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Extensions of the Matrix Form of Double Entry*

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90-64

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Introducing Account Balances into the Square Matrix Form

The matrix form of double entry has a *major deficiency*. . There is no appropriate place for account balances. At present, to include these, the original matrix form must be augmented in some fashion. This can be done to the physical form by adding additional rows and columns.[1] In the analytical form, where indexing alone is used in place of physical cells, additional variables such as beginning balance account ———— must be used. This defect can be remedied by introducing into the set of permissable notations the minus sign and by utilizing the unused cells of the matrix which constitute the diagonal.

The diagonal of the matrix form of double entry is always empty for the simple reason that the same account is generally[2] not debited and credited in any one transaction. On the other hand anything assigned to a diagonal cell means that the same account is to be both debited and credited in that amount. This is equivalent to doing nothing or adding zero to the account. In order to use the diagonal, and if it is to be used at all as a repository for account balances, some means of identifying the account balance must be devised. This can be done by the following convention: In the diagonal of a matrix form of traditional (debit-credit) double entry a minus sign indicates a credit balance and the absence of a minus sign indicates a debit balance.[3] Thus a_{kk} means a debit balance of amount a in account k and $-b_{hh}$ means a credit balance of b in account h. It has been emphasized elsewhere that accounting data is signless but that the

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[1] See for example A. Charnes, W. Cooper, and Y. Ijuri, "Breakeven Budgeting and Programming to Goals," Journal of Accounting Research (Spring 1963), pp. 35; and A. Wayne Corcoran, "Matrix Bookkeeping," The Journal of Accountancy (March 1964), pp. 60-66.

[2] The exception is very unusual,(e.g., a transfer between the subordinate accounts of a control account).

[3] Any set of indicators can be used instead of plus-minus. However the plus-minus set has the additiona useful characteristic of indicating the signed result of the account balance calculation.

debit and credit mechanism in conjunction with the accounts indicate whether a change in an account is an increase or a decrease.[1] Thus the minus sign has no significance in the traditional form of double-entry and an arbitrary definition can be assigned to it.

In addition to the minus convention it is necessary to modify some of the operations on the matrix. The diagonal cell must be omitted from every row or column summation. This can be done in standard notation as follows:

$$\sum_{\substack{i=1 \\ i \neq k}}^{n} a_{ik} \qquad \text{or} \qquad \sum_{\substack{j=1 \\ j \neq k}}^{n} a_{kj}$$

Now every matrix form of traditional double entry can show beginning balances of accounts as well as the partially aggregated transactions contained in each cell. The rule for calculating a new account balance a_{kk} is:

$$a'_{kk} = a_{kk} + \sum_{\substack{i=1 \\ i \neq k}}^{n} a_{ik} - \sum_{\substack{j=1 \\ j \neq k}}^{n} a_{kj} \tag{1}$$

The first term is the old account balance, the second the debits to the account, and the third the credits to the account. The new balance can be either positive or negative. This is completely controlled by the sign and magnitude of a_{kk} and the magnitude of the other two terms to the right of the equality.

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[1]See James J. Linn, "An Analysis of Double Entry," (Working Paper 87-64, Alfred P. Sloan School of Management, Massachusetts Institute of Technology, 1964), p. 10.

Transforming the Square Matrix Form into a Traingular Matrix Form

An interesting characteristic of the matrix form of traditional double entry is that the only difference between any cell except those in the diagonal and its mirror image across that diagonal is that the indices are reversed. This characteristic and an extension of the minus convention can be employed to reduce a square matrix to a traingular matrix.

The minus sign will have the following meaning: Any figure with a minus indicates an index (debit-credit) interchange. Thus

dr	-10	means	cr	10
cr	-10	means	dr	10

Letting a_{ij} stand for debit account i and credit account j in the amount of a, this can also be stated as $-a_{ij} \rightarrow a_{ji}$. This extension and the original minus convention together make up the extended minus convention. The original convention[1] permits the introduction of account balances into either a square or traingular matrix form of traditional double entry, and the extension of this convention enables the square matrix form to be transformed into a triangular matrix form.

The rule for this, based upon the convention extension is:

$$a_{ij} = -a_{ji} \quad \text{for all} \begin{cases} i = 1,\ldots, j-1 \\ j = 1,\ldots, i-1 \end{cases}$$

Once this transformation has occurred all transactions normally placed in that part of a square matrix below the diagonal will be placed in that part above.[2] The original and new transactions are distinguished by the new being preceded with a minus sign. This is based upon the presumption that the transactions remained separated when assigned to a cell. If they are accumulated, the integrity of the cells has been violated. The total of each cell of the triangular matrix, in such a case,

[1]See p. 1.

[2]On the other hand, the portion above the diagonal could be mapped onto the portion below. However, a triangular matrix is conventionally that portion of a square matrix above the diagonal.

is the net of the debits to account i and credits to account j less the debits to account j and the credits to account i. It may be possible to accumulate, for each cell, the positive and negative amounts separately. For some sets of implementing operations this is tatamount to using a square matrix.

A square matrix which contains account balances can be transformed into a triangular matrix because the diagonal is unaffected. The extended minus convention is applicable since both parts of the convention occur after the transformation. Assuming the transactions were at most accumulated by sign within each cell, account balances and full detail can be calculated.

A new account balance (a'_{kk}) is calculated as follows:

$$a'_{kk} = a_{kk} + \sum_{i=1}^{g} a_{ik} - \sum_{j=h}^{m} a_{kj} \qquad i > k > j \tag{2}$$

$$i, j = 1, 2, \ldots, k-1, k, k+1, \ldots, n$$

$$\sum_{i=1}^{g} a_{ik} = a_{1k} - a_{1k} + a_{2k} - a_{2k} + \ldots + a_{k-1,k} - a_{k-1,k} \tag{3}$$

$$\sum_{j=h}^{m} a_{kj} = -a_{k,k+1} + a_{k,k+1} - a_{k,k+2} + a_{k,k+2} - \ldots - a_{kn} + a_{kn} \tag{4}$$

The even terms (2nd, 4th, etc) of (3) and (4) are debits and the odd terms (1st, 3rd, etc.) are credits.

This is illustrated in Figure I. The $-a_{ji}$ would be a_{ij} in a square matrix.

Substituting (3) and (4) into equation (2),

$$a'_{kk} = a_{kk} + [a_{1k} - a_{1k} + \ldots + a_{k-1,k} - a_{k-1,k}] - [-a_{k,k+1} + a_{k,k+1} - \ldots - a_{kn} + a_{kn}]$$

$$= a_{kk} + a_{1k} - a_{1k} + \ldots + a_{k-1,k} - a_{k-1,k} + a_{k,k+1} - a_{k,k+1} + \ldots + a_{kn} - a_{kn} \tag{5}$$

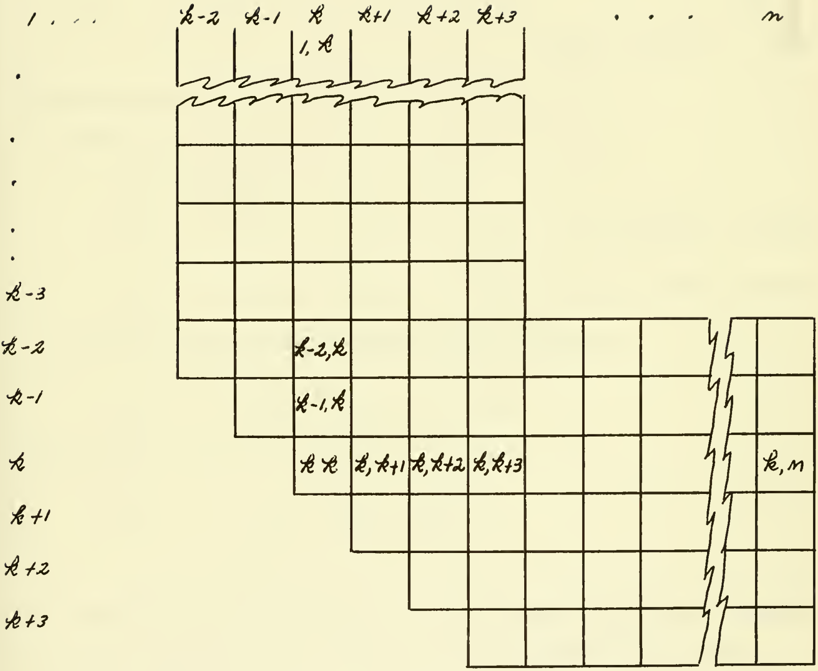

Figure I

then by rearranging terms this equation becomes

$$= a_{kk} + a_{1k} + \dots + a_{k-1,k} + a_{k,k+1} + \dots + a_{kn} - a_{1k} - \dots - a_{k,k-1} - a_{k,k+1} - \dots - a_{kn} \quad (6)$$

Now the equation can be divided into three sections.

These are:

a_{kk} the beginning balance (debit)

$a_{1k} + \dots + a_{k-1,k} + a_{k,k+1} + \dots + a_{kn}$ all the debits to account k

$-a_{1k} - \dots - a_{k-1,k} - a_{k,k+1} - \dots - a_{kn}$ all the credits to account k

In square matrix notation the debits would be written as

$$a_{1k} + \dots + a_{k-1,k} + a_{k+1,k} + \dots + a_{nk}$$

and the credits would be written as

$$-a_{k1} - \dots - a_{k,k-1} - a_{k,k+1} - \dots - a_{kn}$$

with $a_{k} = 0$ omitted.

Term by term these are identical. In the triangular matrix it was necessary to reverse some of the indices. Those that were reversed turn out to be the last half of the debits and the first half of the credits. This, notation, is the only difference between the two sets of data.

Equation (1) expanded and rearranged represents the new (debit) balance of an asset account. For an equity account the notation and signs would be identical except for the beginning balance a_{kk}; this would be $-a_{kk}$ (a credit). Thus the new balance would be $-a_{kk}$ if it were also a credit balance.

Summary

The diagonal of a square matrix form of double entry can be used to record account balances by the introduction of a sign convention to distinguish between debit and credit balances. The plus-minus sign convention is suitable and has the additional advantage of being consonant with the signs of the operations of the process by which account balances are calculated.

By extending the plus-minus convention the square matrix form of double entry can be transformed into a triangular matrix form. While interesting this transformation appears to have limited usefulness since it is necessary, in every cell but those in the diagonal, to distinguish between those elements which were present before the transformation and those which are present after the transformation.

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