WORKING PAPER

SCHOOL OF INDUSTRIAL MANAGEMENT

The Debt-Equity Ratio, the Dividend Payout Ratio, Growth and the Rate at Which Earnings Are Capitalized: An Empirical Study

52-64

William L. White

MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE 39, MASSACHUSETTS

This working paper is one of a set of publications by the School of Industrial Management at M.I.T. reporting on the research currently being pursued by its staff. This investigation was originally submitted as a thesis as partial fulfillment of the requirements for the degree of Ph.D. in Industrial Management at M.I.T. Since it will form a continuing part of the research of the author, who is now an Assistant Professor of Finance, these first results are being published with the hope of stimulating comments and suggestions.

The Debt-Equity Ratio, the Dividend Payout Ratio, Growth and the Rate at Which Earnings Are Capitalized: An Empirical Study

52-64

William L. White

Most grateful thanks are extended to the Harold Stonier Foundation and the American Bankers Association as well as to the Ford Foundation's grant to M.I.T.'s School of Industrial Management for Research in Business Finance for the generous support which made this work possible. Professors Eli Shapiro, Edwin Kuh, and Daniel M. Holland also deserve thanks for their guidance and assistance.

CHAPTER 1

INTRODUCTION

One of the most important problems facing a financial manager is that of choosing from all the available investment projects that set whose acquisition will enhance the value of the firm. That is, he must select that set of proposals whose elements have income streams with a capitalized value greater than their cost. Conversely, his problem is to choose those projects whose cash flows, when discounted at that capitalization rate, have a positive present value.

This problem can be separated into two parts - the determination of the cash flow associated with each project and the discount or capitalization rate to be used in computing the present value of these cash flows. The major concern of this thesis is not with the former but with the latter. Its purpose is to identify the factors determining the capitalization rate and, more specifically, to separate these factors into those associated with the physical assets in which the firm invests and the income streams they generate, and those having their origin in the method used to finance those assets.

If it would be possible to ascertain what forces influenced the rate at which income streams to a specific firm were capitalized and the mechanisms through which these forces acted, a major step would have been taken toward the solution of the capital budgeting problem. Thus, this thesis has as its focus the capitalization rate. It is meant to be both a theoretical and empirical study, drawing on the relevant parts of

economic analysis to construct a logical and (hopefully) relevant model, and relying on statistical tests to determine its real world validity.

Much has been written concerning the manner in which firms ought to make investment decisions. While there is some difference of opinion as to how the problem is to be formulated, there is widespread agreement on the need for a required or cut-off rate of return. Thus, although the linear programming approach to the capital budgeting problem was developed primarily to deal with multiperiod outlays and the analysis of sets of investments, and the straightforward computation of present values usually considers investments as individual ventures, both formulations of the problem require a rate of return as an input.[1/]

This required rate of return or cut-off return for investment projects is the weighted average cost of the debt and equity funds employed to finance the project. Clearly, if the project generates a rate of return in excess of the cost of funds to finance its purchase, the value of the firm will be increased. Projects yielding less than their cost have the opposite effect. Therefore, the implementation of capital budgeting

---

1/See Weingartner, Martin, Mathematical Programming and the Analysis of Capital Budgeting Decisions, Prentice-Hall, 1963; or, Charnes, A., etal., "Application of Linear Programming to Financial Budgeting and the Costing of Funds," Journal of Business, January, 1959, for a presentation of the linear programming view; the discussion in The Capital Budgeting Decision by H. Bierman, Jr., and S. Smidt, Macmillan, 1960, is representative of the analysis of independent projects with single period outlays.

theory requires a knowledge of how this weighted average cost is determined and what its magnitude is. For the individual firm, the crucial problem is isolating the effects on the capitalization rate of actions and variables it can control. Specifically, the important question is whether the financial mix employed affects the weighted average cost of capital, or whether this average is independent of the mix and solely determined by the kind of physical assets in which the firm invests. That is, given the type of assets in which the firm invests, is it possible for the management of a firm to make a set of financial decisions which will influence the weighted average costs of the debt and equity funds employed by them? Restated in even another way, is there any such thing as an optimal debt-equity ratio or an optimal dividend payout policy? These are the questions to which this thesis is addressed.

Several studies have already been undertaken with these questions in mind. They have been both empirical and theoretical. The theoretical work of Modigliani and Miller[2/] has shown that in a world with riskless debt, no growth, and no taxes, the financial managers cannot influence the weighted average cost of capital by altering their debt-equity ratio. The average cost of capital in their model is independent of financial structure. In this thesis, the assumption of riskless debt is rejected in favor of a debt instrument which has associated with it some

---

[2/] Modigliani, Franco, and Merton H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review, Vol. XXVIII, June, 1958, pp. 261-297.

risk for both issuer and acceptor. These risks are developed in Chapter II. The introduction of these risks creates a model in which there is an optimal debt-equity ratio in that there is now a debt-equity ratio which minimizes the weighted average cost of debt and equity.

As soon as the phenomenon of growth is introduced, there is another financial decision which may influence the value of the firm. It is the portion of earnings paid as dividends. Two views have been advanced concerning the importance of this variable. Miller and Modigliani[3/] have presented a model in which the value of a firm is independent of the dividend payout rate chosen. This may be alternately stated as showing that the dividend payout ratio does not influence the rate at which earnings are capitalized. Others, notably, Gordon[4/] and Lintner,[5/] have expressed a quite different view - one which does involve a relation between dividend payout ratios and the value of the firm. For reasons developed in Chapter II, this latter view is adopted in this thesis. In addition to hypothesizing that the dividend payout affects the capitalization rate, a variable is introduced in an attempt to account for the effect of the difference between the personal income tax and the capital gains tax.

[3/] Miller, Merton H., and F. Modigliani, "Dividend Policy, Growth and the Valuation of Shares," Journal of Business, Vol. XLIX, September, 1959.

[4/] Gordon, Myron J., "The Investment, Financing and Valuation of the Corporation," Homewood, Illinois: R. D. Irwin, 1962.

[5/] Lintner, John, "Dividends, Earnings, Leverage, Stock Prices and the Supply of Capital to Corporations," The Review of Economics and Statistics, Vol. XLIV, August, 1962, pp. 243-269.

With these variables we construct a model for the capitalization rate for earnings having three terms which, taken together, imply an optimal dividend policy.

Thus, our interest in the rate at which the earnings of a firm are capitalized arises from the desire to be able to implement the theory of capital budgeting. We are interested both in the magnitude of the capitalization rate and the extent to which it can be influenced by financial decisions. With these goals in mind, a model is constructed which specifically incorporates the influences of the risks which are thought to be associated with debt as well as the advantages and disadvantages of dividend payments.

Once the model is developed, statistical tests are undertaken to assist in assessing the validity of the model as well as to provide estimates of the values of the parameters. These estimates are then to be used in the determination of optimal policies for debt and dividends. The first of these tests is a cross sectional analysis of the firms in five different industries in each of the fifteen years between 1946 and 1960 inclusive. In these tests the equation specified by the model is estimated for each industry in each of the years. These tests show the model "fits" the data well but indicate the colinearity in part of the data makes it impossible to split the influences of debt and dividends into the separate parts necessary for the determination of optimal policies for these variables.

That is, as originally formulated, the total influence of debt is composed of one element which causes the capitalization rate to fall as the debt-equity ratio is increased and one which makes it rise. At low debt-equity ratios the first influence is thought to be the more important while at higher debt-equity ratios, the force of the second influence predominates. This means that at some intermediate debt-equity ratio the combination of the two forces has its minimum. As it turned out to be statistically impossible with the chosen formulation to split the total force into its two components, it was also impossible to determine empirically optimal strategies under the assumption that both forces existed in reality. That is, the inability to partition the total force into the hypothesized components was thought to be derived from problems with the data (strong colinearity) and perhaps from the exact way the equation had been specified but was not taken as necessarily implying that the separate forces did not exist.

However, it was possible to estimate equations containing only one term for debt and one term for dividends instead of the two hypothesized in the initial structure. While the estimates from these latter regressions are of little use in determining optimal debt and dividend policies if the originally hypothesized influences are thought to exist, they still are of some interest. They can be used to test whether the capitalization rate is at all influenced by the debt-equity ratio or the dividend payout ratio chosen by a firm. A statistically significant relation between the capitalization rate and either of these variables would imply that capitalization rates were at least not independent of debt or dividend policy.

When these equations containing only one term each for the influence of debt and dividends were estimated it was found that in only a minority of cases did these two variables significantly influence the capitalization rate. It was also found that a variable introduced to estimate the influence of the differential between the income tax and the capital gains tax did not have a constantly significant slope coefficient. Most of the explanatory power of the model came from a growth variable which was originally introduced as part of an attempt to adjust short-run expectations to long-run expectations. It was also found that rapidly growing firms had higher capitalization rates than slowly growing firms. According to the original hypothesis, the capitalization rate, defined to be the ratio of dividends per share to price per share plus the growth rate of dividends, was independent of the magnitude of the growth rate. It was hypothesized that higher growth rates would imply share prices enough higher to assure that the ratio of dividends per share to price per share plus the growth rate of dividends was the same for firms equivalent in all other respects. These cross section regressions indicated that as growth rates increased, prices did not increase enough to keep the capitalization rate constant, and seemed to imply that rapidly growing firms had higher capitalization rates - lower prices - than the original model predicted.

Summarizing the results of both of the sets of cross section regressions, it was found that the managerial implications which had motivated the thesis could not be extracted from the data, that there was little influence on the capitalization rate for any debt and dividend terms and that growth seemed the most important variable determining capitalization rates.

Having been unsuccessful with the estimation of the parameters of the original model, but suspicious of the lack of results, it seemed wise to examine the statistical properties of the model more closely. The residuals computed from the estimated equations were the object of immediate interest. Autocorrelated residuals, the plague of time series analysis, have their counterpart in cross section analysis. In cross section analysis the problem is that each firm being studied may have associated with it a particular unexplained effect which is present each time a cross section is estimated using that firm as a data point. This means that the error variances in each of the cross sections have common components arising from these firm effects. These effects when present and not specifically estimated, create, among other things, biased slope coefficients. A tentative test showed evidence of such influences associated with most of the firms which were included in the study.

Considerable theoretical interest has been shown in the estimation of equations containing firm effects, but due to computational difficulty, little empirical work has been possible. A method of specifically estimating firm effects which eliminates the biases introduced when they are ignored has been developed by A. H. Carter.[6/] He has shown that an adaptation of the original hypothesis which includes the "firm effects" by introducing dummy variables, one for each firm, will lead to unbiased estimates of slope coefficients and error variances. The availability of these relatively

[6/] Carter, A. H., "The Estimation and Comparison of Residuals Regressions Where There are Two or More Related Sets of Observations," Biometrika, Vol. 36, 1949, pp. 26-46.

new and untried statistical techniques together with the computational ability of the IBM 7090, and the evidence of the potential usefulness of such an analysis in this study, lead to the adaptation of the original cross section hypothesis and to a test which pooled the annual cross sections for an industry into a single regression estimating the influences of debt, dividends, and growth along with a new set of variables, the firm effects.

As there was no reason to believe that the effects of the colinearity encountered earlier would be diminished by the introduction of firm effects, the analysis included only one term each for debt and dividends. This meant that although the equation had better statistical properties than the original cross sections, it was like the second set of cross section regressions in that it would produce few managerial implications for debt and dividend policy. This failing of the new formulation was not considered critical as there was more concern that the earlier cross sections had shown no influence of debt and dividends at all, than that it had been impossible to split these influences into two parts. Hopefully, this new approach would imply at least some influence of debt and dividend policies on equity prices. This hope, and the implementation of an interesting statistical tool, created the interest in the pooled regressions.

Initially three hypotheses were to be tested. First, were there significant firm effects and did their inclusion alter the earlier slope coefficients; second, was growth per se still important; and, finally, could debt and dividend policy be found to influence the rate at which earnings were capitalized?

Significant firm effects were found. The significance was far beyond the .001 level. Also, the slope coefficient for growth was reversed - it had been positive and significant in the cross sections and it was now negative and significant in four of the five industries examined. The positive sign had implied that the original model had overstated the influence of growth on equity prices. In the cross section regressions, as growth rates increased, prices seemed not to rise as much as the model predicted. The negative sign in the pooled regression implied just the opposite, however. In this statistically more correct adaptation of the hypothesis, as growth rates increased, prices increased faster than the original model had predicted. Although the debt term still proved either to be insignificant or to have the wrong sign, the dividend term was significant with higher dividend payout ratios implying higher equity prices. Thus these pooled regressions had proved most successful. They made it clear that the cross section analysis had presented a misleading influence for growth and also restored the dividend payout ratio to a place of importance in determining capitalization rates.

At the time the pooled regressions were to be run, notice was taken of another hypothesis concerning the behavior of equity prices. It had been conjectured that the equity prices of rapidly growing firms might be reduced by the payment of dividends while the share prices of less rapidly growing firms would be little influenced by such an effect. Noting that both growth rates and dividend payout rates were important in the hypothesis being tested in this thesis, the question was raised as to whether it would be possible to test the validity of the conjecture. That is, if the hypothesis tested in this

thesis showed growth and dividends to be important, could it find any statistical verification for an interaction between the two? A rather neat test was possible and the hypothesized interaction was found to be significant. As will be explained more fully in Chapter VI, the elasticity of equity price with respect to changes in the growth rate is cut approximately in half by the introduction of this interaction. In addition to being significant in the sense that it significantly reduced residual variance, this interaction also improved the stability of the coefficient of the dividend payout ratio taken alone. This was taken as further evidence of the validity of the interaction.

Thus, although no specific conclusions directly relevant to a firm's financial policy were possible, two inferences of a managerial sort could be made. First, the original model of the capitalization rate understated the influence of growth on equity prices and second, there was substantiation of a significant interaction in the way that the dividend payout and growth rates affected the capitalization rate. The first inference means that, as the growth rate increases, share prices rise more than enough to keep the ratio of dividends to price plus the growth rate constant - the capitalization rate falls as the growth rate rises. The second inference is that the dividend payout rate ought not to be chosen without consideration of the growth rate - higher growth rates should be accompanied by lower payout rates.

The major statistical or methodological conclusion was that the original cross section analysis was inappropriate and had led to serious bias in the estimation of slope coefficients. In addition it was concluded that the inability to estimate an influence for debt arose from the fact that the method

of allowing for risk was inadequate. Each industry had been hypothesized to contain firms with identical risk associated with the streams of income arising from their physical assets. While this assumption may have been valid enough to measure the influences of growth and dividend policy on capitalization rates, it does not seem to have been valid enough to estimate the influences of debt. This can be seen by noting that debt was thought to influence the capitalization rate by adding some financial risk to the existing risk associated with the physical assets. If the asset risk were not strictly homogeneous within the industries, differences in risk between firms would not result solely from differences in debt-equity ratios. This mixing of different asset risk and different financial risk would make it difficult to isolate the influence of debt alone on financial risk. The reliability of the coefficients of the growth and dividend terms is also diminished by this heterogeneity of risk within industries. However, as the growth and dividend influences do not depend so crucially on homogeneous asset risk, the slope coefficients are not likely to be so seriously affected as in the case of debt. This _a priori_ belief is reinforced by the stability of the coefficient of the growth term and by the favorable results of the test for the influence of an interaction of dividend policy and growth. Thus, with respect to the assumption that the industry classification chosen contained firms with homogeneous asset risk, it is concluded that it is a valid enough risk characterization for the estimation of the influences of growth and dividends but not a valid enough characterization to estimate the influence of debt. To specifically deal with debt, some better way has to be found to standardize for risks other than those arising from the debt itself.

CHAPTER II

The Variables Influencing the Capitalization Rate

This chapter develops the variables thought to influence the rate at which the earnings of a firm are capitalized. In order to clarify the issues and to construct a framework for evaluating the model once constructed, it is useful first to comment upon several other studies concerned with the general problem of valuation.

Much of the empirical research to date on the problem of the valuation of the firm has been primarily concerned with attempts to explain the price at which the equity of a firm is sold. This is usually done by arrying those variables which are thought to affect price on the right hand side of a regression equation and proceeding with a least squares estimate of the slope coefficients. Thus M. J. Gordon[1/] has models of the form:

$$\frac{P}{W} = \alpha[(1-b)\frac{Y}{W}]^{\alpha_1} \alpha_2^{br} S^{\alpha_3} \alpha_4^{W} L^{\alpha_5}$$

where

$\frac{P}{W}$ is year end price/book value per share

$\frac{Y}{W}$ is income/book value per share

b is the retention rate, (income - dividends)/income

1/ Gordon, Myron J., The Investment, Financing and Valuation of the Corporation, Homewood, Illinois: Irwin, 1962.

r is the return on net worth

S is a size index

w is an uncertainty index

L is the debt-equity ratio

Here, s, w, and r measure the influences of the composition and the total of the firm's assets, while b and L are concerned with the financial mix employed to finance them.

Again concentrating on price, Durand[2] estimates

$$\log P = K + b \log B + d \log D + e \log E$$

where P is price, B is book value (or capital per share), D is dividends per share, and E is earnings per share. He also states "... a number of others were tried in the course of the study, and these included: total capital as a measure of size ..., several ratios of assets to capital, a lagged variable consisting of average past dividends, and some variables relating to the growth and stability of earnings. None of these additional variables, however, significantly reduced the residual variance..."

However, several attempts have been made along another track. In these studies, earnings or dividends are explicitly capitalized to obtain price. Thus Durand[3] uses a capitalization process and

---

[2] Durand, David, "Bank Stocks and the Analysis of Covariance," Econometrica (January 1955).

[3] Durand, David, "Cost of Debt and Equity Funds for Business: Trends and Problems of Measurement," Conference on Research in Business Finance, pp. 215-47, New York: National Bureau of Economic Research, 1952.

also discusses several problems associated with both the amount to be capitalized and the capitalization rate. Gordon and Shapiro[4] capitalize, at a rate p, a dividend stream growing at a rate r per year and comment that both the dividend rate and the debt-equity ratio may affect p. Modigliani and Miller[5] also use a capitalization procedure. Their model and conclusions are quite different from those of Durand and Gordon and Shapiro, however. In "The Cost of Capital, Corporation Finance and the Theory of Investment" they explore a model in which the value of a firm is independent of the debt-equity ratio and in "Dividend Policy, Growth, and the Valuation of Shares" another in which it is independent of the dividend payout ratio and is a function only of the market discount for the risk associated with the streams of income arising from the physical assets the firm holds. Neither Durand nor Gordon and Shapiro present any tests of their proposition that the capitalization rate depends in part upon the financing decisions of the firm. Moreover, the Modigliani and Miller assumptions rule out any such effect. Although

---

[4] Gordon, Myron and Eli Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," Management Science, III (1956) pp. 102-110.

[5] Modigliani, Franco, and Merton Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review (1958); and, "Dividend Policy, Growth, and the Valuation of Shares," Journal of Business, Vol. XXXIV, No. 4.

Gordon[6/] is concerned with capitalizing streams, the model he develops is one explaining price and not the capitalization rate.

The purpose of this thesis is to combine the various elements of the regression analysis with the capitalization rate as the variable to be explained. That is, the value of a firm will be derived as the capitalized value of its income stream, but the capitalization factor will be dependent upon the factors thought important in previous studies and by the present author. A model will be developed expressing the manner in which these factors are thought to influence the capitalization rate. Regression analysis will then be undertaken to see if the hypothesized equation determining the capitalization rate can be maintained in a statistical sense. As will be seen, this choice of dependent variable creates several advantages over earlier regression studies, both in interpretation and estimation.

Capitalization streams requires knowledge of two things - the size of the streams and the capitalization rate. Since the analysis which follows will consider the effects of risk, taxes and growth on the value of the firm, the effects of each of these elements upon the size of the stream and the capitalization rate must be determined.

[6/] Gordon, Myron J., The Investment, Financing and Valuation of the Corporation, Homewood, Illinois: Irwin, 1962.

First, to risk. A precise definition of what is meant by the term risk is difficult to phrase. That the risk associated with a situation arises from the degree of variability of its possible outcomes seems a reasonable approximation. However, translating this statement into an exact, measurable quantity is quite difficult. In this thesis, two sources of uncertainty or variability will be distinguished. Both will be called risk, but neither has an unambiguous mathematical definition.

The first source of variability arises from the asset side of the balance sheet and the production process it represents. The model of this process which is adopted in this thesis is the engineer's construct of a "black box". This conception assumes no internal characterisitics need be known and is only interested in the output obtained from a given input. The way input is transformed into output - the transfer function - is the criterion used to characterize the process. Here, the production process is thought of as an activity - a black box - into which are placed current and capital inputs. Out of this process come income streams with a certain variability or predictability. This variability is what characterizes the process. Equivalent processes - equivalent firms - are firms which produce income streams with the same probability distribution of outcomes in response to a unit input.

This technique for dealing with that part of the total uncertainty associated with a firm which arises from the production process, while having considerable intuitive appeal is difficult to implement. Very few actual firms produce a single output or even the same proportions of a set of different outputs. Thus; each firm is a combination of "black boxes", each with possibly different characteristics. In addition, it is almost impossible to precisely define a unit of input. Even if you could find two firms with identical production demands, various combinations of the materials required in production are possible - e.g., excess productive capacity to meet temporary changes in demand as opposed to higher average inventory levels and less excess productive capacity. Each of these combinations of inputs may introduce different kinds and different amounts of variability into the income streams produced. Despite these ambiguities, the intuitive appeal of the black box construct is strong enough, so that this conception will be used here. This source of uncertainty will be called the "asset risk" associated with the firm.

If this were the only uncertainty introduced into the analysis - that is - if the only uncertainty is that associated with the production process, it can be shown that the financing mix - the debt-equity ratio - has no influence on the value placed on the income stream generated by the assets in which the firm invests. If, for instance, a firm shifted its financing mix from all equity to a combination of debt and equity, the value of the equity would have to fall from its original amount by just the increase in debt. The value of the firm -

the total of debt and equity would be unchanged. Modigliani and Miller[7/] have shown that if this were not so, arbitragers could, by levering their own portfolios, continually make profits. The actions of these arbitragers assures that equity prices would respond to increases (or decreases) in debt in such a way as to keep the value of a firm independent of the financial structure and entirely dependent upon the size and uncertainty of the income streams produced by its assets.

If we now explicitly recognize two kinds of financial risk associated with debt, these conclusions are blunted. First, for the arbitrager, the undoing of any gains accruing to a levered stock involves the assumption of debt on personal account. The Modigliani and Miller proposition requires the unlevered equity to be offered as collateral. But the value of this equity at future points of time is uncertain[8/] while the value of the debt obligation is certain. Moreover, since the investor does not have limited liability as does the corporation, this self-levering action commits the arbitrager to a more risky position than when he held the stock levered by the corporation. Thus, he will undertake the arbitrage operation only as long as the value of the levered corporation is enough in excess of that of a similar but unlevered company to compensate him for

---

[7/] Modigliani, Franco, and Merton Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review (1958).

[8/] At the moment, this uncertainty, which is associated with both the expected earnings and the capitalization factor, is merely asserted. Its origins will be discussed more fully in the section on the role of dividends.

undertaking the risk of self-leverage. The levered company then sells at a premium over the unlevered one, or alternatively, the value of the levered company has risen with the rise in its debt-equity ratio.

It follows, therefore, that the increment in value to a corporation arising from increasing its debt-equity ratio rises as the physical assets in which the firm invests generate more variable income streams. For then, the value of the pledged unlevered equity is more variable, and thus the portfolio position of the arbitrager becomes more risky, so the premium he demands is larger in amount. The larger the premium, the greater the gain to the corporation arising from corporate leverage.

Other quite different kinds of reasons may be advanced to support the view that arbitragers would not completely undo any gains from corporate leverage. Legal restrictions on the actions of many large institutional investors do not allow these institutions to undertake the self-levering arbitrage operations, placing a large burden upon the individual arbitragers. Also, borrowing rates may not remain at the same level for corporations and arbitragers who undertake large positions to undo leverage gains. Higher borrowing rates for arbitragers would make it impossible for them to assure that the value of a firm was independent of its debt-equity ratio. Although it might be possible to lever their personal portfolios to match that of any corporation, their returns would be less than those of the owners of the levered corporation as their interest payments

would be larger. Finally, margin requirements may not allow the self levering process to take place without the committment of collateral in addition to the shares involved in the levering process. These infringements on the personal portfolio of the arbitrager keep the actual market from being as perfect at the strict Modigliani and Miller model requires.[9/]

This introduction of the idea of the risk on personal account associated with incurring debt supported by equities of uncertain value has led to the expectation of an increase in the value of a corporation with an increase in its debt-equity ratio. It has made leverage on personal account more risky than leverage on corporate account.

In addition to this personal risk associated with debt, there is another aspect of risk which appears to be of importance. This time it is a risk on corporate account. If, the income stream is composed of elements with different stability, increasing the debt-equity ratio will cause mounting interest payments which will leave the residual stream composed more and more of the risky elements. In addition, a sequence of years with low earnings may not allow the payment of interest and lead to the danger of insolvency. To prevent

---

[9/] A more extended discussion of the validity of the assumptions made by Modigliani and Miller may be found in A. Bargess, The Effect of Capital Structure on the Cost of Capital, Prentice-Hall, 1963.

such an occurrence, the bondholders may impose constraints on dividend policy or investment behavior. Any such limitation, as well as the increase in the risk of the residual stream caused by the increase in the debt-equity ratio, leads to a decline in the value of a unit of the equity.

While the earlier risk on personal account increased the value of the corporation more as it added more debt, this risk on corporate account decreases the value of the corporation as the debt-equity ratio rises. If either of these forces completely outweighed the other - outweighed it for all debt-equity ratios - it would then be wise to either issue no debt at all or to totally finance with debt. For if the influence of risk on personal account dominated the influence of risk on corporate account, then the addition of debt would always increase the value of the corporation and would suggest an all debt financial structure. Similarly, if the influence of risk on corporate account completely dominated the influence of risk on personal account, any addition of debt would decrease the value of the corporation and would lead to an all equity financial structure.

Since we observe no firms with "all" debt, many firms with some, and many with none, the complete dominance by either one of these risk factors seems most unlikely. It suggests that the influence of risk on personal account dominates for "low" debt equity ratios causing the value of the firm to increase as debt is first added, while the effect of risk on corporate account predominates at "high

debt-equity ratios causing the value of the firm to decrease as debt is continually added. Thus we hypothesize that the value of a stream of earnings generated by assets financed with a mixture of debt and equity rises as debt is increased from zero, but falls as "too much" debt is added to the capital structure.

Two sorts of risks have been distinguished - asset risk and financial risk. Asset risk influences value by making firms with more variable outputs per unit of input - firms having production processes generating outputs with greater dispersion - have lower equity prices than firms with less variable outputs. Financial risk has a more complicated effect. The two risk characteristics of debt cause the value of the firm to rise as debt is initially added but lead to a fall in the value of the firm as continual amounts of debt are added.

Next, to the effects of growth on the capitalization rate. Here there are several issues. First, is it earnings or dividends that get capitalized and second, do dividends "count" - that is, is the capitalization rate affected by the dividend payout rate?

In a world without taxes and with no uncertainty associated with the production process, there is a neat, well-defined answer to both

questions. As has been shown by Lintner[10] or Miller and Modigliani[11], capitalizing earnings available to stockholders, or dividends paid to stockholders is equivalent. There is some dispute still, but it revolves around the issue of whether dividends are valued in and of themselves or because they are formally equal to the earnings which are being capitalized. Furthermore, Miller and Modigliani in "Dividend Policy, Growth, and the Valuation of Shares" have shown that with a given investment policy, the dividend payout has no influence on the valuation of the earnings. When there is no uncertainty associated with the outcome of the production process, any dividend now with investment given implies a smaller ownership in the capital gains accruing to the firm - smaller in value by just the value of the dividend.

Introducing uncertainty about the outcome of the production process dispells the simplicity (and certainty) of the above results.[12] It is still true that long-run dividends and long-run earnings available to the shareholders are equivalent, so there seems little problem over what gets capitalized. The exact role of dividends is now more complicated, however.

---

[10] Lintner, John, "Dividends, Earnings, Leverage, Stock Prices and the Supply of Capital to Corporations," The Review of Economics and Statistics, Vol. XLIV, August, 1962, pp. 243-269.

[11] Modigliani, Franco, and Merton Miller, "Dividend Policy, Growth, and the Valuation of Shares," Journal of Business, Vol. XXXIV, No. 4.

[12] Lintner, John, "Dividends, Earnings, Leverage, Stock Prices and the Supply of Capital to Corporations," The Review of Economics and Statistics, Vol. XLIV, August, 1962, pp. 243-269, contains an elaborate analysis of the effects of various types of uncertainty in a Modigliani and Miller context. This piece should make it clear that there is as much uncertainty about how uncertainty ought to be introduced as there is about its implications.

Because of the prevalence of behavior rules for the payment of dividends which relate dividends to smoothed past or to expected future profits, a change in dividends often indicates a revision of earnings expectations on the part of the managers of the firm.[13/] The knowledge of such rules leads to the movements of share prices with changes in dividend rates. From the point of view of this thesis such movements reflect changes in expectations about future earnings and do not affect the process by which earnings are capitalized. Thus, one must show care on the issue of what gets capitalized to distinguish between the information in dividend behavior and the value associated with the actual dividend.

Although uncertainty doesn't affect the amount that should be capitalized, it does seem to alter the influence of the dividend payout rate on the capitalization rate. Presumably, the purpose of investing in securities is to allocate resources over the lifetime of the saver - to enable him to achieve a desired consumption pattern. This being the case, the value of the original investment as well as the returns from the investing are meant to be claimed at some date.[14/] If the behavior

---

[13/] An example of this type of behavior can be found in John Lintner, "Distribution of Incomes of Corporations Among Dividends, Retained Earnings and Taxes," American Economic Review, XLVI, May, 1956, pp. 97-113, where he proposes a dividend rule of the following sort:

$$\Delta D = c(rP_t - D_{t-1})$$

Here, the change in dividends is a fraction, c, of the discrepancy between desired dividends, rP_t, and the actual dividends last period, D_{t-1}. This equation implies that dividends are a weighted average of past profits.

[14/] Modigliani, F., and R. Brumberg, "Utility Analysis and the Consumption Function: An Interpretation of Cross Section Data, in Post Keynesian Economics, K. Kurihara, Editor, Rutgers University Press, 1954.

of the stock prices is somewhat unpredictable - as in the fashion of a random walk around a more or less predictable trend[15/] - the actual price at which securities may be sold at any specific time is unknown. This uncertainty about prices introduces some risk into the realization of capital gains. The ability to purchase many different securities - to construct a portfolio - would lessen this risk of having to sell any particular security at a temporarily low price as it would create more freedom of choice as to the security to sell at any particular time. To the extent that security prices tend to move together in the short run, however, the usefulness of a portfolio in creating predictable capital gains income without capital losses is diminished. In addition, if relatively small or steady amounts of income are desired, the transaction costs involved in the selling of securities may well lead the investor to prefer some dividends. This risk associated with the realization of capital gains along with the inconvenience and charges required to complete the transaction, may well lead the investor to prefer a share paying out some of its earnings as dividends to one which allows the investor to realize his income only in the form of capital gains. This implies that the capitalization rate depends, in part at least, on the amount of earnings paid as dividends.

[15/] Evidence for the validity of this conception of the movements of security prices can be found in S. S. Alexander, "Price Movements in Speculative Markets: Trends or Random Walks," Industrial Management Review, School of Industrial Management, M.I.T., Vol. 2, No. 2, May 1961, pp. 7-26; or, P. S. Cootner, "Stock Prices: Random vs. Systematic Changes," Industrial Management Review, School of Industrial Management, M.I.T., Vol. 3, No. 2, Spring, 1962, pp. 24-45.

The next element to be considered is taxes. Three types of influences, at least, ought to be distinguished. The fact that interest payments may be deducted as business expense when calculating income for tax purposes while dividend payments may not be deducted, clearly ought to shift financing preferences toward debt. In fact, in the context of the Modigliani and Miller models of "The Cost of Capital, Corporation Finance and the Theory of Investment" this deductibility causes the addition of debt to constantly increase the value of the firm. They resolve this dilemma by introducing a ceiling to the debt-equity ratio - a "maximum allowable leverage". Within the framework of the model being constructed here, however, the influence of a ceiling to the allowable amount of debt is not abrupt but gradual and continuous. At first debt is thought to increase the value of the firm and then to decrease it as the residual income becomes more risky. Taxes, through the deductibility of interest, decrease the former disadvantages of debt relative to equity by creating a larger after tax income stream. This shifts the balance of forces towards allowing more debt than in the no-tax situation, but the risk on corporate account still keeps continuous additions of debt from increasing the value of the firm.

In addition to this differential deductibility and its effects on the decision between debt and equity, the differential taxation of personal income and capital gains influences the choice within equity between new issues and retentions as a source of funds.

The value of earnings claimed through capital gains is taxed at rates, at most half as large as those applicable to dividends and no higher than 25%. This makes capital gains desired as against dividends,

and works in an opposite direction to the preference for dividends as against capital gains derived from the uncertainty associated with stock prices and the costs associated with selling shares.

Another aspect of this capitalization problem can be understood if one considers the decision of whether to finance an investment from new issues or from retentions. If profits are retained, the stockholder has the income taxed only at capital gain rates while if some of the profits are paid as dividends and then new shares are issued to the old shareholder to enable him to retain his share of the firm, he loses the personal income tax on the dividend and still pays the gains tax on the future earnings. Thus financing with retentions is made less expensive - more valuable to the shareholders - than financing with new issues.

Thusfar, we have discussed the influence of financial structure on the capitalization rate. It is now necessary to examine the factors affecting this rate which arises from the mix of physical assets in which the firm invests. These factors are thought to exert their influence by affecting the probability distribution of the revenue stream arising from a unit input to the production process. Assuming investors to be risk averters, more risky streams of income sell for lower prices than less risky streams with the same expected value. If it were possible to characterize the riskiness of the income streams associated with the assets of a particular firm it might then be possible to estimate the effect of this risk on the capitalization rate. That is, if the transfer function which characterizes the production process could be exactly defined,

allowance for the risk of each firm would be possible. Most previous attempts at such characterization have not been very successful. Gordon[16/] tried variance of earnings and annual percentage changes in sales as measures of risk in his estimating but found either they had insignificant slope coefficients or incorrect signs. Some rather extensive analysis has been performed on the same data which will be used in this thesis in another study conducted by Cootner and Holland at M.I.T.[17/] They attempted to ascertain what variables were proxies for risk by correlating them with rates of return. Their hypothesis was that higher than average rates of return could be justified only by higher than average risk. The difficulty with the hypothesis is that it is difficult to decide among alternatives which is a better measure of risk. Certainly, there is more to variations in rates of return than variations in risk, so the maximum expected correlation is less than unity, but how large must it be before one is convinced that any "risk" variable truly measures the risk associated with that particular firm?

An alternate approach is suggested in the writings of Modigliani and Miller[18/]. They imagine all firms to have risk characteristics such that they may be grouped into classes within which each firm has identical risk. Apart from saying that it depends on the probability distribution of the income stream, no attempt is made to define exactly

---

[16/]Gordon, Myron J., The Investment, Financing and Valuation of the Corporation, Homewood, Illinois: Irwin, 1962.

[17/]Risk and Rate of Return: A Study by Paul H. Cootner and Daniel M. Holland, Sponsored by the American Telephone and Telegraph Company, Massachusetts Institute of Technology, DSR Project No. 9565, March, 1963.

[18/]Modigliani, Franco, and Merton Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review (1958).

what risk is, only that it is the same for all firms in that class. Their empirical tests implicitly assume that the "industry" is a definition suitably close to a risk class.

Because of the difficulties encountered by Cootner and Holland in defining risk variables and those reported by Gordon when he unsuccessfully used variance of earnings and percentage changes in sales as risk variables, no attempt will be made to associate risk with any specific continuous variable associated with each firm. Instead, an industry classification will be thought to contain firms with largely identical risk characteristics. That is, the risk class concept of Modigliani and Miller with industry groups as risk classes, will be the technique employed to account for asset risk. Thus, all firms within an industry classification will be assumed to have the same transfer function and thus the same asset risk adjustment to their capitalization rate and this adjustment will be different for each industry.[19/]

All the elements which will be included in the analysis as used by others have been discussed. The manner in which they are thought to exercise their influence will now be devloped.

[19/] Durand, David, "Bank Stocks and the Analysis of Covariance," Econometrica (January 1955), seems to employ this technique, as no attempt was made to split the effects of financing on risk from the effects arising from the assets in which the firm invests. As data in his study were drawn only on banks, it appears to have been assumed that the risks associated with the assets of each bank were not very different. Because he had but one risk class, it is difficult to use the results of his study to test the usefulness of this technique as a method of standardizing for risk.

CHAPTER III

The Statistical Specification of the Model

Thusfar we have presented the variables thought to influence the capitalization rate and the types of forces they are thought to exert. After deriving the influence of the capitalization rate on equity prices, this chapter develops the statistical specification of the manner in which debt and dividends are thought to influence this capitalization rate. The results of the tests of this specification are presented in the next two chapters.

The analysis begins by considering the case where no uncertainty is associated with the income streams produced by the assets and there are no taxes. In such a case, there is no need for two financial assets - no need for both debt and equity - and no financial risks need be considered. To see how the capitalization rate influences equity prices it is necessary to consider a firm earning amount Y_o which continually reinvests a portion k of these earnings into assets which yield a rate of return of r. The increment in earnings from such an investment is rKY_o. Thus

$$\frac{dY_o}{dt} = rkY_o \qquad \text{or} \qquad Y = Y_o e^{rkt}$$

Earnings grow at a rate kr per period. Reinvesting a portion k of earnings implies that the portion (1-k) is paid as dividends. Thus at each point in time dividends are $(1-k)\, Y_o\, e^{rkt}$. If the discount for the futurity of these dividend payments is at a rate ρ_o, the

present value of the stream of dividends is

$$V = \int_0^\infty (1-k)\, Y_o\, e^{-(\rho_o-kr)t}\, dt$$

$$= \frac{(1-k)\, Yo}{\rho_o - kr} = \frac{DIV_o}{\rho_o-kr}$$

Alternately, if one wishes to develop the relation between the capitalization rate and equity prices from the point of view of earnings instead of dividends, it is only necessary to note that the return to the owners is a perpetual stream of magnitude Y_o, plus the streams of future earnings arising from the constant reinvestment of a fraction k of earnings. The value of the earnings generaged by the present assets is

$$V' = \int_0^\infty Y_o\, e^{-\rho_o t} = \frac{Y_o}{\rho_o}$$

In addition, each time an amount kY(t) is reinvested, it contributes an increment to the value of the firm which, at that time, has a value

$$\Delta V(t) = \int_0^\infty rkY(t)\, e^{-\rho_o \tau}\, d\tau - kY(t)$$

$$= kY(t)\, \frac{(r-\rho_o)}{\rho_o}$$

The present value of this future increment in earnings is

$$kY(t)\,\frac{(r-\rho_o)}{\rho_o}\,e^{-\rho_o t}$$

The present value of all such future increments is

$$\int_o^\infty kY(t)\,\frac{(r-\rho_o)}{\rho_o}\,e^{-\rho_o t}\,dt$$

$$= \int_o^\infty kY_o e^{krt}\,\frac{(r-\rho_o)}{\rho_o}\,e^{-\rho_o t}\,dt$$

$$= \frac{kY_o\,(r-\rho_o)}{\rho_o(\rho_o-kr)}$$

Thus the total present value of the firm is

$$V = \frac{Y_o}{\rho_o} + \frac{kY_o\,(r-\rho_o)}{\rho_o\,(\rho_o-kr)}$$

$$= \frac{(1-k)\,Y_o}{\rho_o-kr}$$

which is identical with the value obtained from capitalizing the dividend stream.

If the rate of growth of earnings and of dividends, kr, is called g, this may be written

$$\frac{DIV}{V} + g = \rho_o$$

Letting DIV become dividends per share and P be the price per share of the equity, this may be rewritten as

$$\frac{DIV}{P} + g = \rho_o$$

The total yield, dividend plus capital gain must be a constant ρ_o, the discount for futurity. Higher growth rates, dividends per share unchanged, imply higher prices.[1/]

Introducing uncertainty into the income stream generated by the assets brings two changes. The discounting of future revenues to determine present values has complete justification only in the case of certainty. However, most attempts at incorporating uncertainty do assume that correct results are obtained by treating the discount rate as a composite of both time discount and risk discount. The value of an uncertain future stream is then computed by discounting the expected value of the stream at each point in time by this double-purpose discount rate. This approach to valuation under uncertainty is adopted here. Thus, the first change in the certainty model arising from the consideration of uncertain income streams is that ρ_o is now the discount for futurity <u>and</u> risk.

[1/] This result may be found in Myron Gordon and Eli Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," <u>Management Science</u>, III (1956), pp. 102-110; and, Franco Modigliani and Merton Miller, "Dividend Policy Growth, and the Valuation of Shares," <u>Journal of Business</u>, Vol. XXXIV, No. 4.

The consideration of uncertain income streams also introduces the financial problems which are the concern of this thesis. It is now useful to distinguish between a security promising relatively sure income streams - prior claims - called debt and a security representing residual claims - equity. In Chapter II, two influences of debt were noted, one which would lead to an increase in the value of the firm and one which led to a decrease as the debt-equity ratio was increased. These forces are thought to influence the value of the firm by affecting the capitalization rate - by causing it to differ as different debt-equity ratios are used.

To allow the influence of increasing the debt-equity ratio to be first in one direction and then in another, a quadratic effect is specified for debt. That is, the debt adjusted capitalization rate is thought of as

$$\rho = \rho_o \, e^{a(b-\frac{D}{E})^2}$$

where $\frac{D}{E}$ is the debt-equity ratio.

This functional form for the effects of debt allows the addition of debt first to decrease the effective value of ρ and then to increase it as $\frac{D}{E}$ begins to exceed b. Figure 1 shows the behavior of

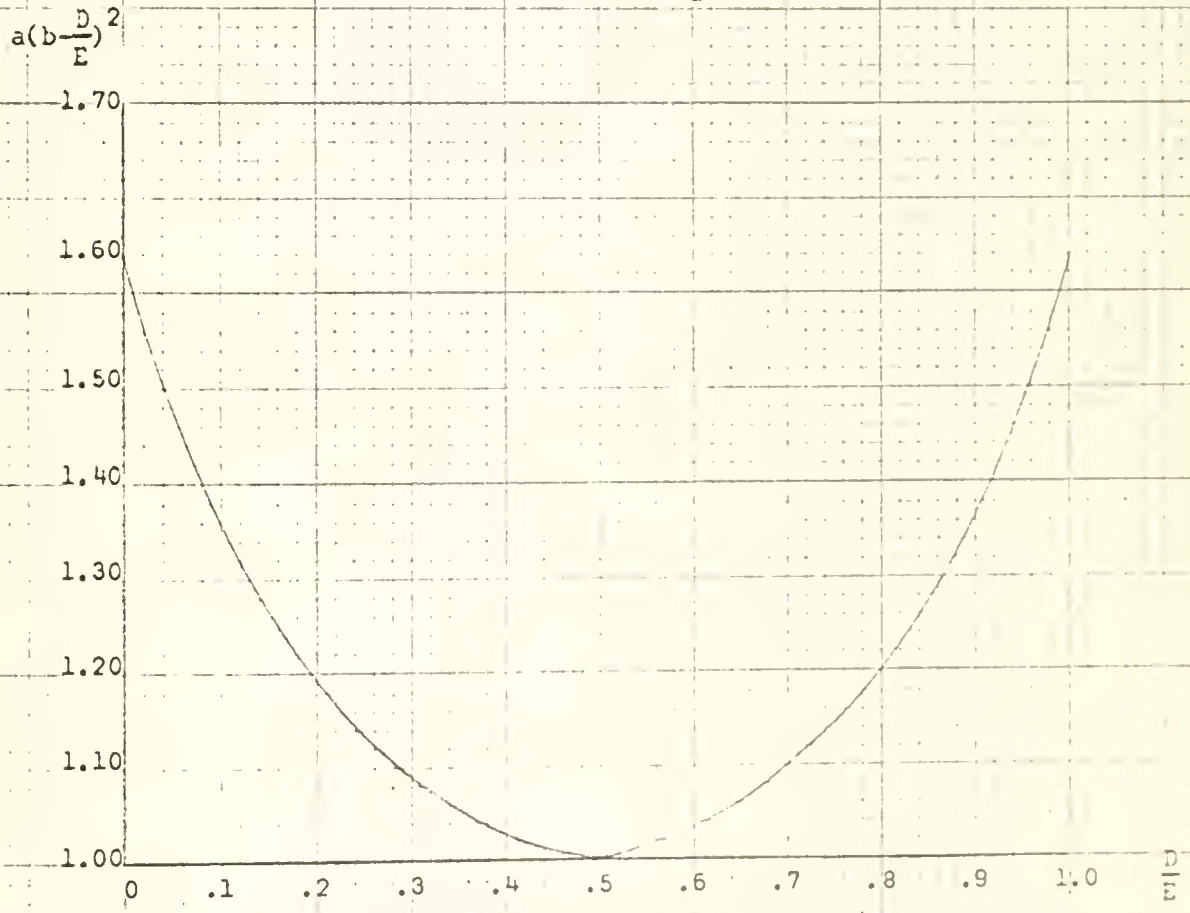
$$\rho_o \, e^{a(b-\frac{D}{E})^2}$$

as function of $\frac{D}{E}$. The coefficient <u>b</u> locates the bottom of the curve, while <u>a</u> determines its steepness. The specification allows debt first to increase the value of the equity and then to decrease it.

FIGURE 1

A Plot of $e^{a(b-\frac{D}{E})^2}$ as a Function of $\frac{D}{E}$.

$a = 1, \quad b = \frac{1}{2}$



As discussed in Chapter II, uncertain share prices and costs of realizing capital gains are thought to make dividends raise equity prices as the dividend payout ratio rises from zero but forgone capital gains make equity prices fall as too large a portion of earnings are paid as dividends. Thus the influence of the dividend payout ratio is two sided - depending upon its magnitude, just as was the influence of debt. To account for this effect of the dividend payout ratio, the capitalization rate is adjusted as follows

$$\rho = \rho_o \, e^{c(d-\frac{DIV}{PROF})^2}$$

where $\frac{DIV}{PROF}$ is the dividend payout ratio used by the firm. Dividends raise equity value until $\frac{DIV}{PROF}$ equals <u>d</u> and decrease it for further increases in the payout rate.

The influence of the differential gains and income tax on the relative value of earnings financed by retentions and new issues will be handled by supposing that earnings financed by retentions are capitalized at a lower rate - have higher value - than earnings financed by new issues. That is, retained earnings, because they are not subject to the personal tax, can purchase more assets than earnings paid out as dividends, taxed at the personal level and reobtained through new issues. The differential effect will be approximated by treating the capitalization rate for retentions as a fraction, f, of the capitalization rate for new issues. The capitalization rate for the total will be taken as the geometric average of these two elements.

Thus, if ρ_o is the rate at which earnings from new issues are capitalized, $f\rho_o$ is the capitalization rate for retentions and

$$\rho = [\rho_o^{NI} (f\rho_o)^R]^{\frac{R}{R+NI}} = \rho_o f^{\frac{R}{R+NI}}$$

R = Retentions

NI = New issues

is the weighted capitalization rate. If f is unity, retentions are no more valuable to the stockholders than new issues and ρ is unaffected by the portion of equity that is composed of retentions. To the extent that f is less than unity, increasing R as a portion of R+NI lowers the weighted average.[2/]

Thus, the influences of uncertainty, debt, dividends and taxes have been specified and the final specification of the financial influences on the capitalization rate becomes

$$\rho = \rho_o e^{[a(b-\frac{D}{E})^2 + c(d-\frac{DIV}{PROF})^2]} f^{\frac{R}{R+NI}}$$

However, two sources of uncertainty were introduced at the outset of Chapter II. Equation 1 shows how allowance has been made for the financial risks. To account for the uncertainty arising from the assets in which the firm invests, it was decided to use the risk class concept with industries as risk classes. To account for the

---

[2/]Substantially the same result would be achieved if an arithmetic averaging were used. The result would be

$$\rho_o[1 - (1-f) \frac{R}{R+NI}]$$

Each of these specifications causes ρ to decrease with $\frac{R}{R+NI}$. As the geometrically weighted average performs better at the extremes and is closer in spirit to the specifications chosen for the other variables, it was chosen over the arithmetic average.

variability in risk between classes, ρ_o will be considered the average market rate of capitalization and will be adjusted in an exponential way to account for the particular risk associated with any risk class. Thus, for the k^{th} risk class, the capitalization rate adjusted for asset risk will be thought of as $(\rho_o)^{r_k}$ with r_k adjusting the average rate, ρ_o, to the special characteristics of the k^{th} class.

Two types of reasons for inter-firm differences in capitalization rates have been discussed - asset differences and financial differences. In addition, there is reason to believe that capitalization rates differ over time. That is, the average market capitalization rate changes through time reflecting changes in the level of interest rates and yields of securities of all varieties. It is a macro-economic variable beyond the control of any firm. Thus, the capitalization rate during any period depends upon the assets in which the firm invests, the debt and dividend policy it pursues, and the average market capitalization rate for that time period.

The following type of market process emerges as the mechanism determining the capitalization rate for each firm. In each period, an average market rate of capitalization, ρ_o is determined by matching the demand and supply of all investors and savers acting together. To any firm in the k^{th} class, the capitalization rate, unadjusted for debt and dividends is $(\rho_o)^{r_k}$. For that period, for each firm in the k^{th} class, the appropriate capitalization rate is

$$(1) \qquad (\rho_o)^{r_k} \, e^{[a(b-\frac{D}{E})^2 + c(d-\frac{DIV}{PROF})^2]} \, f^{\frac{R}{R+NI}}$$

For estimating purposes, it will be written

$$\left(\frac{DIV}{P} + g\right) = (\rho_o)^{r_k} e^{\left[a\left(b-\frac{D}{E}\right)^2 + c\left(d-\frac{DIV}{PROF}\right)^2\right]} f^{\frac{R}{R+NI}} e^{\varepsilon}$$

This is equivalent to

$$\left(\frac{DIV}{P} + g\right) = \rho\, e^{\varepsilon} \qquad \text{or} \qquad \frac{\frac{DIV}{P} + g}{\rho} = e^{\varepsilon}$$

The market process envisioned attempts to determine a price such that the ratio is unity; thus e^{ε} measure the divergence from this value.

Taking logarithms, the expression becomes

$$(2) \quad \log\left(\frac{DIV}{P} + g\right) = r_k \log \rho_o + a\left(b-\frac{D}{E}\right)^2 + c\left(d-\frac{DIV}{PROF}\right)^2 + \frac{R}{R+NI} \log f + \varepsilon$$

Before explaining the tests which were performed to test the validity of this model, it is useful at this time to assess this model in terms of those mentioned earlier, both to compare its managerial implications and to contrast its statistical formulation.

As stated in Chapter I, interest in this capitalization rate arose from an interest in capital budgeting and the need for a required or cut-off rate of return. Projects earning more than this required rate increase the value of the firm, projects earning less decrease the value of the firm. This required rate of return is also called the cost of capital as it is the return which must be earned before increments in value are possible. When both debt and equity are used to finance investments, the cost of capital is the weighted average of the costs - the required return - of the amounts of debt and equity used.

For debt the required return is the interest rate, r, while for equity the required rate of return is the capitalization rate, ρ.

The average of these costs is

$$c/c = \frac{rD + \rho E}{D + E}$$

The literary exposition in Chapter II and the specification in equation (2) present a model in which there is a dividend policy which minimizes the value of ρ with respect to the dividend payout ratio as well as a debt-equity ratio which minimizes the cost of capital with respect to the debt-equity ratio.[3/] Thus, there are implicit two optimal financial policies. It is therefore in conflict with the work of Modigliani and Miller in both "The Cost of Capital, Corporation Finance and the Theory of Investment" and "Dividend Policy, Growth, and the Valuation of Shares," and these differences arise almost entirely from the choice of assumptions with respect to the riskiness of debt and the influence of uncertainty on the value of dividend payments.[4/] The models of Gordon also imply an optimal debt and dividend policy while that of Durand does not include debt and forces dividends to influence share prices in a monotonic way precluding any optimal policy different from distributing either no part or all of the profits.

[3/] The exact implications are discussed more fully in Chapter VI.

[4/] Modigliani, Franco, and Merton Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review (1958); and, Franco Modigliani and Merton Miller, "Dividend Policy, Growth, and the Valuation of Shares," Journal of Business, Vol. XXXIV, No. 4.

Concerning the statistical differences, the major one is the choice of a dependent variable. From the point of view of estimation, it is important to correctly specify the variable which is influenced by the risks associated with debt and the advantages and disadvantages of dividends. The view taken in this thesis is that it is not the share price that is directly affected by debt and dividend policy but that the reactions to these financial choices is on the rate at which earnings are capitalized. Debt and dividend policy are thought to affect the value of the firm by changing the capitalization rate and thus the capitalization rate is the variable to be explained - the statistically correct dependent variable.

One of the difficulties with the Gordon analysis was that cross sections run in different years produced different slope coefficients in each of these years. The variation in these slope coefficients was due to changing circumstances regarding the valuation of each of the independent variables, but was also due to the fact that market rates of interest and equity yields in general were changing from period to period. The formulation presented here with its use of a market determined rate appropriate to each year allows the separation of these two effects and therefore promises a more careful interpretation of the results.

Another advantage of the specification presented here is that it allows certain of the explanatory variables to influence the capitalization rate first in one direction and then in another. The debt-equity ratio and the dividend payout rate are specified to increase the value of the firm as the variables are increased from zero and

then decrease the value as they are continually increased. None of the other statistical studies have been formulated in this manner. In addition, there is little reason to believe that the error terms are heteroschedastic; i.e., that firms with high equity prices per share will have correspondingly high errors of estimation, as is the problem with some of the models explaining price. In these models, there is concern that the magnitude of the errors depends upon the magnitude of the dependent variable. To avoid these difficulties some sort of deflating is necessary.[5/] In the model presented here, however, the dependent variable is already normalized, it is a rate of return and the independent variables are either ratios or growth rates. Thus, there seems little reason to believe that the errors are heteroschedastic.

While the concepts of a risk class and an average market capitalization rate at each point of time aid in analysis of sets of cross section data, they are not an unmixed blessing. Commenting on the risk class concept as a classification scheme for asset risk, Durand[6/] has remarked ... "To the practically minded, it is unthinkable to postulate the existence of two or more distinct corporations with income streams that can fluctuate at random and yet be perfectly correlated from now until doomsday..." Yet to believe that such a

---

[5/] Gordon, Myron J., The Investment, Financing and Valuation of the Corporation, Homewood: Irwin, 1962. Gordon comments on this difficulty in connection with one of his earlier studies, and later deflates prices by book values.

[6/] Durand, David, "The Cost of Capital in an Imperfect Market: A Reply to Modigliani and Miller," American Economic Review (September, 1959).

classification can be a useful theoretical device is even less difficult than to expect to be able to define some empirically useful catagorization in reality. As may become clearer, this disadvantage may well be the most serious difficulty with the model as formulated here.

A second kind of drawback is that, although the effect of the $\frac{D}{E}$ and the $\frac{DIV}{PROF}$ are not linear, they are both quadratic and thus symmetric. If, in fact, the response to increments in debt is small until a certain debt-equity ratio is reached and then becomes very pronounced, any symmetric specification will not fit very well.

The hypothesis is that three types of variables influence capitalization rates; asset risks, financial variables and time. The kinds of tests to determine if the specified relations are correct seem to be of two types. The first is a set of cross section estimates of the coefficients a, b, c, d and f in equation (2) for each risk class in each of several years. These should be inspected for goodness of fit, stability across time, and consistency with related estimates. In each of these regressions the term $r_k \log \rho_o$ would be estimated as the constant, allowing no estimate of r_k. If each of the regressions could be pooled together into a single regression, and a time series for ρ_o introduced, it would be possible to estimate r_k.

In addition, it would be useful if some tests could be performed to ascertain the degree of heterogeneity of risk between classes as well as possible nonlinearity in the relation between dividend payout and growth.

To test these hypotheses, data on several risk classes are required with enough firms in each class to allow cross section regressions to be estimated. The more firms in a class the better the statistical significance in the cross sections, but also the greater doubt as to the homogeneity of the asset risks of each of the firms. As the design of the tests required comparison of cross section estimates in several years, a long time series on each firm was desired, as more years of data would lead to more cross sections to compare. Finally, for all the firms to be analysed it was necessary to have both balance sheet and income statement data. At the outset of the thesis, the available data which best satisfied all these requirements was some prepared by the Studley-Shupert advisory service. The Studley-Shupert data were punched-card balance sheet and income statement data on approximately 400 firms in 40 industries. Five of these industries included a number of firms in excess of 15. These were Building Materials, Chemicals, Drugs, Machinery-Industrial, and Oils. These five industries were chosen as risk classes to test the financial influences hypothesized in the thesis.

The development of the theory, although considering growth, was not entirely dynamic - that is, all the variables were treated as though they were not changing through time ($\frac{D}{E}$, $\frac{DIV}{PROF}$, g, $\frac{R}{R+NI}$) or growing at a steady rate (Y, DIV, V, P). It was the steady state or target $\frac{D}{E}$ ratio and the long-run $\frac{DIV}{PROF}$ ratio as well as the long-run rate of growth which were the variables of the system. With this in mind, the data to enter the regression had to be the best estimates available of these target variables. To this end, the annual data were smoothed in the following fashion.

For the dividend which was being capitalized, a simple average of this year's and the next two years' was taken. This implies that on-the-average the market is able to estimate with no error the magnitude of the dividend, and that it concerns itself with these three years' information. This expectation assumption was made about several of the variables and deserves some comment.

The alternative ways available to deal with the problem of expectations were to build a model explicitly concerned with the transformation of current and past data into expectations on future data, or to assume that, however expectations were formed, they were on-the-average correct. This latter assumption allows the actual data occurring to be used in cases which require expectational data to be employed. It was decided that since the emphasis in the thesis was confined to the manner in which this expectational data would be <u>used</u> in a model predicting price and not on their <u>formation</u>, this simplifying assumption that expectations are formed by a process with zero average error would be made.[7/]

Because of the prevalence of dividend rules relating dividend payments to smoothed profits, and the need for an estimate of the long-run payout ratio used by each firm, the sum of dividends in this

---

[7/] In "Rational Expectations and the Theory of Price Movements," <u>Econometrica</u>, Vol. 29, No. 3, July 1961, J. F. Muth advanced the same idea when he hypothesized that "...expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the objective probability distribution of outcomes)."

and the next two years was divided by the sum of profits in each of these years. An alternate method of smoothing would have been to average the dividend profit ratios in each of these years but this places more weight on a single low profit - high payout ratio year than does the method chosen.[8/]

The same smoothing technique was used to estimate the long-run debt-equity ratio. It should be noted here that debt was defined as bonds maturing in more than one year plus preferred stock while equity was capital stock and capital surplus retentions at book value. Each was defined in an attempt to partition the claims into fixed and residual.

Some care must be shown when estimating the rate of growth of dividends. Assuming again a dividend rule which relates dividends to smoothed past or expected profits, and short term variations in

---

[8/] Suppose

$$\frac{DIV(1)}{PROF(1)} = \frac{DIV(2)}{PROF(2)} = a, \quad \frac{DIV(3)}{PROF(3)} = Ka, \quad K>1$$ because PROF(3) is

abnormally low. The method chosen to derive the long-run payout results in

$$a[1 + (k+1)\frac{PROF(3)}{\Sigma PROF}]$$

The alternative method yields $a(1 - (k-1)\frac{1}{3})$. With PROF(3) abnormally low relative to PROF(1) and PROF(2), the chosen method yields an estimate closer to a. For K<1, the alternate method is better, but the most cause of variation in K would seem to arise from low profits, and as they appear in the denominator of the ratio, would result in K>1.

the rate of growth of profits, the rate of growth of dividends will be a weighted average of past rates of growth of profits. Thus, when the object is to estimate the long-run rate of growth of dividends apart from the influence of the response coefficient, the rate of growth of profits is an unbiased estimator.[9/]

Two techniques were adopted to estimate this growth rate. A five year growth rate generated from the earnings for the current year and the two years on either side of it was computed as that exponential rate of growth which fits the five periods best in a least squares sense. A fifteen year growth rate of earnings, from 1946 to 1960, was also computed according to the same procedure.

The divergence of the set of short term rates for any specific firm from its long term rate led to an alteration of the original hypothesis to incorporate the possible transient effects of short-term variations in the rate of growth of earnings. That is, if the growth rate of dividends used in the capitalization procedure was taken to be the short-term growth, g_s, and if this deviated from the long-term rate of growth, g_L, for that firm, it was expected that the market would discount the difference. When the short term rate

---

[9/] If earnings have been growing at a steady rate for a long time, but at time t_o undergo an alteration in rate of growth to g', a Lintner-type dividend policy results in a rate of growth of dividends which is

$$g' \left[\frac{1 - e^{-(g+c)(t-t_o)}}{1 + g' e^{-(g+c)(t-t_o)}}\right]$$

This approaches g' at a rate which depends upon g' itself and upon c, the response coefficient in the dividend rule.

of growth was in excess of the long, the price was expected not to be as high as the model would predict. Thus, $\frac{DIV}{P} + g_s$ would exceed that which would have been predicted had the short and long-term rates of growth been equal. This led to the presumption that when growth was defined as short-term growth, the difference between the long and short-term growth rates should enter as an independent variable with a sign predicted to be negative.

A similar expectation was held when the long-term growth estimate was used as the rate at which dividends were expected to grow. Here, though, when short-term growth was in excess of long-term growth the expectation was that the price would be higher than predicted and thus $\frac{DIV}{P} + g_L$ would fall short of predicted and require a positive sign on the difference between the long and short rates of growth.

If in an attempt to allow for this influence, only the difference between the short and long-term rates of growth were used it would force their slope coefficients to be identical. To avoid this, the long-term rate of growth was added as an additional independent variable. The final specification becomes

$$\log\left(\frac{DIV}{P} + g\right) = r_k \log \rho_o + h(g_L - g_S) + h'g_L + a\left(b - \frac{D}{E}\right)^2 + c\left(d - \frac{DIV}{PROF}\right)^2 + \frac{R}{R+NI} \log f + \varepsilon$$

Two types of difficulties with this model should be noted. First, there may be some simultaneity inherent in the debt-equity variable. That is, firms with less asset risk may face supply schedules for debt which differ from those for firms with more asset

risk. High debt equity ratios may not indicate financially risky firms, but rather firms with such low asset risk that they are allowed at low cost to finance themselves with large amounts of debt.

Also, it has been postulated[10] that dividend payments made by firms with very profitable potential investments would penalize their share price more than payments made by firms without such profitable opportunities. To the extent that high rates of growth are associated with high profitability, one would then expect the slope coefficient on the dividend payout ratio would be different as growth was different.

This concludes the specification of the model and the definition of the variables. The next chapter presents the results of the tests which were performed to test the validity of these specifications.

---

[10] This conjecture was made by Professor E. Kuh.

CHAPTER IV

The Cross Section Regressions

This chapter presents the results of the cross section estimation. Four regressions were run on both of the dependent variables. The independent variables in each regression were chosen to test the various possible combinations of effects possible with the $\frac{D}{E}$ and $\frac{DIV}{PROF}$ variables. In each regression the long-term rate of growth and the difference between the long and short-term rates of growth was included. The first regression had $\frac{D}{E}$ and its square in addition to these, and the second had $\frac{DIV}{PROF}$ and its square, the third had both $\frac{D}{E}$ and $\frac{DIV}{PROF}$ and their squares, while the fourth had $\frac{D}{E}$ and $\frac{DIV}{PROF}$. In each of these $\frac{R}{R+NI}$ was added as an explanatory variable at the last step.

Three things became apparent immediately. Most of the explanation of the dependent variable had its origin in the growth variables; the high intercorrelation between $\frac{D}{E}$ and its square and between $\frac{DIV}{PROF}$ and its square made it impossible to separate the effect of these variables into the two parts hypothesized; and the coefficient of $\frac{R}{R+NI}$ was for the most part never as great as its standard error, and when it was, it was as often positive as negative.[1/]

[1/] As this coefficient is log f, and f is less than unity, it should be negative.

In an attempt to ascertain the significance of any given slope coefficient, it was decided to accept significance if the coefficient was as large as its standard error. Since the null hypothesis is that the coefficient differs from zero, a one-tail test is appropriate. One standard error leaves approximately a 20% chance of accepting the hypothesis when it should be rejected when there are 15 degrees of freedom, and an 18% chance with 20 degrees of freedom. This is the range of the degrees of freedom resulting from most of the cross section estimates.

Using this rule, the significance of the coefficient of $\frac{R}{R+NI}$ is shown in Table 1. It can be seen that no consistent influence can be attached to $\frac{R}{R+NI}$. This poor result was feared as the data were being gathered. Since the original S-S data did not allocate equity into the two components, R and NI, reference was made to Moody's Manual and the annual balance sheets. The ratio was treated as a constant for each firm over the whole period, so it was estimated as that ratio occurring in the middle of the period, that on December 31, 1952. While compiling the raw data, it could be seen that reference was frequently made to notes to the financial statements explaining that an amount had been transferred from retentions to capital stock due to the issuance of a stock dividend, or that other accounting practices had led to transfers between the accounts. These indicated that although the total equity might be consistently estimated, its allocation between new issues and retentions varied from firm to firm largely due to differing accounting practices. As the purpose is to treat the parts of equity as they are differently

TABLE 1

Frequency of Significance of Coefficient of R/R+NI

$(\frac{DIV}{P} + g_L)$

	A			B			C		
INDUSTRY	NS	S-	S+	NS	S-	S+	NS	S-	S+
Bldg. Mat.	8	3		7	4		7	4	
Chemicals	8		3	7	1	3	8		3
Drugs	10	1		11			11		
Mach.-Ind.	6		5	7	1	3	6		5
Oils	3		8	6	3	2	9	1	1

$(\frac{DIV}{P} + g_S)$

	A			B			C		
INDUSTRY	NS	S-	S+	NS	S-	S+	NS	S-	S+
Bldg. Mat.	10		1	7	2	2	7	2	2
Chemicals	4	2	5	6	2	3	4	3	4
Drugs	7	2	1	7	3		6	3	1
Mach.-Ind.	4	1	5	6	2	2	5	2	3
Oils	9		2	10	1		9	2	

A is regression with growth variables, $\frac{D}{E}$, and $\frac{D}{E}$ squared.

B is regression with growth variables, $\frac{D}{E}$ and $\frac{DIV}{P}$ and their squares.

C is regression with growth variables, $\frac{D}{E}$ and $\frac{DIV}{PROF}$.

NS not significant, S+ significant and positive, S- significant and negative.

affected by the personal income tax, what is needed is an internal-external division of the equity account. It was hoped that these accounting differences would not so distort the comparability of the financial accounts so as to render them useless for a test of the hypothesis at hand. The lack of any consistent influence for this variable seems to indicate that this hope was in vain. That is, the inability to find an influence for the differential treatment of income and capital gains is taken as implying that the data were as poor as feared and is not taken as conclusive evidence that the tax effect was non-existent. Because of these difficulties with the data, all future comments on the results of regressions concern equations in which the term $\frac{R}{R+NI}$ is not included.

Table 2 shows the high intercorrelation between the level and the squares of both the $\frac{D}{E}$ and $\frac{DIV}{PROF}$ variables. It is uniformly quite high and makes attempts to split the influence of either of these variables into two parts very difficult. Some of this correlation could be expected on a priori grounds. The expected value of the correlation coefficient between values drawn from a rectangular distribution and their squares is $\frac{15}{16}$ or .9375. If the distribution is triangular upward, Figure 2, or triangular downward, Figure 3, the correlation is .960 or .956 respectively. Thus although the specific distributions of the independent variable is not known, if they have shapes at all like the three mentioned, the correlation has a high expected value. That such high correlation may lead to unstable

TABLE 2

Correlation of Level and Square of the Debt-Equity Ratio

and the Dividend Payout Proportion

	DEBT/EQUITY					DIVIDEND/PROFIT				
	Building Mat.	Chemicals	Drugs	Mach.-Ind.	Oils	Building Mat.	Chemicals	Drugs	Mach.-Ind.	Oils
1948	.955	.937	.953	.951	.947	.996	.992	.989	.993	.980
1949	.965	.958	.964	.961	.952	.995	.977	.975	.991	.978
1950	.961	.929	.959	.957	.958	.995	.978	.973	.982	.967
1951	.957	.921	.956	.958	.955	.997	.981	.979	.979	.962
1952	.972	.940	.951	.982	.946	.997	.976	.986	.983	.957
1953	.962	.920	.958	.958	.940	.995	.971	.990	.990	.972
1954	.954	.917	.958	.936	.933	.996	.969	.990	.992	.971
1955	.951	.949	.961	.929	.937	.994	.973	.990	.996	.952
1956	.964	.970	.953	.930	.940	.994	.971	.992	.990	.938
1957	.967	.934	.940	.997	.963	.996	.973	.992	.937	.966
1958	.968	.952	.937	.986	.964	.999	.971	.991	.955	.974

FIGURE 2

Triangular Upward Distribution

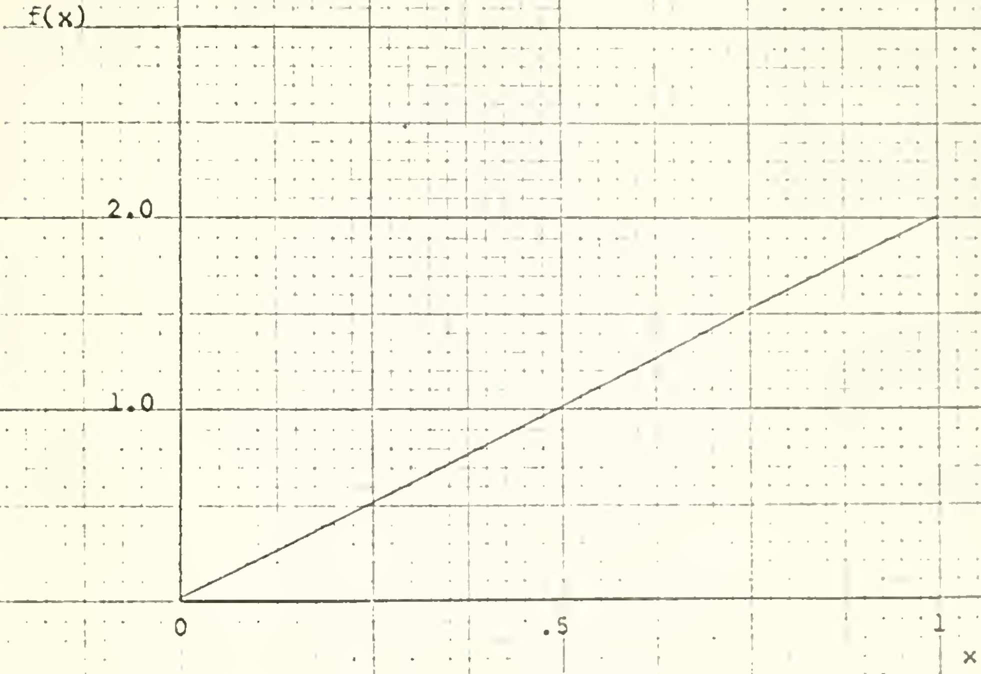
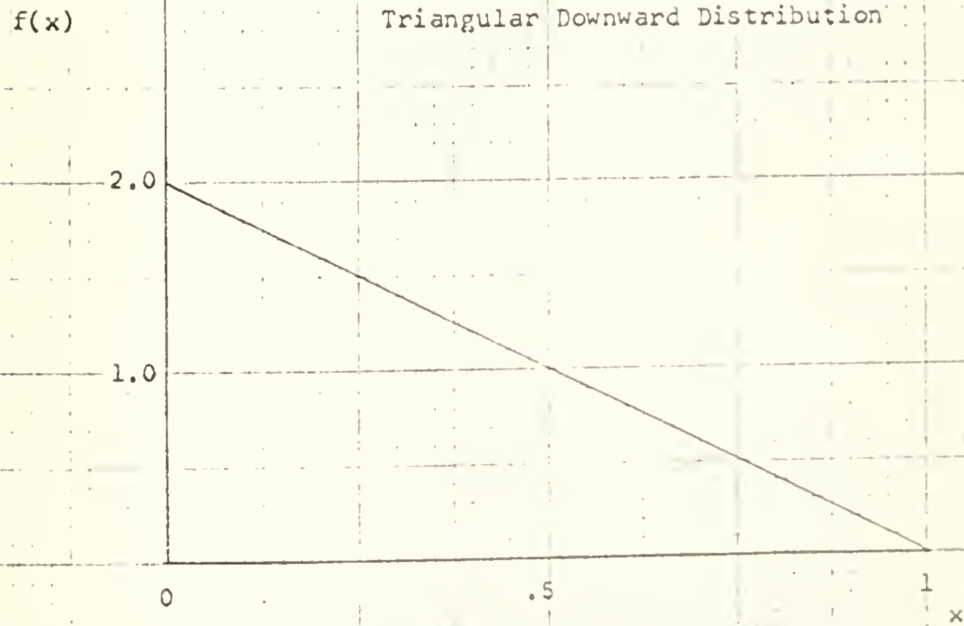


FIGURE 3

Triangular Downward Distribution



slope coefficients can be seen from the following example.

Let X_1[2/], X_2, and Y be vectors whose elements are the values of the level of the independent variable, the squares of that variables and the dependent variable respectively.

Let b_1^1 be the coefficient of X_1 taken alone

b_2^1 be the coefficient of X_2 taken alone

b_1 be the coefficient of X_1 when both X_1 and X_2 are considered together

b_2 be the coefficient of X_2 when both X_1 and X_2 are considered together

Then

$$b_1^1 = \frac{X_1'Y}{X_1'X_1} \qquad b_2^1 = \frac{X_2'Y}{X_2'X_2}$$

And

$$b_1 = \frac{b_1^1 - b_2^1 \frac{X_1'X_2}{X_1'X_1}}{1 - \frac{(X_1'X_2)^2}{(X_1'X_1)(X_2'X_2)}} \qquad b_2 = \frac{b_2^1 - b_1^1 \frac{X_1'X_2}{(X_1'X_1)}}{1 - \frac{(X_1X_2)^2}{(X_1'X_1)(X_2'X_2)}}$$

When there is a high correlation between X_1 and X_2 the denominator is close to zero. To the extent that X_1 and X_2 correlate differ-

---

2/ All variables will be treated as deviations from means.

ently with Y, i.e., to the extent that b_2^1/b_1^1 is not unity the numerator differs from zero. Thus the ratio is sensitive to either small changes in the correlation of X_1 and X_2 or to small changes in the correlation between X_1 and Y and X_2 and Y. In cases where b_2^1 and b_1^1 are approximately equal, b_1 and b_2 will also be close to the same value.

The standard errors of these slope coefficients are

$$S^2_{b_1} = \frac{(X_2'X_2)}{1 - \frac{(X_1'X_2)}{(X_1'X_1)(X_2'X_2)}} \sigma^2_\varepsilon \qquad S^2_{b_2} = \frac{(X_1'X_2)}{1 - \frac{(X_1'X_2)^2}{(X_1'X_1)(X_2'X_2)}} \sigma^2_\varepsilon$$

These need not be quite so unstable as the slope coefficients, for although the denominator is the same as for the slope coefficients and close to zero, the numerator is not necessarily small as it is in the case of the slope coefficients. Instability will still be troublesome to a considerable extent, however, due to the division by the small denominator.

What occurs in the regressions at hand is that both b_2^1 and b_1^1 are small (≈ .10 to .20) and scarcely as large as their standard errors but occasionally b_2 and b_1 are much larger (≈ 10. to 20.), almost equal, and of opposite sign, with relatively much smaller standard errors. The sum of b_2 and b_1, however, is less than the standard error of either, implying that taken together the contribution of the level and the square is not significant. Most of the times when the coefficients of the level and square of the variable are significant they have the wrong sign. That is, they imply that

paying dividends from a position of no payout increases the capitalization rate, but, beyond a point, increasing $\frac{DIV}{PROF}$ lowers it. This would imply an optimal dividend payout rate of unity. Table 3 presents the frequency with which the coefficients of both the level and square were significant while Table 4 presents, for the dividend term, the set of implied sets of b's and a's - the extreme point and the coefficient of steepness. Negative a's imply "inverted" curves with b locating its maximum, and would lead to a minimization of the capitalization rate - a maximization of value - at a dividend payout rate of unity. As can be seen in Table 4, when the slope coefficients on the dividend terms are significant they imply maximum points and coefficients of steepness that vary quite widely over time. This instability of the extreme points and the steepnesses along with their questionable implications is taken as evidence that the possibly misleading statistical results which arise from highly correlated variables mentioned before have in fact occurred.

Because of the intercorrelation and frequent occurrence of incorrect and unstable signs on the slope coefficients an attempt was made to redefine the basic variables. The smoothing definitions were changed to include not just one period on each side of the current one, but to include two on either side - 5 in total. Thus the smoothed debt-equity ratio had five terms in both numerator and denominator as did the dividend-profit ratio. Regressions were run but the intercorrelation problem was as severe as ever. Because of these problems, no further attempt was made to split the effect of debt and dividends into two parts.

TABLE 3

The Frequency With Which Both the Level and the Squared Terms Had a Significant Sign*

Coefficients of Debt/Equity

	$(\frac{DIV}{P} + g_s)$		$(\frac{DIV}{P} + g_L)$	
	"Wrong"	"Correct"	"Wrong"	"Correct"
Bldg. Mat.		2	4	2
Chemicals	3	2	1	
Drugs		1	2	1
Mach.-Ind.		3	5	
Oils	4	1	1	

Coefficients of Dividend/Profit

	$(\frac{DIV}{P} + g_s)$		$(\frac{DIV}{P} + g_L)$	
	"Wrong"	"Correct"	"Wrong"	"Correct"
Bldg. Mat.	6		5	
Chemicals	4	1	4	1
Drugs	7		6	1
Mach.-Ind.	2	2	6	
Oils	9		6	

*The number of cross section regressions run on each industry and each variable was 11.

TABLE 4

Implied Extreme Point, (a), and Steepness, (b), from Equations With Significant Coefficients on Level and Squared Terms

Coefficients of Dividend/Profit

With $\frac{DIV}{P} + g_s$ as independent variable

Bldg. Mat.		Chemicals		Drugs		Mach.-Ind.		Oils	
b	a	b	a	b	a	b	a	b	a
.50	-15.46	.37	4.74	.53	-3.54	.48	4.97	.45	-2.07
.43	-19.80	.51	-3.46	.51	-13.20	.66	3.19	.44	-4.16
.55	-26.06	.51	-1.37	.54	-2.86	.51	-4.64	.69	-2.70
.64	-3.84	.51	-5.90	.54	-4.88	.47	20.03	.58	-1.41
.52	-18.97	.61	-1.17	.49	-5.70			.79	-2.73
.54	-43.64			.50	-3.58			.60	-3.20
				.56	-4.17			.90	-4.09
								.50	-27.08
								.66	-6.61

TABLE 4 (Continued)

With $\frac{DIV}{P} + g_L$ as independent variable

Bldg. Mat.		Chemicals		Drugs		Mach.-Ind.		Oils	
b	a	b	a	b	a	b	a	b	a
.58	-6.59	.45	2.57	.57	-2.08	.41	-1.24	.55	-2.82
.61	-7.05	.61	-1.16	.69	-1.74	.42	-1.79	.58	-2.82
.64	-5.90	.55	-1.74	.58	-3.13	.44	-2.60	.65	-2.04
.65	-2.07	.52	-1.90	.55	-4.34	.44	-2.14	1.04	-.49
.64	-4.22	.55	-1.36	.55	-4.49	.50	-3.14	.79	-.96
				.54	-2.88	.68	-2.52	.95	-.62
				.35	2.02				

These difficulties with the data have the following implications. The empirical determination of an optimal dividend policy is now impossible. Having only a linear term in dividends causes the dividend variable to either continually increase or continually decrease the capitalization rate as the dividend payout rate is increased depending upon whether the slope coefficient is positive or negative. Similarly, the estimation of the two-fold effect of debt is no longer possible. Having lost the term $\frac{R}{R+NI}$ as mentioned earlier in this chapter means that the postulated influence of taxes cannot be maintained.

However, further cross section regressions are able to generate evidence about the influence of growth and whether there is any influence of debt and dividend policy on the capitalization rate. Two questions arise with respect to the growth variables. Is it the long or short-term rate of growth which is capitalized and is there any influence of growth per se. As for debt and dividends, the question remains, do they influence capitalization rates at all?

Dropping the terms in the squares of $\frac{D}{E}$ and $\frac{DIV}{PROF}$, and the $\frac{R}{R+NI}$ term, the model becomes,

$$\log \left(\frac{DIV}{P} + g\right) = r_k \log \rho_o + h(g_L - g_S) + h'g_L + a\frac{D}{E} + c\frac{DIV}{PROF} + \varepsilon$$

Cross section regressions were run for the five industries with this specification. Table 5 presents the coefficients of the two growth variables when $\left(\frac{DIV}{P} + g_S\right)$ was the dependent variable.

TABLE 5 PART 1 OF 5

Slope Coefficients of Growth Variables in Final

Cross Section Specification With $(\frac{DIV}{P} + g_s)$ as Dependent Variable

Building Materials Industry

Year	$(g_L - g_s)$	g_L	R*
1948	-4.54 (.46)	4.72 (1.36)	.963 (.953)
1949	-6.01 (1.08)	13.51 (2.89)	.956 (.944)
1950	-12.38 (1.67)	14.14 (5.12)	.942 (.924)
1951	-9.56 (1.92)	14.08 (4.34)	.925 (.898)
1952	-10.69 (2.90)	11.24 (3.06)	.934 (.905)
1953	-7.42 (1.16)	5.33 (2.08)	.953 (.941)
1954	-4.98 (.89)	5.80 (1.13)	.970 (.962)
1955	-6.97 (.91)	6.59 (.97)	.968 (.960)
1956	-9.86 (.70)	7.46 (1.08)	.968 (.959)
1957	-8.64 (1.37)	3.26 (3.21)	.901 (.865)
1958	-10.23 (1.31)	4.30 (3.55)	.932 (.905)

Coefficients in brackets are standard errors.

*After inclusion of the two growth variables alone.

TABLE 5 PART 2 OF 5

$(\frac{DIV}{P} + g_s)$

Chemical Industry

Year	$(g_L - g_s)$	g_L	R*
1948	-5.23 (.43)	5.34 (.90)	.972 (.967)
1949	-4.74 (.45)	4.50 (1.00)	.952 (.944)
1950	-6.81 (.69)	6.12 (1.02)	.926 (.913)
1951	-18.54 (1.64)	14.57 (2.39)	.937 (.926)
1952	-17.01 (2.16)	13.93 (2.87)	.936 (.920)
1953	-7.60 (.72)	4.73 (1.07)	.962 (.955)
1954	-4.55 (.59)	6.16 (1.16)	.955 (.947)
1955	-6.03 (.80)	7.16 (1.50)	.939 (.929)
1956	-10.70 (1.17)	5.99 (1.81)	.928 (.916)
1957	-14.48 (1.46)	11.04 (1.79)	.921 (.907)
1958	-12.42 (1.26)	10.86 (2.03)	.957 (.949)

*After inclusion of the two growth variables alone.

TABLE 5 PART 3 OF 5

$$\left(\frac{DIV}{P} + g_s\right)$$

Drugs Industry

Year	$(g_L - g_s)$	g_L	R
1948	-13.57 (2.19)	10.72 (2.37)	.932 (.906)
1949	-10.51 (1.63)	11.74 (3.05)	.904 (.874)
1950	-7.97 (3.67)	2.52 (7.76)	.872 (.818)
1952*	-23.46 (6.34)	16.15 (2.25)	.932 (.905)
1953	-9.24 (2.03)	4.97 (1.51)	.892 (.863)
1954	-4.02 (.57)	3.60 (.63)	.952 (.943)
1955	-3.67 (.36)	2.96 (.49)	.957 (.948)
1956	-5.10 (.66)	3.61 (.66)	.935 (.921)
1957	-7.34 (.59)	7.36 (.65)	.960 (.952)
1958	-7.63 (.72)	8.31 (.77)	.961 (.952)

*1951 had insufficient firms as many had highly negative short term rates of growth.

TABLE 5 PART 4 OF 5

$$(\frac{DIV}{P} + g_s)$$

Machine Industrial

Year	$(g_L - g_S)$	g_L	R
1948	-4.76 (.55)	2.41 (1.34)	.959 (.949)
1949	-4.81 (.49)	3.85 (.77)	.941 (.927)
1950	-6.40 (.78)	1.51 (3.86)	.841 (.803)
1951	-4.36 (.76)	1.91 (2.70)	.907 (.886)
1952	-6.32 (1.12)	6.73 (2.64)	.919 (.898)
1953	-5.96 (.47)	6.11 (.95)	.970 (.963)
1954	-5.64 (.47)	5.02 (.66)	.966 (.958)
1955	-6.53 (.48)	5.40 (.93)	.967 (.960)
1956	-6.73 (.80)	5.74 (1.14)	.954 (.943)
1957*	-9.46 (1.90)	7.90 (3.40)	.941 (.918)

*1958 had insufficient firms as many had highly negative short term rates of growth.

TABLE 5 PART 5 OF 5

$(\frac{DIV}{P} + g_s)$

Oil Industry

Year	(g_L-g_s)	g_L	R
1948	-3.95 (.28)	2.69 (.60)	.946 (.938)
1949	-7.64 (1.01)	5.58 (1.97)	.881 (.863)
1950	-9.47 (1.12)	6.33 (2.25)	.874 (.848)
1951	-7.57 (.66)	7.02 (1.13)	.932 (.921)
1952	-10.12 (1.32)	12.33 (1.98)	.899 (.884)
1953	-9.33 (1.46)	10.25 (1.77)	.871 (.851)
1954	-8.37 (.69)	7.35 (.91)	.939 (.930)
1955	-10.87 (1.26)	7.04 (1.61)	.904 (.890)
1956	-13.16 (1.80)	12.12 (2.93)	.859 (.829)
1957	-8.87 (3.95)	17.63 (13.85)	.790 (.706)
1958	-11.63 (1.26)	17.66 (3.03)	.849 (.803)

It can be seen that the coefficient on the difference in the rates of growth is, as predicted, negative and highly significant. Also, the coefficient on the long-term growth rate is also significant, but it is positive. This seems to imply that short-term deviations from the long-term growth rate are discounted and that high rates of growth result in lower prices than the model predicts, possibly because they are thought not to be sustainable.[3/]

Using long-term growth as the element being capitalized, results in the coefficients shown in Table 6 for the difference between long and short-term rates of growth. Here the difference does not appear to have a significant slope coefficient reinforcing the earlier conclusion that long-term rates of growth determine price.

Because of this lack of significance, the regressions were rerun on this variable, $(\frac{DIV}{P} + g_L)$, excluding $(g_L - g_s)$ as an independent variable. Table 7 shows the slope coefficients which resulted, and Table 8 summarizes the frequency with which debt and dividends had significant slope coefficients in both these regressions and the earlier ones run on $(\frac{DIV}{P} + g_s)$.

[3/]This latter result will be reversed in later regressions.

TABLE 6

Slope Coefficient of $(g_L - g_S)$ with $(\frac{DIV}{P} + g_L)$

as the Dependent Variable

Year	Bldg. Mat.	Chemicals	Drugs	Mach.-Ind.	Oils
1948	-.54 (.35)	.02 (.40)	-.65 (.37)	-.35 (.29)	-.53 (.45)
1949	-.04 (.60)	-.18 (.38)	-.16 (.66)	.23 (.41)	-1.11 (.62)
1950	-.04 (.53)	-.04 (.42)	-.13 (.61)	.55 (.22)	-1.01 (.50)
1951	.29 (.66)	-.28 (.50)	.08 (.42)	-.04 (.23)	-.89 (.52)
1952	-.29 (.85)	-.25 (.49)		-.40 (.35)	-.47 (.75)
1953	.31 (.56)	-.52 (.49)	-.89 (.63)	-.21 (.36)	.04 (.44)
1954	.04 (.81)	-.66 (.37)	.32 (.63)	-.76 (.31)	-.19 (.35)
1955	-.69 (.65)	.43 (.42)	.17 (.43)	-.68 (.31)	.42 (.53)
1956	-1.10 (.48)	-1.06 (.25)	.13 (.60)	-.19 (.37)	.15 (.40)
1957	-.76 (.46)	-.84 (.53)	-2.08 (.60)	-.39 (.46)	-.06 (.28)
1958	-.53 (.23)	.09 (.48)	-1.63 (.39)	1.48 (.80)	-.04 (.36)

Coefficients in brackets are standard errors.

TABLE 7 PART 1 OF 5

Slope Coefficients When Long Term Growth, Debt/Equity and Dividend/Profit Are the Independent Variables and $(\frac{DIV}{P} + g_L)$ if the Dependent Variable

Building Materials Industry

Year	Debt	Dividends	g_L	R	Constant
1948	.20 (.14)	1.15 (.36)	6.97 (1.09)	.97	-3.08 (.13)
1949	.17 (.18)	1.14 (.44)	7.28 (1.30)	.95	-3.07 (.19)
1950	-.12 (.15)	1.08 (.40)	7.32 (1.19)	.95	-3.05 (.18)
1951	-.20 (.16)	.71 (.42)	7.74 (1.20)	.93	-2.99 (.21)
1952	.01 (.17)	.90 (.43)	7.04 (1.22)	.93	-3.08 (.22)
1953	-.04 (.16)	.41 (.34)	7.16 (.93)	.94	-2.76 .18
1954	-.05 (.16)	.03 (.35)	7.48 (.88)	.94	-2.66 .19
1955	-.00 (.12)	.27 (.27)	7.57 (.68)	.96	-2.88 .14
1956	.22 (.14)	.77 (.33)	6.19 (.83)	.95	-3.13 .17
1957	.12 (.13)	.41 .27	6.19 (.83)	.94	-2.87 (.14)
1958	.04 (.10)	.38 (.21)	7.43 (.68)	.96	-3.00 .12

TABLE 7 PART 2 OF 5

$$\frac{DIV}{P} + g_L$$

Chemical Industry

Year	Debt	Dividends	g_L	R	Constant
1948	.04 (.11)	.48 (.24)	7.00 (.80)	.93	-2.80 (.15)
1949	.04 (.14)	.46 (.24)	5.77 (.84)	.88	-2.66 (.17)
1950	-.01 (.06)	.31 (.24)	6.31 (.72)	.90	-2.68 (.17)
1951	-.04 (.05)	.23 (.24)	7.10 (.64)	.94	-2.81 (.17)
1952	-.03 (.05)	.16 (.22)	7.51 (.67)	.94	-2.81 (.16)
1953	-.10 (.07)	.07 (.22)	7.47 (.75)	.93	-2.69 (.16)
1954	-.13 (.08)	.12 (.20)	7.83 (.70)	.94	-2.79 (.14)
1955	-.22 (.12)	-.11 (.25)	4.92 (.41)	.94	-2.46 (.14)
1956	-.18 (.12)	-.10 (.22)	4.86 (.35)	.95	-2.49 (.11)
1957	-.11 (.12)	-.07 (.20)	4.67 (.34)	.96	-2.49 (.11)
1958	-.14 (.13)	-.09 (.20)	4.79 (.38)	.95	-2.49 (.18)

TABLE 7 PART 3 OF 5

$(\frac{DIV}{P} + g_L)$

Drugs Industry

Year	Debt	Dividends	g_L	R	Constant
1948	-.06 (.14)	.34 (.23)	7.25 (.96)	.93	-2.80 (.17)
1949	-.01 (.15)	.53 (.27)	8.01 (.96)	.94	-2.98 (.20)
1950	.14 (.17)	.64 (.27)	8.27 (.99)	.94	-3.17 (.21)
1951	.03 (.17)	.48 (.28)	8.83 (1.04)	.93	-3.19 (.22)
1952	-.07 (.19)	.14 (.29)	7.66 (.87)	.94	-2.86 (.21)
1953	.20 (.19)	.26 (.33)	6.65 (.69)	.95	-2.86 (.23)
1954	.25 (.19)	.42 (.31)	6.74 (.59)	.96	-2.96 (.20)
1955	.31 .20	.51 (.30)	6.53 (.53)	.96	-3.00 (.19)
1956	.28 (.22)	.42 (.29)	6.38 (.51)	.96	-2.94 (.18)
1957	.22 (.27)	.37 (.31)	6.38 (.54)	.95	-2.92 (.20)
1958	.23 (.29)	.26 (.37)	6.64 (.55)	.96	-2.98 (.18)

TABLE 7 PART 4 OF 5

$(\frac{DIV}{P} + g_L)$

Machinery-Industrial

Year	Debt	Dividends	g_L	R	Constant
1948	-.20 (.11)	-.67 (.16)	5.61 (.50)	.96	-2.00 (.09)
1949	.05 (.14)	-.85 (.23)	3.99 (.65)	.94	-1.74 (.14)
1950	.18 (.13)	-.78 (.21)	3.48 (.66)	.95	-1.83 (.13)
1951	.19 (.15)	-.60 (.20)	4.15 (.76)	.95	-2.07 (.14)
1952	.10 (.19)	-.57 (.26)	4.37 (.98)	.93	-2.08 (.17)
1953	-.05 (.15)	-.27 (.25)	5.06 (.75)	.95	-2.21 (.15)
1954	-.15 (.15)	-.11 (.32)	5.75 (.90)	.94	-2.37 (.18)
1955	-.19 (.14)	.08 (.35)	6.67 (.87)	.95	-2.59 (.19)
1956	-.21 (.12)	.29 (.25)	7.93 (.74)	.97	-2.89 (.14)
1957	-.23 (.17)	.08 (.28)	7.52 (.84)	.25	-2.73 (.18)
1958	-.29 (.19)	-.14 (.19)	7.13 (.73)	.96	-2.56 (.14)

TABLE 7 PART 5 OF 5

$(\frac{DIV}{P} + g_L)$

Oil Industry

Year	Debt	Dividends	g_L	R	Constant
1948	.18 (.15)	1.08 (.20)	5.78 (.90)	.88	-2.92 (.10)
1949	.32 (.22)	1.02 (.26)	5.43 (1.24)	.81	-2.87 (.15)
1950	.31 (.22)	.95 (.24)	6.82 (1.23)	.84	-3.00 (.15)
1951	.34 (.18)	.92 (.19)	8.23 (1.01)	.90	-3.21 (.12)
1952	.26 (.17)	.92 (.20)	9.09 (1.05)	.90	-3.30 (.13)
1953	.17 (.18)	1.03 (.20)	8.43 (1.04)	.90	-3.21 (.12)
1954	.13 (.15)	.92 (.17)	8.64 (.94)	.92	-3.23 (.11)
1955	.11 (.11)	.75 (.13)	9.24 (.78)	.94	-3.26 .09
1956	.14 (.10)	.59 (.12)	9.78 (.77)	.95	-3.31 .09
1957	.18 (.11)	.53 (.12)	8.99 (.81)	.93	-3.23 .09
1958	.12 (.11)	.39 (.10)	9.25 (.76)	.94	-3.17 .08

TABLE 8

Frequency of Significant Coefficients for Debt and Dividends in Final Cross Section Specification

Coefficients of Debt/Equity

	$(\frac{DIV}{P} + g_s)$			$(\frac{DIV}{P} + g_L)$		
	NS	S-	S+	NS	S-	S+
Bldg. Mat.	7	3	1	7	1	3
Chemicals	6	4	1	5	6	
Drugs	4		6	5		5
Mach.-Ind.	7	1	2	6	3	2
Oils	9	1	1	2		9

Coefficients of Dividend/Profit

	NS	S-	S+	NS	S-	S+
Bldg. Mat.	5	4	2	2		9
Chemicals	7	1	3	5		6
Drugs	5		5	1		9
Mach.-Ind.	5	1	4	5	5	1
Oils	1		10			10

NS not significant

S- significant and negative

S+ significant and positive

The conclusions to be drawn are four; in only two industries, Drugs and Oils, were debt and dividends significant in a majority of the cross sections. Long-term growth as the growth element being capitalized results in more significance for both debt and dividends than does short-term growth, the $\frac{DIV}{PROF}$ variable is more often significant than the $\frac{D}{E}$ ratio, and growth seems important per se.

The inclusion of growth on both the left and right-hand side of the regression equation may cause some suspicion of spurious correlation. This usually arises when an independent variable is added to both sides of a regression equation. If, for example, in the simplest of cases, $y = \alpha x + \varepsilon$ is the correct specification, but $y + x = \alpha' x + \varepsilon$ is the specification used in the regression, the estimate of $\underline{\alpha}'$ is $1+a$ if $\underline{a}$ is the estimate of $\underline{\alpha}$. Thus no real problem need occur in the estimation of the slopes and the residuals are unchanged. The problem of spurious correlation arises as the correlation from the second specification exceeds that in the "true" specification. It is

$$1 - \frac{\Sigma\varepsilon^2}{\Sigma y^2 + \Sigma x^2(a+1)}$$

instead of

$$1 - \frac{\Sigma\varepsilon^2}{\Sigma y^2}$$

Therefore, if the relationship in this model was really between $\log \frac{DIV}{P}$ and g, $\frac{D}{E}$ and $\frac{DIV}{PROF}$, the addition of g to both sides would introduce some spurious correlation. However, the model presented

here states that $(\frac{DIV}{P} + g)$, the total yield on the equity, is a constant, unless there is truly some influence of g. That is, if the true relationship between $(\frac{DIV}{P} + g)$ and g is zero, the expected value in these regressions of the covariance of $(\frac{DIV}{P} + g)$ and g is zero. This arises from the fact that the model states that high growth rates, dividends given, imply just enough higher prices to make $(\frac{DIV}{P} + g)$ independent of g. Thus, under this hypothesis, any significant correlation between $(\frac{DIV}{P} + g)$ and g implies g really influences $(\frac{DIV}{P} + g)$ and does not occur simply because g was included on both sides of the regression equation.[4/]

[4/] This is analogous to the statement that dividing variables on both sides of a regression equation by a scale variable introduces spurious correlation if the model was developed on the unscaled variables, but not if the model explicitly included the scaled variables. See for example, J. Meyer and E. Kuh, "Correlation and Regression When the Data are Ratios," Econometrica, October 1955.

CHAPTER V

The Pooled Regressions

The cross section regressions have presented some unexpected results. Little, if any significance can be found for the variables originally thought to determine capitalization rates, yet high correlations are obtained, with the growth variable providing almost all the explanation. Because of these results and also because it is good econometric practice, the statistical assumptions of the cross section analysis will be examined to see if they could explain the peculiar lack of influence for debt and dividend policy.

In this chapter statistical problems with the cross sections are discovered and the original model is altered to avoid these problems. The new tests show the implications of the cross section regressions to be quite misleading and restore one of the independent variables to a place of importance.

All econometric studies make assumptions about the statistical properties of the residual or error terms in the equation being estimated. In time series analysis a major problem with the errors is that of autocorrelation. This phenomenon has its counterpart in cross section analysis. The cross sections assumed that the error terms were independent drawings while in fact each cross section contained the same firms each year - each drawing. To the extent that there was any special characteristic associated with a specific firm, its influence would exist in each of the cross sections, rendering them no longer independent. To test this possibility, residuals

were compiled for both dependent variables using g_L, $\frac{D}{E}$, $\frac{DIV}{PROF}$ as variables to explain $(\frac{DIV}{P} + g_L)$, and g_L, $(g_L - g_s)$, $\frac{D}{E}$, $\frac{DIV}{PROF}$ to explain $(\frac{DIV}{P} + g_L)$. Table 8A shows the results for $(\frac{DIV}{P} + g_L)$. It can be seen that for many firms, the errors are consistently or predominantly in one direction in each of the years. In the Building Materials industry, for example, 12 of the 15 firms have residuals of the same sign in 10 of the 11 years in which cross sections were run.

Because of the suspicion that effects peculiar to each firm were present in each of the cross sections, the model was revised to estimate these influences explicitly. If these effects were substantial, failure to introduce them specifically would result in erroneous estimates of the standard errors of slope coefficients as well as effectively overstate the degrees of freedom when evaluating variance ratios.[1/] It has been shown by A. H. Carter[2/] that altering the original hypothesis to include a dummy variable for each firm will eliminate these problems and result in unbiased estimates of the slope coefficients and error variances. The inclusion of firm effects requires a method of combining the individual cross sections into one single grand regression employing all the data. The single regression estimates at one time all the slope coefficients on the debt, dividend and growth terms, a set of firm effects, and the slope coefficient r_k.

[1/] This problem is discussed more fully in E. Kuh, Capital Stock Growth: A Macroeconometric Approach, North Holland Publishing Co., 1963.

[2/] Ibid.

TABLE 8A

RESIDUALS*

BUILDING MATERIALS

FIRM	YEAR											
	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	Total
1	.032	.132	.158	.141	.129	.138	.196	.122	.033	.039	.006	1.130
2	.149	.226	.110	.016	.006	.004	.024	-.026	-.070	-.002	.010	0.448
3	.091	.091	.117	.067	-.060	-.021	.047	.049	.006	.050	.106	0.547
4	-.118	-.134	-.163	-.255	-.230	-.070	.062	-.030	-.147	-.135	-.115	-1.339
5	-.135	-.175	-.103	-.109	-.146	-.132	-.115	-.083	-.036	-.083	-.073	-1.195
6	.047	.054	.154	.186	.153	.127	.031	-.003	.050	.118	.075	0.994
7	-.121	-.098	-.153	-.168	-.138	-.103	-.047	-.057	-.086	-.103	-.115	-1.193
8	.091	.024	.003	.072	.113	.060	.069	.112	.157	.136	.128	0.970
9	-.051	-.068	-.089	-.092	-.111	-.131	-.120	-.077	-.141	-.170	-.108	-1.163
10	-.019	.066	.069	.044	-.021	-.047	-.177	-.115	-.024	.063	.049	-0.110
11	-.096	-.103	-.025	.039	.057	-.004	-.016	.040	.127	.052	.005	0.076
12	.055	-.027	-.069	.000	.147	.175	.106	.064	.098	.089	.027	0.667
13	.012	-.121	-.089	.015	.057	-.029	-.051	-.031	.092	.027	-.000	-0.120
14	.083	.098	-.002	-.058	-.018	-.042	-.075	-.078	-.058	-.085	-.070	-0.307
15	-.020	.034	.082	.101	.063	.077	.067	.114	-.002	.003	.074	0.594

*Computed with $(\frac{DIV}{P} + g_L)$ as the dependent variable and $\frac{D}{E}$, $\frac{DIV}{PROF}$, and g_L as the independent variables.

TABLE 8A continued

RESIDUALS

DRUGS

FIRM	YEAR 1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	Total
1	-.366	-.361	-.344	-.315	-.394	-.378	-.405	-.367	-.305	-.307	-.379	-3.926
2	.016	-.072	-.041	-.005	.104	.145	.120	.117	.104	.094	.125	0.709
3	-.027	.031	.075	.024	.037	-.009	-.020	.054	.061	.004	.059	0.291
4	.040	.078	.109	.155	.153	.140	.092	.041	.077	.140	.140	1.168
5	.124	-.018	-.118	-.193	-.272	-.208	-.186	-.178	-.180	-.198	-.200	-1.632
6						-.171	.036	.134	-.036	-.080	-.082	-0.199
7	.178	.164	.181	.203	.167	.070	.038	.011	-.038	-.059	-.031	0.886
8	-.036	.008	.023	.096	.169	.154	.108	.090	.085	.024	.111	0.834
9	.102	.062	-.063	-.130	-.038	-.089	-.053	-.033	.043	.029	.047	-0.124
10	.008	.057	.103	.033	-.041	.012	.083	.068	.126	.195	.131	0.777
11	.046	.159	.088	.052	.037	.071	.094	.088	.138	.096	.065	0.939
12	-.136	-.130	-.068	-.032	-.001	.066	.101	.092	.097	.161	.052	0.203
13						-.240	-.170	-.169	-.197	-.232	-.253	-1.265
14				-.011	-.002	.052	-.004	-.034	-.083	-.066	.040	-0.109
15	-.038	-.058	-.114	-.161	-.009	.161	.106	.075	.065	.084	.117	0.228
16	.060	.003	.004	.070	.141	.102	.047	-.040	-.055	-.008	-.035	0.291
17	.027	.074	.166	.216	.146	.201	.141	.148	.188	.208	.166	1.685
18					-.197	-.079	-.129	-.097	-.091	-.085	-.076	-0.758

TABLE 8A continued

RESIDUALS

MACHINERY-INDUSTRIAL

FIRM	YEAR											
	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	Total
1	-.026	-.091	-.027	.011	.071	-.022	-.049	-.102	-.075	-.150	-.089	-0.552
2	.021	.024	-.028	-.039	-.033	-.056	-.034	-.046	-.089	-.073	-.061	-0.416
3	.142	.097	.075	-.081	-.093	-.063	-.076	-.077	-.047	-.073	-.036	-0.234
4	-.043	.053	-.057	-.018	-.050	-.057	-.102	-.062	-.020	-.002	-.012	-0.374
5	-.066	-.052	.006	.062	-.053	-.055	-.021	-.015	-.009	.011	.057	-0.135
6	-.029	.045	.113	.103	.101	.122	.093	.091	.072	.115	.043	0.872
7	.051	.098	.080	.049	.080	.071	.097	.143	.114	.061	.034	0.881
8	.011	-.094	-.185	.047	.185	.033	-.147	-.104	-.097	-.075	.013	-0.414
9	-.010	.014	.105	.066	.022	.007	-.040	-.034	.046	.029	-.016	0.189
10	-.072	-.135	-.099	-.107	-.116	-.069	-.047	-.073	-.071	-.091	-.068	-0.951
11	-.140	-.166	-.122	-.084	-.019	.013	.014	.016	.049	.082	.023	-0.333
12	.175	.188	.141	.198	.202	.156	.040	.010	.032	.054	.056	1.258
13	.028	-.074	.052	.160	.141	.092	.221	.265	.058	-.051	-.001	0.893
14	-.110	-.168	-.176	-.293	-.311	-.288	-.253	-.235	-.287	-.328	-.371	-2.825
15	.105	.157	.009	-.075	-.188	-.012	.062	-.004	.056	.141	.145	0.397
16	.015	.038	.047	-.036	-.014	.037	.191	.159	.075	.065	.112	0.691
17	-.139	-.098	-.057	.007	.036	.029	-.010	.021	.006	.229	.163	0.187
18	.087	.163	.124	.028	.039	.061	.061	.048	.185	.057	.007	0.866

TABLE 8A continued

RESIDUALS

OILS

FIRM	YEAR											
	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	Total
1	-.148	-.235	-.234	-.283	-.319	-.327	-.275	-.165	-.109	-.137	-.107	-2.343
2	.169	.297	.251	.226	.187	.160	.139	.138	.154	.039	.154	1.919
3	.034	-.024	-.085	-.125	-.214	-.211	-.156	-.183	-.179	-.152	-.065	-1.365
4	-.008	.123	.264	.138	.138	.070	.040	-.014	-.055	-.101	-.074	0.522
5	.088	.071	.073	.073	.094	.132	.124	.085	.039	.042	.009	0.834
6	.010	-.038	-.154	-.101	-.117	.049	.010	-.017	-.019	.038	.027	-0.312
7	-.283	-.285	-.279	-.246	-.232	-.223	-.164	-.110	-.117	-.111	-.096	-2.151
8	-.028	.126	.088	.093	.068	-.017	-.061	-.027	-.025	-.000	.017	0.234
9	-.004	.014	.020	.006	-.059	-.091	-.051	-.039	-.054	.003	.048	-0.207
10	.004	.023	.009	-.058	-.047	.015	.063	.048	.023	.046	-.064	0.063
11	-.004	-.000	-.018	.006	-.014	.026	.026	.051	.078	.110	.050	0.313
12	-.042	-.091	-.088	-.092	-.057	-.108	-.106	-.072	-.026	-.003	-.001	-0.691
13	.093	.097	.017	.057	.049	.052	-.002	.001	-.013	-.004	.015	0.363
14	.114	.154	.158	.042	.049	.082	.108	.004	-.058	-.087	-.094	0.476
15	-.026	-.114	-.030	.013	.016	.034	.000	.009	-.015	-.072	-.079	-0.266
16	.118	.251	.246	.229	.208	.202	.163	.133	.102	.071	.087	1.813
17	.130	.094	.100	.119	.124	.101	.115	.096	.111	.119	.091	1.203
18	-.144	-.139	-.126	-.102	-.128	-.118	-.103	-.099	-.109	-.081	-.089	-1.243
19	.096	-.114	-.126	-.013	.100	.003	-.014	.015	.145	.247	.207	0.547
20	.011	.083	.172	.110	.054	.070	.075	.034	.002	-.019	.001	0.598
21	-.011	.062	.044	.018	.040	.119	.183	.201	.172	.132	.145	1.108
22	-.189	-.223	-.181	-.101	-.086	-.120	-.092	-.086	-.071	-.109	-.110	-1.372
23	-.081	-.164	-.163	-.092	-.037	-.072	-.093	-.077	-.072	-.088	-.136	-1.079
24	.155	.180	.122	.054	.076	.088	.086	.025	.067	.092	.051	1.002
25	-.053	-.147	-.081	.025	.106	.079	-.013	.048	.030	.025	.012	0.032

TABLE 8A continued

RESIDUALS

CHEMICALS

FIRM	YEAR											
	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	Total
1	-.020	-.036	.021	.101	.112	.118	.163	.211	.199	.131	.102	1.106
2	-.145	-.186	-.170	-.100	-.094	-.096	-.150	-.269	-.286	-.210	-.158	-1.869
3	.161	.174	.139	.133	.022	-.011	-.038	-.264	-.092	-.113	-.209	-0.097
4	.127	.128	.083	-.006	-.049	-.019	.035	.238	.217	.182	.150	1.087
5	.032	.050	.081	.080	.124	.183	.077	.044	-.009	-.032	.015	0.647
6	.006	.060	.063	.114	.117	.133	.086	.082	.038	.065	.084	0.853
7	-.070	-.003	.081	.074	.022	.099	.123	.131	.069	.100	.130	0.758
8	-.027	-.083	-.096	-.062	-.087	-.088	-.068	.037	.006	.002	-.013	-0.481
9	.055	-.023	-.040	-.036	.048	.047	.010	.044	.043	.065	.062	0.278
10	-.056	-.110	-.082	.003	.020	-.020	-.020	-.022	-.014	-.036	-.046	-0.384
11	.164	.171	.126	.066	.078	.013	-.009	.045	.015	-.044	.109	0.736
12	.044	.108	.137	.063	.135	.107	.084	-.069	-.066	.008	.037	-.591
13	-.079	-.065	-.062	-.027	-.013	-.021	-.051	-.160	-.165	-.141	-.175	-0.964
14	-.073	-.045	-.057	-.059	-.078	-.063	-.081	.179	.168	.201	.215	0.305
15	.102	.126	.029	-.154	-.143	-.126	-.051	-.077	.017	-.008	-.027	-0.314
16	.077	.130	.184	.151	.148	.202	.226	.171	.120	.157	.116	1.687
17	-.082	-.084	-.075	-.058	-.076	-.086	-.051	.062	.078	.077	.088	-0.208
18	-.142	-.155	-.221	-.207	-.163	-.114	-.074	-.063	-.087	-.145	-.158	-1.535
19		-.031	-.097	-.101	-.143	-.226	-.212	-.101	-.086	-.058	-.073	-1.132
20			.008	.043	.095	.051	.051	.070	.065	.032	.012	0.431
21	-.092	-.116	-.085	-.050	-.064	-.074	-.048	-.043	-.086	-.085	-.057	-0.805
22	.015	-.006	.033	.032	-.012	-.007	-.000	-.046	-.081	-.068	-.084	-0.227

The cross section regressions showed that long-term growth as the element being capitalized gave better results than the short-term rate, and that when the short-term rate of growth was used it had to be adjusted to the long-term rate. For this reason, no further statistical tests will be performed with the short-term rate of growth and effort will be centered on the long-term rate of growth. As there is no reason to believe that the difficulties arising from the colinearity between the level and the square of both the debt and dividend term would be any less than before, further regressions will include only the levels of these variables. This means that few managerial implications will be forthcoming. However, it seems more serious to find no effect for debt and dividend policy in the cross sections than to be unable to split it into two parts. Thus, the present tests are run in the hope of discovering a statistically significant influence for debt and dividends on the capitalization rate.

The final cross section specification had excluded the difference between long and short-term rates of growth and was written

$$\log\left(\frac{DIV}{P} + g_L\right) = hg_L + a\frac{D}{E} + c\frac{DIV}{PROF} + r_k \log \rho_o + \varepsilon$$

with $r_k \log \rho_o$ being estimated as the intercept in each of the cross section regressions. If the data now pooled, $\log \rho_o$ treated as a time series, and dummy variables introduced as firm effects, the specification becomes

$$\log\left(\frac{DIV}{P} + g_L\right)_{ikt} = r_k \log \rho_{o_t} + h_k\, g_{Likt} + a_{kt}\left(\frac{D}{E}\right)_{ikt} + b_{kt}\left(\frac{DIV}{PROF}\right)_{ikt} + D_i + \varepsilon_{ikt}$$

where

$(\frac{DIV}{P} + g_L)_{ikt}$	is the capitalization rate for the i^{th} firm in the k^{th} class in year t.
ρ_{o_t}	is the "average" capitalization rate prevalent in the market at time t.
r_k	is the industry asset risk adjustment factor.
$(\frac{D}{E})_{ikt}$ $(\frac{DIV}{PROF})_{ikt}$ g_{Likt}	are the $\frac{D}{E}$, $\frac{DIV}{PROF}$ and long-term growth rate of the i^{th} firm in the k^{th} class respectively.
D_i	is a dummy variable associated with the i^{th} firm.

Written this way, the model now attempts to explain the price of each firm's share as a function of its risk class, the average capitalization rate for all firms that year, its debt-equity and dividend-payout ratios, its growth rate, and a constant associated with that firm. The inclusion of a constant for each firm assures that the residuals associated with any firm add to zero if summed over all years. That is, these constants specifically estimate those peculiarities associated with each firm which are not explained by the other variables.[3/]

[3/] If only one slope coefficient for debt and dividends rather than 11 was allowed, the introduction of dummy variables would be exactly equivalent to running a regression with all the variables treated as deviations from firm means.

Because of the inclusion of the long-term growth rate as an independent variable, it is necessary to measure all the firm effects from that of one of the firms. That is, the vector of long-term growth rates would be a linear combination of the dummy vectors if each firm were allowed a non-zero dummy. One data vector being a linear combination of another leads to a singular moment matrix - a zero determinant - and does not allow the matrix inversion required to estimate the slope coefficients. Measuring all firm effects as deviations from one firm, does away with the problem.[4]

This specification requires the addition of another independent variable - a time series for log ρ_o. To this end, the capitalization rate for all the firms in the k^{th} class in the t^{th} year was computed. In year t, log ρ_o was defined as the average of the average capitalization rates for all five industries. It is thus an average of all the actual capitalization rates. Under perfectly atomistic competition, although any one firm's rate was included in its computation, it would compose such a small part of the total that the average rate would be a truly independent variable uncorrelated with any residual. Although it is difficult to decide if the present conditions approximate such atomistic competition, the statistical test are performed assuming the average rate to be a truly independent variable.

---

[4] See D. B. Suits, "Use of Dummy Variables in Regression Equations," Journal of American Statistical Association, 1957, Vol. 52, p. 548; for further comments on the use of dummy variables to test for shift variable effects.

Five kinds of results are looked for with this regression. Does the new specification significantly reduce the residual sum of squares; what happens to the coefficients of debt and dividends; does growth still have a strongly positive sign; is it possible to use the specification to test the hypothesis that the dividend coefficient depends upon the growth rate; and is it possible to see if the risk classes actually discriminate between firms with different risk characteristics?

The pooled regression against which the cross sections were tested contained the term log ρ_o, one growth variable and eleven terms - one for each year - for both dividends and debt. Table 9 shows the residual sums of squares associated with each, the F statistic on their difference and the critical values of the F distribution for the relevant degress of freedom. It can be seen that the difference is highly significant implying that the new specification fits the data significantly better than the old.

Table 10 shows the slope coefficient for the log ρ_o term. These are the industry adjustments to the mean capitalization rate for any year and are meant to measure the relative riskiness of each industry and were expected to be around unity - slightly less for the less risky industries and slightly more for the more risky. They are not at all as expected. If the average capitalization rate for all industries is .10, an r_k of .33 implies an average rate for that industry of $(.10)^{.33}$ or .465, while a coefficient of 3.25 implies a cost of less than .001.

TABLE 9

Residual Sums of Squares -- I

	Original Cross Sections	Pooled* Regression	F	$F_{.01}$	$F_{.05}$
Building Equip.	1.58773	.63419	37.6	2.96	2.17
Chemicals	2.78046	.75370	523.5	6.83	3.85
Drugs	3.57060	.60292	241.5	3.17	2.29
Machinery-Ind.	2.31796	1.46341	91.7	3.95	2.60
Oils	3.78000	1.01463	153.4	3.32	2.37

*Containing the capitalization rate as the dependent variable with log ρ_{ot}, one growth term, eleven dividend and eleven debt terms and the firm effects as independent variables.

TABLE 10

Coefficients of Log ρ_o (All Firms)

	Coefficient	Standard Error
Building Equipment	-.67	.44
Chemicals	.29	.27
Drugs	.41	.27
Machinery-Industrial	2.87	.41
Oils	.57	.21

Coefficients of Log ρ_o

(Excluding High Growth Firms)*

	Coefficient	Standard Error
Building Equipment	-.18	.55
Chemicals	.33	.41
Drugs	1.52	.37
Machinery-Industrial	3.25	.44
Oils	.57	.23

*Justification will be presented later for the lack of interest in very rapidly growing firms.

Because the unsuccessful inclusion of this variable may have interfered with estimation of the other slope coefficients, another model was considered in which this time variable was not included. In the new model, each observation on the dependent variable was treated as the deviation from the year's mean for that industry. This specification avoids the explanation of the mean price in any year, and only concerns itself with the deviation of each firm from the industry mean for that year.

Table 11 compares the residual sums of squares from this regression with those obtained from the original 11 cross sections. Again, the introduction of firm effects significantly reduces the residual error.

In Table 12, the slope coefficient of growth and the dummies are presented for the equation with the dependent variable as deviations from year means. The influence of growth is drastically different from that obtained with the cross sections. In all but one case, growth now has a large and significantly negative coefficient while before it always had a positive one. Also, the one positive slope coefficient is insignificant. A careful examination of the data on growth, shown in Table 13, reveals that in those industries having larger coefficients for growth a few firms had growth rates far in excess of the others. This is especially true in the drug industry.

This led to the presumption that there might be a single data point or a few data points, far removed from the main cluster of the data, which were dominating the determination of the slope coefficients. This also would explain the large positive firm effects which appeared,

TABLE 11

Residual Sums of Squares -- II

	Original Cross Sections	Pooled* Regression	F	F^*_{01}	F^*_{05}
Building Equipment	1.58773	.71233	36.0	3.48	2.45
Chemicals	2.78046	.79924	41.4	2.37	1.78
Drugs	3.57060	1.45549	27.0	2.78	2.08
Mach.-Ind.	2.31796	1.56053	23.6	2.77	2.08
Oils	3.78000	1.03331	41.2	2.25	1.75

*Containing the deviation of the capitalization rate from the year's industry mean as the dependent variable with one growth term, eleven dividend and eleven debt terms along with the dummy variables as independent variables.

TABLE 12

Slope Coefficients for Growth and Dummy Variables

	Building Equip.	Chemicals	Drugs	Mach. Ind.	Oils
Slope Coef. for Growth (all firms)	-6.64	.12	-81.65	-6.74	-7.16
	(1.80)	(.63)	(7.64)	(3.19)	(.73)

Dummy Variable Slope Coefficients

Firm No.	Building Equip.	Chemicals	Drugs	Mach. Ind.	Oils
2	.71	-.57	12.34	2.67	.40
3	.45	-.59	4.56	.93	.01
4	-.51	.31	5.83	.30	.53
5	.94	.08	2.70	1.19	1.00
6	.95	.10	3.95	.87	.18
7	1.19	-.05	1.59	1.26	.63
8	.59	-.02	8.01	.39	-.35
9	-.13	.01	9.501	.50	.61
10	-.41	-.09	5.53	1.54	.20
11	.11	.16	6.94	1.30	.41
12	.50	-.09	12.97	.77	.89
13	-.30	-.26	24.66	.41	.45
14	.29	.36	7.50	-.37	-.36
15	.54	-.48	13.84	.14	-.40
16		.14	3.68	.20	.41
17		.04	8.83	.23	.63
18		-.42	21.17	.64	-.40

TABLE 12 Continued

Firm No.	Building Equip.	Chemicals	Drugs	Mach. Ind.	Oils
19		.18			-.19
20		-.38			.73
21		-.37			.11
22		.19			-.22
23					.45
24					.99
25					.28

TABLE 13

Growth Rates in Total Sample

Firm No.	Building Equip.	Chemicals	Drugs	Mach. Ind.	Oils
1	.042	.085	.012	.017	.059
2	.103	.036	.146	.237	.061
3	.077	.003	.058	.092	.048
4	.027	.134	.073	.045	.063
5	.129	.101	.041	.110	.113
6	.118	.099	.053	.076	.057
7	.142	.082	.024	.107	.095
8	.086	.106	.097	.047	.021
9	.049	.093	.114	.053	.082
10	.022	.086	.070	.144	.032
11	.060	.109	.085	.120	.063
12	.080	.074	.153	.063	.099
13	.037	.066	.289	.039	.073
14	.075	.159	.093	.002	.013
15	.089	.030	.164	.023	.029
16		.096	.044	.024	.063
17		.111	.106	.030	.081
18		.053	.248	.056	.032
19		.144			.022
20		.046			.085
21		.1251			.043

TABLE 13 Continued

Firm No.	Building Equip.	Chemicals	Drugs	Mach. Ind.	Oils
22		.055			.053
23		.068			.094
24					.104
25					.059

especially in the drug industry. For, given a slope coefficient on growth of -80., a growth rate of.05, and a relatively small correction for debt and dividends, the only way to estimate the deviation of $\log(\frac{DIV}{P} + g)$ from the industry year mean, which is or order of magnitude -1 to +1, is to have a large positive constant. Thus, it was hoped that exclusion of the most rapidly growing firms would assure that a few firms did not dominate the data on growth.

Table 14 shows the results of a regression in which firms with growth rates in excess of 10% were excluded. The slope coefficients are somewhat lower; in all but the drug industry, the firm effects are smaller. Since these firm effects measure that part of the deviation of the capitalization rate from the industry year mean which arises from otherwise unexplained peculiarities, it was expected that they be a fraction of the total difference. Since the difference is about one, firm effects of six or seven seemed unreasonable while those around unity or less seemed more likely to be accurate estimates of the true firm effects. Thus, except for the drug industry, a linear approximation for the influence of growth seems to fit the data well and results in consistent slope coefficients and reasonable dummies.

Having found that the very rapidly growing firms tended to distort the determination of the dummy variables it was thought that the inclusion of these very rapidly growing firms may have been the reason for the lack of success in estimating the coefficient r_k. Thus, the term $\log \rho_o$ was reintroduced as a dependent variable and the regressions rerun excluding those firms with growth rates in excess of 10%

TABLE 14

Slope Coefficients for Growth and Dummy Variables

	Building Equip.	Chemicals	Drugs	Mach. Ind.	Oils
Slope Coef. on Growth	-2.50	.11	-70.12	-4.12	-4.91
(High Growth Firms excluded)	(2.34)	(.72)	(10.91)	(3.33)	(.80)

Dummy Variable Slope Coefficients

Firm No.	Building Equip.	Chemicals	Drugs	Mach. Ind.	Oils
2	.15	-.58	3.93	.75	.39
3	-.52	-.60	5.15	.29	.03
4	.25	.08	2.28	1.01	.51
5	-.27	.09	3.46	.73	.18
6	-.49	-.05	1.39	1.02	.55
7	-.07	-.02	6.97	.33	-.27
8	.20	.00	4.85	.46	.55
9	-.20	-.10	6.041	.67	-.14
10	-.01	.16	6.56	.41	.40
11	.37	-.09	2.94	-.32	.80
12		-.27	7.68	.17	.43
13		-.48		.18	-.26
14		.14		.23	-.33
15		-.43		.61	.40
16		-.38		.94	.58
17		-.37			-.34

TABLE 14 Continued

	Building Equip.	Chemicals	Drugs	Mach. Ind.	Oils
Firm No.					
18		-.19			-.12
19					.67
20					.15
21					-.21
22					.38
23					.28

per year. The resulting slope coefficients are shown in Table 10.* Again, as before, the coefficients have too wide a spread to be considered as risk adjustment factors. Because of this, all further regressions were run with deviations from industry year mean as the dependent variable - no further attempt was made to predict that year mean.

Thus, the first result of the pooled regressions is that they have reversed the sign of the slope coefficient on the growth term. The negative slope coefficient on growth implies that the original model understates the influence of growth on equity prices. In the original model, as the growth rate increased, the price had to rise enough to keep $\frac{DIV}{P} + g$ a constant. The negative sign for growth, when it is added as an independent variable, means that as growth increases, the sum of $\frac{DIV}{P} + g$ declines. An increase in the growth rate now forces the price to rise more than enough to keep $\frac{DIV}{P} + g$ constant. This can be stated more succinctly by examining the derivative of price with respect to the growth rate. If

$$\frac{DIV}{P} + g = K$$

$$\frac{dP}{dg} = (1 - \frac{dK}{dg})\frac{P^2}{DIV}$$

When K is a constant, $\frac{dK}{dg}$ is zero. When $\frac{dK}{dg}$ is negative, as it is when the slope coefficient on growth is negative, however, the change in P resulting from the change in g is composed of two terms and causes $\frac{dP}{dg}$ to be larger than before.

---

*This may be found on page 91.

This extra interest in growth in the postwar period may be evidence of another implication of taxes for the capitalization rate. Taxes were specifically introduced in connection with the portion of equity obtained through retentions. No recognition was given to the fact that the two components of the return - the dividend yield and the growth in price - were taxed at different rates. Since the gains tax is always lower than the income tax (except at zero tax rates), returns obtained through gains should be more valuable. If they were as certain as the dividend income these capital gains are clearly more valuable. This differential taxation may well explain a large part of the extra interest in growth.

The fact that the inclusion of these firm effects significantly reduces residual variance and reverses the sign of the slope coefficient on the growth variable attests to the importance of specifically estimating these firm effects when there is any evidence of their existence.[5/] Until now, much of the work in the analysis of covariance has had to be of an analytical nature and the empirical work has had to make many approximations as the techniques require the inversion of matrices of large order. Each dummy variable, in this case each firm effect, increases the order of the matrix by one,

---

[5/] For example, Myron Gordon in The Investment, Financing and Valuation of the Corporation, p. 153, reported that one of his industries, Food, showed "highly significant" evidence of firm effects, but he performed no analysis specifically including these effects. For a much more extensive analysis of the whole field of the analysis of covariance, see A. M. Mood, Introduction to the Theory of Statistics, New York: 1950; or Edwin Kuh, Capital Stock Growth: A Macroeconometric Approach, North Holland Publishing Co., 1963.

so that allowing different slope coefficients for each variable in each year and estimating firm effects required the inversion of matrices of order 40-50. With an original equation containing 3 or 4 variables, 25 firms and 10 years, the order could easily reach 80-90, and often much more. While computational problems of time and accuracy have been unsurmountable in the past, the availability of a very high speed computer with a large memory capacity such as the IBM 7090 has made the work in this thesis possible. Also the moment matrices for dummy regressions are relatively easy to partition so that inversion routines could be developed using present machine capacity to invert rapidly and accurately the matrices necessary for a regression for say, 40 firms in each of 20 years with 5 variables in the equation.

As stated in Chapter II, it has been conjectured that it is likely that the equity price of a rapidly growing firm would suffer more from dividend payments than that of a more slowly growing one. This effect is thought to occur if rapid rates of growth imply high profitability of investment. Retained earnings would thus generate large capital gains which because of the differential taxation of income and capital gains would be more valuable than dividends. The present model specification allows a rather neat test of this hypothesis. It may be formulated in the following manner.

Let the slope coefficient of the dividend profit rate depend upon growth, being larger for larger rates of growth, i.e., let $a \frac{DIV}{PROF}$ become $(a+bg_L) \frac{DIV}{PROF} = a \frac{DIV}{PROF} + bg_L * \frac{DIV}{PROF}$.

Since this results in the old specification plus the term $bg_L * \frac{DIV}{PROF}$, it is possible to perform an F test for the significance of an added variable.[6]

Table 15 shows the coefficients and the result of the F test for each of the industries. Three times the interaction is significant at at least the 5% level. In all but one case, it has the predicted positive sign, causing the price to fall as the product of growth rate and dividend payout rises.

In addition to increasing the explanatory power of the model for certain industries, the term $g * \frac{DIV}{PROF}$ alters the influence of $\frac{DIV}{PROF}$ in these industries. In Oils, adding $g * \frac{DIV}{PROF}$ changes the coefficient on $\frac{DIV}{PROF}$ alone from positive to negative, in industrial machinery the coefficient, initially negative, is made more negative and in drugs a mixed coefficient is made much more stable and negative in all but one year. Table 15A shows the slope coefficients with and without the inclusion of $g * \frac{DIV}{PROF}$. This increase in stability and significance of the coefficient of $\frac{DIV}{PROF}$ brought about by the inclusion of the term $g * \frac{DIV}{PROF}$ is taken as further evidence of the validity of the interaction.

Thus far, three of the hypotheses which were advanced on page 89 have been tested. There are substantial firm effects, their omission did lead to a misestimate of the influence of growth and, finally,

---

[6] In the analysis of the cross sections it was difficult to measure significance except in terms of standard errors, but this single grand regression enables the easy inclusion and exclusion of sets of variables facilitating F tests on the differences of variances, and allows more complete statements to be made concerning statistical significance.

TABLE 15

Significance of the Slope Coefficient for $g^{*}\frac{DIV}{PROF}$

	Coefficient	F	$F^{*}_{.01}$	$F^{*}_{.05}$
Building Material	-4.50 (3.58)	1.58	7.00	3.95
Chemicals	2.50 (3.05)	6.53	6.75	3.90
Drugs	9.24 (5.21)	3.15	6.95	3.95
Machinery-Industrial	12.45 (3.03)	16.90	6.86	3.92
Oils	11.96 (2.10)	32.34	6.70	3.88

TABLE 15A

Coefficients of $\frac{DIV}{PROF}$ With and Without the Inclusion of $g\frac{DIV}{PROF}$ as an Independent Variable

	Drugs		Mach. Ind.		Oils	
	w/o	w	w/o	w	w/o	w
1948	.00	-.72	-.56	-1.15	.28	-.30
1949	.01	-.69	-.58	-1.16	.20	-.37
1950	-.03	-.70	-.60	-1.17	.17	-.40
1951	.00	-.65	-.59	-1.14	.14	-.43
1952	-.14	-.79	-.55	-1.10	.12	-.46
1953	-.23	-.88	-.50	-1.07	.16	-.41
1954	1.49	.82	-.51	-1.10	.13	-.44
1955	-.25	-.93	-.54	-1.14	.08	-.50
1956	-.23	-.92	-.53	-1.12	.03	-.57
1957	-.19	-.86	-.48	-1.07	.04	-.57
1958	-.17	-.83	-.41	-1.01	.05	-.55

there is a significant interaction between growth and the dividend payout. Before moving on to the remaining two hypotheses, it is useful to look more carefully at the influence of growth and the interaction between growth and dividends.

In order to see if there was any change in the importance of growth or the interaction of growth and dividends over the eleven years for which data were available, the regressions were rerun including eleven coefficients for both the growth and the growth times dividend payout rate - one for each year. The resulting coefficients were then examined to determine if the influences showed evidences of changing over time.

In only two industries did the estimation of one slope coefficient per year for the term $g*\frac{DIV}{PROF}$ significantly reduce the error variance below that obtained when only one slope coefficient for all years was allowed. Table 16 shows the time series of slope coefficients for the $g*\frac{DIV}{PROF}$ term for these two industries. In both cases, the difference arises from smaller coefficients in 1948 and 1949 with essentially similar ones in the other years. It appears that the joint influence of growth and dividends was not so strong in the immediate postwar years as it was later on. This is consistent with the differential tax explanation, if one believes it took several years to be assured that personal taxes would after the war remain at their relatively high levels and would not fall to pre-war levels.

TABLE 16

	Coefficients of $g_{\frac{DIV}{PROF}}$		Coefficients of g_L	
	Chemicals	Oils	Chemicals	Oils
1948	3.02	7.59	-2.20	-7.98
	(3.32)	(4.36)	(.82)	(.85)
1949	.91	7.18	-3.30	-8.27
	(3.16)	(4.03)	(.87)	(.87)
1950	2.55	10.74	-2.16	-6.90
	(3.06)	(3.88)	(.85)	(.89)
1951	4.33	12.83	-1.20	-5.67
	(3.04)	(3.77)	(.92)	(.87)
1952	6.15	13.09	-.22	-5.57
	(3.07)	(3.71)	(.88)	(.84)
1953	7.38	11.71	.41	-6.38
	(3.18)	(3.74)	(.78)	(.83)
1954	8.09	12.34	.48	-5.88
	(3.27)	(3.71)	(.72)	(.84)
1955	7.34	13.43	-.13	-4.75
	(3.28)	(3.59)	(1.72)	(.84)
1956	4.33	14.00	-1.47	-4.16
	(3.21)	(3.23)	(.79)	(.85)
1957	3.61	10.36	-1.74	-5.06
	(3.01)	(2.93)	(.80)	(.87)
1958	4.89	10.14	-1.03	-4.84
	(2.78)	(2.68)	(.84)	(.87)

With the hope of learning more about the behavior over time of the response to the growth variable, a similar regression was run allowing instead of one, eleven coefficients for growth - one each year. Of the four industries in which the growth variable yielded reasonable estimates, Building materials and industrial machinery showed no evidence of a change over the period. For oils, the slope coefficient of the growth term was somewhat higher in the immediate postwar period than later but showed little change after 1950. Chemicals, however, had a quite different pattern. The coefficient moved from between -2 and -3 in the beginning of the period to +.5 in 1954 and then returned to the range -1 to -2 in the last years.[7/] Except for the chemical industry, there seems little change in the slope coefficient over time. This result is consistent with the feeling that the interest in growth per se arises from the differential taxation of income and capital gains. Since this tax differential did not change drastically over the period, one wouldn't expect the interest in growth to change.[8/]

[7/] Concerning the results in the chemical industry, these may well arise from differences of opinion during the period as to the long-term growth rate. Although earnings had grown sharply from 1946 to 1950, there was considerable concern in 1952 and 1953 that the reduced military requirements were producing a buyer's market while the industry had considerable excess capacity. (See, for instance, "Facts, Fears, and the Future," K. H. Klipstein, Chemical and Engineering News, May 4, 1963, pp. 1854-1856.) In fact, earnings did not perform as well in the years 1951 through 1954 as they had in the earlier period or as they did in the latter part of the period. The "bearish" outlook in the early 1950's may have led to a discounting of the expected long-term rate of growth. If the growth rates actually being capitalized were less than those used in the regressions with the most rapidly growing firms being the most highly discounted, the positive slope coefficients for growth found in the regressions would be explained.

[8/] Rising income levels and a progressive tax system do result in higher average personal tax rates, but this effect is not thought too serious in the present connection.

This concludes the analysis of the influences of growth and the growth-payout rate interaction. There is some evidence that the interaction was more important after 1950 than before, while there is little evidence that the influence of growth alone changed over the period.

Having explored the influences of growth and the product of growth and dividend payout, the significance of the dividend payout was next tested. In chemicals and drugs it was found not to contribute significantly to the explanation of the dependent variable. In the three other industries, however, $\frac{DIV}{PROF}$ does contribute significantly. In oils and industrial machinery it has a negative sign implying dividends alone (not including their interaction with growth) raise equity prices. In building materials, however, the sign is positive implying dividends reduce equity prices in that industry. Table 15A gives an indication of the size of these dividend coefficients.

The next move was to remove the $\frac{D}{E}$ ratio from the regression to allow a test for the significance of its inclusion. In only two industries did the residual variance obtained by including the debt-equity ratio differ significantly from that obtained without use of the $\frac{D}{E}$ term. These were building materials and oils. In both, the coefficient was negative implying that equity prices rose as the $\frac{D}{E}$ ratio rose.

At this stage a recapitulation and some analysis is necessary to avoid too much confusion from all the tests. The first problem was to see if the pooled regressions did a better job of explaining the data than did the individual cross sections. The conclusion was an

unqualified yes. There was significant evidence of substantial firm effects. Then the interaction of growth and dividend payout was tested and three industries, chemicals, industrial machinery, and oils showed significant evidence of such an effect. The sign of this influence agreed with the a priori belief. When this influence was allowed to vary over the period, chemicals and oils gave evidence of a slight increase over the period in the penalty attached to a high growth firm paying out a large fraction of its earnings as dividends. When the influence of growth was allowed to vary over the period it was found that building materials and industrial machinery showed no evidence of change. There was only a slight increase over time in this influence in the oil industry. The chemical industry, however, showed considerable variation over the period. This variation was thought to have arisen from the unsettled outlook for continued earnings growth after the Korean War. After exploring the influence of growth, both by itself and in conjunction with the dividend payout, the partial influence of the dividend payout alone was examined. In three cases, building materials, industrial machinery and oils, it contributed significantly to the results. For two of these industries, the partial influence of dividends increased share prices.

The test, which is in some ways the most crucial of all, was then performed - the test for the significance of debt. If the industries chosen truly represented a risk class in the sense discussed in Chapter II, there was the strong presumption that higher debt-equity ratios would imply lower equity prices. This would arise either from

increasing financial risk or from some sort of arbitrage process keeping the value of the firm from being completely dependent on the capital structure.[9/] The results showed, however, that only in two industries did debt add significantly to the analysis. Building materials and oils were the two, and the sign was opposite to that predicted in both. Earlier, mention had been made of the fact that if the industries chosen did not represent risk classes there would be the problem that high-debt firms may not be firms with high financial risk as much as firms with low physical asset risk. This appears to be the case in these industries.

Still rejecting, on *a priori* grounds, the hypothesis that debt does not influence equity prices, the evidence that it does not add significantly to the explanation of capitalization rates in the chemical, drug and industrial machinery industries and has the wrong sign in the building materials and oil industries casts doubt on the validity of the assumption that these industry classifications contain firms with common enough risk characteristics to be useful as a risk class. This is not to say that there are no purposes for which an industry classification as a broad measure of risk would be useful, but that it appears that industry is an inadequate proxy for risk when the influence of debt on equity prices is to be separated from the influence of the riskiness of the physical assets in which the firm invests.

[9/] There was the thought that a kind of arbitrage process as discussed in Franco Modigliani and Merton Miller's "The Cost of Capital, Corporation Finance and the Theory of Investment" might be operative even in a world of growth, taxes and risky debt.

One may well quarrel with this interpretation. We have said that, since debt should have a positive coefficient and was found not to have one, the industries must not be risk classes. An alternate view could be that they truly are risk classes and debt has no influence on equity prices. Although there is only inferential and no direct evidence, the first view seems more likely.

Having said that the risk class-industry connection was a poor one for estimating the influence of debt on capitalization rates, one is left with the question of its usefulness in estimating the influences of growth and dividends. Here again the answer is more difficult to give.

One test can be performed, however, to help determine if there is any difference among the industries. This is not a test to see if the firms within an industry have identical risk characteristics but rather a test to see if there is any difference among industries. It concerns the average capitalization rate each year. If average capitalization rates for all industries showed no variation in any given year it would seem likely that the industry classification scheme was not able to discriminate carefully among firms. If, however, it could be shown that the average capitalization rate differed from industry to industry, this information could be taken

as inferential evidence that there was some merit in the classification system. For, although the classes may not be homogeneous within, they are at least heterogeneous with respect to each other.[10/]

The test performed was a standard analysis of variance with a two way classification.[11/] The data were arrayed in a matrix with industries across the top and years down the side. As there was the presumption that year means would vary across years, the test was to determine if there was a significant difference between rows (years) and columns (industries). Table 17 shows the results. With very high confidence, the assumption that there is no difference between industries is rejected. Similarly, the years appear to be different, as suspected. This conclusion supports the belief that the industry classes while perhaps not satisfying the criterion that firms within be homogeneous with respect to risk, at least satisfy the assumption that there is a difference between the classes.

Aside from this evidence that the industries were not identical to each other, the reliability and stability of the results obtained should also be considered when evaluating the classification scheme. In three industries there is a significant contribution arising from

---

[10/] As many variables influence the capitalization rate a classification scheme able to discriminate extremely well with respect to risk might effectively group firms into risk classes, but these firms could have other characteristics making their capitalization rates equal on the average to those in another risk class. This exact counterbalancing of other influences against physical asset risk appears highly unlikely, however.

[11/] See A. Mood, Introduction to Statistics, p. 331 for a lucid explanation of both the theory and the practice of analysis of variance tests.

TABLE 17

Analysis of Variance of Mean Capitalization Rates

Source	Sums of Squares	Degrees of Freedom	Mean Square	F Ratio
Mean	242.29135	1		
Year Effect	.29890	4	.07473	22.61
Ind. Effect	.27832	10	.02783	8.42
Deviations	.13218	40	.00331	
Total	243.00075	55		

$F^{*}_{.005\ 4,40} = 19.8$

$F^{*}_{.005\ 10,40} = 5.00$

the $g*\frac{DIV}{PROF}$ term and in four it has the correct sign. This term also increases the importance and renders more stable the influence of $\frac{DIV}{PROF}$ alone as attested by Table 15A. Thus this interaction seems quite well documented. Also the coefficient of growth, when both $\frac{DIV}{PROF}$ and $g*\frac{DIV}{PROF}$ are included, is significantly negative in four of the five cases. This result is taken as substantial evidence that the original model understated the influence of growth on equity prices.

Thus, although we conclude that the industries are not homogeneous enough risk classes to enable estimating the influences of debt, the analysis has shown four things. Cross section analysis appears to have serious shortcomings because of substantial firm effects.[12/] Pooled regressions seem to be a much more efficient and appropriate way of approaching the problem of explaining capitalization rates over time and between firms. That the original model understated the influence of growth and that high payout ratios along with rapid growth penalize stock prices also seem to be substantiated. In addition, there is some evidence that dividends apart from their interaction with growth raise equity prices.

[12/] With this in mind, and with Gordon's recognition of potential firm effects in "The Investment, Financing and Valuation of the Corporation," his results should be interpreted with caution.

CHAPTER VI

Implications of the Model and of the Statistical Tests

This chapter spells out the managerial implications of the original model and contrasts these with the influences statistically estimated. It conludes that the major findings of the thesis are methodological and concern the variables which ought to be included and the types of test which seem relevant for further examination of the influence of risk, growth and taxes on share prices.

As originally formulated, the model had important managerial implications. It prescribed a dividend policy and a debt-equity mix which would maximize the value of the firm. This is best seen by considering the weighted average cost of capital implied by the original model. Let c/c be defined as this weighted average. Then $c/c = \frac{rD+\rho E}{D+E}$ with D and E being debt and equity, while r and ρ are the interest rate and the capitalization rate respectively.

The capitalization rate was hypothesized as

$$(3) \qquad \rho = (\rho_o)^{r_k} \, e^{[a(b-\frac{D}{E})^2 + c(d-\frac{DIV}{PROF})^2]} \, f^{\frac{R}{R+NI}}$$

If, in the context of raising a given amount of equity, it is desired to determine that $\frac{DIV}{PROF}$ which minimizes ρ, it is possible to rewrite this as:[1]

[1] R has been written as $(1-\frac{DIV}{PROF})$, the retention rate, times aggregate profits represented as $\Sigma\pi$.

$$\rho = K\, e^{c\left(d-\frac{DIV}{PROF}\right)^2} f^{\left(1-\frac{DIV}{PROF}\right)\frac{\Sigma\pi}{R+NI}}$$

as these are the only elements which depend on $\frac{DIV}{PROF}$. This yields a minimum $(\frac{DIV}{PROF})$ of

$$\left(\frac{DIV}{PROF}\right)^* = d + \frac{\log f}{2c}\,\frac{\Sigma\pi}{R+NI}$$

Since f is less than unity, log f is negative and $\frac{DIV}{PROF}^*$ is less than d by an amount proportional to the income tax gains available through retentions. If, however, large amounts of new issues have to be sold because profits provide only a small part of total desired equity, the old stockholders have to forego some of the gains of retentions in order to raise the equity price to dilute their share in the equity as little as possible while obtaining the new equity.

The model is also useful to determine that $\frac{D}{E}$ ratio which minimizes the cost of capital. If we assume the total of D and E to be fixed and seek to find the optimal portion of debt, it is necessary to differentiate c/c with respect to debt, imposing the constraint that dD = -dE.[2/] This yields

$$\left.\frac{d\ c/c}{dD}\right|_{dD\,=\,-dE} = r-\rho\left\{1+2a\left(1+\frac{D}{E}\right)\left(b-\frac{D}{E}\right)\right\}$$

Being quadratic in $\frac{D}{E}$, this expression has two zeros -- that is, there are two extreme points. Only one of these has any real meaning,

---

2/ A somewhat similar approach is adopted by E. Kuh in "Capital Theory and Capital Budgeting," Metroeconomica, December, 1960.

however, as the other implies that a maximum cost of capital is obtained at a negative debt-equity ratio.[3/]

If it had been possible to estimate the model in its original form, it would be useful to explore the implications for $(\frac{D}{E})^*$ of changes in a and b. However, the expression for $(\frac{D}{E})^*$ is quite complicated and having no estimates of a and b gives such an analytic exploration little use.

As estimated, however, the model does not contain all the terms in equation (3). Rather, it has become

$$\rho = K e^{(a+b_{gL}) \frac{DIV}{PROF}} e^{cg_L}$$

with the coefficients a and b significant in three of five industries, and c in four of the five.

The $\frac{DIV}{PROF}$ variable enters linearly, not quadratically, in the logarithm of ρ. This makes the $\frac{DIV}{PROF}$ which minimizes log $(\frac{DIV}{P} + g)$ be either zero or unity. It is zero if $\frac{DIV}{PROF}$ has a positive sign as

---

[3/] Setting $\left.\frac{d(c/c)}{dD}\right|_{dD = -dE}$ equal to zero results in a quadratic expression for $(\frac{D}{E})^*$. To see where this expression has zeros, it is necessary to note that when $\frac{D}{E}$ is zero, the function has a negative value. For large $\frac{D}{E}$, the function is clearly positive. With a little thought it can be seen that if a quadratic function is non-zero, has a negative value at zero, and a positive slope for large value of the variable, one of its zeros must be left of the origin. This being the case, only one of the zeros of $\frac{d(c/c)}{dD}$ implies a positive $(\frac{D}{E})^*$ and this $(\frac{D}{E})^*$ must be a minimum.

then dividends continually lower price and it is unity if $\frac{DIV}{PROF}$ has a negative sign. The magnitude of the sign is (a + bg) with a negative and b positive. As stated before, high growth makes dividends less valuable.

If the data on $\frac{R}{R+NI}$ had been more accurate and had led to a reliable estimate of log f, the difficulty still would not be resolved. For then the derivative of log $(\frac{DIV}{P} + g)$ with respect to $\frac{DIV}{PROF}$ would be a + bg -log f $\frac{\Sigma\pi}{R+NI}$. This is still a constant - not a function of $\frac{DIV}{PROF}$ - and thus still prescribes a polar dividend policy.

The policy suggestion for dividends from the present model is not too unreasonable for rapidly growing firms as it prescribes no dividends at all, but for those growing at less rapid rates, the conclusion that continual increases in $\frac{DIV}{PROF}$ would increase equity prices seems unreasonable. Dividends were thought to be valuable because they could produce more steady income than capital gains and also avoided the transaction costs involved in claiming capital gains. As the payout rate becomes larger and larger, the differential between the income tax and the capital gains tax rates should overwhelm any interest in dividends arising from these two sources. Any possible counterbalancing effect at high $\frac{DIV}{PROF}$ ratios was lost

when the $(\frac{DIV}{PROF})^2$ term was indistinquishable from the $\frac{DIV}{PROF}$ term itself. Losing this term caused the loss of any reasonable managerial application in the field of dividend decisions.[4/]

Since the influence of debt was either non-existent or was thought to arise from the heterogeneity of risk within the industry groups, no managerial implications at all are forthcoming in this area.

Although, it seems that problems with the data and with the formulation have kept the model from attaining certain of its original goals, it is useful to examine one further aspect of the final specification. These are the estimates of the elasticities of share price with respect to the growth rate and the dividend payout rate.

The final specification was

$$(\frac{DIV}{P} + g_L) = K\, e^{(a+bg_L)\frac{DIV}{PROF}}\, e^{cg_L}$$

which implies

$$\frac{DIV}{P} = K\, e^{(a+bg_L)\frac{DIV}{PROF}}\, e^{cg_L} - g_L$$

[4/] Adding the term $bg_L * \frac{DIV}{PROF}$ to the original specification would have led to an optimal $\frac{DIV}{PROF}$ of $d + \frac{\log f}{2c}\,\frac{\Sigma\pi}{R+NI} - \frac{b}{2c}\,g_L$.

Altering the growth rate has three influences on share prices, one from the $e^{bg_L \frac{DIV}{PROF}}$ term, one from the e^{cg_L} term and one from the $-g_L$ term. Thus, computing the elasticity of price with respect to growth results in the three terms shown in Table 18. This elasticity computation is useful to determine the relative size of the different influences of growth on share prices. The effect of the interaction of growth and payout rate is about one-half as large (and opposite in sign) as the other two influences of growth. Thus, the total elasticity of share price with respect to growth is substantially diminished by this interaction.

It is also possible to compute the elasticity of share price with respect to the payout ratio. These results are presented in Table 18. Again, it can be seen that although dividends by themselves raise share prices, the joint influence of the growth rate and the payout rate is considerable. For the oil industry, this interaction is sufficient to make the net effect of increasing the dividend payout be a reduction in share prices.

These elasticities should be interpreted with care for, as has been said many times, the final specification is not thought to include all the influences of the independent variables. The slope coefficients measured are the best estimates available of the effects, but there is the strong belief, for instance, that the one coefficient which resulted for the payout rate measures the composite of both the originally hypothesized effects. This being the case, the estimates of the elasticities should be treated as only gross indications of the magnitude of the influences of the dividend payout or share prices. Similar reservations apply to the growth elasticities.

TABLE 18

Elasticities

Growth Elasticities

	Total	$-g_L$	e^{-ag_L}	$e^{bg*\frac{DIV}{PROF}}$
Industrial Mach.	.504	.785	.324	-.605
Oils	1.325	1.572	.733	-.980

Dividend Payout Elasticities

	Total	$e^{-c\frac{DIV}{PROF}}$	$e^{bg*\frac{DIV}{PROF}}$
Industrial Mach.	.607	1.212	-.605
Oils	-.420	.570	-.990

Building Materials was excluded from this table as it had no significant effect for the $g*\frac{DIV}{PROF}$ term; Chemicals because there was no significant effect of growth; and Drugs because the slope coefficients for both the dummies and the growth term remained quite large even with the elimination of the most rapidly growing firms, casting doubt on the validity of these estimates.

Concerning the influence of taxes, the results are mixed. The differential between the income tax and the capital gains tax led to the hypothesis that the variable $\frac{R}{R+NI}$ should have a negative influence on the capitalization rate. The size of this coefficient was to depend on the magnitude of the difference between these two tax rates. This influence could not be substantiated. The feeling was that this lack of positive results arose from difficulties with the data and should not be taken as evidence that no such effect existed. The influence of growth per se, however, was taken as evidence of an effect on share prices of the differential between the income and gains tax. The fact that the capitalization rate fell as the growth rate increased was thought to arise from a greater interest in price appreciation than in dividend payments. Thus although taxes did not enter as originally expected, they did show some effect in the final specification.

This concludes the analysis of the results of both the cross section regressions and the pooled regressions. Few of the originally specified managerial implications can be found. Everything is not lost, however. The interaction of high growth and high dividend payouts which had been discussed elsewhere has been substantiated, along with evidence that the original model understated the total influence of growth.

The implication of the interaction between the growth rate and the dividend payout rate as well as the greater interest in price appreciation than in dividends is that there is a bias toward the retention of earnings created by the differential between the income

tax and the capital gains tax. This tax induced bias shifts the allocation of saving more toward that by corporations and may induce a different composition of investment than would occur under a different tax law. Such a statement, however, is more academic than realistic, as a change in the gains - income tax relationship would have so many influences on both saving and investing habits that to measure just this one effect would give no estimate of the direction of the net change, let alone its amount.

Also, in a quite different vein, the evidence that short-term deviations from long-term rates of growth were discounted implies that the market has good enough forecasting ability to stabilize price movements in the face of fluctuating short-term growth rates.

Rather than managerial or macroeconomic implications, what do seem to be forthcoming from this thesis are some indications of the kinds of variables with which further research into the influences of debt, dividends, taxes and growth on stock prices must be concerned. These are the emphasis on dummy variable regressions which allow for firm effects and the avoidance of cross section analysis, along with an extreme interest in growth, both alone and in connection with the $\frac{DIV}{PROF}$ variable.

Most importantly, there is the need for a much better way of dealing with risk in order to measure the influence of debt. This last requirement seems to be the most difficult of all, especially given the difficulty Gordon had with variance of earnings, and that encountered by Holland and Cootner in defining a risk variable.

Both these studies attempted to find a variable associated with each firm which measured its risk. Neither claimed much success. The alternate approach adopted here yielded no more, and worse than that, implied that until the problem was adequately treated, the influence of debt could not be measured.

Aside from this problem, at least three others need to be explored. The attempted use of the average market capitalization rate with an industry adjustment for risk proved unsuccessul. Some way should be found to avoid having to estimate deviations from industry means. Also, the dummy slope coefficients should be analysed to determine what influences they measure. Finally, the data on $\frac{R}{R+NI}$ were thought to be poorly adapted to the hypothesis. Since the tests were run, it has been suggested that a better estimate of internal versus external equity could have been obtained by simply adding up retentions in the period as the interval funds, and subtracting this total from the total increase in equity to obtain the estimate of external funds. This would avoid many of the various accounting practices applying to stock dividends, swaps, and revaluations.[5/]

[5/] The Studley-Shupert data make such an analysis possible, and it will form a part of the author's further research in this area.