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DYNAMICS OF PRICE ELASTICITY AND THE PRODUCT LIFE CYCLE - AN EMPIRICAL STUDY*

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ABSTRACT

The author presents a product life cycle model which incorporates carryover-effects and obsolescence and allows for time-varying price responses. An empirical study of 35 products reveals typical changes in price elasticity over the product life cycle and casts doubt upon the hypotheses prevailing in the marketing literature. Some important implications for strategic pricing and anti-trust issues are being discussed.

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INTRODUCTION

In the marketing literature it has frequently been alleged that marketing strategy should vary over the product life cycle (Kotler 1971, Lambin 1970, Levitt 1965, Wasson 1974, see also Dhalla and Yuspeh 1976). Such allegations presuppose a certain knowledge on the efficiency of various marketing instruments at different stages of the life cycle. In fact, very little is known about this issue. In support of the allegations, reference is usually made to Mickwitz (Kotler 1971, p.62; Lambin 1970, p.15; Parsons 1975) who - back in 1959 - presented some theoretical considerations on the changes in marketing elasticities over the life cycle, but did not give any empirical evidence of his hypotheses.

Too often no clear distinction between the life cycle of a particular product and the life cycle of a whole product class has been made, two exceptions being the studies of Polli and Cook (1969) and Dhalla and Yuspeh (1976). The present study is clearly confined to single products, no conclusions on whole product classes will be drawn. Throughout the paper, the term product life cycle (PLC) denotes the time series $q_{i,1},\ldots,q_{i,T}$ of quantities sold of a particular product or brand i. The PLC-concept is not understood as an ideal-type model.

We focus on price and on the changes in price elasticity over time. According to Mickwitz (1959) and his followers price elasticity increases over the first three stages of the PLC (introduction, growth, maturity) and decreases during the stage of decline.

The first part of this hypothesis seems to be supported by some findings of diffusion research according to which early adopters of new products typically have higher incomes and pay less attention to price than later adopters do (Robertson 1967, Rogers 1968). The hypothesis is also confirmed by a General Motors study on the price elasticity of automobile demand for the years 1919 - 38 to which Dean (1950, p.227) refers. One should note, however, that both the diffusion studies and the GM study are concerned with product classes and

do not readily allow conclusions for single products or brands.

As for single products , a great many empirical tests of dynamic sales response functions have been conducted, almost all of which are, however, related to advertising. Clarke (1976) reviewed about 70 of these studies, further reviews can be found in Parsons and Schultz (1976) and Dhalla (1978).

Relatively few studies include price as an explanatory variable (Telser 1962, Lambin 1970, Houston and Weiss 1974, Wildt 1974, Lambin,Naert,and Bultez 1975, Moriarty 1975, Lambin 1976, Prasad and Ring 1976). All of these studies assume time-invariant price response or price elasticity coefficients and, therefore, do not permit any conclusive inference on the changes in price response or price elasticity over the PLC. The only models which include time-varying sales responses are limited to advertising issues (Beckwith 1972, Parsons 1975, Wildt 1976, Winer 1976, Erickson 1977).

Wittink (1977a,b) has presented a model in which price elasticity varies with respect to the advertising expenditure, but nevertheless is constant with respect to time. A decrease in the magnitude of price elasticity over time is produced in the well-known competitive simulation model of Kotler (1965). This model, however, can hardly be tested empirically and yields - due to the fact that price elasticity approaches the zero level - strategic recommendations which cannot be considered as reasonable; this is shown in Simon (1978).

This short survey, thus, leads to the conclusion that no convincing empirical or theoretical evidence of the changes in price elasticity over the PLC and of the marketing efficiency of price at different stages of the PLC is available to date. It is the purpose of this study to shed some light on this point.

THE DATA

Data on prices and quantities sold of 43 products (brands) on 7 different markets were available for this study. All data are of most recent origin (all after 1970) and refer to the West German market. They were supplied by large German corporations on a confidential basis so that the product identities cannot be revealed. The most important data characteristics are given in table 1.

INSERT TABLE 1 HERE

All products represent frequently purchased branded items. On each market, products at different stages of their individual PLC's are represented. All markets had been established before the period under investigation so that our analysis applies and is limited to products which are introduced onto markets with existing substitutes, it does not apply to generically new products. We are not aware of any single study which includes a greater number of products.

The data show enough variation to admit an examination of the dynamic relationships between prices and sales. The managers concerned with the products consider price (besides quality which remained unchanged over the period under investigation) as the most important marketing variable.

Even in the case of the detergents, the absence of non-price data doesn't seem to be too serious a problem. This is in particular true for advertising data due to two reasons. On the one hand, advertising is much less important in Germany than in the U.S.; this is mainly due to strict limitations of TV-advertising (only 20 min. per weekday, no adv. on sundays and holidays; in 1977 the advertising budget of Procter & Gamble (USA) alone amounted to 93.8% of the total amount spent on TV-adv. in Germany).

On the other hand, the managers hold that advertising spending for detergents is rather evenly distributed over the year and hasn't changed much over the period under investigation, so that the impact of advertising is likely to be adequately reflected in the constant term of the sales function.

MODEL SPECIFICATION

The empirically tested dynamic sales response models usually have the form

$$q_{i,t} = a_1 + a_2\, q_{i,t-1} + f(p_{i,t}, \bar{p}_{i,t}) \qquad (1)$$

where $q_{i,t}$ product i's sales in period t (either units or market share)

$p_{i,t}$ product i's price in period t

$\bar{p}_{i,t}$ some weighted average price of products competing with product i in period t

$f(\cdot)$ price response function

a_1, a_2 parameters

The sales and price variables are either in natural or in logarithmic dimension. Typically all functional relationships in (1) are assumed to be time-invariant. Hence, for constant prices and $|a_2| < 1$, function (1) can only describe the approach of $q_{i,t}$ towards an equilibrium level of sales. The dynamics of (1) do not allow for a representation of a life cycle curve with an ascending and a descending branch if prices remain unchanged. Moreover, the time-invariant price response presupposed in this function must be considered as a very restrictive assumption.

Within the last few years a number of advertising models which allow for time-varying coefficients of both advertising and the lagged sales variable, the so called "carry-over effect", have been proposed (Beckwith 1972, Parsons 1975, Wildt 1976, Winer 1976). The results of these few studies as to the carry-over

effect are not unequivocal. Parsons (1975), for instance, presupposed an increase in the carry-over effect over time and Wildt (1976) investigated industry sales and not product sales. The results of Beckwith (1972) and Winer (1976) both of whom studied the Lydia Pinkham data indicate a downward tendency of the carry-over effect. Product life cycle theory indeed suggests that the abilitiy of a product to retain its customers from period to period should decrease in the course of time due to the introduction of new competitive products which, in a dynamic market, are likely to be superior either technologically or "psychologically" (fashion, taste etc.).The erosion or "obsolescence" of the old products and the diffusion of the new products, however, occur gradually and not immediately. It seems reasonable to assume an exponential pattern of the decrease in the carry-over in order to account for this phenomenon.

Thus, we obtain for the non-price terms in (1), for which we write $A_{i,t}$

$$\text{A1:} \qquad A_{i,t} = a_1 + a_2 \cdot q_{i,t-1} \cdot (1-a_3)^{t-\bar{t}_i} \qquad (2)$$

where $0 \leq a_3 < 1$ can be interpreted as 'rate of obsolescence' and $\bar{t}_i$ denotes the period of introduction of product i. For $t=\bar{t}_i$ we have $A_{i,t}=a_1$, hence a_1 represents product i's initial demand potential.

The results of Winer (1976) indicate that not only the carry-over effect but also the initial demand potential may be subject to the obsolescence phenomenon. Assuming the same rate a_3 we obtain as an alternative model to (2)

$$\text{A2:} \qquad A_{i,t} = (a_1 + a_2\, q_{i,t-1})\, (1-a_3)^{t-\bar{t}_i} \qquad (3)$$

It should be noted that A1 and A2 include the function with constant parameters as a special case where $a_3=0$.

A great variety of possible life cycle curves can be represented by means of these simple functions. This flexibility is highly important since empirical

PLC's tend to have very different shapes (Cox 1967, Polli and Cook 1969, Wasson 1974, Dhalla and Yuspeh 1976). Figure 1 gives an illustration of this flexibility [f(.)=0].

INSERT FIGURE 1 HERE

Some of the products under investigation show seasonal sales patterns which are due to season-related diseases in the case of the drugs and to certain habits of German housewives in the case of the detergents (draperies etc. are typically laundered in spring and fall). Both managerial experience and visual inspection of the sales curves indicated that only two types of seasonal patterns existed so that one dummy variable $D_t = \{0,1\}$ is sufficient to account for the seasonalities. Adding the seasonal term to A1 and A2 respectively we obtain.

$$\text{A3:} \quad A_{i,t} = a_1 + d.D_t + a_2\, q_{i,t-1} \cdot (1-a_3)^{t-\bar{t}_i} \tag{4}$$

$$\text{A4:} \quad A_{i,t} = (a_1 + d \cdot D_t + a_2 q_{i,t-1})\, (1-a_3)^{t-\bar{t}_i} \tag{5}$$

In a few cases, a further version A5 which is equal to A1 with $a_1=0$ has been tested.

It seems reasonable to assume that product i's sales depend both on the absolute level of its price $p_{i,t}$ and on the differential between $p_{i,t}$ and the prices of competing products.

In the absence of evidence to the contrary, we hypothesize and test a linear relationship between $q_{i,t}$ and the absolute price $p_{i,t}$.

$$B_{i,t} = b \cdot p_{i,t} \tag{6}$$

As to the sales effect of the price differential we adopt a hypothesis which was first proposed by Gutenberg (1955, 1976) and has found wide acceptance in the European marketing literature. According to this hypothesis a relatively small price differential is assumed to have an underproportional sales effect, whereas a relatively great price differential is assumed to produce an over-

proportional sales response. This hypothesis is based on the experience that only very few customers are likely to switch from their accustomed brand to another brand if the price differential changes by e.g. 1% or 2% only, whereas the number of brand switchers typically grows overproportionally when the price differential increases for instance to 20% or 30%.

A nonlinear relationship of this type can be represented by a sinh-function (sinus hyperbolicus, Albach 1973). We consider two versions of sales response to price differentials, the first being

$$\text{C1:} \quad C_{i,t} = c_1 \cdot \sinh\,(c_2\, \Delta p_{i,t}) \tag{7}$$

where $\Delta p_{i,t} = (\bar{p}_{i,t} - p_{i,t})/\bar{p}_{i,t}$ is the price differential,

$\bar{p}_{i,t} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{m_{j,t} p_{j,t}}{(1-m_{j,t})}$ is the market share ($m_{j,t}$) weighted average price of products competing with i,

c_1, c_2 are parameters.

In the version C1 the price response is time-invariant. The second version to be tested is based on the assumption that the sales response on a price differential is proportional to the total market demand hitherto effective.

$$\text{C2:} \quad C_{i,t} = c_1 \sinh\,(c_2\, \Delta p_{i,t})\, \bar{q}_{t-1} \tag{8}$$

where $\bar{q}_{t-1} = \sum_{j=1}^{n} q_{j,t-1}$ is the total market demand in t-1.

The version C2 meets in particular the requirement of Parsons and Schultz (1976, p.158) that a time-varying response should rather be explained by marketing variables than merely by time.

The terms $A_{i,t}$, $B_{i,t}$, and $C_{i,t}$ can be linked either additively or multiplicatively. We hold that a multiplicative linkage is less appropriate in our case since

it implies that the price response, i.e. the derivative $\partial q_{i,t}/\partial p_{i,t}$, develops proportionally with the non-price term $A_{i,t}$ so that the price response would be affected by the obsolescence effect in the same way as the carry-over effect. This would, in fact, amount to a predetermination of the question to be investigated. Therefore, the assumption of independence between the non-price influences and the price influences is made so that a linear function is obtained.

$$q_{i,t} = A_{i,t} + B_{i,t} + C_{i,t} + u_{i,t} \tag{9}$$

where $A_{i,t}$ is either A1,....,A5; $C_{i,t}$ is either C1 or C2; and $u_{i,t}$ is the error term.

In anticipation of the detailed regression results we note here that the influence of the absolute price, $b \cdot p_{i,t}$, did not prove significant for any of the products. This result coincides very well with the managerial opinion that primary demand for the products under investigation has not been affected by changes in the <u>absolute price levels</u> (since 1970). This applies both to the detergents and to the pharmaceuticals.

Due to this outcome, we can confine subsequent attention to $A_{i,t}$ and $C_{i,t}$. The solid line in figure 2 gives a graphical illustration of the price response function (with $A_{i,t}=1$, $B_{i,t}=0$, $c_1=.1$, $c_2=10$, $\bar{p}_{i,t}=1$)

INSERT FIGURE 2 HERE

The price elasticity denotes the percentage change in sales induced by an incremental (or 1%-) change in price and is mathematically defined as

$$\varepsilon_{i,t} = \partial q_{i,t}/\partial p_{i,t} \cdot p_{i,t}/q_{i,t} \tag{10}$$

For the two versions C1 and C2 of our price response function we obtain

C1: $$\varepsilon_{i,t} = -c_1\, c_2 \cosh(c_2\, \Delta p_{i,t}) \frac{p_{i,t}}{q_{i,t}\, \bar{p}_{i,t}} \tag{11}$$

C2: $$\varepsilon_{i,t} = -c_1\, c_2 \cosh(c_2\, \Delta p_{i,t}) \frac{p_{i,t}\, \bar{q}_{t-1}}{q_{i,t}\, \bar{p}_{i,t}} \tag{12}$$

The equations (10) - (12) show that the dimensions of prices and quantitities are eliminated when $\varepsilon_{i,t}$ is computed. Hence, price elasticity is a dimensionless measure of price response and can readily be compared for different products.

The proposed price response function and its price elasticity have the following properties:

(1) The function gives economically reasonable values within a certain interval only. It doesn't make any sense to compute the expected sales effect of an arbitrarily large price differential (e.g. 1000%) by means of this function. According to Kotler (1971) this property applies to most marketing response functions.

(2) The magnitude of price elasticity increases for increasing positive and negative deviations of $p_{i,t}$ from $\bar{p}_{i,t}$; this is a necessary consequence of our basic assumption that sales response increases overproportionally with $\Delta p_{i,t}$. The price elasticity values are given by the dotted line in figure 2.

(3) The function allows for any development of price elasticity over time; $\varepsilon_{i,t}$ may decrease, increase , remain constant, or develop irregularly over time. Some examples which give evidence of this flexibility are depicted in figure 3 (the parameter values can be found in table 2).

INSERT FIGURE 3 HERE

(4) Since the absolute price level has turned out to have no significant influence on sales, the direct price elasticity $\varepsilon_{i,t}$, the cross-price elasticity $\varepsilon^C_{i,t} = \partial q_{i,t}/\partial \bar{p}_{i,t} \cdot \bar{p}_{i,t}/q_{i,t}$, and the respective market share

elasticities have the same magnitude. Therefore, we need not distinguish between direct and cross elasticities (though they have different signs) and can confine ourselves to the discussion of their common magnitude.

REGRESSION RESULTS

Since market shares do not necessarily show a PLC-pattern (e.g. if market sales and product sales develop proportionally $\rightarrow m_{j,t}$=const.) sales units were considered as the more appropriate dependent variable for our purpose.

The different versions of (9) are nonlinear with respect to the obsolescence parameter a_3 and the price parameter c_2. Therefore, the nonlinear least squares estimation technique of the TSP-program (a Gauss-Newton algorithm) was applied. The results of these estimations, however, proved highly unsatisfactory due to the following reasons (ranked according to their importance):

- though convergence was achieved in most cases the coefficients were almost invariably insignificant.
- the rate of obsolescence a_3 often had a negative sign which is economically unreasonable since it implies an unlimited growth of the carry-over effect.
- in about 20% of the cases no convergence was achieved.

These results suggested to attempt a different approach in which a_3 and c_2 were prefixed so that the sales function became linear in the remaining parameters and ordinary least squares (OLSQ) estimation procedures could be applied.

The search for the obsolescence parameter a_3 was limited to the interval (0, .1) since a_3 can reasonably be assumed not to exceed .1 for the given data intervals (quarters and bimonths).

A similarly apparent interval for reasonable values of c_2 is not available. For a given $\Delta p_{i,t}$, this parameter determines the magnitude of the argument of sinh and

thereby, the degree of nonlinearity of price response. One can easily realize this relationship in figure 2 by considering $\Delta p_{i,t}$ as given and c_2 as variable. For $|c_2 \Delta p_{i,t}| < 1$, sinh is almost linear; for $|c_2 \Delta p_{i,t}| > 1$, sinh becomes increasingly nonlinear. Thus, by prefixing different values of c_2 we can account for different degrees of nonlinearity in the sales response to price differentials.

In the estimations we usually prefixed three values in the following way (x_i denotes the maximal magnitude of $\Delta p_{i,t}$ over all periods)

case	value of c_2	range of argument of sinh	maximum of sinh	competitive price effect (shape of sinh within range)
1	$c_2=1/x_i$	-1 +1	1.17	quasi-linear (proportional)
2	$c_2=2/x_i$	-2 +2	3.62	medium nonlinear
3	$c_2=3/x_i$	-3 +3	10.01	highly nonlinear

In this way, both a quasi-linear and various nonlinear patterns of sales response to price differentials were admitted. In a few cases, where the results indicated that smaller or greater values of c_2 would improve the estimation some additional prefixations of c_2 were tested.

For each product, about 20 - 25 estimations with different combinations of a_3 and c_2 were run, the total number of regressions amounting to about 5000. The detailed results are reported in table 2.

INSERT TABLE 2 SOMEWHERE HERE

(Footnote to table 2)

Column (1) gives the product number (first digit: market, second digit: product). OLSQ in column (2) means ordinary least squares and CORC stands for the Cochrane-Orcutt iterative technique - a generalized least squares method - which was applied when the Durbin-Watson statistic (DW) of the OLSQ-

estimate fell into the inconclusive range or indicated autocorrelation. This enforced criterion has been suggested (Schneeweiss 1974, p.244) since DW is of limited reliability when one of the regressors is the lagged dependent variable (Durbin 1970). Durbin's H which would be appropriate in this case is not provided in the TSP-program of MIT-Harvard by means of which the estimations were made.

Column (10) gives the introduction periods $\bar{t}_i$, a negative number indicates that the product has been introduced before the period under investigation. In the cases marked by an asterisk the true introduction periods were not available, and $\bar{t}_i$ was set equal to 1. The numbers in parentheses are the t-statistics and a, b, c, and d denote significance at 1%, 5%, 10%, and 25% respectively (one tailed test).

(End of footnote table 2).

Reasonable results have been obtained for 35 out of the 43 products. A summary of the statistical criteria of the regressions is given in table 3.

INSERT TABLE 3 HERE

Thus, 82% of the coefficients were significant at 90% or more and 83% of the coefficients of determination R^2 exceeded 0.60. These results give strong empirical support to the hypotheses underlying our model. Both the PLC-dynamics and the competitive price effects appear to be adequately represented.

PRICE ELASTICITIES

From the regression equations, we computed price elasticities for all products and all periods. For this purpose the actual values of prices and quantities were inserted into (11) and (12) respectively.

In order to obtain condensed and comparable measures of the magnitude and the development of each product's price elasticity the median $\bar{\varepsilon}$ and the average growth rate $\bar{g}$ of each time series $\varepsilon_{i,t}$, $t = \bar{t}_i, \ldots, T$ were calculated. In this case, the median is the appropriate measure of the average magnitude of price elasticity since it excludes the influence of outlyers which were not infrequent. The average growth rate $\bar{g}$ is obtained as the geometric mean of the time series of elasticity growth rates. Note that the arithmetic mean would be inappropriate when applied to growth rates. The values of $\bar{\varepsilon}$ and $\bar{g}$ are given in columns (3) and (4) of table 4.

INSERT TABLE 4 SOMEWHERE HERE

One readily recognizes from column (3) in table 4 that the elasticity medians of the two product groups are considerably different. Almost all of the price elasticities of the pharmaceutical products (markets 1 - 4) are smaller than (or close to) 1, whereas the values for the detergents without exception are greater than 1. This important finding is further clarified in figure 4 where the distributions of the elasticity medians are depicted, separately for the two product groups. Only cases with significant price influence are included in figure 4.

INSERT FIGURE 4 HERE

The graphical illustration gives even stronger evidence of the differences in price response between the two product groups, the medians of the two distributions (.44 and 1.88) being significantly different at the 1%-level. Both these differences and the absolute magnitudes of price elasticities coincide very well with the managerial experience. The results are also in good accordance with the findings of other researchers (Telser 1962, Lambin 1976).

The average growth rates $\bar{g}$ in column (4) of table 4 indicate that the price elasticities have frequently undergone considerable changes over time of both positive and negative sign. In order to investigate this issue more deeply and to find out whether the changes in price elasticity show characteristic linkages with certain PLC-stages, we make two types of comparisons.

We first compare the elasticity growth rates of those products which were at the same PLC-stage (introduction, growth, maturity, or decline) during the last quarter or bimonth under investigation.

In addition to this cross-section comparison we study the magnitudes of price elasticity of one and the same product at different stages of this product's PLC. This longitudinal comparison is necessarily limited to products whose sales curve includes at least two PLC-stages; 30 products belong to this group.

Both the cross-section and the longitudinal comparisons require a preceding classification of the actual sales curves into PLC-stages. It is certainly desirable to use objective criteria for this classification. Respective attempts, in which growth rates, moving averages of 2, 3, and 4 growth rates, changes in signs of growth rates, or the stage identification criteria proposed by Polli and Cook (1969) were used, did, however, not prove useful. Polli and Cook state themselves that their criteria "are by no means flawless" and their application would, in fact, have led to stage sequences like e.g. maturity-growth-decline-maturity. The growth patterns in our sample (and probably empirical growth patterns in general) are somewhat different from the regular PLC-schemes usually found in marketing textbooks. Positive and negative growth rates or averages of growth rates actually occurred at all stages, and the magnitudes of growth rates showed enormous irregular variations (see also Dhalla and Yuspeh 1976). Therefore, a standardized classification scheme was not considered as appropriate and we decided to effect the necessary classification on the basis of a visual inspection of the sales curves. The procedure is demonstrated for three of the

products under investigation in figure 5.

INSERT FIGURE 5 HERE

Though this method may seem somewhat arbitrary we consider it as justified and appropriate in this case. On the one hand, the resulting classification is not likely to differ significantly from person to person, as discussions of the author with both managers and scientists have shown. Even if there are slight deviations in the classification they are not likely to affect the results. It should also be noted that this way of classification fully corresponds to the way in which the manager has to determine at which stage of its PLC a product actually is.

To a certain degree, the appropriateness of our classification is confirmed by a comparison of the relative average duration of each stage with the frequency distribution of stages obtained by Polli and Cook (1969) for brands. This comparison reveals a considerable conformity.

	Introduction	Growth	Maturity	Decline
Relative average duration (%)	11.2	29.2	33.1	26.3
Frequency distribution (%) Polli and Cook (1969)	n.a.	37	36	27

The results of the cross-sectional and the longitudinal comparisons are summarized in table 5 and columns (5) - (12) of table 4 respectively. Table 5 gives the average growth rates of price elasticity of all products arranged according to their PLC-stages during the last quarter or bimonth under investigation.

INSERT TABLE 5 HERE

Some striking characteristics are revealed:

- the magnitudes of $\bar{g}$ show a considerable uniformity within the various stages,
- all signs of $\bar{g}$ within the growth stage are negative,
- all signs of $\bar{g}$ within the decline stage are positive,

- with only two exceptions (5.1 and 7.3) the following relation proves true

$$\bar{g}_{Growth} < \bar{g}_{Maturity} < \bar{g}_{Decline}$$

Thus, we can conclude from the comparison of the price elasticities of various products being actually at different stages of their life cycles:

(1) Changes in price elasticity over the PLC seem to have a rather uniform pattern.

(2) Price elasticity of growth products decreases over time.

(3) Price elasticity of decline products increases over time.

(4) The rates of change in price elasticity are not uniform in sign for products being at the maturity stage. These rates, however, seem to be smaller in magnitude than both the rates of growth products and decline products.

In columns (5) - (12) of table 4 the numbers of quarters or bimonths and the elasticity medians of the different PLC-stages are given for each product. If we compare for each product the medians of adjacent stages (thus, only products with at least two stages are included), the following relationships are revealed:

(1) In 18 out of 19 cases (95%) the relation $\varepsilon_{Introduction} \geq \varepsilon_{Growth}$ is confirmed.

(2) In 10 out of 14 cases (71%) the relation $\varepsilon_{Growth} \geq \varepsilon_{Maturity}$ is confirmed.

(3) In 8 out of 8 cases (100%) the relation $\varepsilon_{Maturity} < \varepsilon_{Decline}$ is confirmed.

The plot of the medians of the various stages further elucidates these findings.

INSERT FIGURE 6 HERE

We can summarize our findings as follows:

An empirical investigation of 35 products gives strong support to the hypothesis that price elasticity shows typical changes over the product life cycle. During the introduction and the growth stage, a considerable decrease

seems to prevail. At the maturity stage, price elasticity typically reaches a minimum which is again followed by an increase during the decline stage.

These empirical findings are in contradiction to the hypotheses prevailing in the literature (see introductory section). This contradiction may partially be explained by the fact that usually no clear distinction between the absolute sales effect of a price change, which is given by the derivative $\partial q_{i,t}/\partial p_{i,t}$, and the relative sales effect, which is equal to the elasticity $\varepsilon_{i,t} = \partial q_{i,t}/\partial p_{i,t} \cdot p_{i,t}/q_{i,t}$ has been made.

How can the uniformity of the empirical outcomes be explained in view of the fact that the underlying price response function explicitly allows for different development patterns and does not constrain the results to be as reported. The main reason for the far-reaching uniformity of the elasticity developments has to be seen in the changes in $q_{i,t}$ (appearing in the denominator of the elasticity term) which typically turned out to be considerably greater than the changes in the derivative and in $p_{i,t}$, both appearing in the numerator of the elasticity term. Thus, in a certain sense the development of the sales $q_{i,t}$ tends to determine the changes in $\varepsilon_{i,t}$. Though the derivative $\partial q_{i,t}/\partial p_{i,t}$ typically also increases over the ascending branch of the PLC this increase is almost never so great as to neutralize the reciprocal effect of the growth in sales.

IMPLICATIONS

Since it has been our main objective to measure price elasticity and its changes the managerial and anti-trust implications of our findings shall be outlined in short only. The results seem in particular important for the optimization of the pricing strategy over the life cycle. The optimal pricing strategy is obtained by maximizing the sum of the discounted cash flows over the periods t,...,T (the

product index i is subsequently omitted)

$$\max \pi = \sum_{\tau=0}^{T-t} \{p_{t+\tau}\, q_{t+\tau} - C_{t+\tau}(q_{t+\tau})\}\, (1+i)^{-\tau} \tag{13}$$

where C(q) is the cost function and i is the discount rate.

The maximization of (13) requires a hypothesis on the presumable reaction of competitors to the firm's price setting. This complex issue cannot be discussed in great detail here. It seems,indeed, of minor importance in this case since we are interested less in the absolute levels of optimal prices than in their developments over time. Whereas the former are certainly governed by the competitive reaction pattern the latter are more likely to depend on the changes in price elasticity and cross-price elasticity over time.

Therefore, we consider the assumption that the prices of competing products are treated as givens and not as functions of p_t as not too restrictive for our purpose, which as aforementioned is to gain insights into the <u>development</u> of optimal prices.

Under this assumption the differentiation of (13) with respect to p_t leads to the first order condition

$$\frac{\partial \pi}{\partial p_t} = q_t + (p_t - C_t')\frac{\partial q_t}{\partial p_t} + \sum_{\tau=1}^{T-t} \{p_{t+\tau} - C_{t+\tau}'\}\, \frac{\partial q_{t+\tau}}{\partial p_t}\, (1+i)^{-\tau} = 0 \tag{14}$$

where C_t' denotes marginal cost.

Due to the formulation of $A_{i,t}$ in (3) and (4) we obtain the long-run effect of a price change in t as the product of the short-run price response, i.e. the derivative $\partial q_t/\partial p_t$ and the cumulative carry-over effect.

$$\frac{\partial q_{t+\tau}}{\partial p_t} = \frac{\partial q_t}{\partial p_t}\, a_2^{\tau}\, (1-a_3)^{\tau t + \tau(\tau-1)/2} \tag{15}$$

Inserting (15) into (14), multiplying by p_t/q_t, and solving for the optimal price p_t^* gives

$$p_t^* = \frac{\varepsilon_t}{1+\varepsilon_t} C_t' - \frac{\varepsilon_t}{1+\varepsilon_t} \sum_{\tau=1}^{T-t} (p_{t+\tau} - C_{t+\tau}') \, a_2^{\tau} \, (1-a_3)^{\tau t + \tau(\tau-1)/2} \, (1+i)^{-\tau} \quad (16)$$

Since ε_t still depends on p_t (16) doesn't allow for a straightforward computation of p_t^*. The equation clarifies, however, the following relations:

(1) The optimal dynamic price p_t^* is a compound of the optimal static price, which is given by the first term in (16) - this is the well-known Amoroso-Robinson-Relation - and the present value of the future marginal revenues caused by a price change in t.

(2) If $a_2 > 0$, $0 \leq a_3 < 1$, and $p_{t+\tau} > C_{t+\tau}$, this present value is positive and p_t^* is in all periods $\tau < T$ less than the optimal static price (note that this statement doesn't depend on the assumption on competitive reaction).

(3) If the price elasticity behaves according to our empirical findings (depicted in figure 6) then the optimal mark-up factor $\varepsilon_t/(1 + \varepsilon_t)$ is relatively smaller at the introduction and growth stage and relatively greater at the maturity stage, it again decreases during the decline stage.

(4) Both the long-run price effect and the development of price elasticity give support to a strategy of the penetration type. One should keep in mind, however, that these statements (and our analysis as a whole) apply to products which enter onto a market with existing substitutes and have to be viewed under the limitations of the assumed competitive reaction pattern. The assumption of a different pattern may considerable damp (though not eliminate) the outlined trend in optimal prices.

New products which establish a new market or product class and, thus, have no substitutes at the time of their introduction are in a completely different

situation and, consequently, different strategic recommendations apply (see Simon 1976).

It should also be mentioned that changes in cost have, of course, the same importance for the pricing strategy as the price response factors. If, for instance, marginal cost decreases according to the experience curve concept (Henderson 1972) the optimal prices need not increase over time since the increase in the mark-up factor can be compensated (or even overcompensated) by the decrease in marginal cost.

The optimal pricing strategy for a particular product at a particular time depends on the relative magnitudes of the demand and cost factors. Therefore, no general recommendation as to which type of strategy is optimal can be given, this decision has to be made in each individual case.

The numerical optimization of the pricing strategy is best achieved by means of a branch-and-bound algorithm which optimizes over a finite number of price alternatives within a prefixed price range. In figure 7 the optimal pricing strategy for product 4.2 of our sample is depicted. The actual price of this product remained constant at .71 whereas the price differential Δp_t, being negative for all t,changed from - 48% at t=1 to - 26% at t=10. The firm under consideration usually prices its products above the average prices of competing products. The competitors presumably expect this behavior and are unlikely to react if prices are up to this expectation.

Therefore, the optimization was run over the interval (.48, .80). The marginal cost was assumed to be constant ($C_t' = .20$) and an annual discount rate of 10 was applied, this rate is actually used in investment decisions by the producer of the product. The optimization was carried out for a planning horizon of 10 quarters or 2 1/2 years.

INSERT FIGURE 7 HERE

The resulting optimal strategy confirms the conclusion drawn from equation (16): The initially prices are considerably lower than the prices in later periods (penetration strategy). The fact that the initial prices are also less than the actual prices may be an indication that practitioners don't pay sufficient attention to the long-run effects of pricing. The present value of profits of the optimal strategy exceeds the respective value of the actual strategy by 33.7%.

The limitations of such an optimization have, of course, to be observed. Our model doesn't incorporate any negative goodwill or sales responses which may result from the price increases, the necessity to raise prices several times may well prevent managers from setting a low introduction price. Such considerations can, however, hardly be represented in a quantitative model and should have their proper place at the stage of managerial evaluation of the optimization results.

Further implications of our analysis refer to anti-trust issues. The question whether price competition is workable or not and whether dominant products are subject to substantial competition or not played an important role in a number of recent anti-trust cases (both in Germany and in the European Community).

The discussions on these points have regularly been characterized by a lack of objective information. The methods described in this article represent an appropriate tool for the measurement of competitive intensity and interdependencies under dynamic conditions. Albach (1977) used similar tools to determine the relevant market for pharmaceutical products and to measure the effectiveness of competition. He also extended the concept of the dynamic cross-price elasticity by estimating partial cross-price elasticities between single products or product groups. In this way an objective assessment of a product's competitive position on a dynamic market seems attainable.

SUMMARY

A dynamic sales model which incorporates the product life cycle concept and time-varying price responses has been presented. The model is of a very general nature and includes both time-invariant and time-varying carry-over effects as well as quasi-linear and nonlinear patterns of sales response to price differentials.

An empirical study of 35 products reveals typical changes in price elasticity over the life cycle and gives support to the conclusion that the magnitude of price elasticity decreases over the introduction and growth stage, reaches its minimum at the maturity stage, and again increases during the decline stage.

Though the analysis is subject to limitations (e.g. relatively short periods under investigation, many products included only 2 or 3 PLC-stages) the results cast heavy doubts upon the hypotheses prevailing in the marketing literature. They also call for further research for different product classes.

The findings seem to indicate the optimality of a penetration type strategy for products which are introduced onto markets with existing substitutes. Further implications are related to anti-trust issues.

TABLE 1: DATA CHARACTERISTICS

Market	Product Class	Number of Products	Share of total market represented in last period	Maximal number of observations	basis for price comparison	period length
1	Pharmaceutical	8	70.2 %	24	weight	quarter
2	"	6	83.0 %	24	daily dose	quarter
3	"	6	84.9 %	24	daily dose	quarter
4	"	8	82.2 %	24	daily dose	quarter
5	Detergent	5	69.7 %	18	weight	bimonth
6	"	5	55.5 %	18	package	bimonth
7	Household Cleanser	5	65.3 %	14	weight	bimonth

Product	Estimation Method	Function Type	absulute term a_1	season dummy d	retention rate a_2	obsolescence a_3	price deviation effect c_1	c_2	period of introduction $\bar{t}$	R^2	DW
1.1	OLSQ	A3/C2	690 (1.72)[b]	447 (2.57)[a]	.85 (4.67)[a]	.03	.15 (3.53)[a]	2.28	0	.7604	2.07
1.2	CORC	A3/C2	32.3 (1.07)	76 (3.15)[a]	1.25 (5.54)[a]	.07	.0012 (.138)	2.48	10	.7104	2.08
1.3	CORC	A3/C1	144 (3.84)[a]	19 (.87)[d]	1.18 (5.16)[a]	.05	12.77 (3.33)[a]	8.05	5	.4372	1.86
1.4	OLSQ	A1/C2	67 (3.33)[a]	-	.426 (1.83)[c]	.04	.0125 (1.54)[c]	2.34	15	.7357	2.63
1.5	OLSQ	A3/C2	5.9 (.45)	16.3 (1.45)[d]	.626 (5.19)[a]	.01	.0099 (2.63)[b]	3.53	18	.9718	2.73
1.7	OLSQ	A3/C2	181 (2.62)[b]	121 (3.10)[a]	.98 (3.71)[a]	.06	.0026 (1.46)[c]	2.02	13	.7260	2.08
2.1	OLSQ	A1/C2	466 (8.98)[a]	-	1.06 (13.43)[a]	.01	.001 (1.81)[c]	9.42	19	.9934	2.15
2.2	OLSQ	A3/C1	641 (1.74)[b]	221 (2.40)[b]	1.32 (14.72)[a]	.01	808.6 (1.72)[b]	.538	-16	.9734	2.43
2.3	OLSQ	A3/C2	141 (2.44)[b]	201 (2.27)[b]	1.22 (11.29)[a]	.001	.0619 (1.57)[c]	.525	-16	.9824	1.97
2.4	CORC	A3/C2	4567 (12.81)[a]	681 (9.32)[a]	-.307 (-2.58)[a]	.002	.0702 (.894)[d]	.523	-32	.7847	2.21
2.5	CORC	A5/C2	-	-	.88 (1.42)[c]	.04	.131 (1.50)[c]	1.39	5	.8376	1.63
2.6	OLSQ	A3/C2	64 (2.19)[b]	152 (2.17)[b]	.978 (15.6)[a]	.0015	.0007 (.65)	7.06	12	.9770	2.32
3.2	CORC	A1/C2	19428 (4.20)[a]	-	.336 (.812)[d]	.04	.002 (.448)	7.32	0	.9801	1.79
3.3	CORC	A1/C1	7349 (3.44)[a]	-	.35 (1.61)[c]	.004	1196 (2.05)[b]	2.16	-24	.6675	2.01
3.4	CORC	A5/C2	-	-	1.03 (7.12)[a]	.005	.0007 (.513)	4.85	11	.9477	2.56
3.6	CORC	A5/C2	-	-	.81 (15.2)[a]	.003	.0013 (6.45)[a]	14.74	14	.9131	1.24
4.1	OLSQ	A1/C1	6834 (4.46)[a]	-	.468 (6.24)[a]	.04	378 (2.65)[b]	3.35	17	.9693	3.09
4.2	OLSQ	A1/C1	936 (2.42)[b]		.756 (3.14)[a]	.04	45.7 (1.51)[c]	6.71	14	.8888	1.01
4.5	OLSQ	A1/C2	573 (.268)	-	1.11 (4.04)[a]	.01	.0224 (1.66)[c]	5.66	1	.5074	2.14
4.6	OLSQ	A1/C2	9494 (7.84)[a]		.39 (4.02)[a]	.01	.083 (5.01)[a]	3.85	3	.8594	1.20
4.7	OLSQ	A1/C2	1602 (1.79)[c]		.564 (2.91)[a]	.01	.0005 (.253)	15.58	14	.5215	2.40
4.8	OLSQ	A1/C2	2304 (17.1)[a]		.801 (11.26)[a]	.04	.0077 (1.40)[c]	3.45	0	.9348	1.91
5.1	OLSQ	A3/C2	1160 (19.08)[a]	73 (1.80)[b]	-.38 (-2.81)[a]	.08	.0104 (4.11)[a]	43.09	1*	.6272	1.55
5.2	OLSQ	A1/C1	1142 (9.91)[a]	-	-.136 (- .99)[d]	.08	100.2 (1.40)[c]	9.02	1*	.1102	1.85
5.3	OLSQ	A3/C2	1097 (2.15)[b]	175 (2.52)[a]	.186 (.47)	.01	.0006 (.25)	44.56	1*	.3788	1.85
5.4	CORC	A3/C2	782 (2.92)[a]	125 (2.86)[a]	.28 (1.12)[d]	.045	.0068 (1.37)[c]	27.70	1*	.6356	1.87
5.5	CORC	A4/C2	196 (5.00)[a]	37 (3.19)[a]	.463 (4.09)[a]	.045	.0166 (15.25)[a]	6.65	1*	.8537	2.29
6.1	OLSQ	A3/C1	596 (2.74)[a]	92.8 (2.16)[b]	.264 (1.06)[d]	.0015	126.8 (2.06)[b]	12.36	1*	.5536	1.28
6.4	CORC	A3/C1	750 (3.39)[a]	98 (2.83)[a]	.108 (.45)	.005	79.7 (1.33)[d]	16.13	1*	.6074	2.04
6.5	OLSQ	A4/C2	130 (2.29)[b]	32 (4.62)[a]	.42 (1.75)[c]	.015	.0037 (3.82)[a]	11.26	1*	.6772	2.42
7.1	OLSQ	A1/C2	1023 (2.06)[b]	-	.63 (3.18)[a]	.01	.0028 (1.72)[c]	92.11	1*	.6192	1.32
7.2	OLSQ	A1/C2	445 (2.08)[b]	-	.75 (6.33)[a]	.01	.0169 (1.30)[d]	5.32	5	.8907	2.33
7.3	OLSQ	A1/C2	2396 (3.70)[a]		.164 (.976)[d]	.0025	.0927 (4.80)[a]	8.85	1*	.7709	1.82
7.4	OLSQ	A1/C1	4301 (2.62)[b]		.79 (4.59)[a]	.01	228 (1.48)[c]	20.51	1*	.6777	1.78
7.5	OLSQ	A1/C2	1784 (2.86)[a]		.64 (2.44)[b]	.03	.0494 (2.02)[b]	6.58	1*	.6992	1.87

TABLE 3: SUMMARY OF STATISTICAL CRITERIA

	Coefficients	Significant Coefficients				Equations	R^2				
		1 %	5 %	10 %	25 %		<.60	.60 .70	.70 .80	.80 .90	>.90
n	118	57	23	17	9	35	6	8	6	5	10
%	100	48	20	14	8	100	17	23	17	14	29

TABLE 4: PRICE ELASTICITIES

(1) Product	(2) introduction period	(3) PRICE ELASTICITIES Total period $\bar{\varepsilon}$	(4) Total period growth rate $\bar{g}$	(5) Introduction n	(6) Introduction ε	(7) Growth n	(8) Growth ε	(9) Maturity n	(10) Maturity ε	(11) Decline n	(12) Decline ε	(13) price influence significant at (%)
1.1	0	.37	2.90	4	.33	7	.33	8	.36	5	.46	1
1.2	10	.15	-1.54	2	.29	2	.22	8	.11	3	.16	n.s.
1.3	5	.73	-11,72	2	3.93	5	2.79	12	.64			1
1.4	15	.83	-5.93			3	.98	7	.66			10
1.5	18	1.41	-18.90	2	2.34	5	1.24					5
1.7	13	.84	-15.14	3	2.35	9	.74					10
2.1	19	1.26	-10.39	2	1.26	4	1.24					10
2.2	-16	.34	-9.41			14	.53	10	.26			5
2.3	-16	.41	-4.35			21	.43	3	.39			10
2.4	-32	.05	10.11					24	.05			20
2.5	5	.37	-12.07	5	1.68	15	.33		.			10
2.6	12	.25	-19.71	2	2.48	11	.22					n.s.
3.2	0	.36	-5.35	7	.73	17	.31					n.s.
3.3	-24	.45	3.50					14	.34	10	.62	5
3.4	11	.14	-2.14	2	.15	12	.13					n.s.
3.6	14	.54	-12.95	2	1.62	9	.54					1
4.1	17	3.52	-12.43	2	7.11	6	3.45					5
4.2	14	1.09	-19.89	2	6.88	10	1.01					10
4.5	1	.63	1.14	3	.55	5	.67	13	.63	3	.77	10
4.6	3	.87	.66	6	.89	10	.79	6	.96			1
4.7	14	1.03	-6.24	1	34.0	2	1.13	8	.94			n.s.
4.8	0	.43	4,71			2	.19	7	.41	14	.44	10
5.1	1*	2.85	-5.50					18	2.85			1
5.2	1*	1.22	1.10					18	1.22			10
5.4	1*	1.07	-14.28	7	4.81	11	.94					10
5.5	1*	2.22	4.75					2	1.68	5	2.25	1
6.1	1*	1.92	-1.17					18	1.27			5
6.4	1*	1.83	-3.34	7	2.18	11	1.61					10
6.5	1*	1.45	8.76					11	1.02	7	1.10	1
7.1	1*	4.48	3.77			5	8.70	9	4.40			10
7.2	5	1.82	-6.59	2	3.03	8	1.76					15
7.3	1*	4.73	3.30			7	4.18	2	4.44	3	5.27	1
7.4	1*	1.34	-1.29			10	1.50	4	1.33			10
7.5	1*	3.49	-2.03			11	3.96	3	2.70			5

* true introduction period not known, value set equal to 1

TABLE 5: ACTUAL LIFE CYCLE STAGE AND GROWTH RATE OF PRICE ELASTICITY

	GROWTH		MATURITY		DECLINE	
	Product	$\bar{g}$	Product	$\bar{g}$	Product	$\bar{g}$
PHARMA	1.5	-18.90	1.3	-11.72	1.1	2.90
	1.7	-15.14	1.4	-5.93	3.3	3.50
	2.1	-10.39	2.2	-9.41	4.5	1.14
	2.5	-12.07	2.3	-4.35	4.8	4.71
	3.6	-12.95	4.6	.66		
	4.1	-12.43				
	4.2	-19.89				
	mean	-14.53		-6.15		3.06
DETER-GENTS	5.4	-14.28	5.1	-5.50	5.5	4.75
	6.4	-3.34	6.1	-1.17	6.5	8.76
	7.2	-6.59	7.1	3.77	7.3	3.30
	7.4	-1.24				
	7.5	-2.03				
	mean	-5.49		- .96		5.60

FIGURE 1: PRODUCT LIFE CYCLES

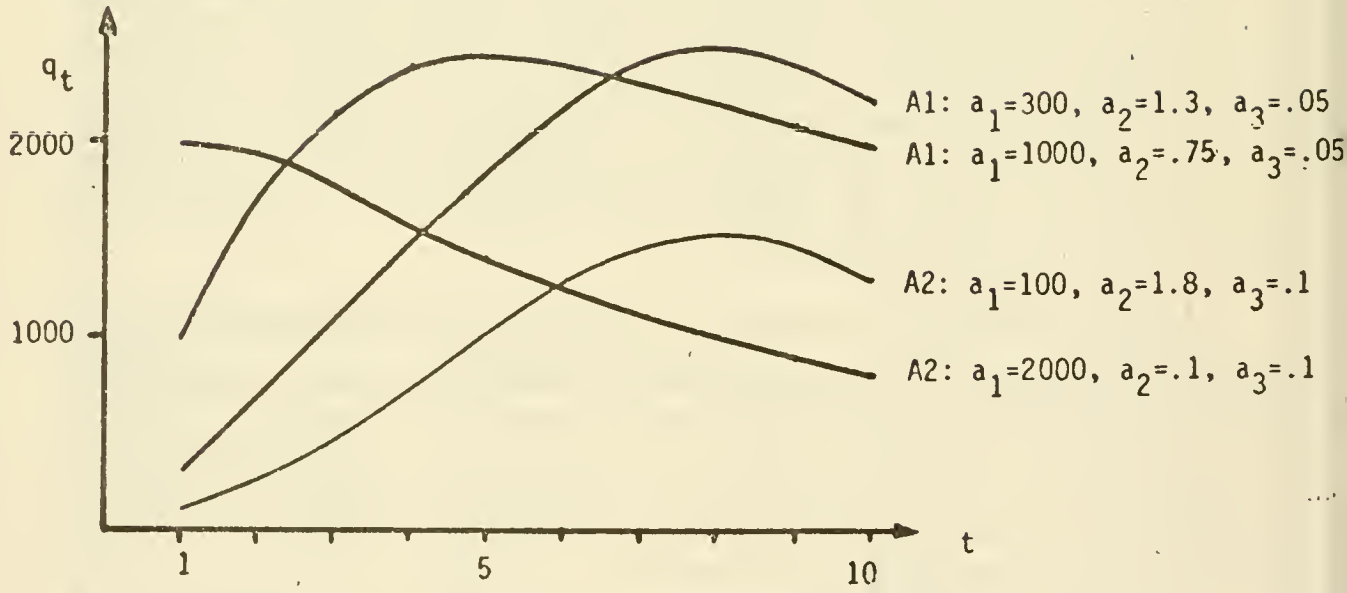

FIGURE 2: PRICE RESPONSE FUNCTION AND PRICE ELASTICITY

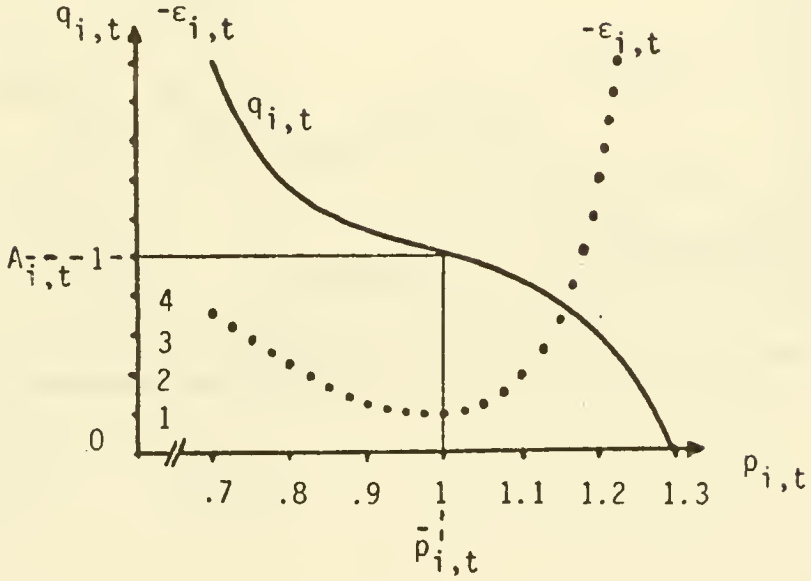

FIGURE 3: EXAMPLES OF PRICE ELASTICITY DEVELOPMENTS

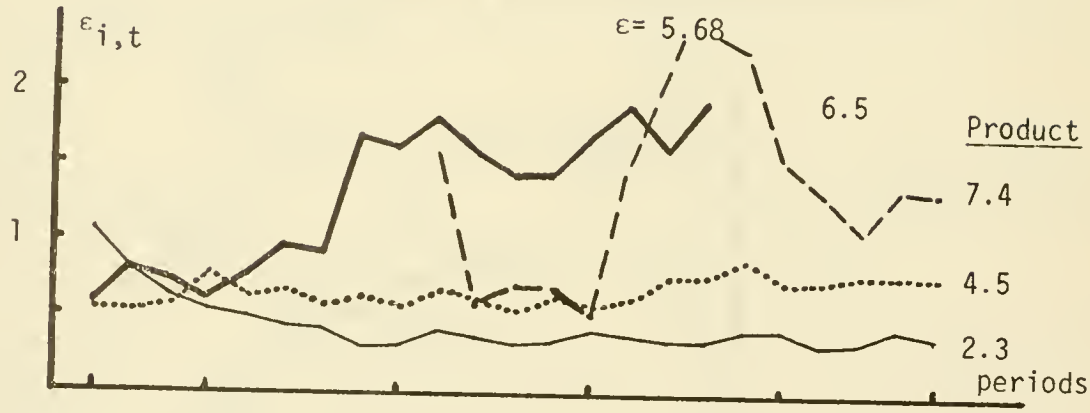

FIGURE 4: DISTRIBUTIONS OF THE MEDIANS OF PRICE ELASTICITY

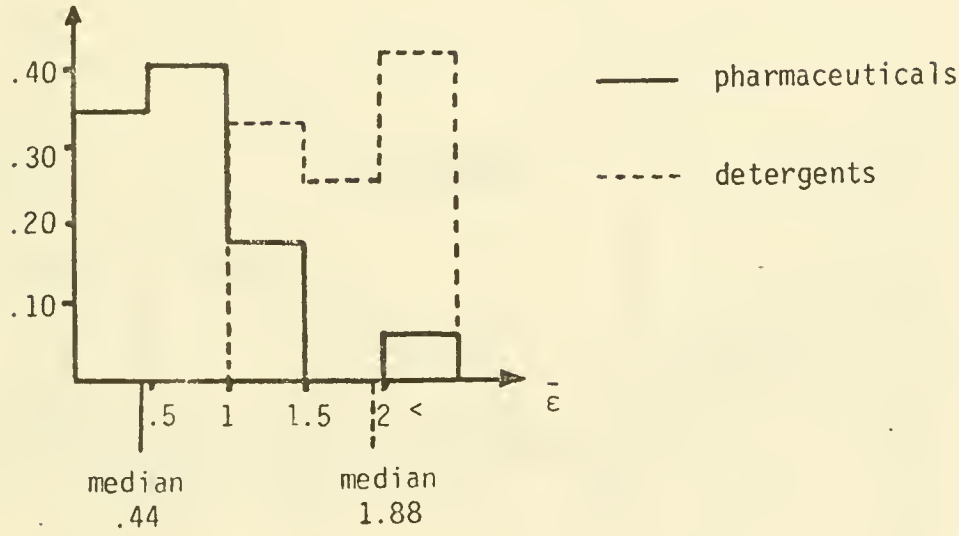

FIGURE 5: EXAMPLES OF CLASSIFICATIONS INTO LIFE CYCLE STAGES

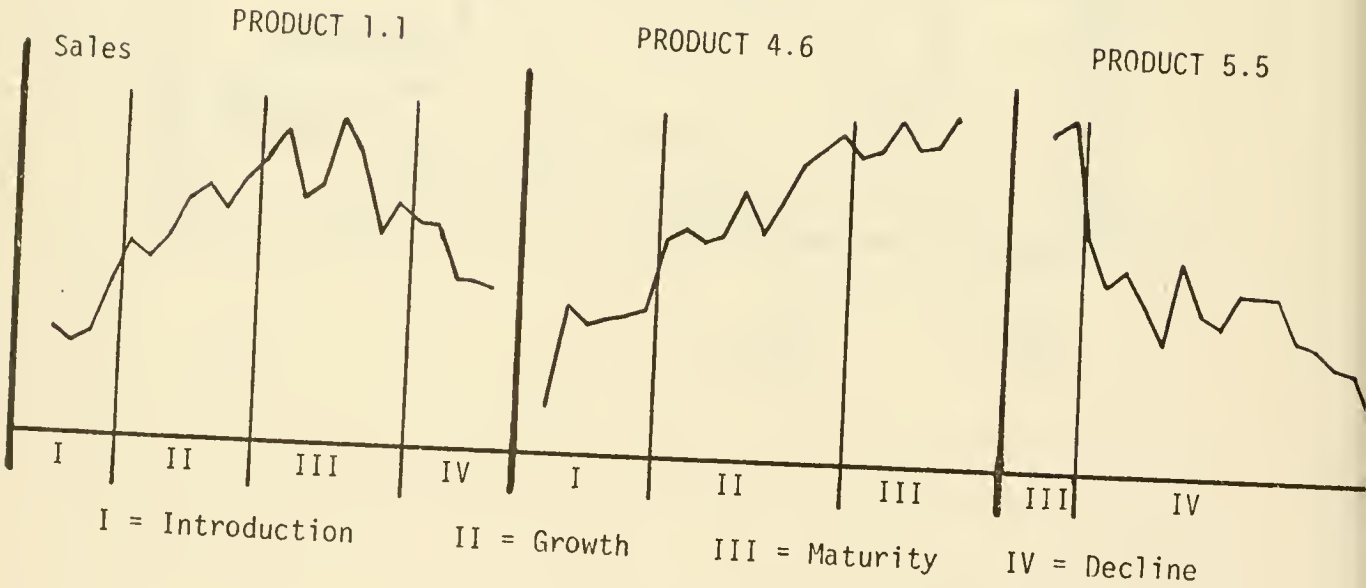

FIGURE 6: AVERAGE PRICE ELASTICITIES AT DIFFERENT STAGES OF THE PLC

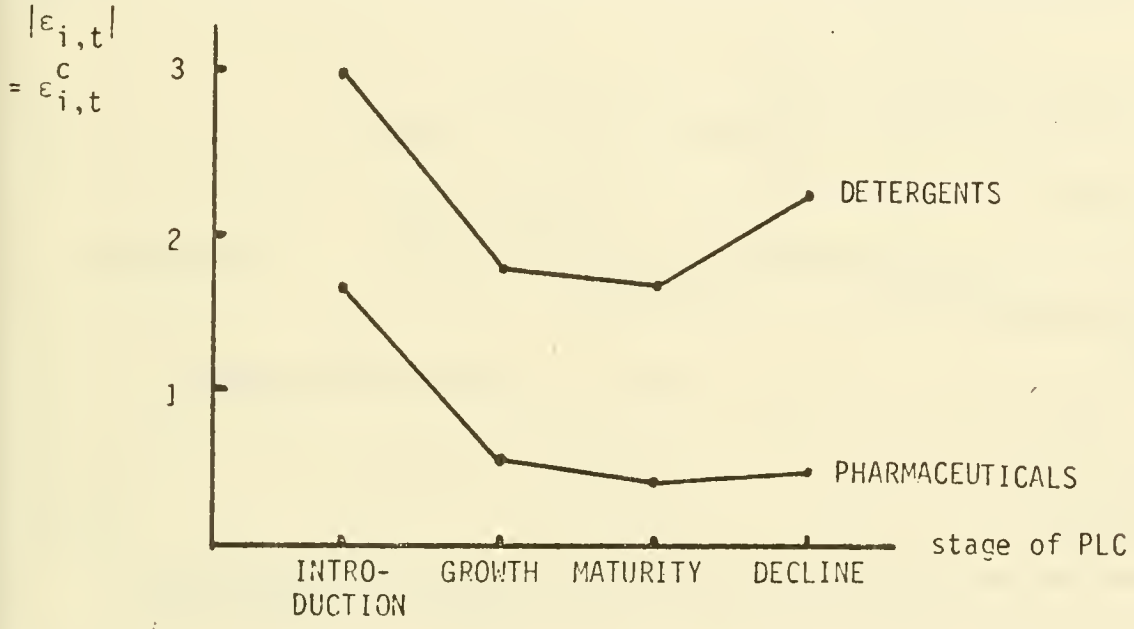

FIGURE 7: OPTIMAL AND ACTUAL PRICING STRATEGY OF PRODUCT 4.2

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