage markup pricing yields more profit than simple marginalist pricing for both producer and retailers for values of markup between 20 and 38 percent. One possible way by which the producer could possibly force retailers to set their

price within that range, is by putting a maximum price on the product. A minimum price is not needed here, since optimally the retailers would add a markup higher than 38 percent. Instead of putting a maximum price on his product, the producer could also put a "manufacturer's suggested retail price" on the product. Retailers will then charge a price equal to or below that suggested retail price, since charging a higher price than the one suggested on the package would be resented by the consumers.

Observe also that for all values of b between 1.0 and 2.31 total profit in the market is larger with constant percentage markup pricing than with simple marginalist pricing. So the possibility exists to improve profit performance for both producer and retailers through some kind of negotiated profit agreement.

In this section we have compared constant percentage markup pricing with simple marginalist pricing in the case where markup decisions are made by retailers. At the end of section II.3 we have seen that the producer will often try to exercise indirect control over pricing by putting maximum or suggested prices on his product. In that sense we can no longer say that retailers have full power to make markup decisions. The logical next step is to consider the case where a producer decides on both the price at which he sells to the retailers and on the price paid by the ultimate consumer.

III. Markup Decisions are made by the Producer

Consider the following market structure: One producer, a group of retailers, and final consumers. The decision variables for the producer are price, advertising, and markup. Demand will be a function of price, advertising and product availability. A consumer cannot purchase a

product unless it is available. Sales volume will therefore be positively related to product availability. The higher markup offered by the producer, the more retailers will be found interested in selling his product. The number of retailers, which will be taken here as a proxy measure of product availability, will thus be a function of markup. Advertising may be used by the producer to put indirect pressure on the retailers to provide shelf space for his product. The number of retailers will also depend on retail price, since other things being equal, a higher price implies a lower potential sales volume. The number of retailers will thus be a function of markup, advertising, and price.¹²

Let d = the number of retailers

$$n_i = \frac{\partial q}{\partial i} \frac{i}{q} \text{ for } i = p, s, d$$

$$n_{dj} = \frac{\partial d}{\partial j} \frac{i}{q} \text{ for } j = p, s, m$$

$$w^0 = (p-m-MC)/p,$$

and let other variables be defined as in section II. The producer's profit function is

$$\pi_M = (p-c-m) q (p, s, d(p, s, m)) - s \quad (30)$$

For the profit maximizing producer, optimal values for price, advertising, and markup are obtained by setting $\partial \pi_M / \partial p = \partial \pi_M / \partial s = \partial \pi_M / \partial m = 0$

$$\begin{aligned} \frac{\partial \pi_M}{\partial p} = q - q \frac{\partial c}{\partial q} \left(\frac{\partial q}{\partial p} + \frac{\partial q}{\partial d} \frac{\partial d}{\partial p} \right) \\ + (p-c-m) \left(\frac{\partial q}{\partial p} + \frac{\partial q}{\partial d} \frac{\partial d}{\partial p} \right) = 0 \end{aligned}$$

Dividing through by $\left(\frac{\partial q}{\partial p} + \frac{\partial q}{\partial d} \frac{\partial d}{\partial p} \right)$, and rearranging terms we obtain

$$-(\eta_p + \eta_d \eta_{dp}) = 1/w^0 \quad (31)$$

Analogously for $\partial\pi_M/\partial s = 0$, and $\partial\pi_M/\partial m = 0$, we find

$$\frac{ps}{s} (\eta_s + \eta_d \eta_{ds}) = 1/w^0, \quad (32)$$

and

$$\frac{p}{m} \eta_d \eta_{dm} = 1/w^0 \quad (33)$$

respectively.

Equating (31) and (32) we find the relationship between the optimal values for markup and price, m^* and p^* .

$$m^* = \frac{\eta_d \eta_{dm}}{-(\eta_p + \eta_d \eta_{dp})} p^* \quad (34)$$

Assume now that marginal cost is constant and also that all elasticities are constant. The term $\eta_d \eta_{dm} / [- (\eta_p + \eta_d \eta_{dp})]$ in equation (34) is then constant, and we find that optimal markup is a constant percentage of optimal price. Note the similarity between this result and equation (2) in section II. In both cases we find that it is optimal to set markup at a constant percentage of price, although the constants themselves are different. We should emphasize, however, that the rationale behind each of these two results is very different. In section II the decision maker on markup is the retailer. In section III it is the producer who makes decisions on both final sales price and markup.

Returning to equations (31) and (33) we can solve for optimal price p^* ,

$$p^* = \left(\frac{\eta_p + \eta_d \eta_{dp}}{1 + \eta_p + \eta_d \eta_{dp} + \eta_d \eta_{dm}} \right) MC \quad (35)$$

Substituting this value for p^* into equation (34) gives

$$m^* = \left(\frac{-\eta_d \eta_{dm}}{1 + \eta_p + \eta_d \eta_{dp} + \eta_d \eta_{dm}} \right) MC \quad (36)$$

From equation (36) we see that if marginal cost increases, markup increases (with higher marginal cost, optimal price is higher, and hence optimal markup is higher). For products with more elastic demand, that is, higher absolute values for η_p , markup is lower. For products that are more sensitive to availability, that is, higher values for η_d , markup is higher. The more sensitive the number of retailers is to markup, that is, higher values for η_{dm} , the higher markup should be.

In some cases the monopolistic producer will charge a price below the optimal price, for example, to prevent entry by competitors. If that is the case, markup as given by equation (36) is no longer optimal. From equation (33) we can derive optimal values for markup (m^o) for any given value of p

$$m^o = \frac{\eta_d \eta_{dm}}{1 + \eta_d \eta_{dm}} (p - MC) . \quad (37)$$

The relevant portion of the function m^o is values of p that exceed marginal cost, since negative markups are impossible.

For p equal to p^* , m^o becomes equal to m^* . Observe that for a given value of p , an increase in marginal cost results in a decrease of markup. This is illustrated in Figure 4. Furthermore, if p is not determined optimally, price elasticity has no direct impact on markup. Only distribution related elasticities affect markup, and they work in the same direction as indicated in the discussion of equation (36).

IV. Summary and Conclusions

In this paper we have shown through a series of examples that in studying decisions on markup one has to start with defining the vertical market structure. That is, we have to know which economic entities deal with the product before it reaches the marketplace. In these vertical structures we have to find out who makes markup decisions, and what the power structure looks like.

First we reviewed the traditional rationalization for constant percentage markup pricing in retailing. The traditional analysis fails to recognize the existence of other economic units in the vertical market structure, and the influence which they may have on the level of markup.

Next we studied a case where markup decisions are made by the retailers. We assumed that the retailers bought from a monopolistic producer. Constant percentage markup pricing was compared to simple marginalist pricing, and it was found that constant percentage markup pricing could be economically more attractive to the retailers. We also examined the effect of ignoring the existence of intermediaries on optimality conditions.

Finally, we looked at a case where markup decisions are made by a monopolistic producer, and where availability is related to markup.

These examples clearly demonstrate that markup should be treated as an explicit decision variable in studies of marketing programming, and hopefully provide some insight in how this should be done.

FOOTNOTES

- ¹For an early discussion of interaction between prices at different levels of a vertical market structure see Hawkins [2].
- ²In the sequel we will refer to "simple" marginalist pricing in order to better distinguish from constant percentage markup pricing. Later we will show that constant percentage markup pricing is equivalent to marginalist pricing where "producer's reaction" is explicitly taken into account, whereas in simple marginalist pricing this is not done.
- ³Throughout this paper profit functions will be exclusive of fixed costs. In the sequel "average cost" will stand for "average variable cost".
- ⁴Second order conditions will be assumed to be satisfied throughout this paper.
- ⁵It is assumed that all retailers charge the same price to the consumers. The assumption is reasonable if the price at which retailers buy from the producer is the same for all retailers and if they use the same markup policy.
- ⁶We could, of course, also work directly with the producer's demand curve. The reason for not doing so is that we want to relate p and p_R to markup policy.
- ⁷See, for example Lambin [3]. Since Lambin does not describe the nature of the vertical market structure we cannot say whether or not retailers make markup decisions as is assumed in this section.
- ⁸In marketing, the demand curve will usually be written $q = k/a - p/a$ since p rather than q is regarded as the decision variable.
- ⁹Alain Bultez first pointed this out to me.
- ¹⁰Of course the retailers still have to be compensated. If their compensation is a fixed cost, then optimal p and q are not affected. If retailer compensation is based on volume the optimal values will be different.
- ¹¹For a general theoretical discussion of channel control and channel choice, see Helmy H. Baligh [1].
- ¹²For a detailed analysis of how the number of retailers depends on the producer's decision variables, see Naert [4] and [5]. The vertical market structure considered there is more complex in that the producers' level is an oligopoly rather than a monopoly.

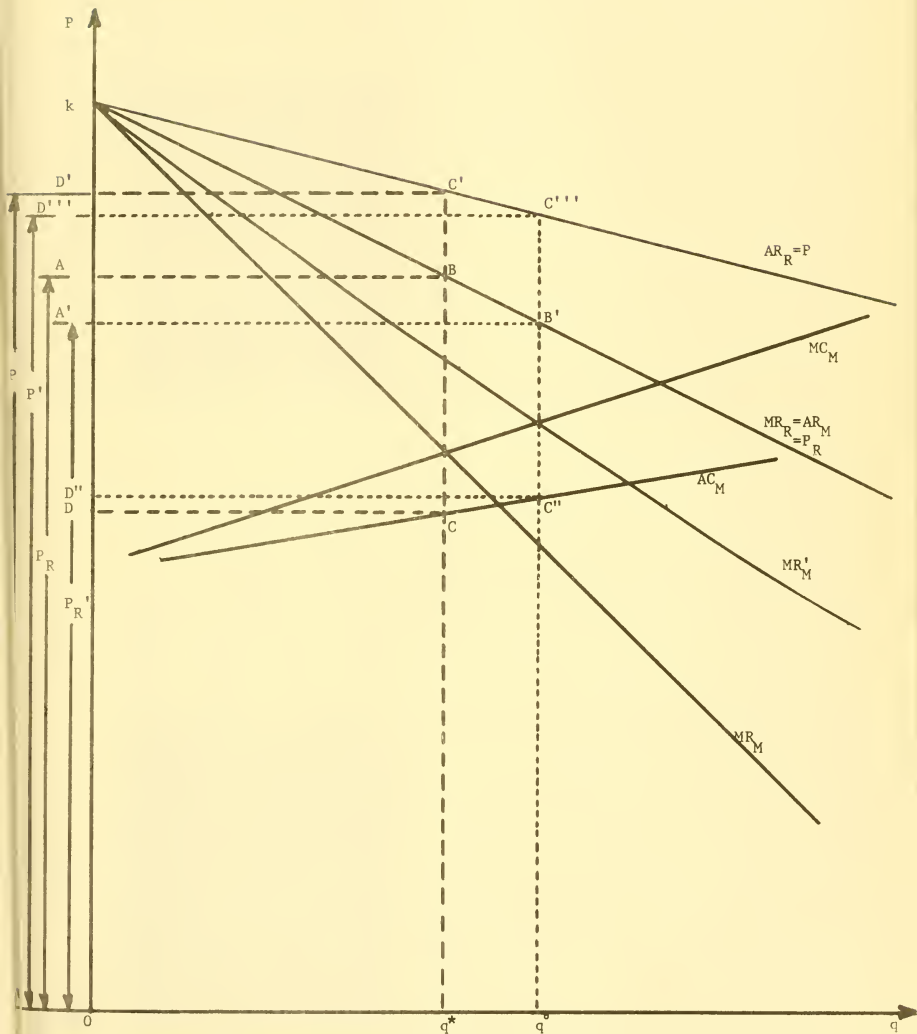
REFERENCES

1. Helmy H. Baleigh, "A Theoretical Framework for Channel Choice", in Peter D. Bennett (ed.), Economic Growth, Competition and World Markets, Chicago, American Marketing Association, 1965, pp. 631-654.
2. Edward R. Hawkins, "Vertical Price Relationships", in Reavis Cox and Wroe Alderson (eds.), Theory in Marketing, Chicago, Richard D. Irwin, 1950, pp. 179-192.
3. Jean-Jacques Lambin, "Optimal Allocation of Competitive Marketing Efforts: An Empirical Study", The Journal of Business, Vol. 43, November, 1970, pp. 468-84.
4. Philippe A. Naert, "Mathematical Models of Vertical Market Structures", Unpublished Ph.D. dissertation, Cornell University, 1970.
5. Philippe A. Naert, "Optimizing Consumer Advertising, Intermediary Advertising, and Markup in a Vertical Market Structure", Management Science, Vol. 18, No. 4, Pt. II, December, 1971, pp. P-90 - P-101.
6. Kristian S. Palda, Pricing Decisions and Marketing Policy, Englewood Cliffs, N.J., Prentice-Hall, 1971.
7. Lee F. Preston, "Markups, Leaders and Discrimination in Retail Pricing", Journal of Farm Economics, May, 1962, pp. 291-306.
8. Louis W. Stern, "Economic Factors Influencing Manufacturers' Decisions Concerning Price Maintenance", Journal of Retailing, Vol. 41, Spring, 1965, pp. 30-37.



Figure 1

Effect of Neglecting the Existence of Intermediaries





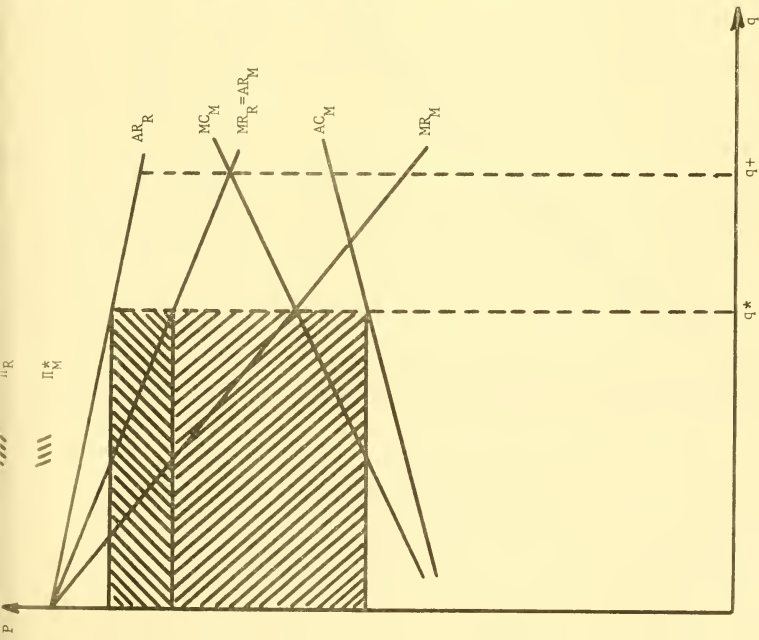


Figure 2a
Simple Marginalist Pricing

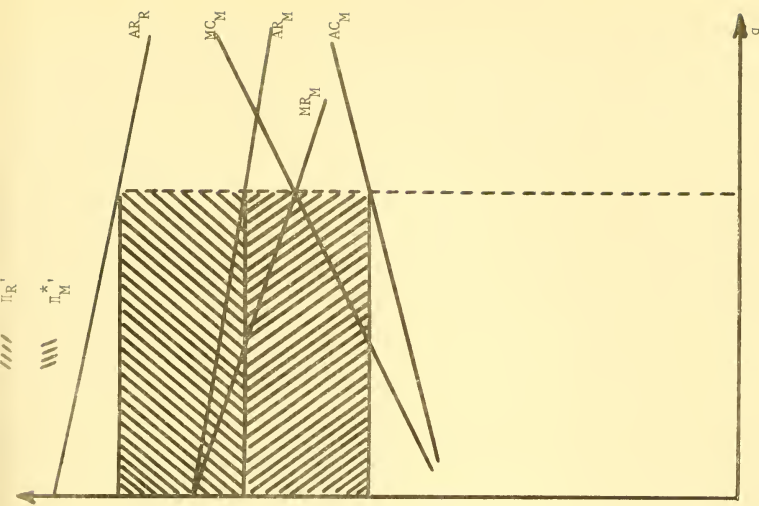


Figure 2b
Constant Percentage Markup Pricing

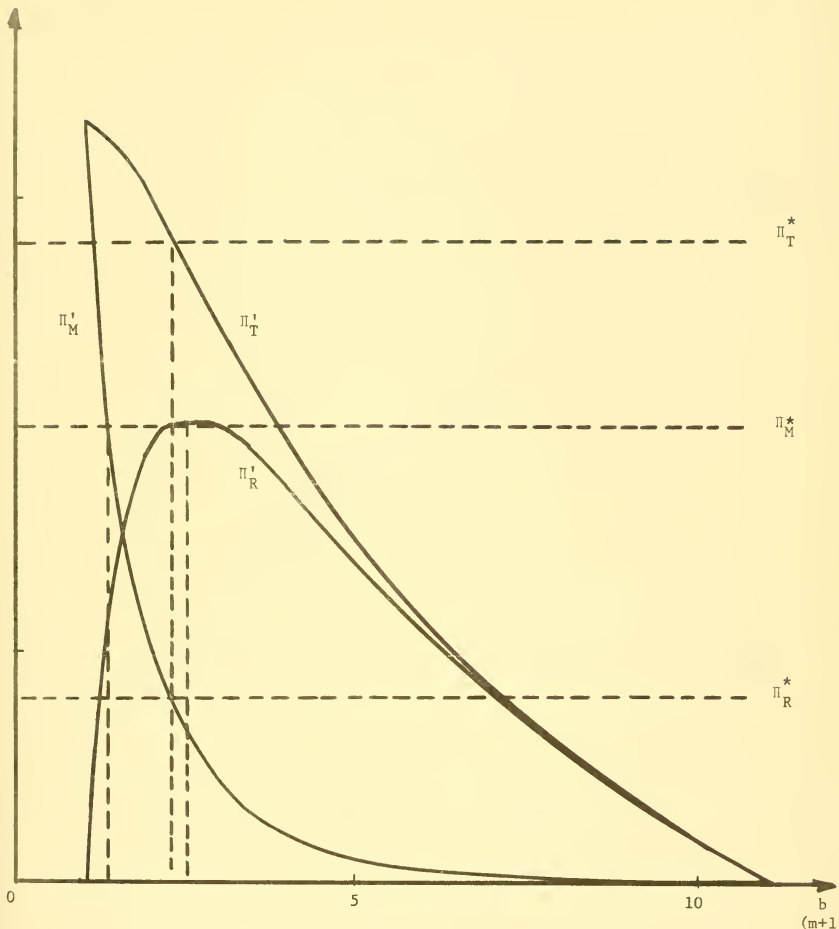


Figure 3

Profit for Various Levels of b

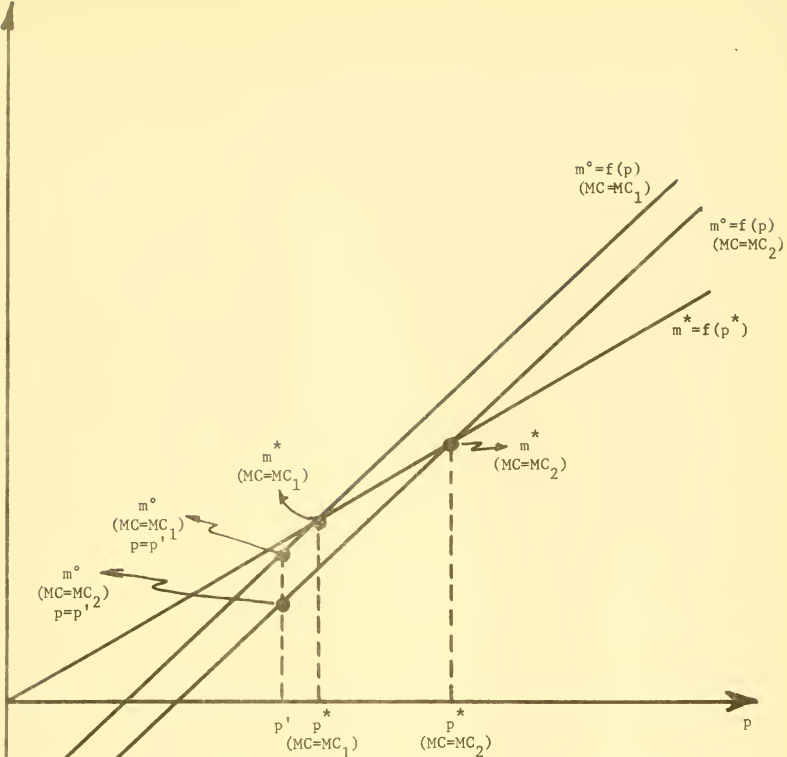


Figure 4

$m^* = f(p^*)$; and
 $m^\circ = f(p)$ for two values
 of MC ($MC_2 > MC_1$)

BASEMENT

DATE DUE

NOV 1975	MAY 20 1990
MAY 21 '76	
FEB 20 '79	AUG 10 1992
MAR 22 '78	MAY 18 1999
APR 12 '79	FEB 10 2000
JUL 1 '79	MAY 18 2000
AUG 13 '81	
NOV 3 '81	NOV 29 2000
MAR 23 '85	SEP 27 2000
JUN 3 '86	
SEP 08 1986	
NOV 27 '87	

MIT LIBRARIES
3 9080 003 701 866

611-72

BASEMENT

MIT LIBRARIES
3 9080 003 701 833

612-72

MIT LIBRARIES
3 9080 003 670 814

613-72

T-J5 143 w no.614- 72
Welsch, Roy E/A modification of the N
636673 D*BKS 00027764

614

3 9080 000 747 904

MIT LIBRARIES
3 9080 003 670 863

-72

T-J5 143 w no.616- 72
Welsch, Roy E/The variances of regres
636653 D*BKS 00027762

3 9080 000 747 854

MIT LIBRARIES
3 9080 003 701 825

617-72

MIT LIBRARIES
3 9080 003 701 874

618-72

MIT LIBRARIES
3 9080 003 701 783

619-72

MIT LIBRARIES
3 9080 003 701 924

622-72

HD28.M414 no.623- 72
Merton, Robert/Fallacy of the log-norm
636656 D*BKS 00027760

623

3 9080 000 747 789

HD28.M414 no.619-72
Plovnick, Mark/Expanding professional
636663 D*BKS 00027761

[620 never issued]

