Competitive Advertising and Pricing in Duopolies:
The Implications of Relevant Set/Response Analysis

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Abstract

Consumers choose from among those brands they consider relevant.
Advertising spending influences whether consumers consider brands as
relevant. Firms set advertising and price and react, re-react, etc., until
an equilibrium is reached. We demonstrate how advertising affects the price
equilibrium, how price affects the advertising equilibrium, and how external
factors affect both. We show how the ability to anticipate competitive
response affects advertising strategies and profits and we compare the
competitive formulation to a noncompetitive formulation. The analysis is
compatible with an evaluation cost theory of consumer behavior. We examine
empirical support for the theories.

Advertising response analysis has been a key component of marketing decision support systems for almost 30 years (Benjamin and Maitland 1958, Benjamin, et al. 1960, Little 1966, 1979a, Lodish 1986). Senior managers at General Electric (Lillis and McIvor 1985), Kodak and General Motors (Barabba 1985), Carnation (Struse 1985), and Mattel (Hatch 1980), describe how such analyses have improved advertising decisions. Eight years ago, Little (1979a) reported that over $400 million was being spent annually on data and experiments for market response analysis. We expect that figure has more than doubled today with the advent of automated data collection via supermarket scanners and split-cable television advertising tests. Academic interest is high as a variety of researchers investigate how to estimate accurately response curves. See, among others, Assmus, Farley and Lehmann (1984), Chakravarti, Mitchell and Staelin (1981), Little (1975), Mahajan, Muller and Sharma (1984), and Rao and Sabavala (1986).

The normal application is to prescribe how a target firm should act. The impact of competitive advertising is included in some models, and, in practice, judgments are sometimes made about how competitors will respond, but it is rare that formal models attempt to predict competitive reaction.

In this paper we examine the competitive implications of one of the most widely applied models of advertising response, the relevant set model. We examine what happens when all firms in a market use response analysis models (which include the effects of competitive advertising) and act accordingly. In our scenario firms react, re-react, re-re-react, etc., until the market stabilizes. We examine the characteristics of the resulting equilibrium (when it exists), show how market share, production cost, advertising effectiveness, and managerial anticipation affect the

equilibrium, and show how the competitive advertising equilibrium affects price elasticity and the competitive price equilibrium.

While our main focus is on strategic implications, our paper would not be complete without suggesting at least one consumer behavior motivation of the relevant set model of advertising response. We provide one such motivation based on a theory that balances the cost of evaluating a product with the expected increment benefits.

Throughout this paper we illustrate key results with advertising response functions taken from published meta-analyses.

After a note on exposition, we describe the relevant set model of advertising response and cite supporting evidence. We next describe the evaluation cost model and then the advertising equilibrium. Three sections describe its affect on price, what happens when price and advertising are set simultaneously, and what happens when firms anticipate one another. We close with a discussion of the strengths and weaknesses of our analysis.

A NOTE ON EXPOSITION

Competitive effects can be complex and fraught with technical conditions. We choose in this paper to illustrate key implications with as little algebra and game theoretic terms as is necessary. We state formal results verbally whenever feasible. In some cases this means a less general result is stated in the text to avoid technical conditions. When this proves necessary, we state the more general result in a supplemental appendix. All proofs are available in that appendix.

The relevant set model is an N-firm model, but its algebraic complexity grows exponentially with the number of firms. Thus, in the belief that many

key ideas become clear when we study two firms, we illustrate the implications of competitive advertising in a duopoly. We believe our qualitative results generalize, but formalization of our beliefs awaits an elegant analytic reduction of the algebraic complexity.

We have discussed our results with practicing managers and consultants. They have asked many questions. We choose to organize our headings and subheadings around the questions that are asked most often.

RELEVANT SET/RESPONSE MODEL IN A DUOPOLY

Advertising is a complex phenomenon that affects consumer response in myriad ways. Many analytical models have been proposed in marketing (see Lodish 1986, Little 1979b, Mahajan Muller and Sharma 1984, Parsons 1975, Sasieni 1971, Simon 1982, Teng and Thompson 1983) and in economics (Dorfman and Steiner 1954, Gould 1970, Milgrom and Roberts 1986, Nelson 1970, Nerlove and Arrow 1962, Schmalensee 1978, Telser 1962, and Vidale and Wolfe 1957). These models are often dynamic, concerned with the speed of response, decay, or carry over from one period to the next.

In our analysis we are concerned with long term, steady state response--that is, we model what happens if firms hold their spending constant long enough for sales to stabilize. In general, response analysis says that there is some relationship, a response function, that allows a manager to predict steady state sales as a function of the advertising spending by both firms in the market. Such response functions are estimated by judgment, experimentation, and/or econometrics. Although there is much debate on how best to estimate response functions (see a review in Rao and

Sabavala 1986), there is little debate that such functions exist and are relevant.

The Relevant Set

The relevant set/response model is one response model that is applied often. A brand is said to be in a consumer's relevant set if it is evaluated seriously. For example, Silk and Urban (1978) define a relevant set as those brands a consumer has used, has on hand at home, would seriously consider using, or would definitely not use. It is related to Howard and Sheth's (1969) concept of an evoked set except it includes brands consumers have evaluated and rejected.

The relevant set is a much stronger requirement than awareness. For example, unaided awareness refers to those brands a consumer can name without prompting by an interviewer. Silk and Urban (1978) reported that if 95 percent of the consumers are aware (unaided) of a brand then it is in the relevant set of only about 50 percent of the consumers. At 70 percent unaided awareness, the relevant set percentage drops by 10 percent and disappears almost entirely if unaided awareness is below 60 percent.

The Response Model

The relevant set/response model states that if a firm invests k dollars per annum on advertising, that firm's brand will be in the relevant set of A(k) consumers, where A(k) is usually stated as a fraction. Typically, a relevant set percentage of 50 percent or more takes at least a few million dollars in advertising spending.

Suppose that firm 1 achieves A_1 as a relevant set fraction and firm 2 achieves A_2. Then, as illustrated in Fig. 1, A_1A_2 consumers consider both brands, $A_1(1 - A_2)$ consider just brand 1, $A_2(1 - A_1)$ consider just brand 2, and $(1 - A_1)(1 - A_2)$ consider neither. In other words, it is as if firm 1 competes in a product/price duopoly for A_1A_2 percent of the market and a product/price monopoly for $A_1(1 - A_2)$ percent of the market. Sales are a combination of these "duopoly" and "monopoly" sales.

Insert Figure 1 About Here

In particular, if s_1 were the sales firm 1 would realize if both brands were in all consumers' relevant sets and S_1 were the sales firm 1 would realize if only brand 1 were in the relevant sets. Then firm 1's sales are given by

(1)   sales of firm 1 = $A_1A_2s_1 + A_1(1 - A_2)S_1$

(Sales of firm 2 are given by a similar equation). Note that s_1 is a function of/both prices, p_1 and p_2, while S_1 is only a function of p_1. As long as the two brands are substitutable, $s_1 \leq S_1$.

In general, the relevant set percentage for firm 1 is dependent on the spending levels of both firms, however many empirical models work quite well even though they emphasize the effect of k_1 on the relevant set percentage of firm 1. See Silk and Urban (1978) and Hauser and Gaskin (1984). For our analysis the effect of k_1 on inducing consumers to consider brand 1 has a much more important effect on strategy than does the effect of k_2. Thus we

choose a strategy of simplified exposition. We choose to emphasize the dominant effect and thus avoid unnecessary algebra and unenlightening technical conditions. In the supplemental appendix we state more general conditions that are sufficient to assure that the results hold when one accounts for secondary effects. Note, however, that even in the simplified formulation, advertising by firm 2 still has a strong effect on the sales of firm 1, the profit of firm 1, and the strategy of firm 1. It is these dominant competitive effects that we explore.

Under the above conditions, profit, π_1 of firm 1, is given by

$$(2) \quad \pi_1 = (p_1 - c_1)\{A_1(k_1)A_2(k_2)s_1(p_1,p_2) + A_1(k_1)[1 - A_2(k_2)]S_1(p_1)\} - k_1.$$

This, and the equivalent equation of π_2, is the relevant set/response model. Note that the profit of firm 1 is dependent on the price and advertising of both firms. Product design and positioning affect profit via effects on s_1 and S_1. The market is called a differentiated market because s_1 and S_1 are continuous functions of prices. (In an undifferentiated market, the lower priced brand would capture all sales.)

Some Evidence

The relevant set/response model implies: (a) consumers do not consider all brands in a market, (b) relevant sets can explain much about observed purchase behavior, and (c) inclusion of a brand in a relevant set is more or less independent of the other brands.

That consumers do not consider all brands as relevant is well documented. Silk and Urban (1978) reviewed numerous studies citing median

(and/or mean) relevant set sizes of about 3-4 brands in categories such as coffee, tea, beer, magazines, and deodorants. Such small relevant set sizes are observed even though there are 20-30 brands available in these categories. We analyzed two categories with more recent data and found the median relevant set size to be three brands in automatic dishwasher detergents and four brands in plastic wraps.[1] Even in automobiles, with over 160 brands available, Hauser, Urban, and Roberts (1983) report a median relevant set size of five brands.[2]

As evidence of its explanatory power we cite that the relevant set/response model is a critical component of the pre-test market evaluation system described in Silk and Urban (1978) and the defensive strategy system described in Hauser and Gaskin (1984). There have been over four hundred applications of the former and about twenty applications of the latter. Predictive accuracy is documented in Urban and Katz (1983) and Hauser and Gaskin (1984). While the accuracy of the relevant set/response cannot be disentangled from the full decision support models, it is doubtful that they would exhibit good predictive accuracy if the embedded relevant set model were not at least a good approximation to reality. Furthermore, in an evaluation of Silk and Urban's (1978) data, Hauser (1978) showed the 78 percent of the explainable uncertainty in the purchase data was explained by

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[1]Data from propriety Information Resources, Inc. data bases. Mean relevant set sizes were 2.9 and 3.8, respectively.

[2]The mean was slightly higher, 8.1, but the mode was only three brands.

the relevant set/response model. Only the residual 22 percent was explained by data on preference within the relevant set.[3]

Finally, we observe that equation (1) implies probabilistic independence, that is, for given levels of advertising spending the probability that a consumer considers brand 1 is independent of the probability that that consumer considers brand 2. In other words, if we know the marginal probabilities, A_j, then we should be able to predict the frequencies observed for specific relevant sets.

To test this assumption we used data on relevant sets in the market for plastic wraps. The data were available for the four largest brands. Since the number of consumers considering zero brands is unknown, we normalized the prediction. The main results are shown in table 1. Treating table 1 as a contingency table with 11 degrees of freedom (15 brands minus 4 marginal probabilities from the data), the resulting χ^2 statistic does not reject the independence assumption at the .10 level. ($\chi^2_{observed} = 14.8$, $\chi^2_{11} = 17.3$.) A similar test in the automatic dishwashing detergent category also did not reject the independence assumption.

While none of the above evidence tests equation (1) directly, we feel that the evidence, plus the wide application of the relevant set/response model, justifies further study.

Insert Table 1 About Here

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[3]A logit model was estimated on constant sum paired comparison preference measures within relevant sets. Explainable uncertainty was measured by an information theoretic statistic, U^2, which was decomposed into components due to the relevant set and preference within the set.

Some Assumptions and Conditions

To proceed further we must make some assumptions about the advertising and price response functions, $A(\bullet)$, $s(\bullet)$, and $S(\bullet)$.

We assume that advertising spending increases the relevant set proportion, but at a decreasing rate. Technically, we assume that $A_j(k_j)$ is nondecreasing and strictly concave for both firms. While this may seem restrictive, after all Little (1976) and Lodish (1986) report s-shaped response functions, profit maximization implies that either a firm should operate on the concave portion of the response curve, or not advertise at all. For our analysis, firms operate on the concave portion of the advertising response curve.

Empirically, an advertising elasticity between 0.0 and 1.0 implies a concave function. In a summary of 128 econometric studies, Assmus, Farley and Lehmann (1984) reported mean short term advertising elasticity[4] of .221 with an ANOVA grand mean of .695. In a similar study of 37 European markets, Lambin (1976, p. 98) reported a long term mean elasticity of 0.228 with all reported long term elasticities less than 1.0.

For the duopoly and monopoly price response we need not assume that s_1 and S_1 (s_2 and S_2) are concave. We need only the less restrictive assumption that firms consider prices in the ranges where the implied duopoly and monopoly profit functions, $\pi_{d1} = (p_1 - c_1)s_1$, $\pi_{m1} = (p_1 - c_1)S_1$, are concave. Such an assumption is reasonable and will apply to a wide range of important markets. For example, Hauser and Wernerfelt (1987)

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[4] These numbers apply to short run elasticities. Long run elasticities are not reported but can be derived, approximately, from their tables. Using a mean carryover effect, their table implies a long run elasticity of 0.415, which is concave.

showed that the assumption applies to <u>any</u> market described by the "Defender" price and positioning model.[5]

Finally, we state two technical assumptions we refer to in our results. (P1) is the standard economic assumption sufficient for a price equilibrium to exist. (A1) is an equivalent assumption for advertising.[6]

These conditions are not necessary for our results, but they are sufficient. Because they occur in many results, we state them in one place for easy reference.

(P1) $\quad |\partial^2 \pi_{d1}/\partial p_1^2| > |\partial^2 \pi_{d1}/\partial p_1 \partial p_2|$

(A1) $\quad |\partial^2 A_1/\partial k_1^2| > |\partial A_1/\partial k_1|^2$

Equivalent conditions apply to brand 2.

Before describing the implications of the relevant set/response models, we consider one potential motivation from a theory of rational consumer decision making. Subsequent sections do not depend on the specific motivation, only the above assumptions. We present the motivation because it ties the relevant set/response model to traditional economic theory.

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[5] The Defender price response is quasi-concave. It is not concave for all prices, but the maximum occurs on the concave portion of the curve.

[6] (A1) applies for all exponential response functions of the form $A(k) = \alpha - \beta \exp(-k/K)$ with $0 \leq \beta \leq \alpha$ and for all (constant elasticity) power response functions of the form $A(k) = (k/K)^a$ with $0 \leq a < 0.5$. (89 percent of Lambin's elasticities fall within this range as does Assmus et al.'s mean elasticity.)

EVALUATION COST MODEL

The key question in this section is whether a model of rational utility maximization implies that consumers evaluate only a small number of the available brands. The basic idea is that consumers balance evaluation cost with potential gains from further evaluation. This idea is not new. It is related to search theories in Gould (1980), Nelson (1970), and Schmalensee (1982). It is consistent with the behavioral science observation that consumers use heuristics to eliminate alternatives (Tversky and Kahneman 1974, Shugan 1980, and Bettman 1979), and it is used prescriptively to select advertising copy (Gross 1972). Our contribution is to formalize the idea in the context of relevant sets and to compare its predictions to data.

To illustrate the model we make specific assumptions about the distributions of utility (normal across brands) and search cost (lognormal across consumers).

Formal Model

For many products, such as the plastic wraps and dishwashing detergents in the above empirical examples, the utility of a given brand varies among uses in response to changes in the context in which the product is consumed. For example, plastic wraps are used for summer picnics and winter leftovers--different foods need to be stored at different times. For these products there are advantages to comparing relevant brands on each purchase occasion. On the other hand, if the number of relevant brands is large the "cost of thinking" (Shugan 1980) may be large. There is also a cost due to adding a brand to the relevant set--the opportunity cost of time, out of pocket costs, the "loss" if the brand does not fulfill expectations, etc. Thus

there is a meaningful tradeoff among (a) the cost of enlarging the relevant set and (b) the expected additional utility of choosing from a larger relevant set.

To formalize this tradeoff consider a consumer with a relevant set of n - 1 brands who is deciding whether to expand his (her) relevant set to n brands. From his (her) perspective the utility, $\tilde{u}_n$, of the nth brand is a random variable. Suppose he (she) believes it is normally distributed with mean, $\bar{u}$, and standard deviation, σ. Suppose further there is some cost, d, associated with evaluating the brand. If E_{n-1} is the maximum utility of the n - 1 brands (we make this an expectation below), then he (she) will evaluate the nth brand if the expected incremental benefit exceeds the evaluation cost. That is, if

(3) $\underline{E}[\max\{\tilde{u}_n, E_{n-1}\}] - d > E_{n-1}$

where $\underline{E}[\cdot]$ denotes probabilistic expectation.

If, a priori, all brands are independently and identically distributed (i.i.d.), then E_{n-1} is just the expectation of the maximum of n - 1 i.i.d. normal variates. It is easy to show, e.g., Gumbel (1958), that this expectation can be written in terms of $\bar{u}$, σ, and a tabled function, e_n, which represents the maximum of n standardized normal variates.[7] That is,

(4) $E_{n-1} = \underline{E}[\max\{\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_{n-1}\}] = \bar{u} + \sigma e_{n-1}$

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[7]For tables of e_n see Gumbel (1958, p. 131), Gross (1974, p. 92), and Urban and Hauser (1980, p. 385).

Recognizing that the expectation in equation (3) now becomes E_n, and rearranging terms, we rewrite (3) as

(5) $\qquad e_n - e_{n-1} > d/\sigma$

Finally, if we define $\Delta e_{n-1} \equiv e_n - e_{n-1}$ and $\lambda \equiv d/\sigma$, and assume λ is distributed across consumers with a lognormal distribution with mean, μ, and standard deviation, Σ, the fraction of consumers, P_n, who have n brands in their relevant set is just

(6) $\qquad P_n = \text{Prob}[\Delta e_n < \lambda \leq \Delta e_{n-1}] = \Lambda(\Delta e_{n-1}|\mu,\Sigma) - \Lambda(\Delta e_n|\mu,\Sigma)$

where $\Lambda(\bullet|\mu,\Sigma)$ is the cumulative lognormal distribution.

Some Implications

Equation (5) is simple but informative. The left side is just the net advantage due to choosing from n brands rather than n - 1 brands. The right side indicates that this advantage must exceed d/σ, the ratio of evaluation cost to variation in utility. As n increases the left side decreases but the right side stays constant--the net benefit decreases with n but the evaluation cost ratio stays the same. Thus, at some n*, the consumer ceases to evaluate additional brands and chooses from within the relevant set of n* brands.

Equation (5) also tells us what a brand must do to gain entry to a consumer's relevant set. One approach is to decrease evaluation cost, d, perhaps by informative advertising, free samples, or other trial

inducements. Another approach is to increase the upper tail of the utility distribution.[8] perhaps by stressing unique selling points or the brand's core benefits. Of course, all of this assumes the brand passes a low-evaluation-cost heuristic screen to be considered for the relevant set. Thus, advertising must also achieve awareness, encourage consumers to consider a brand, and seek to influence the heuristic screen. From the standpoint of response analysis, more dollar spending allows a brand to do all of these more effectively, but as the brand reaches more and more (marginal) consumers, it faces a greater evaluation ratio, d/σ, and the task becomes more difficult--that is, we expect $A(k)$ to be an increasing and concave function.

The last implication of interest is whether it is easier or more difficult to enter the relevant set of a consumer who has more brands in his relevant set. For a given evaluation cost ratio, equation (5) confirms that it is more difficult if n is larger. However, equation (6) also affirms that consumers who evaluate n brands have lower evaluation cost ratios. The net effect turns out to be ambiguous since it depends on a series of joint probability distributions--it becomes an empirical rather than a theoretical question. We feel that the relevant set/response analysis assumption of independence is as good a place to start as any. It is parsimonious and not rejected by the data.

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[8]In equation (5), σ influences the upper tail of the normal distribution. For general density functions we can show it is the upper tail that matters.

Empirical Evidence

Given data on relevant frequencies, equation (6) implies a histogram

for λ. We took the published relevant set frequencies in Silk and Urban

(1978) for deodorants and the frequencies for plastic wraps from the Hauser

and Gaskin (1984) data set and fit lognormal distributions to the implied

histograms. See Figure 2. Since there is no guarantee that the histograms

should have even unimodal shapes, we find the "lognormalness" of the

histograms encouraging. Applying a contingency test to the predicted and

observed frequencies, we found that neither fit could be rejected at the .05

level.[9] (χ^2_5 = 3.65 for deodorants, χ^2_5 = 3.48 for plastic wraps.)

Insert Figure 2 About Here

Summary of Evaluation Cost Model

The evaluation cost model provides one underlying motivation for

relevant set/response analysis. We are encouraged by its agreement with

published empirical analyses. We turn now to the strategic implications of

relevant set/response analysis. The evaluation cost model makes the

strategic implications more believable, but it is not a necessary condition

for their validity.

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[9]The actual and predicted frequencies for deodorants are for n = 2 to
7:/.318, .231., 071., 040, .023 versus .312, .341, .196, .090, .037,. 024.
For plastic wraps: .161., .239, .279, .178, .074, .027 versus .158, .298,
.248, .151, .075, .032. For deodorants, μ = .25, Σ = .13. For plastic
wraps, μ = .195, Σ = .10.

THE ADVERTISING EQUILIBRIUM

The equilibrium concept we use is that of a Nash equilibrium. Formally, the equilibrium (if it exists) can be thought of as independent rational choices by each firm (a single stage game), but we find the following informal description easier to visualize. Suppose that prices, and hence s_j and S_j, are fixed and that, in the first period, the firms spend k_1^1 and k_2^1, respectively, on advertising. In period 2, firm 1 observes its competitor's advertising, k_2^1, and reacts accordingly, choosing k_1^2 to maximize profit. Firm 2 does likewise. This continues for period after period until the market stabilizes at k_1^* and k_2^*, the Nash equilibrium spending levels. Notice that, at equilibrium, k_1^* is the best response by firm 1 to an advertising level of k_2^* by firm 2. Similarly, k_2^* is the best response to k_1^*.

If the equilibrium exists and is unique we can find it in at least two ways. Numerically, we can start at any value of k_1 and k_2, compute the best responses, and continue until the market stabilizes.[10] Alternatively, we can solve simultaneously for k_1^* and k_2^* such that $\pi_1(k_1^*, k_2^*)$ and $\pi_2(k_2^*, k_1^*)$ are maximized. Both methods give us the same answers.

Notice that we have assumed implicitly what game theorists call zero conjectural variations (ZCV). That is, we assume that neither firm anticipates that its change in advertising will cause its competitor to react. At equilibrium, zero conjectural variations are consistent with behavior. ZCV is common in most economic analyses, and our informal

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[10]For exponential or power response functions we have found that the market stabilizes quite rapidly.

experience suggests that it is a reasonable descriptive model. However, we
return to this assumption in a later section to discuss its implications.

Existence and Uniqueness

Result 1. For fixed prices, the advertising equilibrium exists and, when
(A1) holds, it is unique.

Result 1 is proven using Rosen's (1975) theorem. It is useful because
it allows us to talk about the properties of the advertising equilibria and
to find them numerically. The equilibrium may be unique if (A1) does not
hold, but it is always unique when (A1) holds.

Benchmark--Non-Competitive Analysis

To appreciate better the interactive nature of the equilibria, consider
a noncompetitive formulation in which the response function simply scales
s_1, that is

(NC1) $$\pi_1^{NC} = A_1(k_1)(p_1 - c_1)s_1 - k_1$$

with a similar equation for firm 2. The optimality conditions are

(NC2) $$\partial A_1(k_1^*)/\partial k_1 = [(p_1 - c_1)s_1]^{-1}$$

Since $A_1(\cdot)$ is a concave function, smaller slope $(\partial A_1/\partial k_1)$ implies larger
k_1^*. Hence, in this formulation, the larger the value of $(p_j - c_j)s_j$ the

more a firm advertises. If the margins, $p_j - c_j$, are equal, the firm with the largest unadjusted sales, s_j, advertises more.

We do not propose (NC1) as a model of firm behavior since marketing science theory is moving toward game theoretic foundations, but we do note that some recent models (e.g., Kumar and Sudharsen 1986, Rao and Sabavala 1986, and Hauser and Shugan 1983) use conditions similar to (NC1) and (NC2). For our purposes (NC2) serves as a benchmark.

Does Competitive Analysis Make a Difference?

Our first analyses deal with the "comparative statics." That is, we change external conditions, such as the fixed s_1, and see how the equilibrium advertising changes. These results are much more complex than the implications of (NC2). Although they are derived analytically, it is as if we (1) observed the advertising equilibrium for one set of s_1, S_1, etc., (2) changed the values of s_1, S_1, etc., (3) allowed both firms to find a new equilibrium, and (4) observed how they changed. We focus on firm 1; the results are symmetric for firm 2. Condition A1 is assumed.

Result 2. The equilibrium advertising, k_1^*, increases if duopoly sales, s_1, monopoly sales, S_1, price, p_1, or the competitor's cost, c_2, increase (while all else remains unchanged). It decreases if costs, c_1, the competitor's price, p_2, or the competitor's sales, s_2 or S_2, increase.

Result 3. If greater duopoly sales ($s_1 > s_2$) implies greater or equal monopoly sales ($S_1 \geq S_2$), margins, $p_j - c_j$, are equal, and the same response functions apply to both firms, then the firm with the larger unadjusted

market share, $m_j = s_j/(s_1 + s_2)$, advertises more in equilibrium. That is, if $m_1 \geq m_2$, then $k_1^* \geq k_2^*$.

Results 2 and 3 are comfortable results. Both make intuitive sense and do not contradict the simpler, noncompetitive analysis. However, one difference is that the effects of competitive actions (p_2, c_2, s_2, S_2) and monopoly sales, S_1, now influence advertising spending. This explicit dependence is intuitive and opens doors for more complex analysis. Notice also that Results 2 and 3 depend only on (A1) and concavity. The implied qualitative changes can be implemented without specific knowledge of the response functions.

We deal explicitly with price later. For now we caution the reader that Result 2 indicates what happens when either price or sales change but the other does not. For example, Result 2 applies if sales change due to product improvement without a price change, or if price changes but distribution incentives change simultaneously to maintain sales. When price changes affect s_1 and/or S_1, the net effect is ambiguous.

Finally, we comment on Result 3. Suppose firm 1 has a greater unadjusted share, m_j, for reasons unrelated to price or advertising. Perhaps it has a superior product, a better position, or entered the market earlier. Then, even if the response functions favor neither firm, firm 1 will find it in its own best interest to advertise more. The result is not mysterious and needs no complex explanation of market power. The greater unadjusted share simply means that profit maximization leads to greater advertising by the market share leader. Thus, even if the smaller firm benefits more at the margin because its response function is steeper for

smaller k, competitive interaction leads to smaller advertising for the smaller firm.

Result 4. The competitive formulation implies a greater or equal advertising spending than the noncompetitive formulation.

Result 4 cautions the academic marketing analyst that he (she) will likely underestimate advertising spending if competitive interactions are ignored. Result 4 cautions the brand manager that competitive considerations are likely to drive up advertising spending relative to planned levels if the brand plan simply chooses advertising based on current competitive scenarios.

If Advertising Is More Cost Effective, Does One Spend More?

Suppose that the product category becomes more important to consumers. e.g., high fiber cereals following a Surgeon General's report, or media prices suddenly change, e.g., the 1987 tax on advertising by the State of Florida (Agnew, 1987). Then, for a given spending level, a brand can get into the relevant sets of more (or fewer) consumers. We would say that the cost effectiveness of advertising has changed. It is reasonable to wonder whether this change in effectiveness causes brands to spend more or less on advertising.

To model this phenomenon we write $A_j(k_j)$ as $A_j(k_j/K_j)$ where K_j parameterizes the cost (or effectiveness) of advertising. As advertising becomes more effective, K_j decreases and it takes less spending to achieve a given consideration percentage.

The competitive model in equation (2) gives us the mechanism with which to address this question for any specific problem, but, in general, it is difficult to give guidelines without knowing the specific response function and current spending levels. Equilibrium advertising can either increase or decrease as K_j changes.

What we can say is that the competitive formulation gives very different recommendations than does the noncompetitive formulation (equation NC1). We illustrate this difference with a constant elasticity response function, $A_j = (k_j/K_j)^a$ where we choose the long term elasticity, a = 0.415, as computed from the mean in the Assmus, et al. (1984) meta-analysis.[11] (Similar results are obtained for the mean of Lambin's (1984) meta-analysis.) To maintain logical consistency with A_j as a percentage, we allow A_j to saturate for $k_j \leq K_j$. That is, $A_j \equiv 1$ whenever k_j exceeds the cost of saturation, K_j.

Insert Figure 3 About Here

Figure 3a plots optimal advertising (k_1^*) versus the cost of saturation (K_1) for the noncompetitive formulation and figure 3b is an equivalent plot for the equilibrium advertising in the competitive formulation. Notice the qualitative difference. For low saturation cost, both models imply brand 1 will saturate by setting k_1^* equal to K_1, thus as K_1 rises so does k_1^*. For moderate saturation cost, the two models continue to agree. In the noncompetitive formulation the increased efficiency of the media overwhelms

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[11]For illustration we set $p_1 - c_1 = p_2 - c_2 = 1$, $S_1 = S_2 = s_1 = s_2 = 10$, and $m_1 = 0.2$.

the increased cost; the net impact is decreased spending and dramatic decreased awareness. The competitive formulation mimics the noncompetitive formulation in this middle range because in this middle range brand 2, the larger share brand, still chooses to saturate, hence the spending of brand 1 has no impact on the spending of brand 2.

Insert Figure 3 About Here

Competitive phenomena became apparent in the region of higher saturation cost. Here neither firm saturates; the spending of each firm influences that of the other firm. Not only does equilibrium advertising reach a higher level (result 4), but the qualitative implications differ. In the competitive formulation advertising spending rises then falls to an asymptote; in the noncompetitive formulation it falls toward zero.

We feel figure 3 illustrates dramatically how competitive interactions can lead to greater spending and complex response to changes in the media environment. Note also that figure 3 is not a pathological case. The constant elasticity response model is used widely by both academics and business and the key parameter in figure 3 is chosen from a broad-based meta-analysis.

Summary

We have demonstrated that the advertising equilibrium exists, is unique, and has many intuitive properties. Some qualitative results agree with the noncompetitive formulation, however noncompetitive formulations underestimate spending and miss complex, interactive phenomena. We have

also shown that larger share brands advertise more (all else being equal) and advertising effectiveness has complex yet interesting effects on spending.

We now examine how advertising affects the price equilibrium.

DOES ADVERTISING AFFECT PRICE EQUILIBRIA?

Existence and Uniqueness

Result 5. For fixed advertising, the price equilibrium exists and, when (P1) holds, it is unique.

Result 5 says simply that the condition sufficient for a unique duopoly equilibrium (P1) is also sufficient for a unique equilibrium when firms compete in the partial duopoly, partial monopoly implied by the relevant set/response model. Result 5 is not surprising, but it does enable us to examine the price equilibria knowing they exist and are unique.

Modified Price Elasticity

Empirically, models of unadjusted duopoly, $s_1(p_1, p_2)$, such as the Defender consumer model (Hauser and Shugan 1983), give estimates of elasticity that are quite large, say 10.0 or more. However, when embedded in relevant set/response analysis, such models predict well (Hauser and Gaskin, 1984) and give reasonable elasticities. Similarly, Silk and Urban's (1978) price model accurately portrays price response when embedded in a relevant set/response analysis.

One explanation is that relevant set/response analysis moderates price response. Indeed, for two brands, if ϵ_{d1} is the (duopoly) price elasticity and ϵ_{m1} is the (monopoly) price elasticity, then we have:[12]

Result 6. The effective price elasticity, ϵ_r, in relevant set/response analysis is a convex combination of the monopoly and duopoly price elasticities. In particular, for brand 1:

$$(7) \qquad \epsilon_{r1} = \frac{A_2 s_1 \epsilon_{d1} + (1 - A_2) S_1 \epsilon_{m1}}{A_2 s_1 + (1 - A_2) S_1}$$

A similar equation applies to brand 2.

Clearly, $\epsilon_{m1} \leq \epsilon_{r1} \leq \epsilon_{d1}$. Thus, Result 6 explains why, contrary to first examination, the Defender and Assessor price response models give elasticities that are reasonable empirically. For example, if s_1 is one-half of S_1, A_2 is about 0.5, and the monopoly response is more or less inelastic, then the effective elasticity is about one-fourth of the duopoly elasticity. That is, if the consumer model gives a duopoly elasticity of 10, then the effective elasticity is 2.5. This value is well within the ranges reported by Lambin (1976, p. 103) in his meta-analysis of 37 econometric studies.

We feel that Result 6 goes a long way toward explaining why models like Defender and Assessor work in practice.

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[12]By elasticity we mean $\epsilon_{d1} = -(p_1/s_1)\partial s_1/\partial p_1$, $e_{m1} = -(p_1/S_1)\partial S_1/\partial p_1$, and $\epsilon_{r1} = -(p_1/\text{sales } 1)\partial(\text{sales } 1)/\partial p_1$.

Effect of Advertising on Equilibrium Prices

If we compute the equilibrium prices for the noncompetitive formulation (equation (NC1)), it is easy to show that they are independent of advertising levels, k_1 and k_2. In fact, Hauser and Shugan (1983, Theorem 6), formalized this result when one firm responds to another's fixed strategy. Things change in the competitive formulation (equation (2)); changes in advertising affect the equilibrium prices in interesting ways.

Result 7. If condition (P1) holds, then an increase in brand 1's advertising spending, k_1, increases brand 1's equilibrium price, p_1^*, if $\partial^2 \pi_{d1}/\partial p_1 \partial p_2 < 0$. It decreases brand 1's equilibrium price if $\partial^2 \pi_{d1}/\partial p_1 \partial p_2 > 0$.

Result 8. If condition (P1) holds, then an increase in brand 2's advertising spending, k_2, decreases brand 1's equilibrium price, p_1^*.

From Result 7 we get that the effect of advertising on equilibrium price depends on the shape of the demand curve--in particular, the interaction of p_1 and p_2. For example, for the constant elasticity or the exponential demand curves, an increase in firm 1's advertising always increases firm 1's equilibrium price while for the linear demand curve it decreases the equilibrium price.[13]

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[13] By constant elasticity, we mean $s_1 = p_1^{-\varepsilon_{d1}} f(p_2)$, by exponential, $s_1 = e^{-\gamma p_1} f(p_2)$, where $f' > 0$. By linear we mean $s_1 = \alpha - \beta_1 p_1 - \beta_2 p_2$.

Result 7 cautions academic researchers and decision support system users that a seemingly innocent choice of functional form is important. Result 7 suggests that great care should be exercised when choosing the shape of the demand curve, long before specific parameters are estimated.

Result 8 is less sensitive to the shape of the demand curve; an increase in competitive advertising always lowers firm 1's equilibrium price. Recall that Results 7 and 8 refer to the net result of action and counteraction, not just the first response.

We close with a result on how unadjusted market share affects equilibrium price.

Result 9. If $A_1 = A_2$, $c_1 = c_2$, and Condition (P1) holds, then the brand that can achieve more sales for a given set of prices (i.e., $s_1(p,q) > s_2(p,q)$ and $S_1(p) \geq S_2(p)$), will choose a smaller (or equal) price in equilibrium (i.e., $p_1^* \leq p_2^*$).

WHAT IF PRICE AND ADVERTISING ARE SET SIMULTANEOUSLY?

We have seen already that price affects the advertising equilibrium and advertising affects the price equilibrium. In general, firms set price and advertising more or less simultaneously. When this happens the algebra becomes complex, but the equilibrium exists and, under conditions that generalize (A1) and (P1), it is unique. See the appendix for Result 12, a more formal result.

The concepts behind Results 1-9 apply for the more complex case but the technical conditions are much less transparent than (A1) or (P1). After all, equation (2) implies complex interactions, and the technical conditions

deal with these interactions. We feel that Results 1-9 are easier to understand than more technical results. For the interested reader, Result 12 gives a flavor for the more complex results. The proof methods in the supplemental appendix are readily extendable.

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DOES IT PAY TO BE SMART?

Our final topic relaxes the zero conjectural variation assumption to allow firms to anticipate one another. Because anticipation of price response is covered elsewhere (e.g., Friedman 1977), we concentrate on anticipation of advertising response. To illustrate the effect, we consider the case where brands are otherwise symmetric, $s_1 = s_2$, $S_1 = S_2$, and $A_1(k_1) = A_2(k_2)$ when $k_1 = k_2$. We assume, of course, that $s_1 \leq S_1$.

By "smart" we mean that one brand anticipates the other's response. For example, brand 1 may anticipate that a change of one unit of advertising spending will evoke a response of δ_1 units in competitive advertising. Brand 1 would then choose k_1 to maximize profit, but in doing so would take into account brand 2's response.[14] Brand 2 may or may not anticipate brand 1. As before, firms react, re-react, re-re-react, etc., until the market stabilizes at some set of advertising levels.

For comparison we let k_1^* and k_2^* continue to denote the equilibrium under zero conjectural variation. We let k_1^c and k_2^c denote the collusive (or cooperative) spending that would result if the two firms agreed to set advertising at a level which would maximize joint profits. Let k_1^δ and k_2^δ be

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[14]Technically, firm 1 solves the first order conditions. $d\pi_1/dk_1 = (\partial\pi_1/\partial k_1) + (\partial\pi_1/\partial k_2)(\partial k_2/\partial k_1) = 0$ where $\partial k_2/\partial k_1$ is assumed to equal δ_1.

the spending levels when firm 1 anticipates a response of δ units. We will make it clear from context the value of δ_2. Finally, let π_j^*, π_j^c, and π_j^δ be the corresponding profits.

Result 10. When $s_1 = s_2$, $S_1 = S_2$, and $A_1(\cdot) = A_2(\cdot)$, each firm anticipates the other, anticipations are equal, and $0 \leq \delta_1 = \delta_2 \leq 1$, then greater anticipation means less advertising spending and greater profit. Full anticipation ($\delta_j = 1$) implies spending (and profits) at the collusive level.

In symbols, Result 10 implies that $k_1^c \leq k_1^\delta \leq k_1^*$ and $\pi_1^c \geq \pi_1^\delta \geq \pi_1^*$. (By definition $\delta = 0$ implies ZCV.)

Thus, if both firms have equal anticipations, it is better to be "smart." The result on collusive profits should not surprise us since $\delta_j = 1$ implies that each firm anticipates that the other will match its advertising level.

Result 10 seems to imply that smartness pays--but it counts on both firms being smart. Result 11 addresses the question of what happens if only one firm becomes "smart."

Result 11. When $s_1 = s_2$, $S_1 = S_2$, and $A_1(\cdot) = A_2(\cdot)$, condition (A1) holds, firm 1 anticipates firm 2's response, but firm 2 does not anticipate firm 1's response, and $0 \leq \delta_1 \leq 1$, then greater anticipation means: (a) less advertising for firm 1, (b) more advertising for firm 2, (c) less profit for firm 1, and (d) more profit for firm 2.

Interestingly, if only one firm is smart, it pays to be dumb (or perhaps just very clever--its depends upon your viewpoint). When firm 1 unilaterally anticipates firm 2, firm 1 is worse off and firm 2 is better off.

At first result 11 seems counterintuitive--why would competitive intelligence hurt a brand? Result 11 becomes more clear when we realize that the game is being played with respect to advertising spending and that the advertising game is like a prisoner's dilemma (PD). That is, joint cooperation means lower spending and more profit, whereas unilateral cuts in spending decreases profit. Thus, joint anticipation is equivalent to greater cooperation--it helps--whereas unilateral anticipation is like unilateral attempts at cooperation--it may hurt. For more discussion of cooperation in PD-like games see Axelrod (1984) or the so-called "folk theorems" of industrial organization (e.g., Jacquemin and Slade 1987).

Summary

Like competitive advertising itself, anticipation is complex. If both firms are "smart," both are better off. But anticipation can be dangerous. Unilateral anticipation can actually hurt profits.

SUMMARY AND FUTURE DIRECTIONS

We have examined the strategic implications of the widely applied relevant set model of advertising response. Nash equilibria exist and, under reasonable conditions, are unique. In most cases, externally set variables affect these equilibria in intuitive ways. But in some ways the equilibria differ from those implied by the noncompetitive response model.

For example, competitive considerations lead to higher advertising levels and such levels are sensitive to pricing decisions by both firms. Advertising effectiveness has a complex, yet understandable, influence on the optimal advertising spending.

We have shown that relevant set/response analysis has an empirically important moderating effect on price elasticities and that advertising's influence on price depends critically on the shape of the demand curve.

If both firms anticipate, both are better off, but unilateral anticipation makes firms worse off.

We have also provided a theoretical motivation for relevant set/response analysis. That is, consumers balance evaluation cost with expected incremental benefits. This explanation seems to fit observed relevant set frequencies within statistical confidence limits.

We feel that the evaluation cost motivation and the strategic implications help marketing scientists to understand better this relevant set model of response analysis which has had a major impact on marketing decision support systems. But our analyses are just a beginning.

We have used empirically documented models of consumer response and have illustrated their impact with published parameter values. However, there is no guarantee that firms follow the zero conjectural variation or even the anticipation models. Just because such models are "accepted" in economic game theory does not make them valid.

We feel that the next critical research step is to observe markets and see if the advertising and price equilibria behave as predicted by Results 1-11. We feel they will, but science demands the predictions be put to

test. (Although our results are for a duopoly they are extendable, through numerical analysis if necessary, to more than two firms.)

Finally, we feel that the evaluation cost model has many implications beyond relevant set/response analysis. For example, it might be used to study promotion, product sampling, product line phenomena, retail clustering, and first entrant advantages.

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Appendix

The following result gives the more general sufficient conditions for equilibria when advertising and price are set simultaneously.

See Supplemental Appendix I for proofs of results 1 through 12 and Supplemental Appendix II for generalization to the case where the relevant set percentage depends on competitive advertising as well as each brand's own advertising. The qualitative implications of results 1 through 12 do not change, but the technical conditions, A1 and P1, are more complex.

Result 12. If advertising and price are set simultaneously, then the zero conjectural variation Nash equilibrium exists and if conditions (AP1) and (AP2) hold, it is unique.

(AP1)
$$A_2[\partial^2\pi_{d1}/\partial p_1\partial p_2 - \partial^2\pi_{d1}/\partial p_1^2] + (1 - A_2)(\partial^2\pi_{m1}/\partial p_1^2)$$
$$+ (\partial A_2/\partial k_2)|\partial\pi_{d1}/\partial p_1 - \partial\pi_{m1}/\partial p_1| < 0$$

(AP2)
$$(\partial A_1/\partial k_1)^2[A_2(\partial\pi_{d1}/\partial p_2) + (\partial A_2/\partial k_2)(\pi_{m1} - \pi_{d1})] + \partial^2 A_1/\partial k_1^2 < 0$$

Table 1

TESTS OF INDEPENDENCE ASSUMPTIONS FOR PLASTIC WRAPS

Relevant Set Number	Brands Considered*				Predicted Number of Consumers	Observed Number of Consumers
	A	B	C	D		
1	1	1	1	1	8.9	6
2	1	1	1	0	27.3	22
3	1	1	0	1	2.9	4
4	1	1	0	0	8.9	11
5	1	0	1	1	14.1	9
6	1	0	1	0	43.4	46
7	1	0	0	1	4.6	5
8	1	0	0	0	14.1	21
9	0	1	1	1	12.4	11
10	0	1	1	0	38.3	35
11	0	1	0	1	4.6	6
12	0	1	0	0	12.4	20
13	0	0	1	1	19.8	26
14	0	0	1	0	60.9	70
15	0	0	0	1	6.4	6
16	0	0	0	0	no data	no data

*"1" indicates the brand was considered; "0" indicates it was not. The identity of the major national brands are disguised due to confidentiality. Brand D is the store brand.

Figure 1

ILLUSTRATION OF HOW ADVERTISING AFFECTS RELEVANT SETS

Brand 1's
Advertising

A_1

	Duopoly	"Monopoly" for Brand 1
$1 - A_1$	"Monopoly" for Brand 2	Untapped Market

A_2 $1 - A_2$

Brand 2's
Advertising

Figure 2. Comparison of Histograms Implied By the
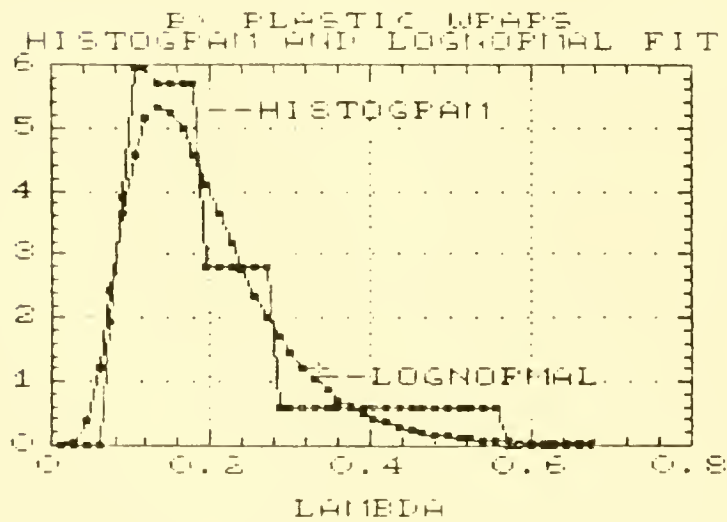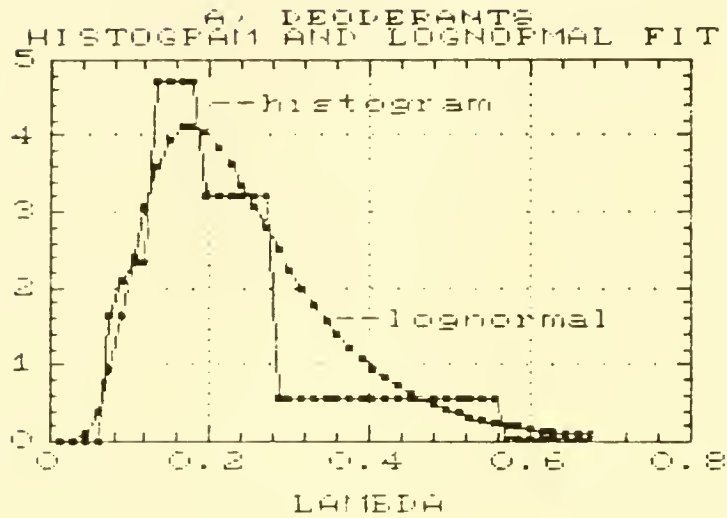Evaluation Cost Model with Lognormal Distributions

A. DEODERANTS
HISTOGRAM AND LOGNORMAL FIT

--histogram

--lognormal

LAMBDA

B. PLASTIC WRAPS
HISTOGRAM AND LOGNORMAL FIT

--HISTOGRAM

--LOGNORMAL

LAMBDA
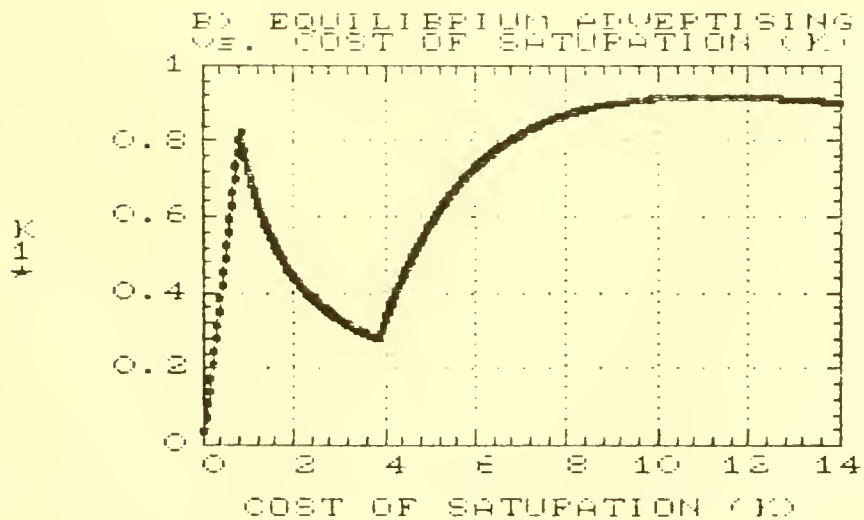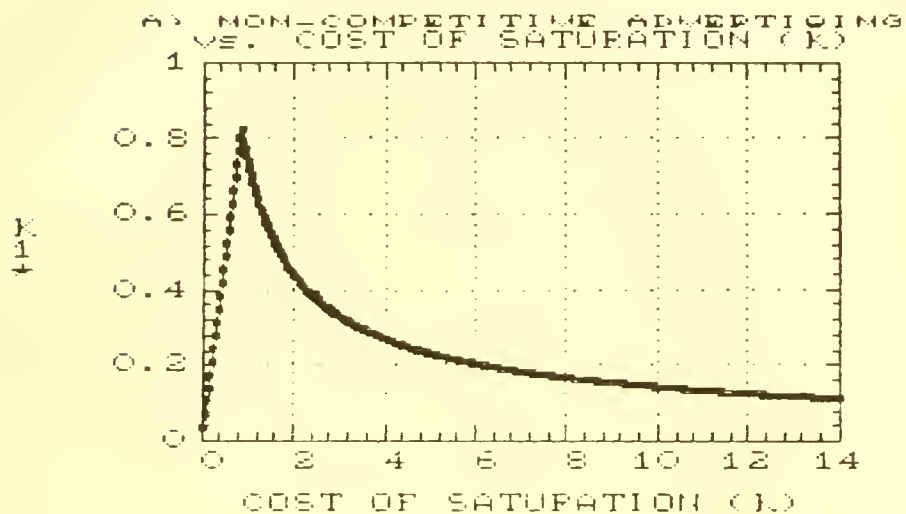
Figure 3. Comparison of Optimal Advertising as Implied by
(a) Non-competitive Formulation and (b) Equilibrium Analysis

Date Due