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CROSS SECTIONAL EXPENDITURE SYSTEM ESTIMATION USING A VARYING SECOND STAGE UTILITY FUNCTION

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Abstract

Consumers are assumed to employ a two-stage utility maximization procedure. Households have identical indifference curves for aggregate commodities but may differ in the intra-commodity allocations of elementary goods. The varying second stage (VSS) induces price variation in the commodities of the first stage, enabling price effects to be estimated from cross-sectional data.

The VSS methodology is demonstrated with a linear expenditure system. Data from a U.S. Consumer Expenditure Survey is used to estimate separate expenditure systems for households with different income levels and ages. The results are interpreted with emphasis on welfare comparisons and price effects.

Cross-Sectional Expenditure System Estimation

Using a Varying Second-Stage Utility Function[1]

by Arthur Lewbel[2]

1. Introduction

A common claim made in consumer demand models is that all consumers face the same prices at the same time.[3] While this is a reasonable assumption for elementary goods, it does not follow that prices for aggregate commodities such as food or clothing will be the same for everyone, since the price measure of an aggregate will depend on the mix of goods comprising it, which may differ from consumer to consumer.

Assume that a fairly homogeneous group of consumers each employs a utility tree, that is, a two-stage utility maximization procedure, to allocate their expenditures. It is plausible that in the first stage, these consumers each employ similar schemes to allocate income to aggregate commodities. In the second stage, it is unlikely that any two consumers would purchase the exact same market basket of goods.

In this paper, it is shown how a two-stage utility function may be specified whose parameters are the same in the first stage, while varying from consumer to consumer in the second stage. Aside from the theoretical advantages of a specification that permits consumers to differ by more than a stochastic component, this type of utility function has the property of permitting both price and income effects to be estimated from purely cross-sectional data.

This methodology differs substantially from the Adult Equivalent scales in [22] and the Household Equivalent scales in [2, 16]. These previous approaches are based on parameterizing differences in households' utility functions, based on demographic characteristics of the household.

The utility tree structure employed in this paper eliminates the need for any such explicit demographic parameterization. A separate expenditure system may be estimated for any chosen group of households, without regard for other groups. In addition, the underidentification of price effects found with family equivalent scales in [16] is not a problem here. The method also does not employ any cardinal utility notions, such as want-independence, as in [10].

In this paper's varying second stages (VSS) model, consumers are assumed to have different intrabranch (or second-stage) budgeting procedures. Stoker [24, p. 32] observed that "when individual allocation processes are allowed to vary across consumers, the aggregate prices also vary across consumers." For Stoker, this price variation was a hindrance to appropriate aggregation of consumers. Here it is recognized as a means of measuring price effects with cross-sectional data.

In the next section, the VSS utility approach is formally presented. The Linear Expenditure System (LES) is shown to be particularly amenable to a VSS specification. The method introduces both simplifications and complications in the estimation of an LES, which are discussed. The remainder of the paper is then devoted to an application. Data from a single U.S. Consumer Expenditure Survey is used to estimate complete Linear Expenditure Systems for some demographic groups, and differences in the estimated parameters are discussed. For example, elderly and high income earners are seen to have generally lower price elasticities than other groups. The use of Frisch's [10] money flexibility as a welfare indicator is discussed in light of these results.

2. The VSS Methodology

Here the method of varying second-stage utility functions is derived and interpreted.

Let X be the set of households under consideration.

T is the set of all time periods under consideration, or more generally, the set of different price regimes under consideration, such as different countries.

J is the set of all aggregate commodities under consideration.

I is the set of all individual goods under consideration.

I_j is the set of all individual goods comprising commodity $j \varepsilon J$.

P_{ij}^{xt} is the price of good i, which is a component of aggregate commodity j, for household x, in time period t.

Q_{ij}^{xt} is the quantity of same.

E_{ij}^{xt} is the expenditures on same.

P_j^{xt} is the price index of the aggregate commodity j, for household x, at time t.

Q_j^{xt} is the quantity measure of same.

E_j^{xt} is expenditures on same.

E^{xt} is total expenditures for household x at time t.

U^x is the utility function of household x.

U_j^x is the second-stage utility function of the aggregate j for household x.

Q_j^{xt} will be defined as the value of a certain monotonic transformation of the function U_j^x evaluated at quantitites of Q_{ij}^{xt} leaving

(1) $P_j^{xt} = E_j^{xt}/Q_j^{xt}$

as a definition for the aggregate price index.

For a utility function that permits strong additive price aggregation, the two-stage budgeting procedure is

(2)

a) Maximize $U^x(Q_j^{xt}, j\varepsilon J)$

subject to $\sum_{j\varepsilon J} P_j^{xt} Q_j^{xt} = E^{xt}$

b) Maximize $U_j^x(Q_{ij}^{xt}, i\varepsilon I_j) \equiv Q_j^{xt}$ for each $j\varepsilon J$

subject to $\sum_{i\varepsilon j} P_{ij}^{xt} Q_{ij}^{xt} = E_j^{xt}$

A necessary and sufficient condition for the above two-stage budgeting procedure to result in a global utility maximum is that U^x be homothetically separable as written and that some monotonic transformation of U_j^x be a homogeneous of degree one function of its arguments.[4] This is the convenient form in which to define Q_j^{xt}, and is what the Gorman Polar form reduces to given strong additive price aggregation.

$$(3) \quad P_{ij}^{xt} = P_{ij}^{yt} \text{ for all } x\varepsilon X,\ y\varepsilon X,\ t\varepsilon T,\ i\varepsilon I_j,\ j\varepsilon J$$

Equation (3) will be assumed to hold in all that follows, so the household superscript can be dropped from elementary good prices.

Now consider a set of households that, because of similarities in some observable demographic characteristics, are assumed to have identical utility functions. Observable differences in purchased quantitites in a given time period will be attributed only to differences in income levels. When data is only available for a single point in time, the expenditure system will reduce to a model of Engel curves.

Now relax the assumption of identical utility functions. Instead assume only that U^x is the same function of aggregate quantities for each household. The aggregate quantities themselves, corresponding to the second-stage utility functions, are permitted to differ from household to household. Second-stage equations will not actually be estimated, so this assumption will not introduce any loss in degrees of freedom from VSS utility function parameters.

It is not contradictory to assume consumers have identical first-stage utility functions while differing in the second stage.

Consumers with identical incomes may have the same general willingness to substitute between gross aggregates such as food and clothing, while differing in opinion on details such as the substitutability of margarine for butter. This is more plausible than assuming that they do agree to the last detail.

This raises the question of how to interpret the parameters of the estimated first-stage utility function. For example, consider an estimated elasticity of food prices on food expenditures. The mix of elementary goods that elasticity corresponds to will vary from person to person.[5] This can be contrasted to estimated utility parameters from aggregate macroeconomic data, where the commodity whose elasticity is being measured doesn't vary, but the household it applies to is a nonexistent representative consumer. Worse than differing from household to household, the usual macroeconomic approach produces elasticities that have no known meaning for an individual consumer or subset of consumers.[6]

Going to the other extreme, ordinary models of household budget data assume consumers with like demographic characteristics have utility functions that differ at most by a stochastic component. To be more than Engel curves, such models require household data under varying price regimes, such as for different time periods, which are frequently unavailable. Furthermore, the assumption that households have completely identical utility functions becomes even less plausible when the sample is drawn from differing geographical regions, times, or other circumstances that provide such alternative price regimes. These assumptions and data requirements are a high price to pay for ease in interpretability.

Estimated parameters of a utility function whose arguments differ from person to person may have less intuitive appeal than more conventional models. Nevertheless, the VSS has the advantages of having a precise meaning for individual consumers, has less restrictive assumptions than the usual cross-sectional household data models (including equivalent scales models) and can reveal price effects without requiring data to span multiple price regimes.

3. A VSS Model

As an example of a VSS utility function, consider the two-stage Linear Expenditure System. The demand equation for each commodity j is

$$(4)\quad E_j^{xt} = b_j(E^{xt} - \sum_{k \in J} c_k P_k^{xt}) + c_j P_j^{xt} + e_j^{xt}$$

This is only the first stage of the utility function, which is all that can be estimated. If the entire utility function (encompassing both stages) is also assumed to be of linear expenditure form, the demand equations for elementary goods would be

$$(5)\quad E_{ij}^{xt} = b_{ij}^{x} (E^{xt} - \sum_{l \in I} c_{lj}^{x} P_{lj}^{t}) + c_{ij}^{x} P_{ij}^{t} + e_{ij}^{xt}$$

An error term e is included in the equations above, although for a given household, a non-zero error cannot be interpreted as an offset to a constant term, since the presence of such a term is not consistent with the utility function that generates the LES.[7]

The utility function parameters in the second stage have a household superscript indicating that they differ from consumer to consumer. One attractive property of the LES is that the parameters in equations (4) and (5) are related in a very simple way, which is

$$(6)\quad C_j = \sum_{i \varepsilon I_j} C^x_{ij}\,,\; b_j = \sum_{i \varepsilon I_j} b^x_{ij} \quad \text{for all } x\varepsilon X,\ j\varepsilon J.$$

This shows the freedom consumers are given to vary from each other within the confines of the common first stage of equation (4).

Another advantage of the LES lies in the ease with which aggregate commodity price indices (distinct for each household) may be specified. The commodity price indices that make equation (4) consistent with equation (5) are

$$(7)\quad P^{xt}_j = \frac{\Sigma P^t_{ij} c^x_{ij}}{\Sigma c^x_{ij}} = \frac{\Sigma P^t_{ij} c^x_{ij}}{\Sigma P^0_{ij} c^x_{ij}} \quad \text{for } x\varepsilon X,\ t\varepsilon T,\ j\varepsilon J \text{ and summations over all } i\varepsilon I_j$$

The second equality stems from prices in the base year being equal to one. Equation (7) can be compared to a Paasche index from which it differs only to the extent that purchased quantity proportions differ from committed quantity proportions. Recall that second stages cannot be estimated because no elementary good price variation by consumer is available. Estimates of C^x_{ij} are therefore not available, so a Paasche index will be used to approximate the exact index of equation (7). Diewart [9] would advocate use of the ideal, rather than the Paasche index for a general linear homogeneous utility function, but in the case of an LES, the Paasche is more readily interpreted. This Paasche approximation is virtually identical to the one given in [7, p. 154] with similar motivations. Note that since the true indices are not actually constructed, model estimates may be interpreted as arising from any utility function that can be cast in hierarchic form with equation (4) as the first stage.

For estimation, equation (4) differs from an ordinary LES equation only in that the prices are known to be measured with some error. Even if estimated aggregate commodity prices from a second-stage model were available, the e_{ij}^{xt} errors would still be present. As it is, the use of a Paasche approximation virtually guarantees some discrepancy from the correct price index.

In general, errors in variables render generalized least squares estimators of the type used to estimate an LES inconsistent. This error in prices is not a peculiar property of the VSS methodology. Every empirically estimated expenditure system that is not completely disaggregated into elementary goods suffers the same defect, by use of statistical rather than true price measures.

Many studies exist on the topic of biases in statistical indices, a recent example being [6]. The general consensus is that deviations from true price indices tend to be small, but systematic.

Motivation for ignoring these errors may be given by appeal to the budgeting process itself. Assume that the two-stage process is not only an econometric nicety, but in fact, represents consumers' actual budgeting procedure. In the first stage, the consumer knows no more than the economist about what his true price indices will be, since they depend on the not yet performed second-stage allocation. Thus, the consumer must settle for a suboptimal first-stage allocation based on an approximation to true prices.

This scenario doesn't eliminate the problem, unless it is assumed that a consumer's approximation to true prices equals the researcher's. Even then, the use of an approximation will mean that the consumer has not in fact maximized utility.[8]

The errors in prices problem is acknowledged here, but not solved. While general techniques do exist for coping with such errors,[9] the derivation of an LES estimator that explicity handles the problem is beyond the scope of this paper. The preceding paragraphs offer only the twin consolations that the errors are probably small, and may be assumed away with a plausible scenario.

Having reduced the problem to one of estimating an ordinary LES with a large number of observations, direct appeal can be made to existing literature on the subject. In this application, Deaton's [7] estimator and algorithm is used.[10] This is not a FIML estimator, since Deaton had insufficient data for full information estimation. In this application, lack of data is not a problem, but computational burden is. Moreover, the bypassed problem of errors in variables would tend to undermine gains from full information.

Before proceeding to empirical results, consideration should be given to any special properties the data for the model may possess. For example, if data from only one time period are available, and this period is chosen to be the base year, then all the price indices (including true indices) will equal one. In this case, in the base year only, the LES equations simplify to

$$(8) \quad E_j^{xt} = b_j(E^{xt} - \sum_{k \in J} c_k) + c_j + e_j^{xt}$$

Estimating the b's and c's in (8) only requires estimating b_j and a_j for each $j \in J$ in the equations

$$(9)\quad E_j^{xt} = b_j E^{xt} + a_j + e_j^{xt}$$

where a_j is a constant. Equation (9) could be estimated separately for each commodity j by ordinary least squares, or the seemingly unrelated regressions model of [29] could be used,[11] if non-zero covariances are believed to exist among commodities. In matrix form, equating (8) to (9) for all j yields

$$(10)\quad a = [I - b\iota']c$$

where I is the identity matrix and ι is a vector of ones. c is then calculated from the estimates of vectors a and b by

$$(11)\quad c = [I - b\iota']^{-1}a$$

This simplified estimator is not used in the current application, because it can only be applied when no time variation in prices is available. The latest U.S. consumer expenditure survey spans two years, so some time variation is available. One of the aims of this inquiry is to get as much untapped information about price effects out of the Consumer Expenditure Survey as possible. It would be counterproductive to ignore what little elementary good price variation there is present in the data, even if doing so would simplify estimation.

Equation (9) is a linear Engel curve, of the type estimated in [1]. The above equations therefore give an explicit utility function interpretation for Linear Engel Curves. It is important to realize that equations (9) and (11) cannot be used as is for forecasting. When prices change from their base year values, the full LES of equation (4), including separate price indices for each household, must be used to forecast budget allocations.

4. Model Construction

The household budget data for this study comes from the most recently available U.S. Consumer Expenditure Survey. This survey was done in 1972 and 1973, and includes more than 20,000 households.[12] Expenditures in each household are grouped into 126 categories, corresponding to elementary goods. The elementary goods are in turn grouped into eleven aggregate commodities. Expenditure levels and Paasche price indices are constructed for each aggregate commodity in each household. Since the survey spans two years, some time variation is present in the prices, in addition to the VSS-induced variation. The households are then grouped on the basis of demographic characteristics, and a seprate expenditure system is estimated from the data for each group.

The major decisions in this process are the choice and composition of aggregate commodities, the selection of demographic characteristics used to group the households, and the choice of functional form of the expenditure systems to be estimated. Each of these decisions is discussed in turn.

Empirical methods for grouping goods into aggregates are few, and are not very rigorously grounded in consumer demand theory.[13] In this application, the large number of elementary goods and observations involved makes such methods impractical in any case. The grouping of elementary goods into commodity aggregates is therefore based on intuitive notions of appropriate commodity definitions rather than any empirical process. Likewise, the composition of elementary goods from the raw Consumer Expenditure Survey data is ad hoc, although the necessity of available price indices to match the elementary goods plays a role in their construction.

The choice of appropriate demographic groupings is a similarly unstructured problem. The criterion here is that groups be homogeneous enough to plausibly assume that all individuals within a group share the same first-stage utility function. Other than this, it is desirable to restrict attention to groupings that are few and populous, so as to minimize estimation variances and keep between-group comparisons to a manageable number.

The choice of functional form for the model is less arbitrary. Expenditure systems consistent with utility theory are desired, and the utility function chosen must also permit the VSS procedure.

Since the second-stage function is arbitrary, any utility function of aggregate commodities can be chosen as the first-stage process. Nevertheless, it is useful to hypothesize a complete utility function, both first and second stages, so that the theoretically exact price indices may be explicitly compared to their statistical approximations, and to assess formally the properties of an estimator of the first-stage function.

It will be important to distinguish empirical results that are caused by innate characteristics of the utility function itself from those arising from the VSS methodology. Therefore, the chosen expenditure system should be one whose properties are well understood, and one that has been used extensively in other investigations, the results of which may be compared to the outcome of this study.

Given the large size of the U.S. Consumer Expenditure Survey, it is necessary to choose a system for which estimation algorithms exist that converge rapidly and have relatively frugal space and computational requirements.

A data set consisting of household budget data, with little or no time variation, does offer some simplifications that help to offset the problem of size. For example, models of household survey data measure long-run effects (see [15]), which in this case eliminates the need for intertemporal utility functions as in [14]. Equilibrium behavior also implies that stock adjustment effects are not likely to adversely affect the estimates, justifying the inclusion of durables as well as non-durables and services in the system.[14]

The LES was shown to be particularly amenable to the varying second-stage approach and also meets all the requirements listed above. It is therefore well suited for this application.

A recurring theme above is the problems of a large data set, which begs the question of why the U.S. Consumer Expenditure Survey was chosen. Since a time series of such cross-sectional data is not available for the U.S., a study of demographic differences in consumer behavior (other than an Engel curve analysis) is not possible by more conventional means. More importantly, a large number of observations will be necessary to get statistically significant parameter estimates. This is because households that are similar enough to have identical first-stage utility functions will tend to have similar second stages as well, limiting the price variation induced by VSS utility.[15] As a result, the statistical advantage of thousands of data points becomes a necessity rather than a luxury. This is also why the small time variation in prices afforded by a survey that spans two years was incorporated into the construction of commodity price indices, thereby further increasing the sought-after price variation.

Before proceeding to the empirical analysis, it will be useful to summarize briefly the required data preparations. Prices for disaggregate, elementary goods (the p_{ij}^{t}) are obtained, which need not vary by household. In this study, U.S. consumption deflators are used. Also required are expenditures by household, both for these elementary goods and summed into aggregate commodities. From expenditures and prices, Paasche approximations to the P_j^{xt} of equation (7) are constructed. These prices are different for each household, consistent with the VSS approach. As a result we have, separate for each household, a price and expenditure number for each aggregate commodity $j \varepsilon J$. Each household may then be used as an observation in the estimation of the first stage, which is just an ordinary LES of the aggregate commodities. The households may be grouped in any way desired, and a model estimated for any sizeable subset of them. In this application, households are grouped by age and income levels, and separate models estimated for these different groupings.

It is worth noting that the data as constructed may be used for a simpler model than an entire expenditure system. The single equation models of one commodity that are common in demand analysis may be estimated cross-sectionally, for different demographics groups, using data as constructed above. However, for this application, the theoretical rigor of a complete expenditure system is maintained.

5. Empirical Comparisons and Welfare

Thirteen models were estimated in all. Model A is a single LES, combining everyone in the survey. While it is unlikely that U.S. households are homogeneous enough to all possess the same first-stage utility function, model A does provide a starting point against which other models, including more traditional time series models, may be compared.

Models B1 to B6 consist of households grouped into six (real) income brackets, with a separate, complete LES estimated for each bracket. Similarly, models C1 to C6 are separate LES's estimated for households grouped by age of head of household. Age and income level should be sufficient to capture many of the lifestyle characteristics likely to cause differences in utility functions. The number of models was chosen to be large enough to show real tendencies and trends associated with age or income, yet small enough to leave a sizeable number of observations in each model. Table 1 gives a summary of each of these models.

One interesting summary statistic of the models is the estimate of what Frisch [10] calls money flexibility. This is the income elasticity of the marginal utility of money in the additive form of a utility function, and is what Muellbaeur [16] advocates using for welfare comparisons between households.

Model A gives an estimate for the general population's money flexibility of -1.3 (see Table 1). This is somewhat lower than Frisch's estimate of -2 for Norwegian data, but in the right ballpark when compared to the range of -0.1 to -10 that Frisch assumes for general populations of varying degrees of wealth. The difference between -2 and -1.3 supposedly represents the welfare advantage of the early 1970s American over the early 1950s Norwegian.

TABLE 1

SUMMARY OF MODELS

Model	Demographic Criterion	Number of Households in Model	Expenditures[a]	Committed[a] Expenditures	Money[a] Flexibility
A	All	9267[b]	9,200	2,100	-1.3
B1	Income: under 5[c]	4509	3,900	2,400	-2.7
B2	5 - 10	4456	6,800	3,300	-2.0
B3	10 - 15	4048	9,600	2,200	-1.3
B4	15 - 20	2850	12,000	590	-1.1
B5	20 - 25	1542	15,000	-59	-1.0
B6	over 25	1493	20,000	17,000	-6.4
C1	Age: under 24[d]	1655	6,500	2,000	-1.4
C2	25 - 34	3767	9,800	2,800	-1.4
C3	35 - 44	3149	12,000	1,700	-1.2
C4	45 - 54	3493	12,000	3,300	-1.4
C5	55 - 64	3050	9,100	2,000	-1.3
C6	over 65	3784	5,400	3,700	-3.2

[a]Numbers in these columns are computed at the means of price and expenditure data in each model, then rounded to two significant digits.

[b]Computer software limitations prevented models with more than 10,000 observations from being estimated. In model A, more than this many observations were available, so a 50 percent sample (stratified by age of head, income bracket, family size, and year of survey) of the available households was taken and used. The actual number of usable households in the survey is approximately twice the number given for model A.

[c]Income per household in thousands of (real) 1972 dollars.

[d]Age of head of household.

More interesting than this overall comparison is how money flexibility varies by income level. Frisch believes it should decline in absolute magnitude as real income increases, although he acknowledges that "some authors argue as if the absolute value of money flexibility should be increasing with real income."

The empirical results of models B1 through B6 support Frisch's beliefs with one glaring exception. The magnitude of money flexilibity declines uniformly from -2.7 to -1.0 as the income brackets increase, but for the wealthiest income bracket it jumps to -6.4. While this result may be called a statistical fluke,[16] it can be explained by appeal to the generating LES model. For the LES, money flexibility depends only on the ratio of committed to total expenditures, given by

(12) $F = 1/(R-1)$

where F is money flexibility

R is the ratio of committed to total expenditures.

If committed expenditures are given Samuelson's [23] interpretation of being subsistence level requirements, then presumably R would decline as income level increased, resulting in F following Frisch's priors. There are some catches in this interpretation of F and R, however. For example, getting F to be less than one, thereby attaining the magnitudes Frisch associates with the wealthy, requires that committed expenditures be negative, where the interpretation of being subsistence level expenditures loses its meaning.

More importantly, F and R are, like other utility parameters, subjective phenomena and as such will depend on the attitudes of the individual consumer. In the LES, committed expenditures are in some sense a pre-allocated portion of the budget, and thereby serve as an origin, from which remaining (supernumerary) income is allocated by Cobb-Douglas functions. For a poor man, this pre-allocation could literally represent survival expenditures, although it need not. For the wealthy consumer it seems unlikely that subsistence allocations would ever be considered. It is better to consider committed expenditures as precisely what the name implies, that is, a base level of purchases the consumer will (or perceives he must) make, below which the utility function as given is undefined. Paying the maid may be just as much a part of a rich man's committed expenditure as a loaf of bread is in the poor man's. In fact, one could argue that it is precisely the rich consumer's attitudes about standards of living and "necessary" expenditures that drives him to gain the wealth required to support perceived committed expenditures of the magnitude found in model B6.

Such excursions into psychological motivations are hardly warranted or verified by the estimates of a single model. Nevertheless, the point that R and F are subjective means that an estimate of $F = -6.4$ in model B6, or the more ordinary estimates in the other models, can be no more dismissed as incorrect than any individual's utility function parameters may be faulted for being bad or good. Such analyses also clarify the dangers of attaching welfare interpretations to cardinal utility constructs like money flexibility.

This does not mean that $F = -6.4$ in model B6 is the correct estimate of the true F, but only that it could be. If it isn't at least approximately right, then either it is a statistical fluke, or the model is misspecified. Arguments for the latter possibility are easy to find. The model may be too simple, or the estimator may be bad. The assumption of a common first-stage utility function may be more appropriate for the poor and middle class brackets than for the wealthy, who, after all, have more choice in what to buy and therefore have more opportunity to differ widely from each other. B6 also encompasses a greater range of expenditure levels than the other income models, perhaps aggravating violation of the first-stage assumptions.

The age models also have one deviation from their norm. Models C1 to C5, encompassing all but retirement age households, show a tight uniformity of F in the -1.2 to -1.4 range. This agrees with model A, showing that age does not appear to be a factor in determining F or R. The exception is model C6, where F = -3.2. As with model B6, this exception may be attributed to statistical anomaly, model inadequacy, or generalizations about the attitudes and actions of elderly consumers. For example, retirement income is often fixed, typically at a lower level than during employment. At the same time, attitudes about committed income levels built up over a lifetime may remain unchanged, or even increase due to special needs associated with old age. Together these factors would result in simultaneously lower total expenditures and higher committed expenditures as indicated in Table 1 leading directly to the high value of F.

On the whole, the estimates of F in all the models have tended to be consistent with each other and with prior beliefs. The two exceptions, those of the highest income and oldest age models, may be attributed to behavioral differences in these extreme groups, though of course verification through further studies would be needed to support such claims.

6. Empirical Demographic Comparisons by Commodity

We now turn to analysis of the aggregate commodities. This study does not attempt to survey the abundant literature of estimated expenditure systems and demand models for individual commodities. Discussion here will be limited to brief and largely speculative interpretations of the estimated models' parameters and elasticities.

Before proceeding, a brief definition of the commodities and expenditures is in order. Total expenditures in these models comprise all uses of money that the consumer expenditure survey classifies as consumption expenditures, rather than asset formation. Examples of the latter would include purchases of stock or bonds, construction of a garage or similar major additions to one's house, and ordinary savings. In general, asset formation expenditures are investments. On the other hand, consumption expenditures consist of purchases of services, non-durables, and some durables, such as furniture or automobiles, that in general depreciate with use.

For this study, total expenditures was divided into eleven aggregate commodities, comprising J in the notation of the previous sections. These commodities are:

1. unprepared foods
2. prepared foods (meals and purchased drinks)
3. shelter
4. fuel and utilities
5. household appliances, furnishings and furniture
6. clothing
7. auto ownership and operation (except fuel)

8. public transportation
9. professional services
10. recreational vehicles
11. other recreation (including miscellaneous gifts)

Figures 1 through 4 depict estimated parameters and elasticities for each commodity in each model, while figure 5 shows actual budget shares of the commodities.[17,18]

1 & 2 Food

Food is divided into two categories. Unprepared food is basically groceries bought to be prepared at home, while prepared food consists of purchases at restaurants, bars, etc. It is to be expected that for most demographic groups, prepared food would have substantially lower committed quantities (c_j)[19] and higher elasticities than unprepared food. c_1 is in fact greater than c_2 in every model, ranging from 2.5 to more than 10 times as large, consistent with prepared food being a luxury. Elasticities are higher for prepared food in all but two models, B5 and B6. In these models, elasticities for prepared and unprepared foods are about equal, and the ratio of c_1 to c_2 is in the 2.5 range. This is explained by the wealthier consumers' tradeoff between prepared and unprepared food depending more on personal taste than financial aspects. The price sensitivity increases uniformly from models B1 to B5 for unprepared, and B1 to B4 for prepared food, indicating an increasing ability or willingness to adjust quality or mix when prices change. The wealthiest group shows a sharply lower price elasticity, in line with the high committed expenditures found for this group, where both indicate an insensitivity

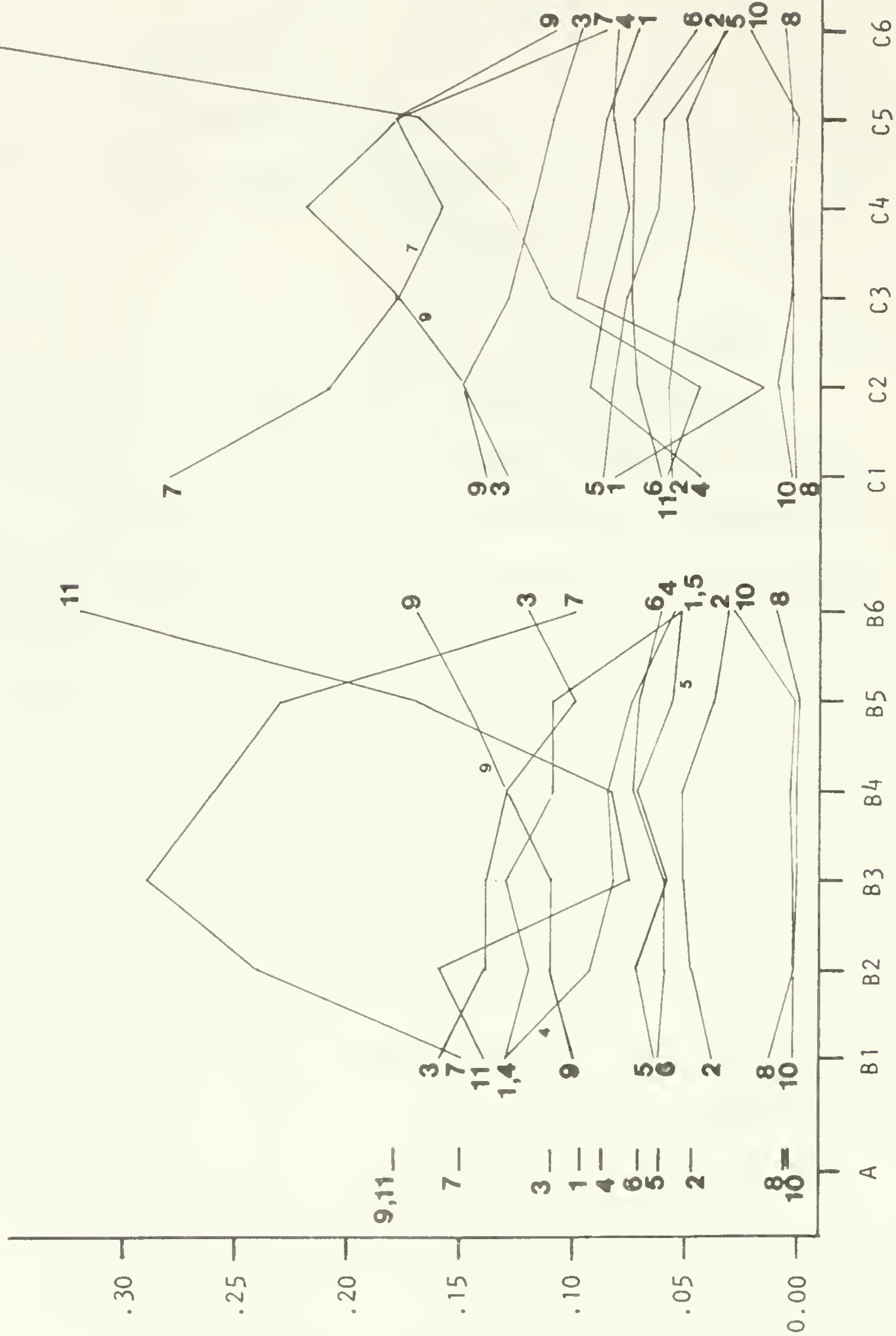

FIGURE 1 b_j PARAMETERS

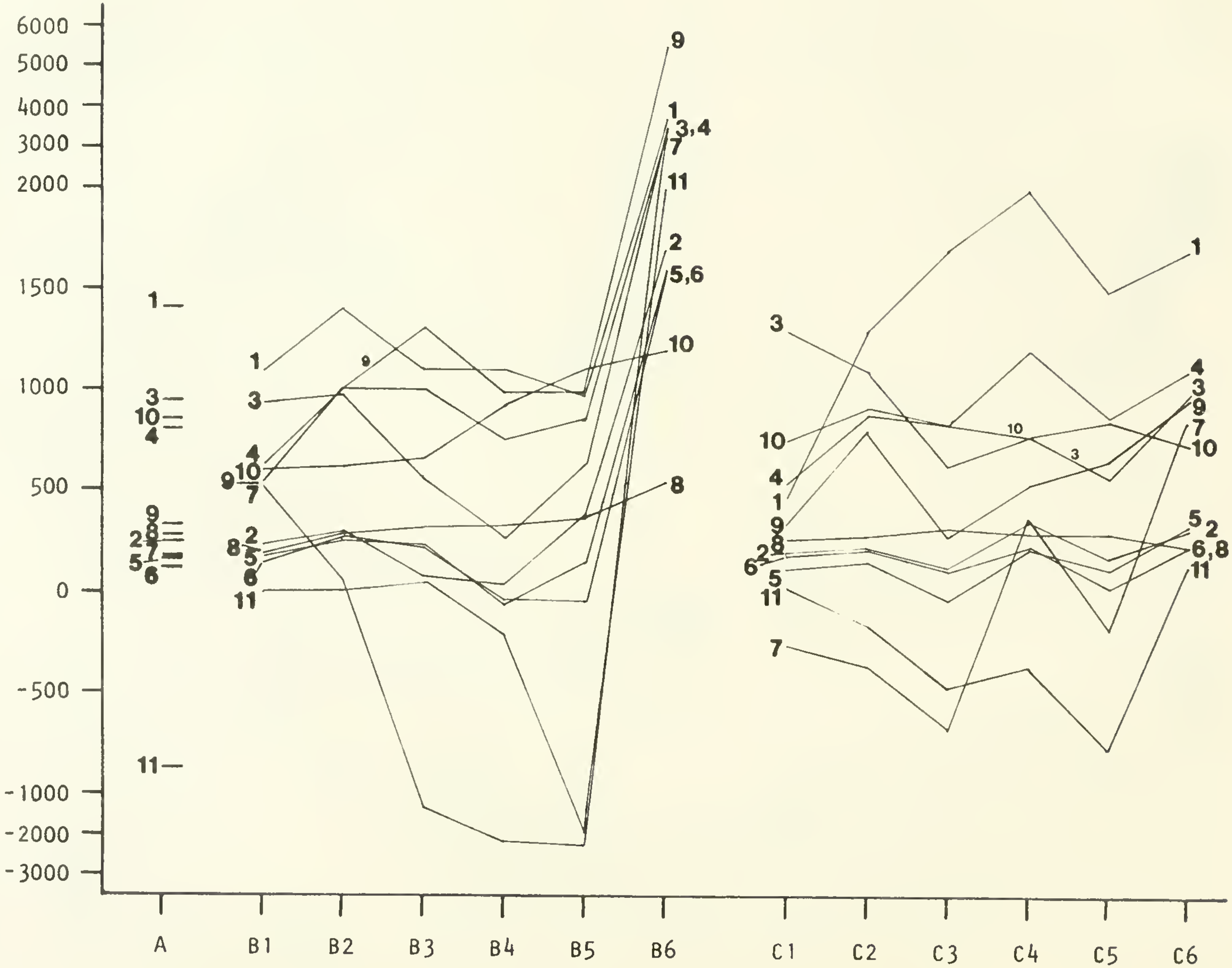

FIGURE 2 c_j PARAMETERS (VERTICAL SCALE FORESHORTENED AT THE ENDS)

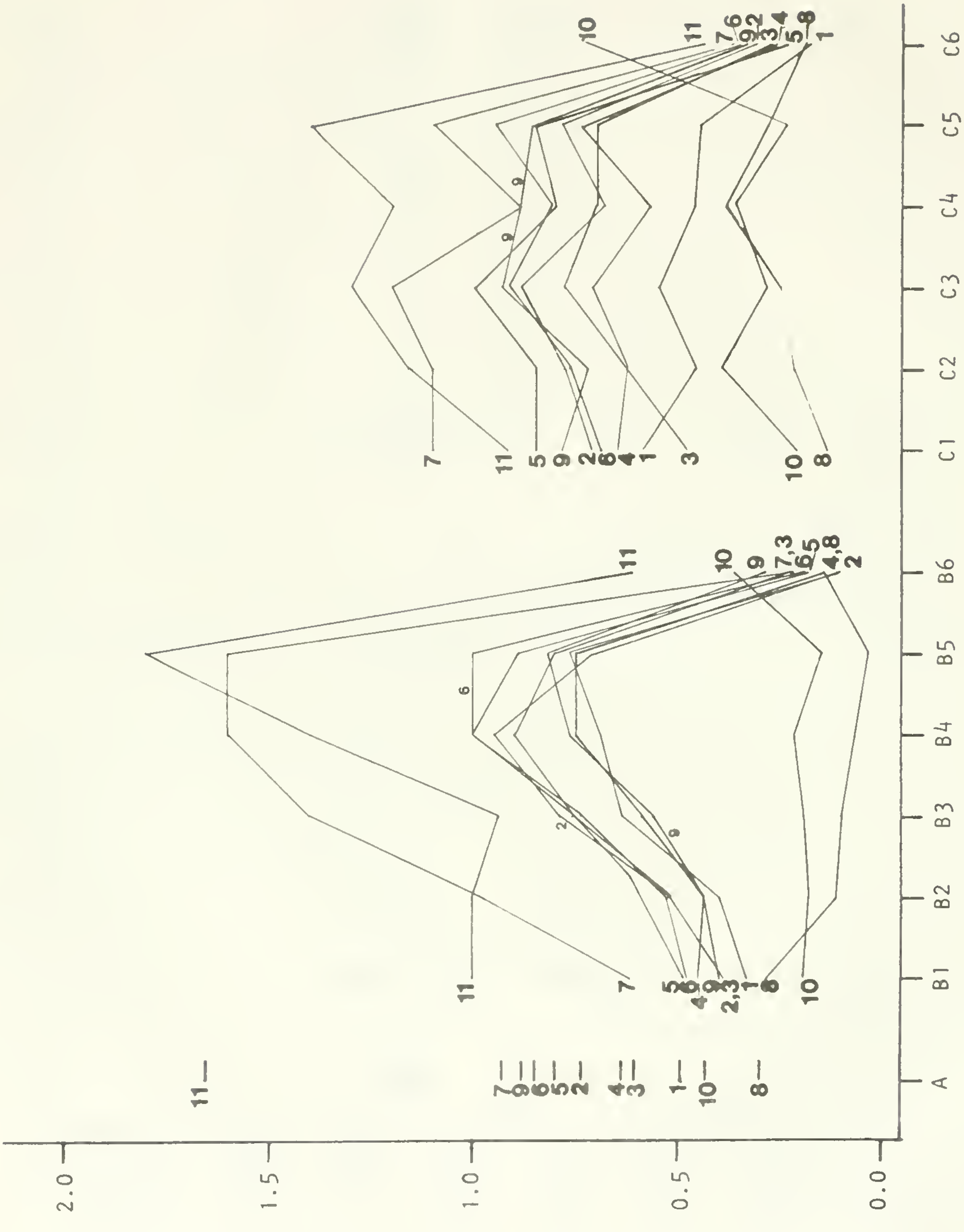

FIGURE 3 PRICE ELASTICITIES

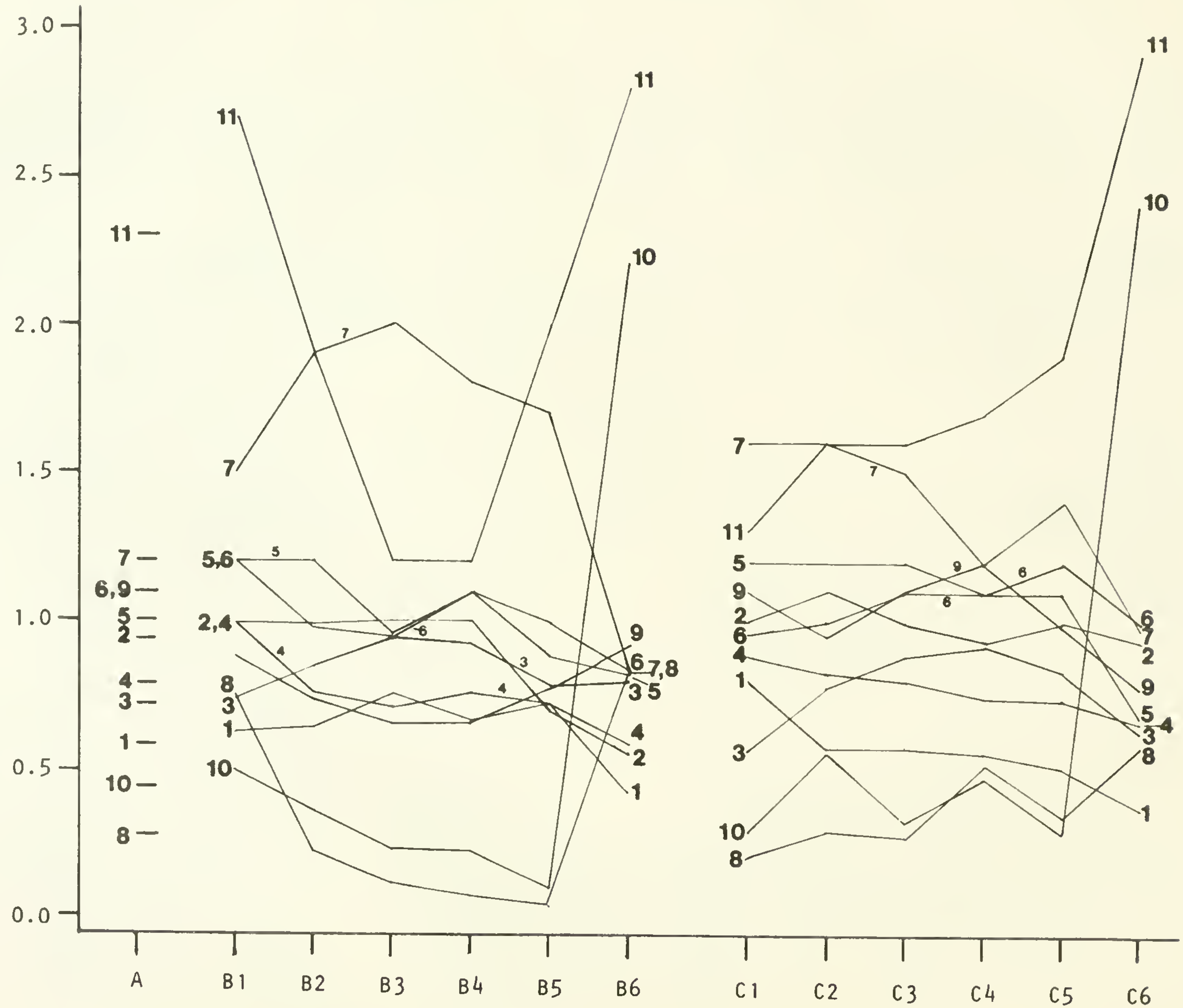

FIGURE 4 EXPENDITURE ELASTICITIES

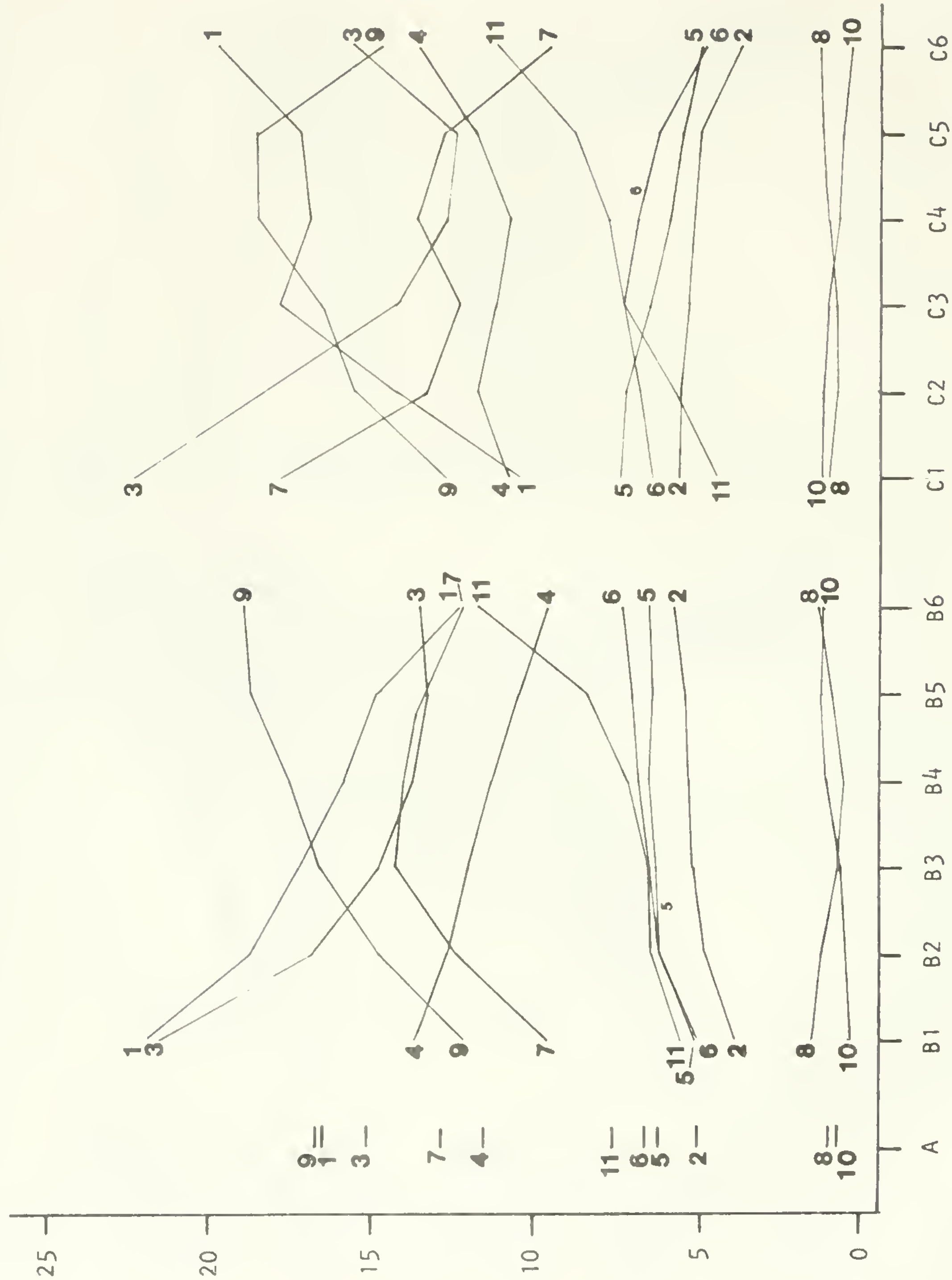

FIGURE 5 PERCENTAGE BUDGET SHARES

to marginal price and income changes induced by strong tastes and preferences rather than by survival necessities.

By age, committed quantities for food increase through C4, then drop back, though this variation may be due more to family size than tastes of different age groups. Price and expenditure elasticities don't show any obvious trends by age, although the youngest households have an above-average expenditure elasticity for unprepared food, and the oldest group have sharply lower price elasticities.

In general, the above results follow food budget shares observed for the groups. The budget share for unprepared foods decreases as income increases, and increases with age. Prepared foods move in the opposite direction.

3. <u>Shelter</u>

The price elasticities of shelter parallel those of unprepared food, both being necessities. For the poor and the young, shelter has a high budget share and, sensibly, a high committed quality.

4. <u>Fuel and Utilities</u>

This category includes gasoline for cars, trucks, etc., as well as home heating oil, gas, electricity, and other utilities. Gasoline is included in this category to bring virtually all home energy consumption into one group. Demographic differences in elasticities and committed quantities for this commodity have a special importance in the current economy, although the limitations of old data (1972-73) must be considered in this regard.

The price elasticity shows a general, although not uniform, increase with income level up to B5, then a large drop for B6. Income elasticities are more stable, staying in the .7 to .8 range for B2 through B5, though being higher for B1, and lower for B6. By age, the elasticities are fairly constant, although the expenditure elasticity declines with age. Once again, the elderly show a sharply greater insensitivity to prices than all other age groups. In terms of budget share, fuel and utilities decrease with income, and show mixed effects with age, although the elderly spend a greater share of their expenditures on it than any other age group.

5. Appliances, Furnishings, and Furniture

Being durables and semi-durables, purchases in this group can usually be deferred, resulting in high elasticities almost across the board. The exception is people over 65. This group may tend to have more fixed incomes and expenditures than other groups, leading to less opportunity or need for deferring expenditures. Price elasticities follow the demographic patterns of the previous commodities, although at a generally higher level.

6. Clothing

As another semi-durable necessity, the parameters and elasticities of this commodity are similar to those of appliances, furnishings, and furniture, and move in the same direction. The younger households have lower elasticitites for clothes than for appliances. The two even out by C4, after which clothing has the higher elasticities.

7. Auto Ownership and Operation (except Fuel)

Being a high cost durable results in the highest elasticities seen so far, tempered only by less deferrable maintenance and operation costs. One odd estimate is the price elasticity in B1.

While it is the second highest price elasticity for any commodity in that model, the magnitude is low relative to the other models. It may be that auto ownership for the poor only occurs when the households have a very strong need or desire for it, thus making the price elasticity smaller than it otherwise might be. Less surprising is the small price elasticity of the wealthiest group.

8. Public Transportation

This commodity is small, and tends to have low elasticities. It also has high committed quantities relative to its budget share. For most consumers, public transportation appears fixed and not readily forsaken. B1 households have higher elasticities than most others, possibly indicating a willingness to give up transportation when necessary. The rich also have a high elasticity, although for them the tradeoffs may be more like forgoing airplane trips, or switching from taxis to private cars.

9. Professional Services

The price elasticity of services follows the theme of increasing with income up to B5, then plummeting for B6. Expenditure elasticities are less consistent in their movement, and in fact tend to stay in the .7 to .8 range. The services elasticities in the A and C models are higher than those of the B models. This may be due to a mix of professional services that differs more substantially by income class than by age.

10. Recreational Vehicles

This small commodity could not be grouped with other recreation because these goods are generally far more durable and higher priced. The other possible place for it is with autos, although the difference between necessity and luxury would almost surely result in different elasticities for the two. Thus recreational vehicles stands alone as a luxury durables commodity. The elasticities for recreational vehicles are uniformly low, with two exceptions, while committed quantities are high across the board.

This is an example of a good whose demand is largely determined in the pre-allocation part of the LES, and is therefore only mildly affected by small changes in price or expenditure level.

The exceptions are once again models B6 and C6, both having expenditure elasticities four to five times higher than those of other models. Before placing too much emphasis on this result, it should be noted that in terms of percent error, recreational vehicles has the poorest fit of all eleven commodities.

With this caveat in mind, we may still speculate on the reasons for the extreme elasticities. For the wealthiest group, it is not hard to imagine incremental income being funneled into recreation, including recreational vehicles. The retired also might be expected to put more incremental dollars into recreation of all kinds, although they display a higher price sensitivity for doing so than the wealthy. Conversely, retirees in need of cash would tend to cut back on expensive luxury items first, again yielding a high expenditure elasticity.

11. Other Recreation and Miscellaneous Gifts

While not entirely discretionary, this commodity includes toys, sporting equipment, magazines, and many other non-essentials.

Unlike recreational vehicles, committed quantities are almost always near zero or negative. Elasticities are quite high. Even the price elasticity in B6, while low compared to every other model, is almost twice as high as the next highest price elasticity in that model. Not surprisingly, the budget share devoted to this commodity increases uniformly with age and with income level.

7. Conclusions

A method of modeling consumer expenditure allocations is proposed, based on the assumption that households have the same aggregate, or first-stage, allocation process but may differ in their demand functions for elementary goods. Models based on the varying second-stage assumption have many attractive properties, including:

1. The implausible assumption required for conventional cross-sectional analyses that demand functions be identical for each consumer up to a stochastic component is relaxed.
2. Own and cross price effects can be estimated even when little or no elementary good price variation is available in the data, as is usually the case for household budget surveys. This is especially relevant for estimating differences in price effects for different demographic groups, for which time series data is nonexistent. In contrast to other proposed techniques for cross-sectionally estimating price effects, neither explicit formulations of how consumers vary across groups, nor cardinal utility

assumptions are required.

3. Unlike conventional analyses with aggregate time series data, the estimated models have a precise meaning for each consumer, rather than representing a fictional "representative" consumer.

4. The methodology shows how some Engel curve models may be reinterpreted to yield information on price effects. In particular, it is shown how LES parameters may be computed from linear Engel curves. Conversely, in some cases estimation of a complete expenditure system may be simplified to Engel curve equations.

The method of VSS utility functions was empirically demonstrated using data from a single U.S. Consumer Expenditure Survey. Thirteen separate Linear Expenditure Systems were estimated, corresponding to groups of households that vary by income level and age of head of household. The results of these models were briefly analyzed, with special attention being paid to how price and expenditure effects vary with age and income, and to welfare comparisons.

On the whole, estimates tended to vary smoothly from model to model, with two exceptions. Those exceptions were the households with the highest income, and households whose heads were of retirement age. These two groups displayed estimated parameters substantially different from those of other households. Using money flexibility as a measure of welfare produced paradoxical results for these two models, which were explained by a careful interpretation of what LES committed expenditures mean for different demographic groups.

The method of varying second-stage utility has a potentially large number of applications, since more conventional methods cannot be used to estimate demographic differences in price elasticities except in the rare cases where consistent surveys spanning a large number of time periods are available, or when time series and cross-sectional data may be successfully combined into one model.

FOOTNOTES

1. This research was done at the Massachusetts Institute of Technology, and was supported in part by the Consumer Economic Services group of Data Resources, Inc., in conjunction with development of a Consumer Allocation Model. All model estimates interpretations, and derivations presented here are entirely my own, and do not reflect the opinions of Data Resources, Inc.

2. I wish to thank Sue Rudd, Thomas Stoker, Dan O'Reilly, Michael Flanagan, Martin Duffy, Robert Pindyck, and Franklin Fisher for their help and advice. Any errors are entirely my own.

3. See, for example, [13].

4. The theorem quoted can be found in [4]. In [12, 25, 26] more general conditions are derived permitting a two-stage procedure, although in the general case, the connection between the first and second stage is more complicated than use of simple aggregate commodity price indices as in equation 2. A summary of such two-stage procedures is [5].

5. In the notation given, it is assumed that consumers agree on what elementary goods constitute each aggregate, and are therefore only allowed to differ on the relative proportions of the elementary goods, that is, they may differ on the functional form of U_j^x, but not on its arguments. This restriction may be

lifted by giving I_j a household superscript, but the added generality of doing so will not be used in all that follows. A more formidable problem would be to empirically construct appropriate I_j's. The issue is thorny, although some proposals are given in [3, 19, and 21]. This study will follow the majority of other empirical attempts by grouping goods in a hopefully reasonable, though not empirically deduced, fashion.

6. This statement assumes that the straight, parallel Engel curves required for exact aggregation (see [11]) are not present. The condemnation of macroeconomic consumer demand models is mitigated to some extent by Muellbauer in [17, 18] who shows how demographic changes may be soaked up by a single parameter in a time series model, the value of which should change with each observation. In the AIDS model [8], this parameter is estimated as a constant, which implies that, as with other macroeconomic models, the effects of demographic differences are relegated to the error term, and the meaning of the estimates for a given consumer are still unknown.

7. Inclusion of a constant term would violate basic utility assumptions. For example, money illusion would be present in such a model. This situation does not make an additive error incorrect. It only means that the error must be interpreted as the net effect of (stochastic) differences in the estimated parameters for each consumer, rather than the effect of omitted variables that would appear additively in an errorless model. Theil [27]

gives a formal interpretation of these errors, which requires a specification directly relating the variance-covariance matrix to the substitution matrix.

8. Perhaps Theil's [27] theory of second moments of the disturbance could be expanded to cover this example, which may be viewed as another case of balancing loss of utility with costs (in this case, of information) of perfect maximization. Also note the similarity to rational expectations models, when the consumer is assumed to know no more or less about the world than the model builder does.

9. See, for example, [28, pp. 607-615] for a brief discussion of such techniques. In general, the usual methods for coping with errors in variables will not be readily applicable both because of the complexity of expenditure system models, and because the difference between true and statistical price measures will tend to be systematic rather than purely random.

10. The estimator is least squares with a heteroscedasticity correction that weights variances and covariances as functions of budget shares. The derivation of this estimator is given in [7, pp. 34-53] and is not repeated here. This is his estimator for the ordinary LES, not the hierarchic LES proposed in a later chapter of his book.

11. OLS will have the desired property that the b's sum to one and the a's (and errors) sum to zero. For seemingly unrelated regressions, singularity in the variance-covariance matrix is

handled by dropping one of the commodity equations.

12. A few hundred households were omitted from this study for failure to supply enough demographic information such as income level and age of head of household, to permit demographic classification of the household.

13. See [7, pp. 169-172] for a discussion of the problem. In the end, he too chooses an arbitrary grouping of goods based on intuition rather than rigor. See also footnote 5.

14. It might be possible to explicitly incorporate stocks of durables using, for example, the utility function described in [20, pp. 183-198] as the first stage. In this application, doing so would entail substantial increases in data and computational requirements that are already very steep without this further complication.

15. The problem is a blessing in disguise, since were it not so, the usual assumption of equal prices paid by all consumers would be a poor one for aggregate commodities, thereby throwing into question the results of all Engel curve analyses based on household budget data.

16. The arguments for a statistical fluke would include reference to the fact that model B6 has fewer observations than any of the other models, and that estimated committed expenditures may be skewed upward by a few very wealthy households in this model.

17. Cross elasticities, which are easily calculated from the

parameter estimates and budget shares, are not presented for lack of space. The parameters, own price and income elasticities, and budget shares for 13 full-size expenditure systems would take up a large amount of space in table form, and are less easily interpreted than the graphs of figures 1 through 5. The tables used to generate these graphs are available from the author on request.

18. Deaton's [7] model is actually more general than has been described here, in that it permits time variation in the b parameters. In particular, b_j of equation (4) is replaced by $b_j + d_j t$, where both b_j and d_j are estimated parameters. The b parameters are thereby permitted to change linearly over time, at the rate d_j. Since the U.S. Consumer Expenditure Survey spans two years, estimation was performed using $t = 0$ for the first year and $t = 1$ for the second. In the majority of commodities and models, the estimated d_j's were zero to two decimal places, and equaled less than 10% of b_j. To avoid needless duplication of tables and calculations, all reported b_j's are a weighted average of b_j and $b_j + d_j$, where the weights correspond to the number of observations (households) in the model from each of the two years.

19. Recall that quantities for the aggregate commodities are in fact quantity indices or measures. Changes in quantities, including committed quantities, may take three forms. These changes can be due to true quantity changes in the elementary goods, changes in the selection of elementary goods, and quality changes in those goods. In the case of committed quantities of food, major

differences in elementary quantities may be due to variation in family sizes, although quality and mix differences are also likely to be present.

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