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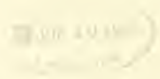
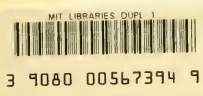
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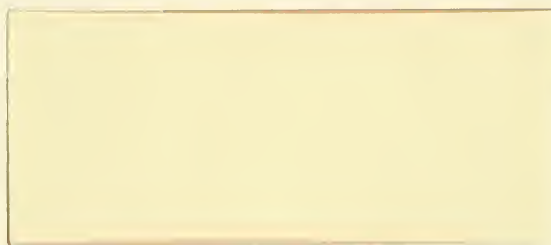
WORKING PAPER
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A COMMODITY LINKED BOND AS
AN OPTIMAL DEBT INSTRUMENT
IN THE PRESENCE OF MORAL HAZARD

Antonio Mello and John Parsons

WP #3035-89-EFA₂ June 1989

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In this paper we communicate two results. First, we construct an example in which a firm strictly prefers to issue a bond linked to the price of a commodity as opposed to a fixed rate bond. The firm with a fixed rate bond chooses a suboptimal investment policy because the outstanding nominal obligation distorts the incentives of the equity owners. The commodity linked bond resolves this moral hazard problem.

Second, we show that in the presence of moral hazard problems the traditional application of contingent claims valuation techniques leads to an incorrect valuation of the the real assets and consequently to an incorrect valuation of the financial liabilities written against those assets. We show how the technique can be adapted to yield correct valuations.

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The use of commodity linked debt has been advocated by Lessard (1977), primarily as an instrument to improve risk sharing between less developed countries that are significantly dependent upon commodity exports and outside investors. Lessard also noted that the improved risk sharing would have consequences for capital budgeting. However, his argument did not focus upon the very direct nature in which a commodity linked bond could improve the equity owner's incentives to choose the optimal investment and operating strategy. Lessard's suggestion has become quite popular, and there now exist many proposals for indexing of less developed country debt to the prices of various commodities or directly to export earnings. However, almost all of the literature continues to focus upon the problem of improved risk sharing. There is almost no attention given to the improved use of the real assets that can result from the proper design of commodity linked debt. In this paper we focus exclusively on the changes in use of the real assets. Our analysis demonstrates, therefore, why a commodity linked bond may be an optimal financing instrument for a project in a developed country as well as a project in a less developed country. It should be noted that the great majority of commodity linked bonds have been issued in developed countries and that many of the issues have been made by industrial corporations involved in mining or processing the relevant commodities as our analysis would predict and in contradiction to the predictions of most other explanations for the existence of these securities. Our analysis calls attention, moreover, to the fact that the optimal commodity linked contract must be tailored to the particular set of real assets being financed.

In the traditional application of the contingent claim techniques it is assumed that the investment and operating decisions of the firm are given and do not vary with the parameters of the liabilities sold against the firm. For example, in Merton (1974) the stochastic process that describes the value of the firm is assumed given, and then the value of any debt claim written

against the firm is derived. In recent work on the valuation of real options the contingent claims techniques are used to derive the optimal investment and operating strategies—for example, in Brennan and Schwartz (1985). In these models, however, the optimal investment and operating strategy is derived under the implicit assumption that the firm is 100% equity owned. It is common to then take the derived optimal investment and operating strategy as the underlying determinant of the stochastic process for the firm value and to then value the various financial claims against this value. However, under some circumstances the structure of financial claims sold against the assets will impact the operating decisions of the management so that the assumed stochastic process for the firm value does not in fact obtain in equilibrium. The problem of moral hazard must be incorporated into the contingent claims models, and we show how this can be done.

1. A Contingent-Claims Analysis of a Mine

Consider a firm that owns a mine that can operate for a total of three more years. To make the problem simple we reduce the management's discretion to a single decision: when should the mine be abandoned. When the mine is open it must produce an amount q at zero average cost.¹ While the mine is still open and operating the owner must pay a per period maintenance cost of A . At any point in time the mine can be abandoned at zero cost, but it cannot be reopened.

The future price of the commodity, s , is a random variable. We model the price using the binomial method described in Cox, Ross and Rubinstein (1979). Accordingly, the price at time $t=0$ is S ; the price at time $t=1$ can take on one of two values, uS and dS . The price at time $t=2$ can take on three values, uuS , udS , and ddS , although conditional on the realization of the price at $t=1$

¹ One can alternatively consider the commodity price given in this paper to be the price net of the constant per unit average cost.

there are only two possible realizations at $t=2$. In Figure 1, we draw the tree that defines the possible paths of prices. Because the payoff to the firm is path dependent in an important way, when we construct a tree to represent the unfolding of uncertainty regarding price we distinguish between the node at time $t=2$ for which $s=udS$ and which was arrived at via $s=uS$ at time $t=1$ and the node at time $t=2$ for which $s=udS$ and which was arrived at via $s=dS$ at time $t=1$. For convenience we denote the four nodes at $t=2$ by $j=1,2,3,4$; a similar notation is used to denote the two nodes at $t=1$; and we may write the price at any node as $s(t,j)$.

[Insert Figure 1 Here]

The risk-free interest rate is r . The term structure of futures prices is determined by the convenience yield which we assume to be proportional to the current price of the commodity, $s(t,j)c$: defining $f(s)$ as the one period future price it follows in our model that $f(s) = s(t,j)(1+r-c)$.

A vector $M = [m(0,1);m(1,1),m(1,2);m(2,1),m(2,2),m(2,3),m(2,4)]$ denotes an abandonment strategy, where $m(t,j) = 0 (= 1)$ means that the firm abandons (continues to operate) the mine in period t when the price is $s(t,j)$. There are 5 undominated strategies: $M_1 = [0;0,0;0,0,0,0]$; $M_2 = [1;1,0;1,0,0,0]$; $M_3 = [1;1,0;1,1,0,0]$; $M_4 = [1;1,1;1,1,0,0]$; and $M_5 = [1;1,1;1,1,1,1]$. Strategies 3 and 4 are the interesting ones that serve as the basis of comparisons made in this paper. In Figure 1 we have written the abandonment decisions at each node of the decision tree for both of these strategies. The reader should note that under strategy 3, if the price at $t=2$ is udS , whether or not the firm is operating depends upon the past history of price: if the price at $t=1$ was uS , then the mine was not abandoned and the mine will also not be abandoned at $t=2$; however, if the price at $t=1$ was dS , then the mine was abandoned and even though the owner might wish to be operating at the new price it is not possible.

It is now possible to derive the value of the mine and to solve for the optimal abandonment policy. Our example is essentially a much simplified case of the natural resource problem analyzed in Brennan and Schwartz (1985)--allowing for the discretization of the problem in our example--and we solve our problem according to the logic used in that paper.

For any arbitrary strategy M it is a straightforward matter to construct the set of cash flows that will be received by the firm at each point in time and contingent upon the realization of the price: $x(M,t,j)$. These cash flows are, however, contingent upon the realization of the stochastic price process and the difficult problem is the proper discount to apply to these cash flows in recognition of the particular risk associated with each. Contingent claims analysis teaches us, however, how to use the futures market combined with borrowing and lending to exactly duplicate the sequence of risky cash flows from the mine under any given strategy. We can determine the amount of cash that would be necessary to open a portfolio of futures contracts and bonds which, when properly rebalanced, perfectly replicates the cash flows from the mine. Under certain assumptions about the completeness of the market in financial and real assets, this must also be the value of the mine under that strategy.

Define $K(M,t,j)$ as the amount of futures contracts that the firm holds in period $t \in \{0,1\}$ if the price is at node j ; and define $B(M,t,j)$ as the amount of one period bonds paying $(1+r)$ in the next period that the firm holds in period $t \in \{0,1\}$ if the price is at node j . For every strategy $M=1,\dots,5$ the portfolio of futures contracts and bonds that the firm must hold at time $t=1$ is the solution to the following set of equations which can be solved pairwise:

$$x(M,2,1) = (u+c-r) K(M,1,1) + (1+r) B(M,1,1),$$

$$x(M,2,2) = (d+c-r) K(M,1,1) + (1+r) B(M,1,1),$$

$$x(M,2,3) = (u+c-r) K(M,1,2) + (1+r) B(M,1,2), \text{ and}$$

$$x(M,2,4) = (d+c-r) K(M,1,2) + (1+r) B(M,1,2).$$

Similarly, the portfolio of futures contracts and bonds that the firm must hold at time $t=0$ is the solution to the following pair of equations, which can be solved once the values from the previous equations are substituted in accordingly:

$$x(M,1,1) = (u+c-r) K(M,0,1) + (1+r) B(M,0,1) - B(M,1,1), \text{ and}$$

$$x(M,1,2) = (d+c-r) K(M,0,1) + (1+r) B(M,0,1) - B(M,1,2).$$

Once this portfolio of futures contracts and bonds is opened, it can be rebalanced in each period as described by the solution to these equations so that it yields exactly the same free cash flow to the firm as would the mine operated according to the strategy M . In order to start the strategy, however, the firm needs to purchase the initial portfolio of futures and bonds. Since the futures contracts require no cash outlays up front, the initial portfolio costs $B(M,0,1)$ and this, then, is the value of the mine operated according to the strategy M .

It is now a straightforward matter to choose the optimal abandonment strategy for the mine: it is simply that abandonment strategy which yields the highest value for the mine:

$$T^* = \max_M B(M,0,1) \text{ and } M^* \in \operatorname{argmax}_M B(M,0,1).$$

It is also possible to find the value of the mine at any point in time as a function of the realized price of the commodity, $T(M^*,t,j) = x(M^*,t,j) + B(M^*,t,j)$.

In constructing this valuation we have ignored the fact that at some point in time and for some realization of price and for some arbitrary planned abandonment strategy the firm might have a negative cash account and a negative value, i.e., we have ignored the problem of bankruptcy.

This is easy to resolve.²

To properly incorporate the problem of bankruptcy we must specify what happens to the firm's

² Note, however, that T^* is always greater than zero, since $T(1,0,1)=0$ is always feasible.

assets in this event. We assume first that the firm does not pay any dividends at $t=0$ and $t=1$: all cash earned is retained and invested in an account earning the riskless rate of interest, r . At the end of period $t=2$ a liquidating dividend is paid. At the beginning of each period, given the realization of the price it is common knowledge whether the firm has enough cash to continue to operate as planned, that is whether it can cover the maintenance cost, A , out of the revenues to be received that period and out of any accumulated cash: if it will not have enough cash, then the firm is declared bankrupt. The firm, less any accumulated cash, is assumed to be liquidated by sale and the assumed sale price is the value of the firm at that point in time and conditional on the current realized commodity price, and under the assumption that the firm is operated from this point on according to the optimal strategy, $T(M',t,j)$, derived above. Define $Y(M,t,j)$ as the balance that would be in the account at the end of a period assuming an arbitrary abandonment strategy M and no bankruptcy; this is the retained earnings of the firm, and at $t=2$ it is the cash available for distribution to equity holders:

$$Y(M,0,1) = 0,$$

$$Y(M,1,1) = Y(M,0,1) (1+r) + x(M,1,1),$$

$$Y(M,1,2) = Y(M,0,1) (1+r) + x(M,1,2),$$

$$Y(M,2,1) = Y(M,1,1) (1+r) + x(M,2,1),$$

$$Y(M,2,2) = Y(M,1,1) (1+r) + x(M,2,2),$$

$$Y(M,2,3) = Y(M,1,2) (1+r) + x(M,2,3), \text{ and}$$

$$Y(M,2,4) = Y(M,1,2) (1+r) + x(M,2,4).$$

The firm is bankrupt at the beginning of period t given a realization of the commodity price if $Y(M,t,j) < 0$. Define $u(M,t,j) \in \{0,1\}$ as a binary operator, where $u=0$ indicates that the firm has gone bankrupt at time t or earlier given the realized path of commodity prices, and define

$w(M,t,j) \in \{0,1\}$ as a binary operator, where $w=1$ indicates that the firm went bankrupt at time t given the realization of the commodity price. Define $Z(M,t,j)$ as the actual balance in the cash account of the firm, given the intended strategy M and the sale of the firm when bankruptcy occurs:

$$Z(M,0,1) = x(M,0,1)u(M,0,1) + T(M',0,1)w(M,0,1)$$

$$Z(M,1,1) = Z(M,0,1) (1+r) + x(M,1,1)u(M,1,1) + T(M',1,1)w(M,1,1),$$

$$Z(M,1,2) = Z(M,0,1) (1+r) + x(M,1,2)u(M,1,2) + T(M',1,2)w(M,1,2),$$

$$Z(M,2,1) = Z(M,1,1) (1+r) + x(M,2,1)u(M,2,1) + T(M',2,1)w(M,2,1),$$

$$Z(M,2,2) = Z(M,1,1) (1+r) + x(M,2,2)u(M,2,2) + T(M',2,2)w(M,2,2),$$

$$Z(M,2,3) = Z(M,1,2) (1+r) + x(M,2,3)u(M,2,3) + T(M',2,3)w(M,2,3), \text{ and}$$

$$Z(M,2,4) = Z(M,1,2) (1+r) + x(M,2,4)u(M,2,4) + T(M',2,4)w(M,2,4).$$

The firm may be valued using a portfolio hedging strategy composed of futures contracts and riskless bonds as described earlier. The only correction that must be made to the hedging algorithm is a recognition that no cash flows actually leave the firm until the end of period 2. Again we define $K(M,t,j)$ as the amount of futures contracts that the firm holds in period $t \in \{0,1\}$ if the price is at node j ; and define $B(M,t,j)$ as the amount of one period bonds paying $(1+r)$ in the next period that the firm holds in period $t \in \{0,1\}$ if the price is at node j . For every strategy $M=1,\dots,5$ the portfolio of futures contracts and bonds that the firm must hold at time $t=1$ is the solution to the following set of equations that can be solved pairwise:

$$Z(M,2,1) = (u+c-r) K(M,1,1) + (1+r) B(M,1,1),$$

$$Z(M,2,2) = (d+c-r) K(M,1,1) + (1+r) B(M,1,1),$$

$$Z(M,2,3) = (u+c-r) K(M,1,2) + (1+r) B(M,1,2),$$

$$Z(M,2,4) = (d+c-r) K(M,1,2) + (1+r) B(M,1,2),$$

$$Z(M,1,1) = (u+c-r) K(M,0,1) + (1+r) B(M,0,1), \text{ and}$$

$$Z(M,1,2) = (d+c-r) K(M,0,1) + (1+r) B(M,0,1).$$

Once this portfolio of futures contracts and bonds is opened, and if it is rebalanced as described by the solution to these equations, it yields exactly the same free cash flow to the firm as would the mine operated according to the strategy M. In order to start the portfolio, however, the firm needs to have cash in the amount $B(M,0,1)$ and this, then, is the value of the mine operated according to the strategy M, $V(M,0,1)$.³

In Table 1 we display the results of this model for a particular set of input values. The reader can see that the optimal abandonment strategy is #3 and that the value of the mine is $V(M',0,1)=V(3,0,1)=\$4.24$.

[Insert Table 1 Here]

At $t=1$ and $j=2$, i.e. when $s(1,2)=dS$, the firm is facing an unsure future: if the price were to increase to uS , it would be profitable for the firm to operate. If the price were to decrease to ddS the firm would choose to abandon the mine rather than incur the operating losses. However, in order to keep this option alive, the firm must keep the mine open today and pay the maintenance costs A . This would be equivalent to choosing strategy #4. The option is valuable: it is worth almost \$0.54. However, the cost of the option is \$0.65, the difference between the current earnings on the commodity and the maintenance costs of keeping the mine open, and so the firm is better off abandoning the mine at $t=1, j=2$.

³ It may be the case that $V(M,0,1) > T(M,0,1)$, the value of the firm calculated in the previous section. This is because V incorporates the possibility of bankruptcy while T does not. $V > T$ when the event of bankruptcy takes the assets away from a management team that is implementing a suboptimal abandonment strategy. Of course, $V(M',0,1)=T(M',0,1)$.

2. Valuing Corporate Liabilities

We now suppose that the firm has outstanding a bond with a promised payment at $t=2$, p , and with no intermediate payments due. The firm is restricted, however, against paying any dividends until the bond is paid. The firm is allowed to use the accumulated cash to pay any operating expenses on the mine at $t=1$ and $t=2$. The accumulated balance in this account is used at $t=2$ to pay off the bond and then the remainder is paid out as a liquidating dividend. No other liabilities exist or may be created.

The definition of the cash available to the firm and the events in which bankruptcy occurs are changed slightly by the existence of the outstanding debt. Define $Y_d(M,t,j)$ as the balance that would be in the firm's cash account at the end of a period assuming an arbitrary abandonment strategy M and no bankruptcy; this is the retained earnings of the firm, and at $t=2$ it is the cash available for distribution to equity holders:

$$Y_d(M,0,1) = 0,$$

$$Y_d(M,1,1) = Y_d(M,0,1) (1+r) + x(M,1,1),$$

$$Y_d(M,1,2) = Y_d(M,0,1) (1+r) + x(M,1,2),$$

$$Y_d(M,2,1) = Y_d(M,1,1) (1+r) + x(M,2,1) - p,$$

$$Y_d(M,2,2) = Y_d(M,1,1) (1+r) + x(M,2,2) - p,$$

$$Y_d(M,2,3) = Y_d(M,1,2) (1+r) + x(M,2,3) - p, \text{ and}$$

$$Y_d(M,2,4) = Y_d(M,1,2) (1+r) + x(M,2,4) - p.$$

The firm is bankrupt at the beginning of period t if $Y_d(M,t,j) < 0$. Define $u_d(M,t,j)$ and $w_d(M,t,j)$ as indicator functions for a bankrupt firm and for the period of bankruptcy, just as before. Define $Z_d(M,t,j)$ as the actual balance in the cash account of the firm, given the intended strategy M and the sale of the firm when bankruptcy occurs:

$$Z_d(M,0,1) = x(M,0,1)u_d(M,0,1) + T(M',0,1)w_d(M,0,1)$$

$$Z_d(M,1,1) = Z_d(M,0,1)(1+r) + x(M,1,1)u_d(M,1,1) + T(M',1,1)w_d(M,1,1),$$

$$Z_d(M,1,2) = Z_d(M,0,1)(1+r) + x(M,1,2)u_d(M,1,2) + T(M',1,2)w_d(M,1,2),$$

$$Z_d(M,2,1) = Z_d(M,1,1)(1+r) + x(M,2,1)u_d(M,2,1) + T(M',2,1)w_d(M,2,1),$$

$$Z_d(M,2,2) = Z_d(M,1,1)(1+r) + x(M,2,2)u_d(M,2,2) + T(M',2,2)w_d(M,2,2),$$

$$Z_d(M,2,3) = Z_d(M,1,2)(1+r) + x(M,2,3)u_d(M,2,3) + T(M',2,3)w_d(M,2,3), \text{ and}$$

$$Z_d(M,2,4) = Z_d(M,1,2)(1+r) + x(M,2,4)u_d(M,2,4) + T(M',2,4)w_d(M,2,4).$$

We are now ready to value firm with debt outstanding and to directly value the debt and equity claims against the firm. The valuation of the firm proceeds just as before, using a portfolio hedging strategy composed of futures contracts and riskless bonds, $K_d(M,t,j)$ and $B_d(M,t,j)$:

$$Z_d(M,2,1) = (u+c-r) K_d(M,1,1) + (1+r) B_d(M,1,1),$$

$$Z_d(M,2,2) = (d+c-r) K_d(M,1,1) + (1+r) B_d(M,1,1),$$

$$Z_d(M,2,3) = (u+c-r) K_d(M,1,2) + (1+r) B_d(M,1,2),$$

$$Z_d(M,2,4) = (d+c-r) K_d(M,1,2) + (1+r) B_d(M,1,2),$$

$$Z_d(M,1,1) = (u+c-r) K_d(M,0,1) + (1+r) B_d(M,0,1), \text{ and}$$

$$Z_d(M,1,2) = (d+c-r) K_d(M,0,1) + (1+r) B_d(M,0,1).$$

$B_d(M,0,1)$ is the value of the mine operated according to the strategy M , $V_d(M,0,1)$.⁴

To value the debt and the equity claims against the firm merely requires correctly registering the payoffs to each claim in each final realization of the commodity price path and valuing the risky

⁴ It may be the case that $V_d(M,0,1) > V(M,0,1)$, the value of the firm calculated when there is no debt outstanding. This is because the possibility of bankruptcy may change the actual abandonment strategy implemented with the real assets when these assets are taken from the original management; and the set of events on which bankruptcy may occur will be different depending upon the actual size of the debt obligation outstanding. This is one example of the 'free cash flow' argument in favor of a large amount of debt that has been made by Jensen (1986). Of course, $V_d(M',0,1) = V(M',0,1)$.

payoff using the appropriate portfolio hedging strategy composed of futures and riskless bonds. The outstanding corporate bond promising to pay p at $t=2$ has the following actual payouts under strategy M:

$$D_d(M,2,j) = p u_d(M,2,j) + Z_d(M,2,j) w_d(M,2,j), \quad j=1,..4.$$

The firm's equity has the following payouts under strategy M:

$$E_d(M,2,j) = (Z_d(M,2,j)-p) u_d(M,2,j) + \max\{Z_d(M,2,j)-p,0\} w_d(M,2,j), \quad j=1,..4.$$

To value these claims we once again construct a hedging portfolio of futures and riskless bonds according to the algorithm used to value the cash flows of the firm as a whole given above.

In Table 2 we display some of the results of this model for the set of input values used earlier in constructing Table 1, with the promised debt payment $p=\$1.35$ at $t=2$. The value of the mine under strategy #3 is $V_d(3,0,1)=\$4.242$. Under strategy #3 and with this size of debt outstanding the firm never goes bankrupt.⁵ The corporate debt is therefore riskless; its value is $D_d(3,0,1)=\$1.076$. The firm's equity has a value $E_d(3,0,1)=\$3.166$.

[Insert Table 2 Here]

3. Moral Hazard and the Revised Valuation of Corporate Liabilities

Although the total value of the firm is less under strategy #4 than under strategy #3, the reader should note that the value of the equity is higher, $E_d(4,0,1) = \$3.20 > E_d(3,0,1) = \3.166 . This is because the outstanding debt obligation of the firm changes the financial incentives facing the equity owners in the event that the price is low. At $t=1, j=2$, the abandonment option is worth $\$0.54$ as we mentioned earlier, but costs $\$0.65$ to keep alive. Keeping the option alive is not worthwhile and the mine should be abandoned. However, the equity holder owns an

⁵ And therefore $T(3,0,1)=V(3,0,1)=V_d(3,0,1)$.

on a different payoff: the returns to the mine in each event less the nominal debt obligation. This equity payoff pattern is riskier than the firm payoff pattern so that the option is valuable to the equity owner even though it is not valuable to the firm as a whole. Consequently, the equity owner chooses to operate at a loss at $t=1, j=2$ in order to keep alive the hope of a large payoff at $t=2, j=3$.

It is therefore incorrect to value the outstanding corporate bonds under the assumption that the firm is managed according to strategy #3 as was done in the previous section and as is often done when various techniques for contingent claims pricing are applied to value corporate liabilities. The actual stochastic process that the value of the firm follows is not independent of the nominal obligations specified in the liabilities because these nominal obligations affect the incentives of the equity owners. One cannot therefore first specify or derive the stochastic process for the value of the firm and then directly calculate the value of any arbitrary financial liability written against that firm.

The usual method for valuing the contingent claims can be easily modified to correctly incorporate the consequence of moral hazard: the modification is as follows. In general, it is necessary to first specify the nominal obligations outstanding on the firm, and then it is necessary to value the equity under all possible strategies. The strategy that will be chosen is the one which maximizes the return to the equity given those financial liabilities. Once this strategy is chosen it becomes possible to value all of the financial liabilities in equilibrium and possible to value the firm as a whole: Define $M' \in \operatorname{argmax} E_d(M,0,1)$, and $V_d' = V_d(M',0,1)$, $D_d' = D_d(M',0,1)$, and $E_d' = E_d(M',0,1)$.

While this modification is straightforward at the level of the logic of the algorithm, in fact it significantly frustrates the use of the mathematical machinery with which these problems have been

solved to date. It is no simple matter to translate this algorithm into a robust valuation technique that is easily usable for a large class of problems. Nevertheless, it is possible to use it to gain some insight into certain important questions as is evidenced in the next section.

4. A Commodity Linked Bond as an Optimal Debt Instrument

Consider a corporate debt obligation in which the firm promises to pay at time $t=2$ an amount that is contingent upon the realized price of the commodity: $p(2,j) = 0.1231 s(2,j)$. If this is the only debt instrument outstanding on the firm, and assuming all other values as given for the previous example, then we can calculate the value of the firm, V_i' , the value of this debt instrument, D_i' , and the value of the equity, E_i' , using the hedging portfolio constructions used above and acknowledging the moral hazard problem as discussed above. The results for abandonment strategies #3 and #4 are displayed in Table 3.

[Insert Table 3 Here]

The important thing to note is that the optimal strategy for the equity holders when the outstanding liability is this commodity linked bond is abandonment strategy #3, the first-best abandonment strategy. The commodity linked debt is, therefore, an optimal debt contract which resolves the moral hazard problem associated with fixed rate debt contracts for this firm. The firm value $V_i' = \$4.24$ is greater than the firm value when a fixed rate debt contract is used, $V_d' = \$4.18$. This commodity linked bond is worth $D_i' = \$0.98$ which is the same as the value of the fixed rate bond, D_d' , and therefore we can say that the firm value is increased by changing the type of debt contract while holding constant the value of the debt outstanding at $t=0$. The value of the equity is higher when the firm has the commodity linked bond outstanding than when it has the fixed rate bond outstanding: $E_i' = \$3.26 > E_d' = \3.20 .

The commodity linked bond increases the value of the firm by changing the equity owner's financial incentives for abandonment when the price of the commodity is low. With the commodity linked bond outstanding it is no longer beneficial for the equity holder to keep the abandonment option open at $t=1, j=2$ and therefore the equity holder will implement the first best strategy #3. The increase in the firm value that follows from this accrues to the equity.⁶

It is worth emphasizing that this result could not be obtained using the conventional techniques of contingent claims analysis, precisely because these techniques do not recognize the moral hazard problems associated with various financial liabilities. The techniques that dominate in valuing the variety of contingent claims that now exist—for example the models of Schwartz (1982) and Rajan (1988) developed for valuing commodity linked debt—ignore the impact that the issuance of a given type of security has on the stochastic process which describes the firm's value. As mentioned before, while the logic of the algorithms is easily reconstructed to incorporate this problem, the mathematical machinery with which the original algorithms have been so fruitfully applied is not so easily reconstructed. Even once a method is devised to value a given set of financial liabilities, acknowledging the problem of moral hazard, a more significant problem lies in devising a means by which to find the optimal set of financial liabilities. It is to these tasks that we intend to return in future research on this problem.

⁶ Although the particular commodity linked bond that we have analyzed here achieves the first-best, it is not the uniquely optimal commodity linked instrument. There is a large set of commodity price contingent debt securities that would also achieve the first-best. In the case at hand, another debt instrument with an option feature would have also achieved the first best. In fact, one would not generally want to make the payment due contingent only upon the final realized price, as we have done in the case of the commodity linked bond used in this example. One would rather have the total value of the promised payment contingent upon the full path of prices in such a fashion that it hedged the total value of production from the mine. The key characteristic of any optimal debt contract for our problem would be that it would lower the required payment in the worst outcome, $s(2,4)$, so that the equity holder would not be tempted to keep the mine open through time period $t=1$.

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Figure 1: the Decision Tree and Associated Parameter Values

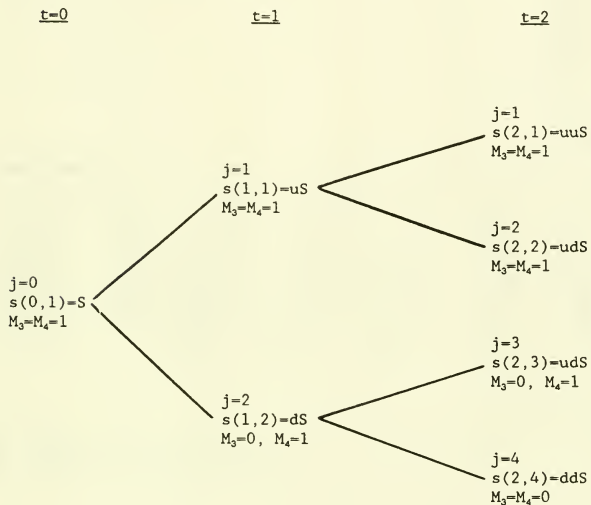


Table 1: Contingent Claim Valuations of the Firm

Model Input Parameters

$s(0,1)= 10.00$
 $u= 1.25$
 $d= 0.80$
 $r= 0.12$
 $c= 0.12$
 $A= 8.65$

Model Results

t, j	$s(t, j)$	$f(t, j)$	$V(1, t, j)$	$V(2, t, j)$	$V(3, t, j)$	$V(4, t, j)$	$V(5, t, j)$
0,1	10	10	0	3.98	4.24	4.19	3.95
1,1	12.5	12.5	0	8.13	8.80	8.80	8.80
1,2	8	8	0	1.51	1.51	1.40	0.92
2,1	15.625		0	12.98	12.98	12.98	12.98
2,2	10		0	6.01	7.36	7.36	7.36
2,3	10		0	1.69	1.69	2.32	2.32
2,4	6.4		0	1.69	1.69	0.97	0.00

$M^* = 3$

$V^* = V(3,0,1) = 4.24$

Table 2: Fixed Rate Debt--Firm, Equity and Debt Values

Model Input Parameters

$s(0,1) = 10.00$
 $u = 1.25$
 $d = 0.80$
 $r = 0.12$
 $c = 0.12$
 $A = 8.65$
 $p = 1.35$

Model Results

t, j	$s(t, j)$	$V_d(3, t, j)$	$E_d(3, t, j)$	$D_d(3, t, j)$	$V_d(4, t, j)$	$E_d(4, t, j)$	$D_d(4, t, j)$
0,1	10	4.24	3.17	1.08	4.19	3.20	0.98
1,1	12.5	8.80	7.59	1.21	8.80	7.59	1.21
1,2	8	1.51	0.31	1.21	1.40	0.38	1.01
2,1	15.625	12.98	11.63	1.35	12.98	11.63	1.35
2,2	10	7.36	6.01	1.35	7.36	6.01	1.35
2,3	10	1.69	0.34	1.35	2.32	0.97	1.35
2,4	6.4	1.69	0.34	1.35	0.97	0.00	0.97

$$M_d' = 4$$

$$V_d' = V_d(M_d', 0, 1) = V_d(4, 0, 1) = 4.19$$

$$E_d' = E_d(M_d', 0, 1) = E_d(4, 0, 1) = 3.20$$

$$D_d' = D_d(M_d', 0, 1) = D_d(4, 0, 1) = 0.98$$

Table 3: Commodity Linked Debt--Firm, Equity and Debt Values

Model Input Parameters

$s(0,1) = 10.00$
 $u = 1.25$
 $d = 0.80$
 $r = 0.12$
 $c = 0.12$
 $A = 8.65$
 $p = 0.1231 s(2,j)$

Model Results

t, j	$s(t, j)$	$V_i(3, t, j)$	$E_d(3, t, j)$	$D_d(3, t, j)$	$V_i(4, t, j)$	$E_d(4, t, j)$	$D_d(4, t, j)$
0,1	10	4.24	3.26	0.98	4.19	3.20	0.98
1,1	12.5	8.80	7.43	1.37	8.80	7.43	1.37
1,2	8	1.51	0.63	0.87	1.40	0.52	0.88
2,1	15.625	12.98	11.05	1.92	12.98	11.06	1.92
2,2	10	7.36	6.12	1.23	7.36	6.12	1.23
2,3	10	1.69	0.46	1.23	2.32	1.08	1.23
2,4	6.4	1.69	0.91	0.79	0.97	0.18	0.79

$M_t' = 3$

$V_i' = V_i(M_t', 0, 1) = V_i(3, 0, 1) = 4.24$

$E_d' = E_d(M_t', 0, 1) = E_d(3, 0, 1) = 3.26$

$D_d' = D_d(M_t', 0, 1) = D_d(3, 0, 1) = 0.98$

