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ESTIMATING THE STRATEGIC VALUE OF LONG-TERM
FORWARD PURCHASE CONTRACTS USING AUCTION MODELS

John E. Parsons

WP #1902-87

Revised June 1987

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Estimating the Strategic Value of Long-Term
Forward Purchase Contracts Using Auction Models

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ABSTRACT

Over the last decade much attention has been focused upon strategic factors influencing corporate financing decisions, especially those relating to informational problems. The results of this research have primarily been suggestive--proposing possible explanations of phenomena, but not providing specific methods for incorporating the strategic factors into quantitative valuation techniques and models. Quantitative models of financial variables have primarily been developed for cases of perfect competition or similar special cases in which the strategic factors are not central. In this paper we provide a model in which the estimation of the value of strategic factors is the objective and for a relationship between prices across time which is at the center of finance theory.

This paper develops the use of an auction model to value long-term forward contracts for the purchase of commodities. Recent theoretical and empirical research has emphasized the danger of ex-post opportunistic bargaining as a primary motivation for the use of long-term forward contracts in preference to a dependence upon spot markets. However, this literature has not developed an operational procedure for assessing the value to the firm of using a forward contract to eliminate this ex-post bargaining problem. Traditional arbitrage methods for valuing forward contracts sold on the organized exchanges ignore the bargaining problem, that is, they assume a competitive market in which the strategic factors creating the bargaining problem and motivating the use of long-term contracts are not present. We demonstrate how auction models can be used to assign a value to the strategic advantage of long-term contracts. This value is shown to depend primarily upon the number of potential buyers in the relevant market and the relation between the lower end of the range of reservation prices of these buyers and the fixed costs of installing the capacity to supply the commodity. Problems with assigning a value to the strategic advantage of long-term contracts are discussed and other important strategic features of long-term contracts which need to be valued are identified.

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1. Introduction

Over the last decade much attention has been focused upon strategic factors influencing corporate financing decisions, especially those relating to informational problems. The results of this research have primarily been suggestive--proposing possible explanations of phenomena, but not providing specific methods for incorporating the strategic factors into quantitative valuation techniques and models. Quantitative models of financial variables have primarily been developed for cases of perfect competition or similar special cases in which the strategic factors are not central. In this paper we provide a model in which the estimation of the value of strategic factors is the objective and for a relationship between prices across time which is at the center of finance theory.

In this paper we develop a model to estimate the portion of a project's value which is secured to a firm through the use of long-term forward contracts for the product of the project's operation. Long-term forward contracts are a typical element of financing for industrial projects in which large amounts of capital must be invested up front to develop production capacity and in which the market for the firm's output consists of a small number of buyers. Firms in such an industry often make the successful negotiation of forward purchase contracts a contingency upon which their decision to install capacity depends. By doing so they incur two advantages relative to firms which foresake the use of forward contracts and which choose instead to first install a given level of capacity and to then seek buyers for their products: i) they gain the information on demand that is revealed in a market price, and ii) they are likely to negotiate a higher sale price for their products since they can avoid the classic 'ex post bargaining problem'. Long-term purchase contracts may, therefore, improve the efficiency of capital investment decisions both directly through the information they yield and indirectly because they permit a firm to appropriate the marginal value of its

investment decision.

The second factor just mentioned, the strategic importance of contractual relations in such environments has been stressed in recent theoretical literature in the field of industrial organization, most notably in the work of Williamson (1975) and of Klein, Crawford, and Alchian (1978), and in empirical work on the US coal industry and electrical utilities by Joskow (1984).¹ The first factor mentioned above, the informational value of these contracts, is not typically discussed in the industrial organization literature. The informational content of forward prices has been emphasized in the literature on competitive equilibrium and rational expectations equilibria, where by assumption all parties are able to utilize the information embodied in the price by mere observation of the price. In Grossman and Stiglitz (1980) the paradox implied by this costless availability of information is discussed. We analyze the case in which the information is available only to those agents actually engaging in forward contracting and in which the market is non-competitive.

Our contribution to this literature is to make operational the

1. In both Williamson (1975) and in Klein, Crawford, and Alchian (1978) the primary comparison is between long-term contracts on the one hand, and vertical integration on the other. Long-term contracts are viewed in these papers as inherently unstable or incompletely enforceable so that they suffer from the ex-post bargaining problem. Vertical integration is the alternative which permits the firm to appropriate the rents or "quasi-rents" generated by its capacity decisions. In this paper we have changed the labels for the relevant comparison. We represent an absence of contracting, i.e. waiting to use the forthcoming spot market, as the initial condition, and we analyze long-term contracts as well specified and enforceable alternatives which permit the firm to appropriate the "quasi-rents".

theoretical insights proposed and popularized in this literature as 'explanations' for these long term contracts. This paper presents a model with which to derive numerical estimates of the strategic value of forward contracts. The model integrates the value of avoiding 'opportunistic' or ex-post bargaining problems with the informational value of long term contracts. By developing a device for estimating the significance of these strategic considerations for any given project, we provide a tool for making specific recommendations as to whether or not the long term contracts should or should not be used to help finance a particular capital project.

The essence of the problem for the firm is that the strategic situation in which it negotiates with its customers is substantially changed by its decision to install costly capacity. The problem is therefore appropriately analyzed as a market game played by the manufacturer and its potential customers or buyers and in which the payoff structure is altered by the decision to install capacity. In this paper we model the market game as the design of an auction by the firm and as the bidding game induced among the firm's potential customers by the choice of the game design. Played prior to the installation of the capacity, i.e., using forward purchase contracts, the game has one expected outcome for the manufacturer. Played consequent to the installation of the capacity, i.e., selling the output on the spot market, the game has a second expected outcome. We define the difference between the expected profits earned by the manufacturer in the former game and in the latter game to be the portion of the value of the project which the forward contracts secure to the manufacturer.

One alternative method for analyzing forward contracts has been pioneered by Black (1976) and by Cox, Ingersoll and Ross (1981), and has been applied by Brennan and Schwartz (1985) to the valuation of long-term contracts in the example of output from a mine. In these papers a stochastic process for the

spot price of the commodity is postulated and the pattern of forward prices which would be consistent with this process is derived. The spot and forward prices are each determined in competitive markets, and the relationship between the spot and forward prices is determined by arbitrage methods. The producer or seller of a commodity cannot influence the information which it obtains from the market for that commodity by its decision to enter the forward market, nor does it influence the expected future spot price by its decision to install capacity. In this arbitrage framework the value to a seller of the forward contract relative to a decision to sell at the anticipated spot price is, by construction, zero: the equilibrium forward price is defined as that price which makes this value zero. Alternatively, for long-term contracts in which the delivery price is fixed and does not correspond to the equilibrium forward price, then the value of the contract is the sum of the difference between this contracted price and the anticipated spot price for each contingent event multiplied by the quantity contracted for delivery and the probability of the contingency.

The critical distinction between the model used in this paper and this arbitrage method is that the model used here involves producers and buyers in non-competitive markets. The producer's capacity decision and the timing for contract negotiations affect the price received. The value to the producer of forward contracts relative to spot transactions may therefore deviate from zero: the problem is to estimate how significant this deviation may be, to determine if these strategic factors are significant. To accomplish this we use the auction analog of the market negotiations and thereby explicitly recognize the different impact which each players' decisions will have on the outcome of the negotiations forward and spot.

These differences between the arbitrage method for valuing forward contracts and auction model reflect the differences between the markets in

which each is relevant. The classical method is most appropriate for commodities sold on the traditional organized futures and spot markets--markets characterized by a high degree of competition and liquidity--and for which forward contracts of less than one year in duration are common. Forward contracts in gold and perhaps petroleum would, for example, fit into this category. The method developed in this paper would be most appropriate for commodities not sold on such liquid markets, for commodities not traded in perfectly competitive markets, i.e., typically for commodities for which forward contracts of between three, twenty and more years are typical. Forward contracts for the purchase of coal and natural gas would be prime examples of this class.

In the next section we present a simple example of the forward and the spot bargaining problems. We will use this example to make clear how fixed costs that must be incurred up-front affect the outcome of the negotiations. In section 3 we briefly present the general concept behind our formalization of the value to a forward contract. In section 4 we present the details of a simple competitive auction and we show how it may be used to operationalize our definition of the value of the forward contract. In section 5 we use this model to assess the value of typical take-or-pay contracts for a natural gas field currently under development, and we show how the model can help to assess when changes in traditional patterns of contracting in an industry might be prudently altered. Section 6 presents some concluding comments.

2. The Value of Forward Contracting: an Example

Imagine a firm or a natural resource owner with a maximal production capacity of one unit. Suppose that the commodity to be produced or the resource to be sold has no personal value to the owner so that the owner's only profit opportunity is the production of the commodity or resource for

sale. The firm must make a decision at an early point in time, $t=1$, to incur an up front capital cost, K , which is directly proportional to the amount of output capacity, Q , which it intends to install, $K=kQ$, $Q \leq 1$. The firm produces the good for sale shortly thereafter, at time $t=2$. The firm then incurs operating costs, V , which are a constant proportion of the quantity, q , actually produced, $V=vq$, $q \leq Q$. For this example we will consider the case in which there is only one potential buyer for this firm's product. The single buyer has a reservation value r for a single unit of the commodity or fraction thereof. We assume that the firm does not know the buyer's reservation value and we represent this uncertainty on the part of the firm as a subjective probability distribution, $H(r)$, over the possible reservations values, $r \in [v, 1+v]$: for simplicity, we assume that $H(\cdot)$ is the uniform distribution.

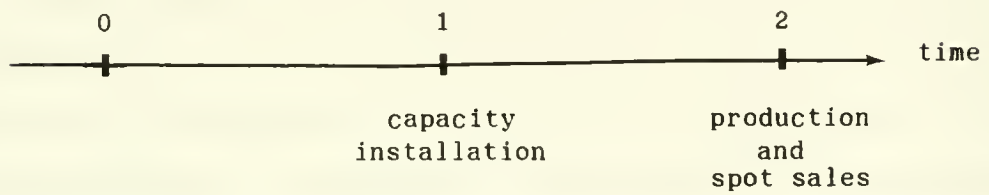
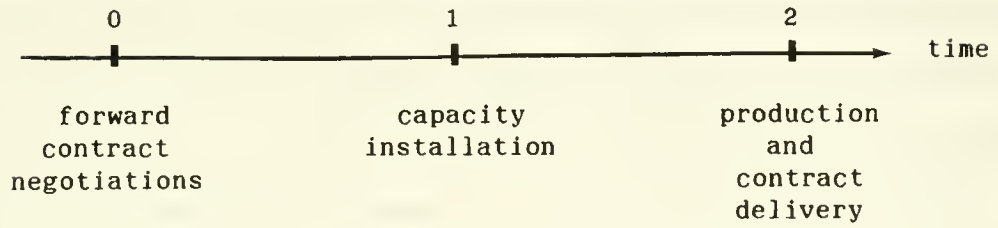
The firm has two choices for negotiating the sale of its product: i) it can choose at time $t=0$ to negotiate with the potential buyer and can then make its capacity and production decision contingent upon the outcome of the negotiations, or ii) it can wait until time $t=2$ to negotiate the sale of its output on the spot market, but to do this it will have had to already install some level of capacity at time $t=1$ prior to the realization of a price on the spot market. The two possible sequences of production and negotiation decisions are illustrated in Figure 1.

[Insert Figure 1]

There are two advantages to negotiating a forward contract instead of first installing capacity and then selling the product on the spot market. The first advantage is the opportunity to make use of the informational content of the forward price structure which in these industries is only observable through the process of negotiating contracts. If the firm makes its capacity installation decision prior to negotiation of a price on the spot market it will install the capacity whenever the average expected profit from

Figure 1

Timing of Decisions under Forward Contracting
and under Production for Spot Sales.



the project is positive. However, it will likely discover that in some cases the actual demand is low enough so that the price received does not cover the costs of production. If the firm bases its decision to install capacity upon the outcome of negotiations over forward contracts, then it can make its installation decision contingent upon receiving a price which is greater than its costs of installation and operation. The process of negotiation with potential customers over price therefore yields to the firm valuable information about the level of demand, and the firm uses this information to improve the optimality of its production decision.

Distinct from the informational advantages of forward contracting are the bargaining advantages which it yields to the firm. In bargaining with a potential buyer over price the firm must make a conjecture about the range of price offers it can expect from the buyer, and it must decide which offers it will accept and which it will reject. A firm which first installs capacity and which then considers an offer from a buyer to purchase the output at a given price must weigh the potentially higher price it can obtain by rejecting this offer with the possibility that it will forego the difference between this offered price and the marginal operating costs. A firm which negotiates over the same price offer before it installs capacity weighs the potential for obtaining a higher price with the difference between the offered price and the marginal costs of both capacity installation and operation. Since the marginal profit is smaller when the marginal cost of capacity is included, since the firm which is forward contracting can avoid this cost in the case that the buyer refuses the higher offer price, it is in a stronger bargaining position than the firm which negotiates after the installation of capacity. A firm which makes the decision to install the capacity and to incur the fixed costs prior to the completion of price negotiations with its potential buyers will have undermined its own bargaining power and it will therefore anticipate

negotiating an average price for its output that is lower than it would receive if it were to negotiate forward purchase agreements prior to or simultaneously with its decision to install the capacity.

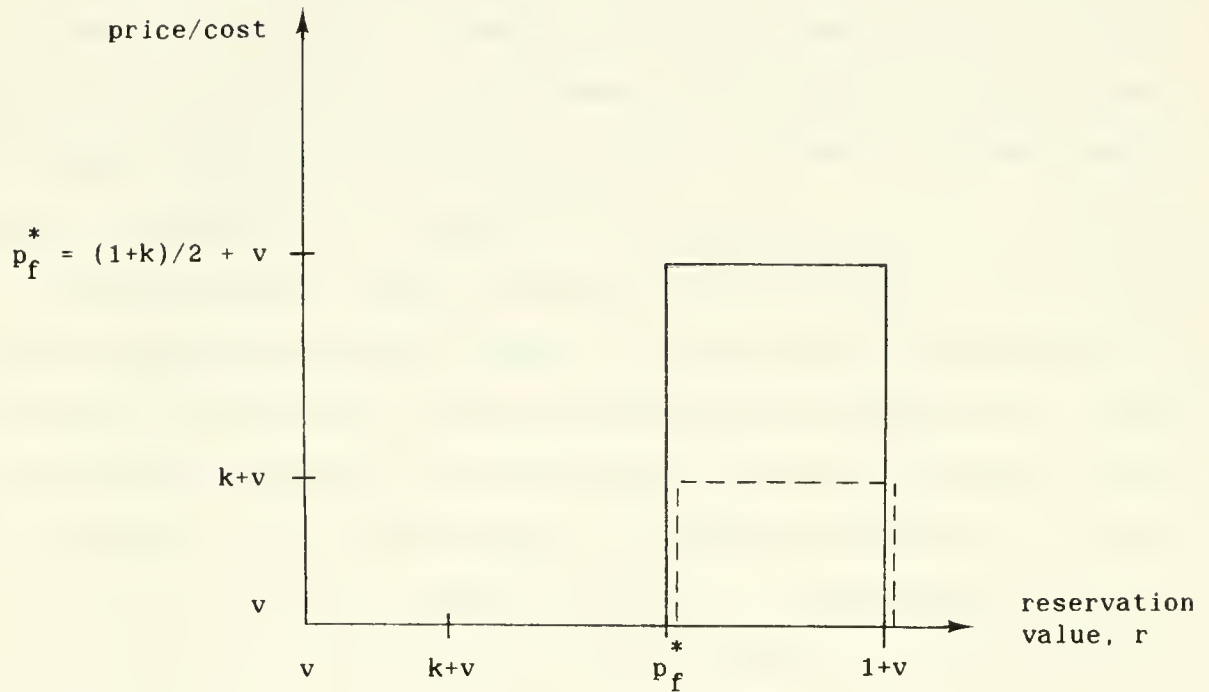
These two effects and their consequences for our example are illustrated in the following series of diagrams: the exact solution to the firm's problem for both forward and spot contracting are given in Appendix 1. In this example we assume that the seller makes a take-it-or-leave-it price offer and the buyer chooses to accept or reject the price and the quantity it wishes to purchase at that price. In Figure 2 the seller's forward bargaining situation is illustrated. The firm chooses a price, p_f , a point along the vertical axis. Buyers with valuations $r > p_f$ will agree to purchase at that price, and because of our particular assumptions on the buyer's preferences they will choose to purchase a full unit. The firm therefore receives the expected revenue net of variable cost which may be represented as the volume of the rectangle bounded by solid lines: above by the price p_f , to the left by the valuation $r = p_f$, to the right by the maximal valuation, and below by the horizontal axis normalized to the variable costs. The firm incurs the expected capital costs that are represented by the dashed rectangle: bounded above by the sum of the marginal fixed and variable cost, on the left by the valuation $r = p_f$, and on the right by the maximal valuation, $r = 1 + v$. The firm's expected profits may be represented as the difference between the two rectangles--the rectangle bounded above by p_f and below by $k + v$: $\pi = [(p_f - k - v)(1 + v - p_f)]$. The optimal price offer is determined to maximize this difference and is therefore $p_f^* = (1 + k)/2 + v$, and the profits at this optimal offer price would be $\pi_f = [(1 - k)/2]^2$.

[Insert Figure 2]

In Figure 3 the firm's negotiating problem at the spot market is described under the assumption that the firm has installed one unit of

Figure 2

Expected Revenue and Profit from Forward Contracting



The producer chooses a price, p_f , marked along the vertical axis. The expected quantity of sales at this price is represented along the horizontal axis as the probability that the buyer has a reservation value greater than the given price. The firm incurs capacity cost, k , represented on the vertical axis, but only for the expected quantity of sales. The large rectangle with solid boundary represents the expected revenue net of operating costs. The smaller rectangle with dashed boundary represents expected costs. The optimal forward price, p_f^* , is chosen to maximize the difference between the two rectangles.

capacity. For a given offer price, p_s , the determination of revenue net of variable cost received by the firm is the same as was described above for Figure 2. At the time of negotiation on the spot market the fixed costs are sunk and therefore the expected fixed costs are independent of the realized reservation value of the buyer: these are represented as the dashed rectangle. Therefore the optimal price offer is determined to maximize the revenue net of variable cost, ignoring the fixed costs. The optimal price offer, $p_s^* = 1/2 + v < (1+k)/2 + v = p_f^*$, is therefore lower than in the forward negotiations and the expected profits from selling on the spot market with this optimal offer price are $\pi_s = 1/4 - k < [(1-k)/2]^2 = \pi_f$.

[Insert Figure 3]

The profit from forward contracting is clearly greater than the profit from spot selling and this difference is illustrated in Figure 4. The area of the rectangle marked by the letter I represents the additional costs associated with the loss of the informational value of the forward contracts. The area represented by the net gain composed of the three rectangles labelled B, $B_1 - B_2 + B_3$, represent the loss of the firm's profit due to the ex-post bargaining disadvantage. The value of forward contracting in our example is then $\Gamma = \pi_f - \pi_s = (k/2)(1+[k/2]) = I + B$.

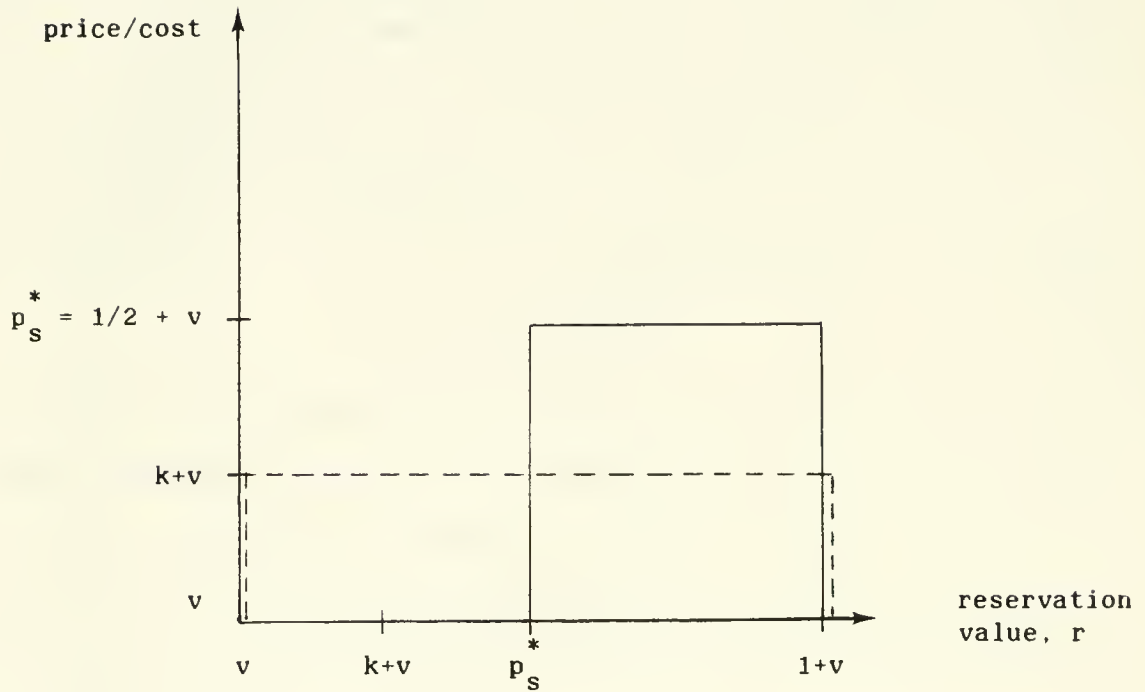
[Insert Figure 4]

In some cases, without the successful conclusion of forward contracts the firm would anticipate bargaining a spot price so low on average as to make the initial installation of the capacity a negative net present value decision and as a result the project would not proceed without the successful completion of the forward contracts. In our example this would arise whenever $k > 1/4$.

To make this exposition concrete, we will put some numbers on our example. Suppose that our producer is a natural gas field developer and is considering the development of a field with a maximal annual capacity of 400

Figure 3

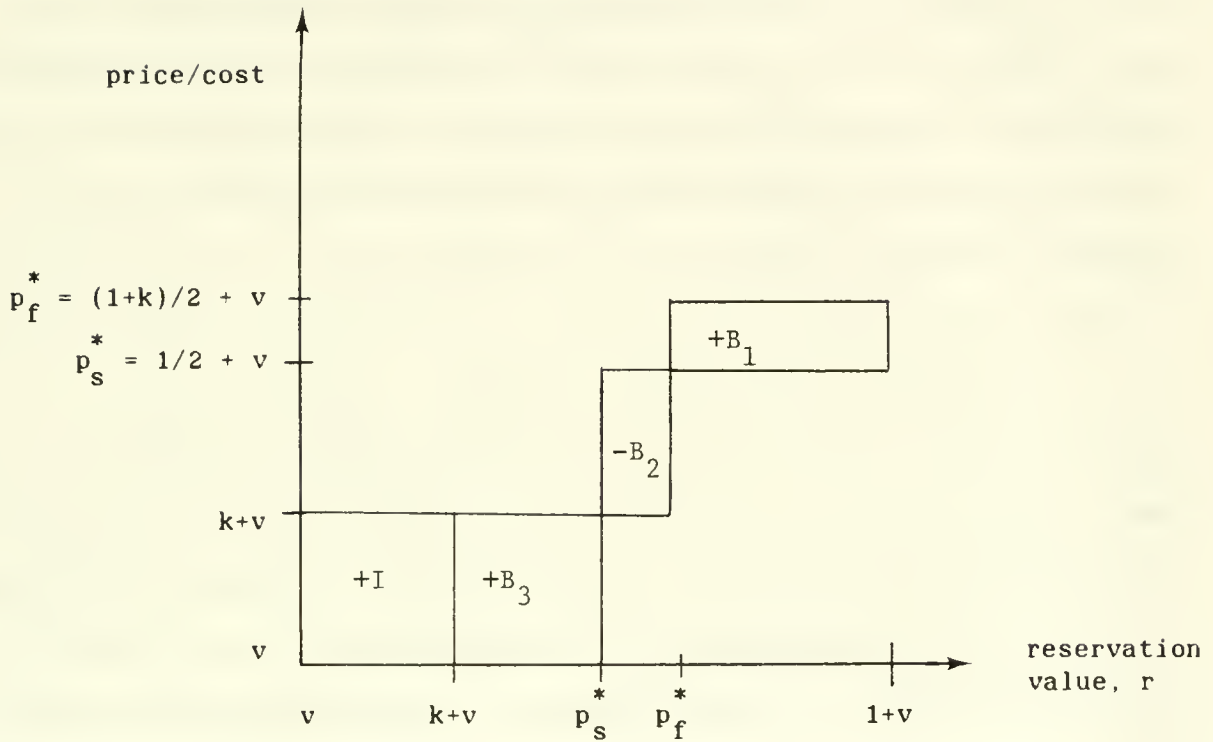
Expected Revenue and Profit from Spot Sales.



The producer chooses a price, p_s , marked along the vertical axis. The expected quantity of sales at this price is represented along the horizontal axis as the probability that the buyer has a reservation value greater than the given price. The rectangle bounded by these two lines represents the expected revenue net of operating costs. The firm incurs capacity cost, k , represented on the vertical axis. Since the capacity must be installed prior to spot sales, the cost for capacity is incurred independently of whether an eventual sale is made and independently of the realized reservation value of the buyer. Expected capacity costs are therefore the large rectangle bounded by the dashed line. Expected profits from spot sales is the difference between these two rectangles.

Figure 4

Comparison of Profits from Forward and Spot Sales



The difference between the profits from forward sales and from spot sales is the sum of the rectangles with the signs as marked. By lowering the price demanded the sales spot lose profits on the quantity of sales expected to be sold forward as indicated in rectangle B_1 . Due to the lower price, however, sales spot are expected to be greater and this increases the unit operating profits which is seen in rectangle B_2 ; this increase in sales is only possible however because the firm has already installed capacity and the costs for this increased capacity is seen in rectangles B_3 and I . Cases in which the reservation value of the buyer lies below $k+v$ are cases in which it is pareto inefficient to install capacity.

Bcf (billion cubic feet) for a total of twenty years. The present value of the capital expenditures necessary to make possible this maximal capacity would be \$3.29 billion. Assuming a constant rate of extraction and a 12% discount rate these capital expenditures amount to \$1.10 per Mcf (thousand cubic feet). The variable or operating costs over the twenty years would amount to \$2.99 billion in present value, or \$1.00/Mcf. We will assume that the range of possible buyer reservation values is from \$1.00/Mcf to \$5.58/Mcf.

These figures would be normalized to conform to the parameters of our example by dividing by 0.2182: $k=0.24$, $v=0.2182$, $r \in [0.2182, 1.2182]$. If the developer negotiated long term purchase contracts with buyers, then it would offer a forward price of $p_f^* = 0.8382$ or \$3.84/Mcf. The offer would be accepted 38% of the time. Total expected profits, recognizing the probability that in some cases the prospective buyer will refuse the offered price of \$3.84/Mcf, are $\pi_f = 0.1444$ or \$0.66 billion. If the developer were, however, to first make the capital expenditures necessary to develop the field, it would enter negotiations for spot or short term delivery contracts with an offer price of $p_s^* = 0.7182$ or \$3.29/Mcf, 14% lower than the offered forward price. This would have yielded a greater probability of successfully selling the available resource, 50%, but a lower expected total profit of $\pi_s = 0.01$ or \$0.05 billion, a decrease of 93% relative to the profits from the forward contracting.

3. The Value to a Forward Purchase Commitment: the General Principle

We formalize this interpretation for a more general case and define precisely a particular measure of the portion of the project value which is secured to the producer by means of a forward contract. The environment is described by: i) the number of buyers in the market, N ; ii) the measurable range of possible Von Neumann-Morgenstern expected utility functions for the buyers, $(u_1, \dots, u_N) \in U^N$, where $u_i: [DxP] \rightarrow R$; iii) a probability measure over the

range of utility functions, $H(u_1, \dots, u_n)$; iv) the producer's capital and operating cost functions chosen from a range of admissible functions, $\{k(Q), v(Q, q)\} \in KxV$; and, v) a set of market rules which map a given set of buyers and utility functions along with the cost functions from the admissible set into an allocation of the commodity to each buyer and a payment from each buyer to the producer, $M: U^N \times [KxV] \rightarrow R^{2N}$.

If the producer were to negotiate a price and quantity of its output prior to incurring the fixed costs then the outcome of the market would be described as $M([u_1, \dots, u_N], k, v) = [(q, p)_{1f}, \dots, (q, p)_{Nf}]$. Then the producer's profits would be written as

$$(1) \quad E_f(\pi_M) = \int_U \left\{ \left(\sum_{i=1}^N p_i \right) - \left(k \left(\sum_{i=1}^N q_i \right) + v \left(\sum_{i=1}^N q_i, \sum_{i=1}^N q_i \right) \right) \right\} dH(u_1, \dots, u_n)$$

Alternatively, if the producer were to negotiate a price and quantity of its output after it had already incurred its fixed costs for a capacity of \hat{Q} , $k(\hat{Q})$, then the outcome of the market would be described as

$$M([u_1, \dots, u_N], k_s, v) = [(q, p)_{1s}, \dots, (q, p)_{Ns}], \text{ where } k_s = 0 \text{ for } Q \leq \hat{Q} \text{ and } k_s = \infty \text{ for } Q > \hat{Q}.$$

In a more general formulation the cost function for installing capital at the late date would not be vertical, and would depend upon the quantity of initial capacity, \hat{Q} . In our current structure the producer's profits for the case of negotiations on price and quantity subsequent to the installation of capacity may be written as

$$(2) \quad E_s(\pi_M) = \max_Q \int_U \left\{ \left(\sum_{i=1}^N p_i \right) - k_s \left(\sum_{i=1}^N q_i \right) - v \left(\sum_{i=1}^N q_i, \sum_{i=1}^N q_i \right) \right\} dH(u_1, \dots, u_n) - k(Q)$$

The difference between the two expressions,

$$(3) \quad \Gamma \equiv E_f(\pi_M) - E_s(\pi_M),$$

is the amount of profit or the portion of the value of a project which the producer secures to itself by virtue of its decision to contract forward for

the sale of its output prior to or simultaneous with its decision to incur the fixed costs.

The value of Γ will depend upon each of the parameters of the environment stated above, most importantly it will depend upon the number of potential buyers in the given market and upon the magnitude of the up-front capital costs relative to the minimum price that the seller can expect to receive in spot sales.

The value of Γ will also depend upon the specification of the market rules, M . In the following section we examine the results for contracting forward and for selling spot using market rules which implement a traditional common price auction.

4. An Auction Model for Forward and Spot Sales

4.1 The Market Rules: a Modified-Vickrey Auction

There exist a multitude of possible market rules which one could use to derive an estimate of Γ . In this paper we propose an auction model as our choice for a set of market rules determining the calculation of Γ . We believe that an auction model is a well defined analog for the negotiation process. The auction model we use has certain attractive properties relative to the class of possible market rules: 1) it is ex post efficient--that is the agents who value the product most receive it; 2) it is time consistent in the sense that the seller does not close the negotiations and leave capacity unexhausted when there exist buyers willing to purchase the product at a price greater than the seller's marginal cost of production; and 3) among the class of selling mechanisms which satisfy the time consistency property it is the method which maximizes the seller's expected revenue. The auction model is also advantageous for our purposes since the bidding rules and equilibrium strategies which yield the reduced form results of the auction have been

derived with explicit attention to the strategic relationship among the buyers and between the buyers and the seller: this is important since the critical factor which distinguishes our analysis of forward contracts from the arbitrage method is that the markets in which the commodities are to be sold are imperfectly competitive and the seller's decision to install capacity endogenously affects the future equilibrium spot price through its effect on the strategic relations in the future negotiations.

The auction model we use is an adaptation of the modified Vickrey auction--itself a variation on the more familiar English or competitive/uniform-price auction. Since the price in this auction will be the same for all buyers and will be set close to the reservation value of the marginal buyer this auction is intuitively analogous to a competitive market. It also corresponds to our notion of a competitive market in the sense that as the number of buyers in the auction grows, the results of the auction approach the results of a perfectly competitive market. The modified Vickrey auction has been formally defined and analyzed in Harris and Raviv (1981).

To discuss this model we first explain the structure of the environment for which it is applicable. We assume that the Von Neumann-Morgenstern expected utility functions for the N buyers take the simple form in which each buyer desires up to a maximum of one unit of the commodity at any price below a given reservation price, r_j , so that $u_j(q,p) = r_j \min\{q,1\} - pq$. The reservation price for each buyer is viewed by the seller as a random variable that may take on any of a finite set of reservation values, $\{R_1, \dots, R_\ell\}$ where $R_{j+1} = R_j + \delta$, for $j=1, \dots, \ell-1$. The probability distribution over this set of reservation values may be defined arbitrarily, but we will use for our examples the uniform distribution, $H(R_j) = 1/\ell$. The producer is characterized by the scalar parameter of its constant marginal cost function, c .

In a Vickrey auction with a capacity of Q units and a constant marginal

cost of production there are two key rules determining the allocation of the output and payments that result from a given realization of the N buyer reservation values: 1) the Q buyers with the highest bids above the marginal cost of production receive the commodity--when fewer than Q bids are above the marginal cost of production, only those buyers bidding above marginal cost receive a unit and some capacity is left unused, and 2) they pay a price equal to the highest bid among those bids not accepted or equal to the marginal cost of production, whichever is greater. Buyers will make their bids based upon their realized reservation values and their assumptions about the probable bids of other buyers. In the symmetric Nash equilibrium to the Vickrey auction the order of the bids will be identical with the order of the buyer valuations and the equilibrium bidding strategy for each buyer is to submit a bid equal to the buyer's valuation so that the equilibrium outcome of the two rules just stated is characterized as follows: 1) the Q buyers with the highest valuations receive the commodity--when fewer than Q buyers have reservation values above the marginal cost of production, then only those buyers with reservation values above the marginal cost of production receive a unit, and 2) each buyer pays a price equal to the highest valuation among those buyers not receiving the commodity or equal to the marginal cost of production, whichever is greater. The price for the output is then determined by competition--when there is excess demand then the price clears the market

This characterization of the equilibrium outcome as a function of the realization of the N random reservation values can be formally defined as follows. Given a specific realization of the N -buyer reservation values, (r_1, \dots, r_N) , define (S_1, \dots, S_N) as the order statistics for the reservation values, so that $S_1 \geq S_2 \geq \dots \geq S_N$, $S_j \in (r_1, \dots, r_N)$. Then the price at which any units of the commodity are sold is

$$(4) \quad p = \begin{cases} S_{Q+1} & S_{Q+1} > c \\ c & S_{Q+1} \leq c \end{cases}$$

and the quantity allocated to each buyer is

$$(5) \quad q_i = \begin{cases} 1 & r_i > \max\{S_{Q+1}, c\}; \\ \gamma & r_i = S_Q = S_{Q+1} \geq c \\ 0 & r_i < \max\{S_Q, c\} \end{cases}$$

where $\gamma = [Q - |\{S_j | S_j > S_Q\}|] / |\{S_j | S_j = S_Q\}|$, i.e. the fractional share obtained by dividing the remaining capacity among the buyers with reservation values equal to S_Q .

In environments similar to ours, when the range of possible reservation values is continuous, and subject to the time consistency condition mentioned above this simple Vickrey auction is optimal for the seller--that is, among the class of feasible, incentive compatible, and time consistent mechanisms it maximizes revenue. However, for an environment such as ours with a discrete range of reservation values we must alter slightly the price rule in order to guarantee revenue maximization--the modification and the reason for it is discussed in Harris and Raviv (1981):

$$(6) \quad p_m = \begin{cases} S_{Q+1} & S_Q > R_h & \& S_{Q+1} = S_Q \\ S_{Q+1} + \delta A(Q+1) & S_Q > R_h & \& S_{Q+1} < S_Q \\ R_h & S_Q \leq R_h \end{cases}$$

for a set of values $(A_1, \dots, A_{\ell-1})$ as defined in Appendix 2, $A(j) = A_j$ when $S_j = R_j$, and where $R_h = \min\{R_j | R_j \geq c\}$. This modified price function differs from the simple Vickrey auction price function defined in (4) in that whenever exactly Q buyers have valuations at least as high as a given price, S_Q , and the valuation of the buyer with the $Q+1$ st highest valuation is below this level,

$S_{Q+1} < S_Q$, the price does not drop all the way to S_{Q+1} , but is set at S_{Q+1} plus a fraction of the increment between S_Q and S_{Q+1} , that fraction being equal to $\delta A(Q+1)$.

The auction rules specified above in equations (5) and (6) differ from the modified Vickrey Auction defined in Harris and Raviv (1981) only in terms of the minimum bid, R_h . The optimal auction for a monopolist typically includes a minimum bid above the cost of production and this is true in the modified Vickrey Auction defined in Harris and Raviv. A minimum bid in an auction with multiple bidders is strategically analogous to a take-it-or-leave-it offer made by a seller bargaining with a single buyer. The imposition of a minimum bid implies that the seller has the power to commit itself to walk away from the auction if no potential buyer is willing to make a bid as high as this minimum; it implies that the seller is able to commit itself to refuse to sell at a lower price once it is revealed that none of the potential buyers is willing to bid the minimum. If the producer has no power to commit itself to a strategy at a later point in time, then it is clear that a minimum bid above the costs of production cannot be a feature of the auction. Without the power to commit itself, if the producer were to set a minimum bid above the marginal cost of production, and if not enough buyers were to bid above this level to exhaust production capacity, then the producer would seek to sell the remaining units by lowering the minimum bid. A negotiation or selling strategy inclusive of a minimum bid above the marginal cost of production would not be a time consistent strategy and therefore not credible. This fact would, of course, be anticipated by the buyers and impact their bidding strategies. The maximal minimum bid which is credible is the minimal reservation value greater than the marginal cost of production, and this is the minimum bid, R_h , which we utilize in our modified Vickrey Auction

as defined in (5) and (6). The auction defined by (5) and (6) then has the property that it maximizes revenue across the set of feasible and incentive compatible selling mechanisms which satisfy time consistency.

In Appendix 3 we provide a display of the outcome of this auction model for a sample set of parameters and the reader is referred there to familiarize him/herself with the equilibrium outcome characterization of the modified Vickrey Auction.

4.2 The Comparison Between Forward and Spot Sales

As discussed in the example given in section 2 and the definition for our measure of the value of forward contracts, Γ , in section 3, the difference between forward and spot contracts follows from the sunk nature of the expenditures for capacity and the weakened bargaining position of the seller that results. The bargaining power lost by the seller as a result of its decision to install the capacity prior to negotiating a sale price enters into the final results of our modified Vickrey auction model through the definition of the minimum bid. The minimum forward price in the modified Vickrey auction defined above is determined as the minimum reservation value greater than the constant marginal cost inclusive of capital costs:

$$(7) \quad R_{hf} = \min\{R_j | R_j \geq k+v\},$$

where k is the scalar parameter of the assumed constant marginal capital cost function and v is the scalar parameter of the assumed constant operating cost function. Once capacity has been installed, however, i.e. for spot sales, the minimum price in the modified Vickrey Auction will be determined as the minimum reservation value greater than the marginal operating cost alone:

$$(8) \quad R_{hs} = \min\{R_j | R_j \geq v\}.$$

The price function for the forward sales, p_{mf} , is then the modified Vickrey

price function from equation (6) where $R_h = R_{hf}$; and the price function for the spot sales, p_{ms} , is the modified Vickrey price function from equation (6) where $R_h = R_{hs}$. The quantity allocations to the buyers are accordingly the quantity allocations from equation (5) where $c=k+v$ and $R_h = R_{hf}$ for forward sales and $c=v$ and $R_h = R_{hs}$ for spot sales.

The profit made by the producer from a forward sale of the capacity is then a simple function of the realization of the order statistics of the buyer reservation values,

$$(9) \quad \pi_f(S_1, \dots, S_N) = [p_{mf}(S_1, \dots, S_N) - (k+v)] \sum_{i=1}^N q_{if}(S_1, \dots, S_N)$$

and the expected profit from the decision to use a forward sale is,

$$(10) \quad E(\pi_f) = \sum_{(S_1, \dots, S_N) \in \Psi} [\pi_f(S_1, \dots, S_N) g(S_1, \dots, S_N)]$$

where Ψ is the event space of possible combinations of order statistics (S_1, \dots, S_N) and $g(S_1, \dots, S_N)$ is the probability function defined on this space from the underlying probability distribution $H(\cdot)$ over the reservation value vectors (R_1, \dots, R_N) .

For spot sales these two functions are,

$$(11) \quad \pi_s(S_1, \dots, S_N) = ([p_{ms}(S_1, \dots, S_N) - v] \sum_{i=1}^N q_{is}(S_1, \dots, S_N)) - kQ$$

and,

$$(12) \quad E(\pi_s) = \max\{0, \sum_{(S_1, \dots, S_N) \in \Psi} \pi_s(S_1, \dots, S_N) g(S_1, \dots, S_N)\}$$

Note that whenever the second argument in the maximum operator is negative the producer can choose to forego any installation of capacity and thereby avoid any expectation of losses.

Our measure of the value to the producer of forward contracts can then be written,

$$(13) \quad \Gamma = E(\pi_f) - E(\pi_s).$$

Theorem 1: $\forall \{N, Q < N, (R_1, \dots, R_Q), k, v, H(\cdot)\} \quad \Gamma \geq 0$, and
 $\exists \{N, Q < N, (R_1, \dots, R_Q), k, v, H(\cdot)\} \text{ s.t. } \Gamma > 0$.

Proof: 1. by eq. (6) $p_{mf} \geq k+v \rightarrow \pi_f \geq 0 \rightarrow E(\pi_f) \geq 0 \rightarrow$ to establish the first part of the proposition it is sufficient to show that

$$E(\pi_f) \geq \sum_{(S_1, \dots, S_N) \in \Psi} \pi_s(S_1, \dots, S_N) g(S_1, \dots, S_N).$$

$$2. \forall (S_1, \dots, S_N) \in \Psi \text{ s.t. } S_{Q+1} \geq R_{hf} \text{ or } S_{Q+1} < R_{hs} = R_{hf}$$

$$\rightarrow (i) \quad p_{ms} = p_{mf}, \text{ and}$$

$$(ii) \quad \sum_{i=1}^N q_{if} = \sum_{i=1}^N q_{is}$$

$$\rightarrow \pi_f = \pi_s$$

$$3. \forall (S_1, \dots, S_N) \in \Psi \text{ s.t. } S_{Q+1} < R_{hf} \text{ and } R_{hs} < R_{hf}$$

$$\rightarrow p_{ms} = \max\{S_{Q+1}, R_{hs}\} < k+v \leq R_{hf} \leq p_{mf}$$

$$\rightarrow \pi_s = [p_{ms} - (k+v)] \sum_{i=1}^N q_{is} - [Q - \sum_{i=1}^N q_{is}]k < 0 \leq \pi_f.$$

4. This last relation combined with that in step 2 guarantees the sufficient condition mentioned in step 1. To establish the second part of the theorem it is sufficient to choose the parameters such

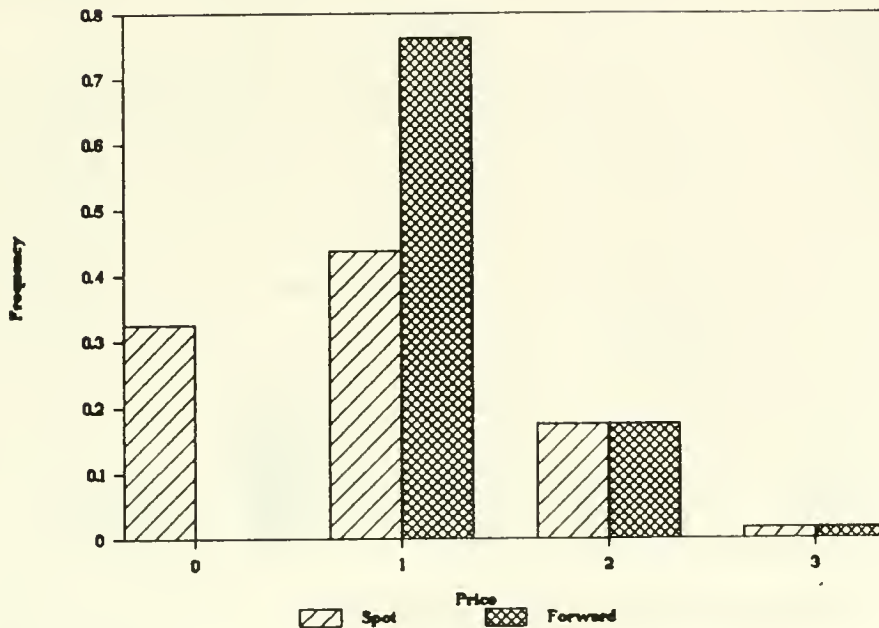
that the case considered in step 3 occurs with positive probability and that $\pi_f > 0$. This completes the proof.

Remarks. Steps two and three in the proof can be restated as follows. Since the minimum price in the forward sales is always greater than the minimum price in spot sales, and since for all prices greater than the minimum price in forward sales the price rules for forward and spot sales are identical, the unit operating profits for all forward sales consummated are weakly greater than the profits for the same spot sales. Of course, since the minimum forward price is typically greater than the minimum spot price there will be some cases in which the producer will sell a larger quantity in the spot market than it will sell in the forward market. This will occur whenever there is excess capacity at the minimum forward price and there are some buyers with reservation values lying between the minimum forward price and the minimum spot price -- this is the case analyzed in step 3 of the proof. However, whenever this is the case, the marginal cost inclusive of capacity are greater than the spot price and the seller faces a negative marginal profit. The seller, therefore, has no regrets at losing the forward sale and avoiding the expenditure on capacity. On the other hand, given that it has already incurred the capital expense--i.e., when engaged in spot negotiations--the extra sale is profitable, but the initial decision to expand the capacity is ex post regrettable. With forward contracting the decision to expand capacity in these events is avoidable.

The result of these two features, that the profit on all sales made using forward contracts are weakly greater than the profits on the same sales made using spot sales, and that the marginal profits are negative on all sales made

Figure 5a

Sample Probability Distribution of Forward and Spot Prices

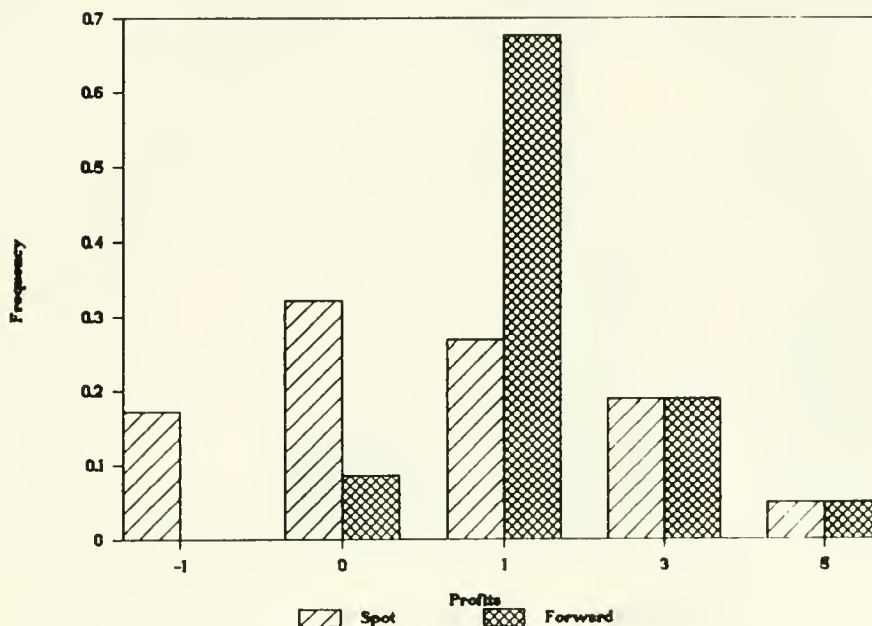


The darker distribution represents the anticipated price distribution from forward sales using the auction rules described in equations (5-7) for the parameters given in Example 2 of Appendix 3; the lighter distribution represents the anticipated price distribution for spot sales using the auction rules described in equations (5-6,8) for the same parameters. To keep the two distributions visually simple and comparable, the probability weights assigned to the modified Vickrey auction prices, $S_{Q+1} + \delta A(Q+1)$, have been reassigned to the prices S_Q and S_{Q+2} so as to maintain the same expected value.

In the forward contract negotiations the producer can credibly refuse a price of zero since the cost of capacity is 0.5. When conducting spot sales, and when facing two or fewer buyers with reservation values above zero, there is not adequate competition to drive the price above zero, and the producer having already installed the capacity cannot credibly refuse a price below the per unit capital charges. Hence, some sales which would occur at a price of one under forward negotiations occur at a price of zero under the spot sales.

Figure 5b

Sample Probability Distribution of Forward and Spot Profits



The darker distribution represents the anticipated profit distribution from forward sales using the auction rules described in equations (5-7) for the parameters given in Example 2 of Appendix 3; the lighter distribution represents the anticipated profit distribution for spot sales using the auction rules described in equations (5-6,8) for the same parameters. Two units of capacity are installed and sometimes two units of the commodity are sold at the prices displayed in Figure 5b.

Zero profits are earned in the forward negotiations in those cases for which there are zero buyers with reservation values greater than zero and hence in which zero capacity is installed. In each of these events spot sales are made at a price of zero and profits are negative since capacity was installed ex ante. Profits are also negative for spot sales whenever capacity was installed and, although there exist buyers with reservation values greater than zero, there does not exist competition to drive the spot price above zero.

spot that are not made forward yields the result that the profit from forward sales is weakly greater than the profit from spot sales, and sometimes strictly greater: the measure Γ will be non-negative.²

The consequences of the weakened bargaining position can be seen in the following pair of diagrams. In the first diagram the probability distributions of forward and spot prices for a sample situation are displayed: the darker distribution represents the distribution of prices which are anticipated as a consequence of forward negotiations--failure to agree on any sale is represented as a zero price; the lighter distribution represents the distribution of prices that are anticipated as a consequence of spot sales--when the price is zero in a spot sale this can represent either a sale at zero price, or no sale, but in either case the producer has already incurred the costs of capacity installation. In the second diagram two probability distributions over unit profits inclusive of unit capital charges are displayed. The darker distribution represents the unit profits from forward contracting; the lighter distribution represents the unit profits from spot contracting.

[Insert Figure 5]

The original literature on opportunistic bargaining yielded the original insight that the distribution for spot sales would be shifted to the left, or to lower unit profits. What was missing, and what the use of an auction model

2. In this paper we have focused exclusively upon the positive value to forward contracts. Presumably these contracts impose burdens and constraints on future transactions which are for various reasons suboptimal and therefore costly. A full treatment of the problem would include a measurement of the strategic costs and the benefits and would determine the optimal contractual design. We consider our estimation of the benefits to be one element of this process and are pursuing research on modeling the strategic costs to long-term contracts and the relation between the benefits and the costs.

promises to give us is a consistent method for assessing the magnitude of this shift for a well specified environment: alternatively, the use of auction models allows us to test if a reasonable range of environments can generate a shift of a given magnitude.

In Appendix 3 we provide two simple numerical examples using our auction model to compare the forward and spot price distributions and to calculate Γ .

5. Natural Gas Take-or-Pay Contracts

As an example of the application of this model for the estimation of the value to a producer of a forward contract we use the take-or-pay contracts which are common in the natural gas industry. Typically, once a natural gas field has been discovered and simultaneous with the decision to develop the field the producer negotiates with prospective buyers long term contracts under which each customer commits itself to pay each year for a given quantity of the gas. These contracts usually run for a duration of fifteen to twenty years. The determination of the exact quantity to be paid for in any given year and the determination of the price can sometimes be subject to complex formulae and may be contingent on various events. Although the quantity to be purchased under the contract is therefore not absolutely fixed, it has been traditional in the natural gas industry for a gas producer to sign forward contracts which almost fully commit the original capacity of a given field. For simplicity, we model the contract as if the quantity to be delivered and the payment in a given year are absolutely fixed at the time of negotiation of the contract.

The magnitude and duration of these contracts has recently been felt by many customers to be a significant element of rigidity in operations, a rigidity which is ultimately costly and therefore which makes the value of the resource that much less. Producers seeking to successfully develop and market

gas properties have therefore been anxious to consider whether or not the take-or-pay burden might be ameliorated, whether or not they can develop a field without having firm commitments from their potential customers to purchase the full output. Producers are actively considering installing capacity and subsequently negotiating the sale of some portion of the gas on a short term basis. This is true especially in western Europe.

Reluctance to relax the magnitude or duration of these contracts is, of course, directly related to the issues at hand in this paper. The forward contract secures to the producer a large portion of the value of the project; without the guarantees offered by the take-or-pay contract the producer must reduce its estimate of the profit which it can anticipate from development of the field, and this lower profit may not warrant development of the field. The estimate derived using our model of the strategic value of the forward contract under varying circumstances will shed light on the cases in which the value secured to the producer by means of the forward contract is relatively small and in which therefore the producer might prudently lower the level of take-or-pay requirement in the interest of successfully negotiation a sale or a higher price for its output.

To illustrate how different features of the market impact the strategic value of the forward contracts, we make two comparisons using the auction model. The first comparison focuses upon the issue of the number of potential customers. We analyze the strategic value of forward contracts for the Venture natural gas field in eastern Canada and compare it with the strategic value of forward contracts for the development of a field in Alberta. The market for the gas from the Venture field consists of a very small set of users in New England. The gas from the Albertan field can be routed to a larger number of users in the midwestern and west coast United States. We will see that the model estimates the strategic value of forward contracts to

be significantly greater for the Venture field than for the Albertan field. This occurs because the competition among the many potential buyers for the Albertan gas makes the capacity installation decision less important for the seller's bargaining power.

The second comparison will focus upon the size of the initial capital expenditures necessary to make the gas deliverable. We analyze the Troll field in Norway and the Soviet gas field in Urengoi. The Troll field reportedly requires relatively low capital expenditures, as natural gas fields go, to deliver the gas to the West European market. The Soviet gas requires large expenditures on pipelines and therefore larger initial capital expenditures to make the gas deliverable. The model calculates the strategic value of forward contracts to be relatively modest for the Troll field, while for the Soviet gas field the value of the forward contracts approaches forty percent of the net present value of development of the field.

5.1 The Number of Buyers: The Venture Field vs. Albertan Fields

To assess the strategic value of a take-or-pay contract for a field we first need to determine the parameter values for the variables of our model. These include the field characteristics: size of the field, the per unit capital costs, and per unit operating costs. These also include the market or buyer characteristics: the number of buyers and the range of possible reservation values for each buyer. We will depend upon figures for the Venture gas field which are taken from Adelman et al. (1985) and use this data to choose the values for our parameters. The numbers and examples used in this paper are meant to be illustrative of our model and we do not detail their derivation nor do we intend them to be definitive for the cases evaluated.

Field characteristics. The Venture field has total reserves of 2.36 TCF (trillion cubic feet) and will be operated at a level of 116.8 BCF/year for a

period of twenty years. We will analyze the value of contracts on an annual basis and therefore the total capacity will be this annual capacity. The capital expenditures necessary to develop the field are \$1837.5 million. Amortized over the life of the field and the quantity to be delivered each year, these expenditures amount to \$2.66/Mcf (\$/thousand cubic feet). The operating costs are \$75 million/year: the field, however, will produce associated gas liquids the sale of which will approximately equal the operating costs, and therefore we will set the operating costs per Mcf at zero.

Market/buyer characteristics. The gas from the Venture field will be sold in the northeastern US market where there are a relatively small number of large buyers. Current sales by Canadian producers to this market are less than double the capacity of the Venture field. We model the negotiations as taking place between the field developer and three buyers, each of which could utilize the full annual capacity of the field: negotiations are therefore modelled as an auction of a single unit--116.8 Bcf/yr of gas--to three bidders. The range of prices to which these buyers might agree in a contract will be determined by the alternative sources of supply that are available to these buyers. Additional western Canadian gas might be available in the near future in this market at a price of approximately \$2.90/Mcf at the border, and at \$3.15 or \$3.40/Mcf at the border in the next decade. Additional gas from Louisiana would be available delivered in Boston at \$4.02/Mcf. Current consumption levels could perhaps be supplied from Louisiana at as low as \$2.80/Mcf delivered. Transportation costs from the Venture field to the border and to Boston could be as high as 60 and 90 cents per Mcf respectively, although figures of 30 and 50 cents are perhaps more realistic. The range of wellhead prices which customers in the northeastern US are likely to accept may therefore lie between \$2.30 and \$3.52/Mcf, or could be as low as \$2.00 to

\$3.10/Mcf. We use the former pair of prices to bound the range of buyer reservation values for our sample calculations in this paper.

These parameter values and the model calculations for expected profit from forward and spot contracting are displayed in Table 1.

[Insert Table 1]

If the developer of the Venture field enters into negotiations for forward commitments on the purchase of gas prior to the expenditure for development and pipeline construction, then the expected annual profits are \$46 million/yr. or \$344 million net present value over the twenty year life of the project. This average incorporates the possibility that in the negotiations the developer may find no buyer willing to commit itself to purchase the gas at the minimum price of \$2.66/Mcf: the probability of this event is 3.7%. If the developer were to install the capacity and subsequently attempt to sell the output 'spot', then the expected annual profits calculated by the model are \$41 million/yr. or \$307 million net present value over the life of the project.

In the case of spot sales, for all events in which no sale was made in forward contracting a sale is made spot and at a price between \$0.10 and \$0.20/Mcf less than necessary to cover capacity and operation charges. For an additional set of cases with a probability of 20% the price of the sale spot is less than the price of the sale forward by about \$0.15/Mcf. Hence, the profits expected from a strategy of development and spot sales are \$5 million less annually than the profits from a strategy of forward negotiations and development contingent on the outcome, a total net present value loss over the life of the project of \$37 million, or a decrease of 10%. This is the portion of the project NPV which is endangered in spot negotiations by the ex post opportunism of the buyers and which is secured by means of forward contracts.

The Albertan Field characteristics. Data on the Albertan fields is also

Table 1

The Strategic Value of Forward Contracts for the Venture Gas Field

Field Size		
Bcf/yr; # yrs. sale unit	116.8; 20 1	
Capital Costs		
\$/Mcf	2.66	
Operating Costs		
\$/Mcf	0	
Buyers		
# reserv. value range, \$/Mcf	3 2.30-3.52	
Expected Profits		
forward	\$46 m/yr	\$344 m total
spot	41	306
difference	5	38
% loss		-10.8%

taken from Adelman et. al (1985). While the Venture data was derived from a field specific source, the Albertan field data is based upon typical costs for a class of fields in Alberta. We will model a field with a 20 year annual production capacity of 50 Bcf. The capital expenditures necessary to develop this field amount to \$300 million over a three year period or an amortized expenditure based upon the planned rate of depletion of \$1.01/Mcf. The operating expenditures necessary are \$0.45/Mcf.

Market/buyer characteristics. The key difference between the Albertan field and the Venture field is that the Albertan field can be connected into various pipeline networks which in turn each serve a broader number of large customers. Access is available both to the dense set of pipelines serving the US midwest and to pipelines serving the US west coast. For illustration we will model the problem as negotiations involving 5 large buyers each of which can consume the full output of the field: hence, the developer is negotiating to sell a single unit to one of five buyers. The range of reservation prices which we will use for each buyer is \$1.00-4.00/Mcf, a range which generates an expected price of \$3.32/Mcf. This price is comparable to that calculated in Adelman et al. (1985) as a likely scenario for exports from Alberta to the midwest and western US.

These parameter values and the model calculations for expected profit from forward and spot contracting are displayed in Table 2.

[Insert Table 2]

If the developer of the Albertan field enters into negotiations for forward commitments on the purchase of gas prior to the expenditure for development and pipeline construction, then the model calculates the expected annual profits to be \$186 million or \$1.39 billion in net present value over the life of the project. This average incorporates the possibility that in the negotiations the developer may find no buyer willing to commit itself to

Table 2

The Strategic Value of Forward Contracts for the Albertan Gas Field

Field Size		
Bcf/yr; # yrs. sale unit	50; 20 1	
<hr/>		
Capital Costs		
\$/Mcf	1.01	
<hr/>		
Operating Costs		
\$/Mcf	0.45	
<hr/>		
Buyers		
# reserv. value range, \$/Mcf	5 1.00-4.00	
<hr/>		
Expected Profits		
forward	\$ 93 m/yr	\$ 695 m total
spot	93	695
difference	0	0
% loss		- 0.2%

purchase the gas at the minimum price of \$1.46/Mcf: the probability of this event is a mere 0.01%, and the probability of all events in which the spot price is less than the forward price is only 0.24%. If the developer were to install the capacity and then attempt to sell the output 'spot', then the expected annual profits calculated by the model are also about \$186 million--the exact difference is two-tenths of a percent of the annual profits. This is the portion of the project NPV which is endangered by the ex post opportunism of the buyers and which is secured by means of forward contracts.

A comparison of the results displayed in Tables 1 and 2 clearly illustrates the difference between the strategic value of the forward contracts for the Venture field and the strategic value for the Albertan field. While the forward contracts secure for the producer nearly 11% of the net present value of developing the Venture field, they are virtually irrelevant to the developer of a field in Alberta. The forward contracts offer little strategic value to the developer in Alberta since with five bidders instead of three the probability is small that there are not at least two buyers with reservation prices above the marginal costs inclusive of capital charges, and therefore there is little probability that competition among the buyers will be absent leaving the seller dependent upon its bargaining power for ensuring a price sufficient to cover the capital and operating expenses.

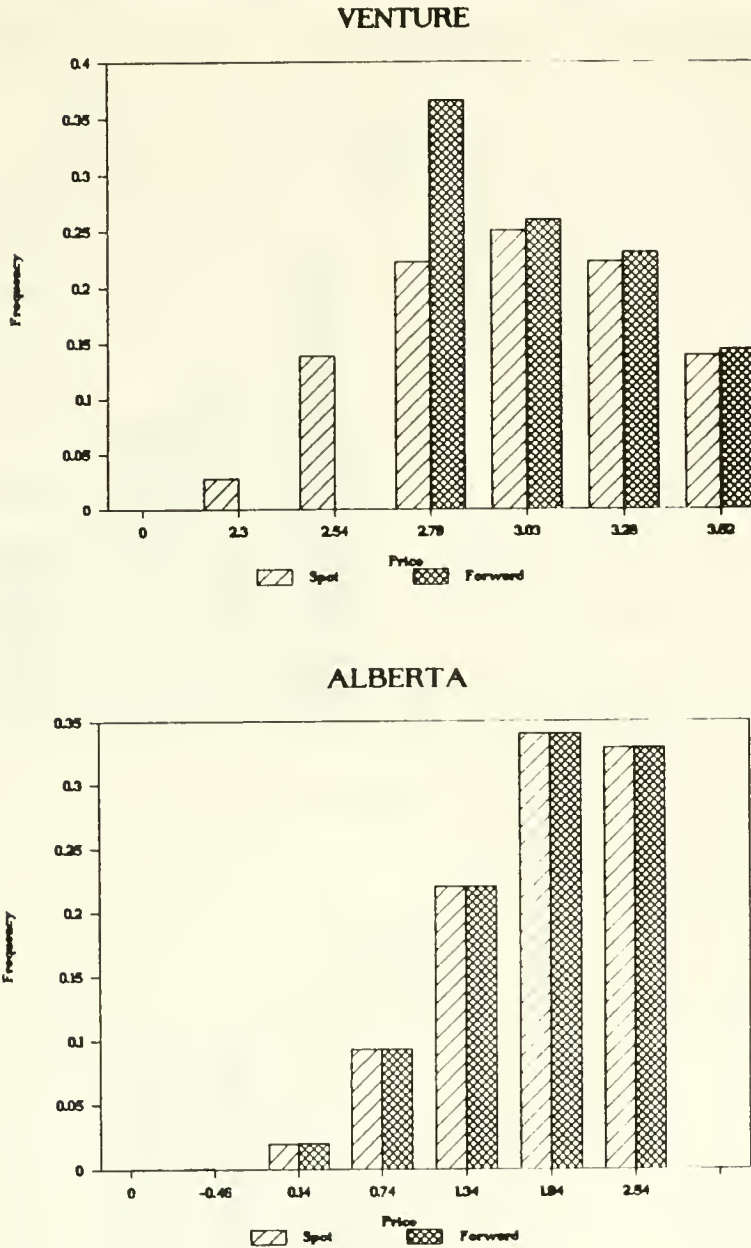
These results can also be seen in a comparison the anticipated probability distributions of forward and spot prices for the Venture field with the anticipated distributions for the Alberta field and in a comparison of the anticipated unit profit distributions forward and spot for the two fields. These distributions are displayed in Figure 6.

[Insert Figure 6]

Assuming that we have captured the central motivation for the forward

Figure 6a

Probability Distribution of Forward and Spot Prices
for the Venture and for the Albertan Gas Fields

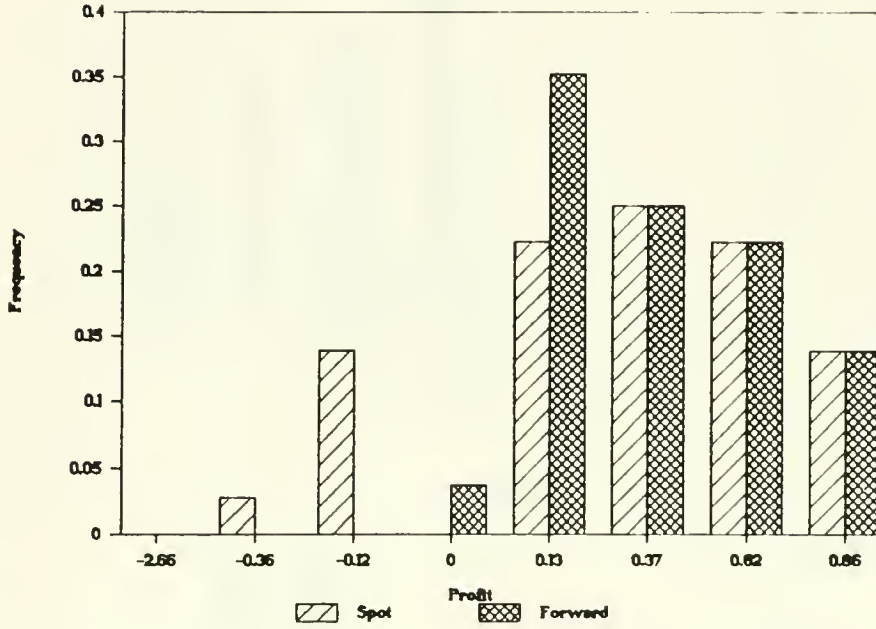


In the first figure or pair of distributions the darker of the two is the anticipated distribution of forward prices negotiated for the Venture gas field and the lighter of the two is the anticipated price distribution for spot sales of the Venture gas. In the second pair of distributions the forward and spot distributions for the Albertan field are displayed. In the case of Venture the forward price distribution is clearly shifted rightward or stochastically dominates the spot distribution and the strategic value to forward contracts can be visually identified in this shift. In the case of Alberta the forward and spot distributions are almost identical and hence the strategic value to forward contracts in the case of Albertan gas is virtually zero.

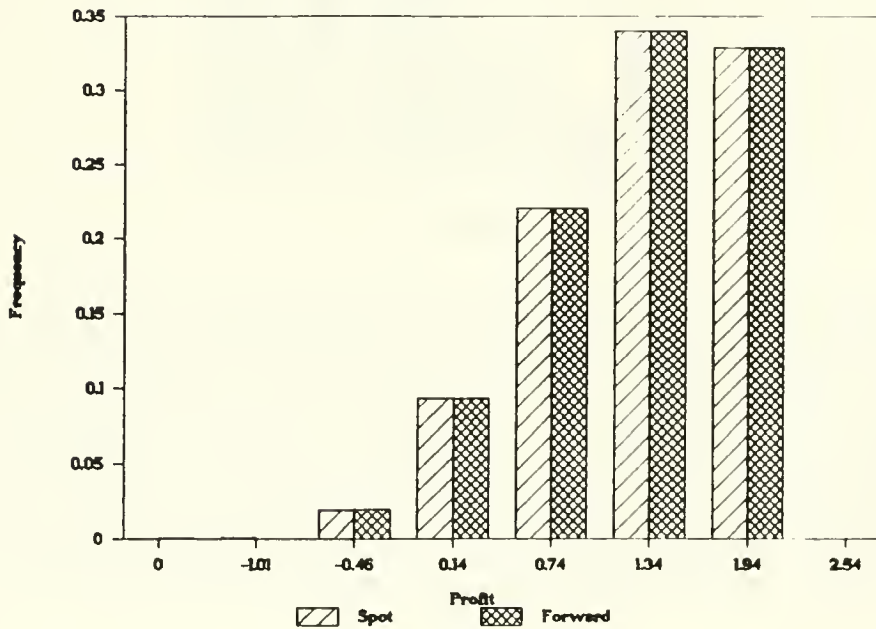
Figure 6b

Probability Distribution of Forward and Spot Profits for the Venture and for the Albertan Gas Fields

VENTURE



ALBERTA



contracts, it would appear from our results that the gradual development of a 'thicker' market in natural gas in the midwestland western United States has significantly reduced the necessity of utilizing the strong take-or-pay contracts that have been common for the past several decades. Several persons in this industry have made such assertions, arguing that an increased reliance upon short term sales is possible: our model calculations support this claim. It is important to note, however, that this possibility is restricted to the particular market: it would not be possible for gas to be marketed on this basis in New England, and as we shall see, is not possible for several other suppliers and markets in the world. This market specific character of changes in contracting is often ignored in the casual prognoses.

5.2 The Proportion of Capital Costs: Troll vs. Soviet Gas

Data on the Troll field is taken from Adelman et al. (1986). Total volume available from the Troll field is 14.2 Tcf or 610 Bcf per year for each of 23 years. Total capital expenditures for development will be \$3.2 billion or \$0.67/Mcf amortized over the schedule of production. Expenditures for pipeline construction will be another \$2.56 billion--\$0.62/Mcf. Per unit capital costs for the Troll field therefore amount to \$1.29/Mcf. Operating expenses for the field will be \$0.23/Mcf: operating expenses for the pipeline will be \$0.18/Mcf. Total operating expenditures are therefore \$0.41/Mcf.³

The gas from the Troll field will be piped into the western European market through two entry points, one in the Federal Republic of Germany and a new one in Belgium. Once landed the gas can be shipped to any of the key national markets, with France and the Federal Republic of Germany being the main buyers and potentially displacing gas to Italy. We will model the gas as

3. The data for Troll used here originated in early published reports and may be relatively optimistic compared to later estimates.

being sold to three buyers. Each buyer is assumed to be able to completely purchase the scheduled annual output of 610 Bcf. The range of reservation values is based upon the demand profile provided in Adelman et al. (1986): \$0.50-4.00/Mcf.

Data on the Soviet Urengoi field and pipeline costs are taken from Adelman et al. (1986). We will examine an expansion of production and transportation capacity of 1412 Bcf/yr producing for 20 years. Per unit capital costs for the Urengoi field are \$1.59/Mcf. Per unit operating costs are \$0.63/Mcf. The Soviet gas will be sold to the same markets that the Troll gas will be sold. We therefore model it as facing the same number of buyers and the same set of reservation values.

These parameter values and the model results for these two fields are displayed in Table 3.

[Insert Table 3]

A comparison of the results for the Troll field with the results for the Urengoi field show the impact on the strategic value of the forward contracts of the higher capital costs of the Urengoi field. Given the same range of reservation values and the same number of buyers, the probability that there exist at least two buyers which value the resource at a price above than the capital expenditures is greater for the field with the lower capital expenditures, Troll, than for the field with the higher capital costs. While it is immediate that the field with the lower costs per unit of production also show a higher profit margin when facing identical sets of buyers, our concern here is not with the absolute profit level, but with the percent of the margin which is secured via contracts.

Again, one can see these results in a comparison of the forward and spot

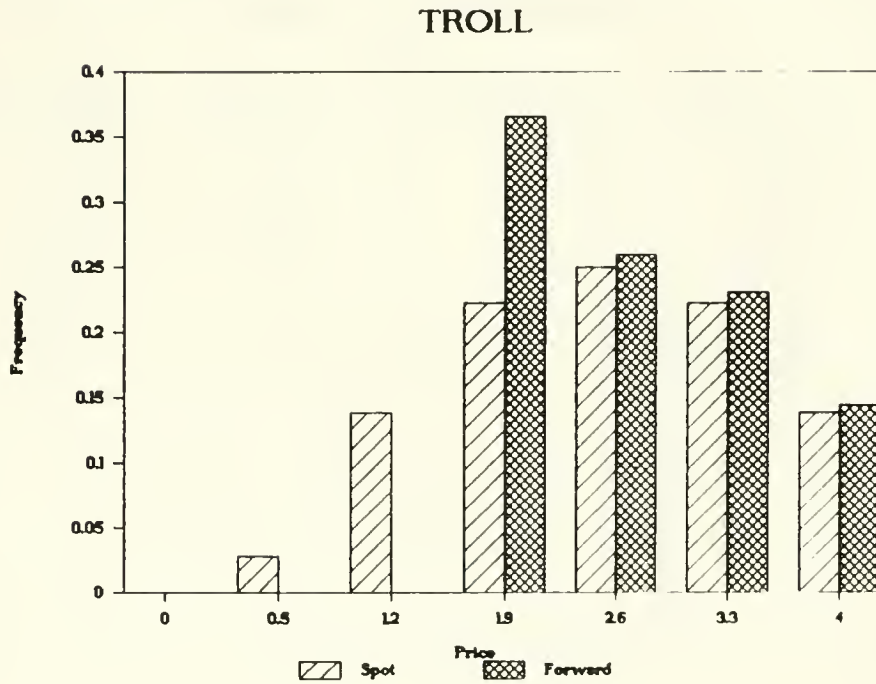
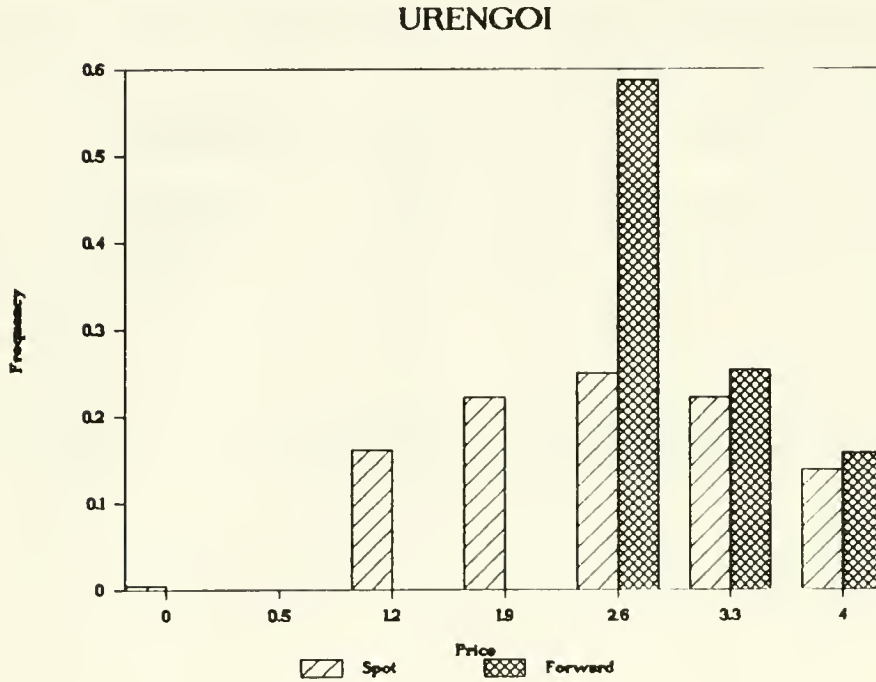
Table 3

The Strategic Value of Forward Contracts for the Troll & Urengoi Gas Fields

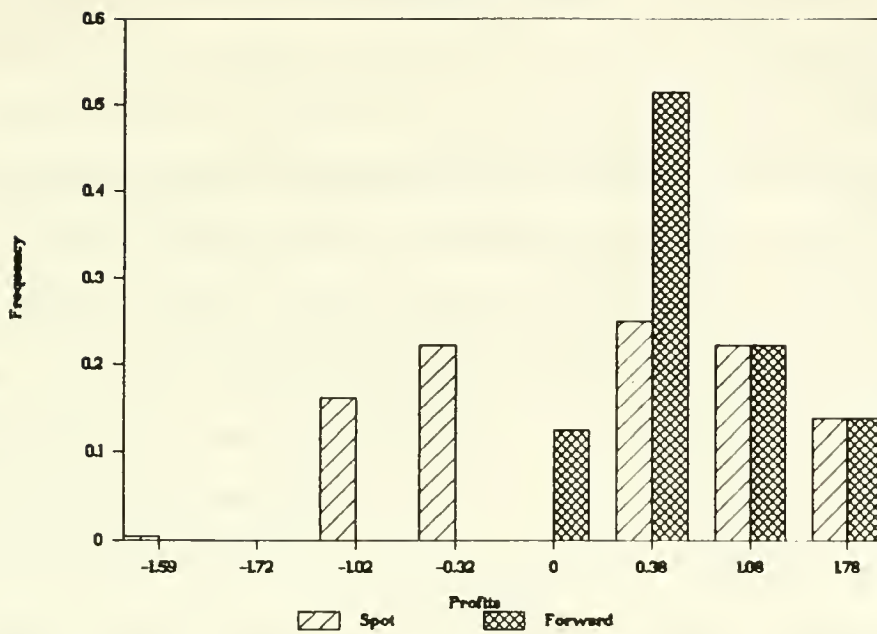
Field Size	Troll		Urengoi	
Bcf/yr; # yrs. sale unit	610;	23	1412;	20
	1		1	
<hr/>				
Capital Costs				
\$/Mcf	1.29		1.59	
<hr/>				
Operating Costs				
\$/Mcf	0.41		0.63	
<hr/>				
Buyers				
#	3		3	
reserv. value range, \$/Mcf	0.50-4.00		0.50-4.00	
<hr/>				
Expected Profits				
forward	\$592 m/yr	\$4,571 total	\$ 964 m/yr	\$7,202 total
spot	514	3,965	478	3,572
difference	78	606	486	3,630
% loss		-13.3%		-50.4%

Figure 7

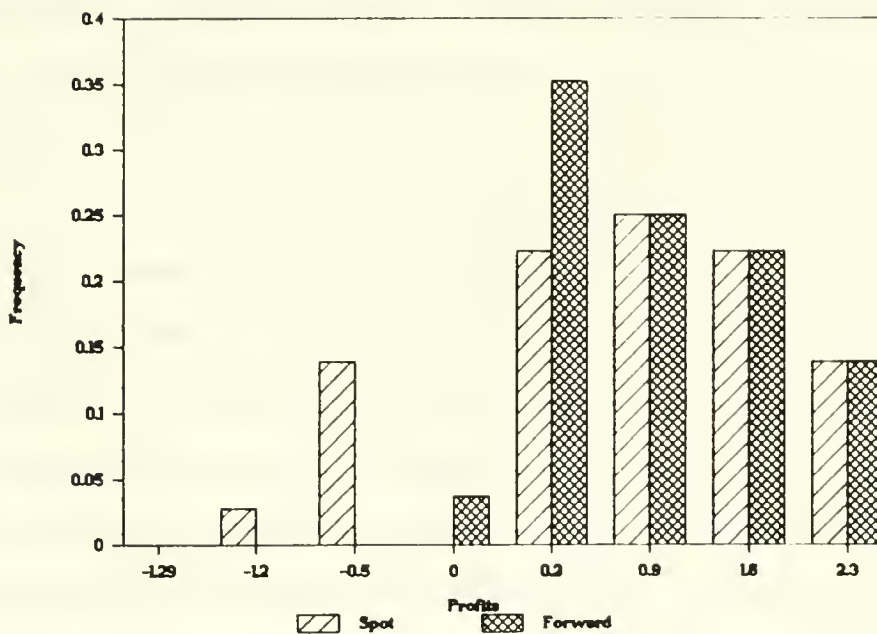
Probability Distribution of Forward and Spot Prices and Profits for Troll and Urengoi Gas Fields



URENGOI



TROLL



price and profit distributions for the two fields as displayed in Figure 7.

[Insert Figure 7]

6. Conclusion

Recent work in the economics of information has emphasized the role of long term contracts in mitigating strategic problems which arise in imperfect or incomplete markets. In this paper we have shown that an auction model can be used to operationalize the results of this body of literature. Using the auction model we can estimate the significance of these strategic problems for a given project and the portion of the project's value which is secured to the producer by means of long term contracts. This estimate allows us to incorporate strategic concerns into traditional and practical project valuation problems.

The strategic problems discussed in this paper are relevant for commodities which are traded in markets that are not perfectly competitive and for which large scale capital investments are necessary. The imperfect competition may arise in some cases due to the fact that any given capital investment must be dedicated to a small set of buyers/sellers in an industry which otherwise includes a large number of buyers/sellers.

The strategic value to long term contracts arises because the equilibrium prices negotiated in forward and spot contracts are influenced differently by the large scale capital investments made by the supplier. In an imperfectly competitive market with large scale capital investment forward supply contracts may be negotiated prior to capital investments and the capital investments may be made contingent upon the results of forward negotiations. Spot contracts, on the other hand, must be made after capital has been irretrievably invested. Forward and spot negotiations are therefore conducted under different strategic considerations and yield distinct distributions of

price and quantity outcomes: the spot distribution is typically biased downward and under forward negotiations no sale is made when the costs inclusive of capital charges would be greater than the negotiated price. The auction model yields a consistent estimate of the divergence between these two distributions based upon fundamental data on the size of the market, the demands of the buyers, and the cost structure for the industry. Other models typically used for analyzing forward contracts, such as the arbitrage technique applied by Brennan and Schwartz (1985) to long term coal contracts, do not incorporate this key influence on the relationship between anticipated forward and spot distributions.

We have applied the auction model to the analysis of long-term take-or-pay contracts used in the natural gas industry. While these long term contract have been typical in this industry for decades we show, for example, that in the midwest North American market the growing number of buyers to which a given seller can route their gas has significantly diminished the strategic value of these contracts. In contrast, where the number of available buyers remains small, as in gas routed to the New England market, the model yields a high strategic value to the traditional take-or-pay contract. In two cases from the European market we show how the strategic value to the contract also depends significantly upon the cost structure of the gas fields, with the strategic value diminishing as the proportion of the costs which must be incurred prior to spot negotiations falls.

Two important avenues of further research immediately propose themselves. First, the structure of the environment analyzed in this paper is extremely stylized: there is only one seller, the demand structures for all buyers are very simple and crude, and the cost structure is simple and restrictive as well. On the one hand, these factors make optimal the use of very simple sale contracts without any contingency or flexibility, and therefore allow us to

view a given problem with a minimum of complication. On the other hand, these restrictions limit the actual situations for which our model will yield accurate results, and moreover, these restrictions prevent us from making any analysis of or recommendations concerning the flexibility and design of more complicated forward contracts which yield the highest strategic value when facing these more complicated environments. We are currently analyzing the application of models of negotiations and bargaining in more complicated environments to the problem of long term contracting. The second area of further research follows from the fact that we have focused upon estimating the strategic factors which motivate or which favor the use of long term contracts. We have left out of the analysis, for example, an estimation of the costs imposed upon both parties to the contract by the inflexibility vis-a-vis production decisions which should be contingent upon information obtained after the signing of the contract. When it is not possible to write complete contingent contracts, these factors may outweigh the strategic value analyzed in this paper. The problem is properly posed as the design of interim efficient contracts in the sense of Holmstrom and Myerson (1981) and is a the subject of current research. The key variables which we are analyzing is the structure of optimal price indexes and the optimal design of interim 'take' or quantity decision rules.

- Adelman, M.A., et al, (1985). Canadian-U.S. Natural Gas Trade, International Natural Gas Trade Project Report, Center for Energy Policy Research, Energy Laboratory, M.I.T.
- _____, (1986). Western Europe Natural Gas Trade, International Natural Gas Trade Project Report, Center for Energy Policy Research, Energy Laboratory, M.I.T.
- Black, F., (1976). "The Pricing of Commodity Contracts," Journal of Financial Economics, 3:167-179.
- Brennan, M. and E. Schwartz, (1985). "Evaluating Natural Resource Investments," Journal of Business, 58:135-157.
- Cox, J., J. Ingersoll, and S. Ross, (1981). "The Relation Between Forward Prices and Future Prices," Journal of Financial Economics, 9:321-346.
- Grossman, S. and J. Stiglitz, (1980). "On the Impossibility of Informationally Efficient Markets," American Economic Review, 70:393-408.
- Harris, M., and A. Raviv, (1981). "A Theory of Monopoly Pricing Schemes with Demand Uncertainty," American Economic Review, 71:347-365.
- Holmstrom, B., and R. Myerson, (1981). "Efficient and Durable Decision Rules with Incomplete Information," Northwestern University CMSEMS DP #495.
- Joskow, P., (1984). "Vertical Integration and Long Term Contracts: the Case of Coal Burning Electric Generating Plants," MIT Economics Department Working Paper #361.
- Klein, B., R. Crawford, and A. Alchian, (1978). "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process," Journal of Law and Economics, 21:297-326.
- Williamson, O., (1975). Markets and Hierarchies: Analysis and Antitrust Implications, esp. pp. 26-30.

The firm can make the buyer an offer to sell any quantity of the commodity at a price p . The buyer can choose to accept or to reject the offer. We assume that once the offer is rejected the possibility for a sale is foreclosed. The buyer's strategic problem is to determine the quantity that it will purchase when confronted with a particular offer for sale of up to q units at price p :

$$(A1.1) \quad d^* \in \operatorname{argmax}_{d \in [0, q]} (r-p)d$$

and the buyer's optimal strategy is to purchase the maximal amount offered whenever the price is below its reservation value:

$$(A1.2) \quad d^*(r, p, q) = \begin{cases} 0 & p > r \\ q & p \leq r. \end{cases}$$

Given the strategy which the buyer will follow it is possible to derive the firm's optimal offer price. The critical question which must be addressed is at which point in time does the bargaining proceed. The results that follow if the firm has already installed a given amount of capacity and then offers a given price will be strikingly different from the results that follow if the firm makes an offer prior to installing any capacity. Consider first the firm's situation if it makes an offer prior to the installation of any capacity. It's problem is to maximize the expected profits from the sale, where the expectation is taken with the respect to the unknown reservation price of the potential buyer:

$$(A1.3) \quad (p_f, q_f) \in \operatorname{argmax}_{p, q} \int_v^{1+v} [p - (k+v)] d^*(r, p, q) h(r) dr.$$

It is straightforward to see that for a given price, p , q_f is given by,

$$(A1.4) \quad q_f = \begin{cases} 0 & p < k+v \\ 1 & p \geq k+v. \end{cases}$$

Then the choice of p_f solves,

$$(A1.5) \quad p_f \in \operatorname{argmax}_p \int_p^{1+v} [p - (k+v)] dr$$

and the solution is,

$$(A1.6) \quad p_f = (1+k)/2 + v.$$

It is a simple matter to calculate the firm's expected profits when it makes this optimal price offer,

$$(A1.7) \quad E(\pi_f(k)) = [(1-k)/2]^2$$

Consider now the firm's situation if it makes an offer after it has

already installed a given amount of capacity, e.g. $Q=1$. Its problem is to maximize the expected profits from the sale, where the expectation is taken with the respect to the unknown reservation price of the potential buyer, and recognizing that the costs of initially installing the capacity are already sunk:

$$(A1.8) \quad (p_s, q_s) \in \operatorname{argmax}_v \int_v^{1+v} \{[p-v] d^*(r, p, q) h(r)\} - kq \, dr.$$

It is simple to show that

$$(A1.9) \quad q_s = \begin{cases} 0 & p < v \\ 1 & p \geq v. \end{cases}$$

Then the choice of p_s solves,

$$(A1.10) \quad p_s \in \operatorname{argmax}_p \int_p^{1+v} (p-v)Q \, dr - kQ$$

$$(A1.11) \quad p_s = 1/2 + v.$$

It is a simple matter to calculate the firm's expected profits when it makes this optimal price offer, and when it chooses its initial level of capacity to maximize its profits in anticipation of the outcome of the spot market,

$E(\pi_s(k)) = \max_Q \{(1/4 - k)Q\}$, and therefore,

$$(A1.12) \quad E(\pi_s(k)) = \begin{cases} 1/4 - k & k \leq 1/4 \\ 0 & k > 1/4. \end{cases}$$

We reinterpret these two bargaining situations, respectively, as i) the anticipated outcome of the decision to forward contract purchase commitments prior to or coincident with the decision to install the capacity and thereby to incur the fixed costs, and as ii) the anticipated outcome of the decision to first incur the fixed costs and to subsequently bargain for the best available sale price on the spot market. The difference between the profits obtained when the producer requires long term purchase contracts on the one hand and the profits from spot sales on the other hand, $E(\pi_f(k)) - E(\pi_s(k))$, is then the portion of the value of the project which is secured to the producer by means of the forward contract, i.e., the value to the producer of negotiating a forward contract:

$$(A1.13) \quad \Gamma = E(\pi_f(k)) - E(\pi_s(k)) = \begin{cases} [(1-k)/2]^2 - [1/4 - k] & k < 1/4 \\ [(1-k)/2]^2 & 1/4 \leq k \end{cases}$$

The increased profits secured to the producer through the forward contract are a result of two distinct underlying forces. One is the information which the producer receives as a result of the forward negotiations, information regarding the reservation value of the buyer. This information permits the producer to make its production decision contingent upon the reservation value of the buyer, and therefore permits more efficient production. In addition, the decision to negotiate forward improves the bargaining power for the producer and thereby increases the price which the seller receives for the output

To isolate the first effect, the value of information derived from contracting forward, consider the following event. Suppose that $k < 1/4$ and that $r < k+v$. The producer which contracts forward will offer to produce and sell the commodity at the price $p_f = (1+k)/2 + v$. Since $r < k+v$, then the buyer will reject the producer's offer, and the commodity will not be produced. This is the efficient production decision when $r < k+v$. The producer which first installs capacity and then sells on the spot market will, when $k < 1/4$ proceed to install the capacity since on average it will be a profitable decision. In the subcase, however, that $r < k+v$, the decision to install capacity is inefficient. The producer will be unable to sell its product at its offer price, $p_s = 1/2 + v$, and will therefore in this event bear the cost k .

To isolate the second effect, consider the complementary situation in which $k < 1/4$, and $r > k+v$. The producer who contracts forward chooses a price which is greater than $k+v$. It raises the price, recognizing that it sacrifices sales to some buyers whose reservation values are greater than the cost of production but less than the producer's offer price. The lost sales are compensated for by the increased price on the remaining sales. The producer's marginal profit increase as a result of a marginal increase in offer price is,

$$(A1.14) \quad \frac{d\pi_f}{dp} = -(p-f) + (1-p)$$

The marginal profit increase for the spot seller is,

$$(A1.15) \quad \frac{d\pi_s}{dp} = -p + (1-p)$$

For the spot seller who has already invested in the capital costs, the marginal profit increase is uniformly less, since it cannot avoid the capital costs for those buyers whose reservation values lie below its increased price. It therefore chooses a lower asking price and earns a lower expected profit as a result of this diminution of its bargaining power.

Appendix 2: Definition of Values used in the Modified Vickrey Auction

In accordance with Harris and Raviv (1981), to define the variables $(A_1, \dots, A_{\ell-1})$ we first define three component variables,

$$a_i = \left(\frac{1}{k}\right)^{N-1} \sum_{j=Q}^{N-1} \sum_{\ell=0}^{Q-2} \binom{N-1}{j} \binom{j}{\ell} (k-i)^\ell (i-1)^{N-j-1} \quad i=1, \dots, k-1$$

$$b_i = \left(\frac{1}{k}\right)^{N-1} \binom{N-1}{N-Q} (k-i)^{Q-1} [i^{N-Q} - (i-1)^{N-Q}] \quad i=1, \dots, k-1$$

$$c_i = \left(\frac{1}{k}\right)^{N-1} \sum_{j=Q}^{N-1} \sum_{\ell=0}^{Q-1} \binom{N-1}{j} \binom{j}{\ell} (k-i)^\ell (i-1)^{N-j-1} \frac{Q-\ell}{j-\ell+1} \quad i=1, \dots, k-1$$

which may then be combined as,

$$A_i = 1 - \frac{c_i - a_i}{b_i}.$$

Appendix 3: Sample Displays of Auction Results and Calculation of Γ

In this section we present three examples of the modified Vickrey auction. The first example illustrates the rules of the auction with a complete listing of the price and quantity allocation for every combination of buyer reservation values in the feasible event space and the calculation of the expected profit. The second and third examples illustrate our calculation for the value of forward contracting. In the second example a complete listing of price and quantity allocations from an auction with the forward contracting minimum bid is displayed. In the third example a complete listing is given for identical parameters, but for an auction with the spot contracting minimum bid. The analysis of these two cases explicates the proof that the value to forward contracting is positive.

Example 1.

Table 4 below displays the full set of possible outcomes for an auction in which the manufacturer can produce a maximum of 1 unit at marginal fixed cost $k=0.5$ and marginal variable cost $v=0$, and in which there are 3 potential buyers, each with possible reservation values $(R_1, \dots, R_4) = (0, 1, 2, 3)$. The first column of the table indexes the possible events and the second column containing the vector of four numbers is a list of each possible combination of numbers of buyers at each of the four reservation prices; the third column is the frequency of that event given our assumed uniform distribution for each reservation value over the range of reservation prices; the fourth column lists the modified-Vickrey auction price that would follow for each event, and the fifth column lists the total quantity that would be sold; the sixth column lists the producer's total profits for each event, and the seventh column lists the probability weighted profits for each event. Displayed at the bottom of the seventh column is the sum of its entries, the expected profit to the producer from this auction sale of forward contracts.

[Insert Table 4]

Referring to the display in Table 4 we will discuss several different possible outcomes for the set of buyer reservation values as a tool for explicating the properties of the auction model and its relation to the likely outcomes from contract negotiations:

Case 1. In the first event there is no buyer with a reservation value greater than zero: the producer will not agree to install capacity for a price less than the total marginal cost, 0.5, and therefore the quantity sold is zero.

Case 2. In the second, third and fourth events there is only one buyer with a reservation value greater than or equal to the total marginal cost and therefore there is no competition driving the price above the minimum: as stated in equation 6 when this is the case the price is set at the minimum reservation price above the marginal cost and therefore the modified-Vickrey price is one.

Case 3. In the fifth event there are two buyers with reservation values equal to one and the price is therefore competed up to one but the producer cannot charge a higher price.

Table 4

Example 1: Display of Event Space of Possible Buyer Valuations,
the Outcome of the Modified Vickrey Auction for each Event,
and the Expected Total Profit

Event	# buyers w/ reserv. val.				prob.	price	total quantity sold	profit	prob. weighted profit
	0	1	2	3					
e	η_1	η_2	η_3	η_4	$g(\eta_1, \dots, \eta_4)$	P_m	$\sum_{i=1}^3 q_i$	π_e	$g\pi_e$
1	3	0	0	0	.01563	1	0	0	0
2	2	1	0	0	.04688	1	1	.5	.02
3	2	0	1	0	.04688	1	1	.5	.02
4	2	0	0	1	.04688	1	1	.5	.02
5	1	2	0	0	.04688	1	1	.5	.02
6	1	1	1	0	.09375	1.6	1	1.1	.10
7	1	1	0	1	.09375	1.6	1	1.1	.10
8	1	0	2	0	.04688	2	1	1.5	.07
9	1	0	1	1	.09375	2.5	1	2.0	.19
10	1	0	0	2	.04688	3	1	2.5	.12
11	0	3	0	0	.01563	1	1	.5	.01
12	0	2	1	0	.04688	1.6	1	1.1	.05
13	0	2	0	1	.04688	1.6	1	1.1	.05
14	0	1	2	0	.04688	2	1	1.5	.07
15	0	1	1	1	.09375	2.5	1	2.0	.19
16	0	1	0	2	.04688	3	1	2.5	.12
17	0	0	3	0	.01563	2	1	1.5	.02
18	0	0	2	1	.04688	2.5	1	2.0	.10
19	0	0	1	2	.04688	3	1	2.5	.12
20	0	0	0	3	.01563	3	1	2.5	.04
Expected Profits = 1.43									

Table 4: The example has the following parameters. There is a maximum capacity of one unit, $Q=1$; the capital cost for installation of the capacity is 0.5/unit, $k=.5$; the operating costs are zero, $v=0$; there are three potential buyers, $N=3$, each with a reservation price which may be chosen from a range of four reservation values between 0 and 3, $\ell=4$ and $\{R_1, \dots, R_4\} = \{0, 1, 2, 3\}$.

Definitions for the variables heading the columns are as follows:

$\eta_i = |\{r_j | r_j = R_i\}|$, the total number of buyers with valuation R_i .

Case 4. In the sixth event there is one buyer with a reservation value equal to two and one with a reservation value equal to one: competition in this event will always drive the price up at least to one. The buyer with the higher reservation value may be forced in some cases to bid a price greater than one in order to obtain the unit of supply, and therefore for this event the expected price in the modified-Vickrey auction is set slightly above one.

Case 5. In the eighth event there are two buyers with reservation values of two and they therefore compete the price up to two.

Example 2.

Table 5 below displays the full set of possible outcomes for an auction in which the manufacturer can produce a maximum of 2 units at marginal fixed and variable costs $k=.5$ and $v=0$, in which there are 3 potential buyers, each with possible reservation values $(R_1, \dots, R_4) = (0, 1, 2, 3)$.

[Insert Table 5]

Example 2 is therefore identical with example 1 except that the capacity of the producer has been expanded from one unit to two units. We can compare examples one and two to see how the relationship between the capacity and the number of buyers affects the outcome of the auction. Events numbered 7 and 8 in both Tables 4 & 5 illustrate this relationship. In both of these events the number of buyers with reservation values greater than marginal cost is two. In Table 4 the resultant prices for these two events were 1.6 and 2. This was due to competition between the two buyers for the one unit of output. In events 7 & 8 in Table 5, although there are two buyers with reservation values above marginal cost, there are also two units of capacity available for sale and therefore there is no competition driving the price up above the minimal reservation value greater than the cost of production, one.

Example 3.

Table 6 displays the results for this example with $Q=2$ under the assumption that the manufacturer installs the capacity and sells the output spot. In Table 6, the sixth column lists the operating profits for each event, i.e. the profits from the spot sale when the sunk capital costs are disregarded. In the seventh and eighth columns the fixed costs are allocated to each event and the net profits and probability weighted net profits for each event are calculated. The total expected profit is the sum of the entries in column eight and is displayed at the bottom of the column.

[Insert Table 6]

A comparison of the results from Table 5 with the results from Table 6 illustrates the difference in profits that a seller can anticipate from using forward versus spot contracts. For example, in events 2-4, there is only one buyer with a valuation above the marginal cost, $k+v$. When the firm is negotiating forward contracts, i.e. as displayed in Table 5, it sells only one unit and incurs the capital cost only for the installation of one unit of capacity. When the firm calculates the expected results from installing the capacity and negotiating spot sales, i.e. as displayed in Table 6, the firm has already incurred the capital cost of installing one unit of capacity and cannot in these cases earn a price which covers the fixed costs. In events 5-10, there are two buyers with valuations greater than the marginal costs of production, inclusive of capital cost. When the firm is negotiating forward contracts, i.e. as displayed in Table 5, it installs the capacity to sell to

Table 5

Example 2: Auction Results for Forward Contracting

Event e	# buyers w/ reserv. val.				prob. $g(\eta_1, \dots, \eta_4)$	price p_m	total quantity sold $\sum_{i=1}^3 q_i$	profit π_e	prob. weighted profit $g\pi_e$
	η_1	η_2	η_3	η_4					
1	3	0	0	0	.01563	1	0	0	0
2	2	1	0	0	.04688	1	1	.5	.02
3	2	0	1	0	.04688	1	1	.5	.02
4	2	0	0	1	.04688	1	1	.5	.02
5	1	2	0	0	.04688	1	2	1.0	.05
6	1	1	1	0	.09375	1	2	1.0	.09
7	1	1	0	1	.09375	1	2	1.0	.09
8	1	0	2	0	.04688	1	2	1.0	.05
9	1	0	1	1	.09375	1	2	1.0	.09
10	1	0	0	2	.04688	1	2	1.0	.05
11	0	3	0	0	.01563	1	2	1.0	.02
12	0	2	1	0	.04688	1	2	1.0	.05
13	0	2	0	1	.04688	1	2	1.0	.05
14	0	1	2	0	.04688	1.6	2	2.2	.10
15	0	1	1	1	.09375	1.6	2	2.2	.21
16	0	1	0	2	.04688	1.6	2	2.2	.10
17	0	0	3	0	.01563	2	2	3.0	.05
18	0	0	2	1	.04688	2	2	3.0	.14
19	0	0	1	2	.04688	2.7	2	4.4	.21
20	0	0	0	3	.01563	3	2	5.0	.08
$\pi_f = \sum_{e=1}^{20} g\pi_e = 1.49$									

Table 5: The example has the following parameters. There is a maximum capacity of two units, $Q=2$; the capital cost for installation of the capacity is 0.5/unit, $k=.5$; the operating costs are zero, $v=0$; there are three potential buyers, $N=3$, each with a reservation price which may be chosen from a range of four reservation values between 0 and 3, $\ell=4$ and $\{R_1, \dots, R_4\} = \{0, 1, 2, 3\}$. Definitions for the variables heading the columns are as defined for Table 4.

Table 6

Example 3: Auction Results for Spot Sales

Event	η_1	η_2	η_3	η_4	$g(\eta_1, \dots, \eta_4)$	p_m	$\sum_{i=1}^2 q_i$	ψ_e	$g\psi_e$	$\psi_e - 2k$	$g(\psi_e - 2k)$
1	3	0	0	0	.01563	0	2	0	0	-1	-.02
2	2	1	0	0	.04688	0	2	0	0	-1	-.05
3	2	0	1	0	.04688	0	2	0	0	-1	-.05
4	2	0	0	1	.04688	0	2	0	0	-1	-.05
5	1	2	0	0	.04688	0.6	2	1.2	.06	0.2	.01
6	1	1	1	0	.09375	0.6	2	1.2	.11	0.2	.02
7	1	1	0	1	.09375	0.6	2	1.2	.11	0.2	.02
8	1	0	2	0	.04688	0.6	2	1.2	.06	0.2	.01
9	1	0	1	1	.09375	0.6	2	1.2	.11	0.2	.02
10	1	0	0	2	.04688	0.6	2	1.2	.06	0.2	.01
11	0	3	0	0	.01563	1	2	2.0	.03	1	.02
12	0	2	1	0	.04688	1	2	2.0	.09	1	.05
13	0	2	0	1	.04688	1	2	2.0	.09	1	.05
14	0	1	2	0	.04688	1.6	2	3.2	.15	2.2	.10
15	0	1	1	1	.09375	1.6	2	3.2	.30	2.2	.21
16	0	1	0	2	.04688	1.6	2	3.2	.15	2.2	.10
17	0	0	3	0	.01563	2	2	4.0	.06	3	.05
18	0	0	2	1	.04688	2	2	4.0	.19	3	.14
19	0	0	1	2	.04688	2.7	2	5.4	.25	4.4	.21
20	0	0	0	3	.01563	3	2	6.0	.09	5	.08

$$\psi_s = \sum_{e=1}^{20} g\psi_e = 1.92$$

$$\pi_s = \psi_s - 2k = .92$$

$$\Gamma = \pi_f - \pi_s = .57$$

$$\% \Delta = -38\%$$

Table 6: The example has the following parameters identical with those used in example 2. There is a maximum capacity of two units, $Q=2$; the capital cost for installation of the capacity is 0.5/unit, $k=.5$; the operating costs are zero, $v=0$; there are three potential buyers, $N=3$, each with a reservation price which may be chosen from a range of four reservation values between 0 and 3, $\ell=4$ and $\{R_1, \dots, R_4\} = \{0, 1, 2, 3\}$. Example 3 differs from example 2 only in that the determination of price and quantity allocation is for spot sales.

Definitions for the variables heading the columns are as defined for Table 4 except for the following addition:

$$\psi_e = \sum_{i=1}^2 [p_m - v] q_i \quad \text{profits on operations, exclusive of capital expenditures}$$

these buyers only because the buyers agree to a price greater than the costs of production. When the firm calculates the expected results from installing the capacity and negotiating spot sales, i.e. as displayed in Table 6, it has installed capacity and cannot therefore force the price up to cover the marginal costs of production; the price is determined exclusively by competition and in these events competition does not drive the price above the costs of production. In the remaining events the price is determined exclusively by competition both for forward contracting and for spot contracting, and therefore the results are identical for the two cases.

In Table 6 the expected profits from selling spot are calculated and then compared with the expected profits from selling forward as exhibited in Table 5. In this example the expected profits from the operations decline by more than 38% when the producer fails to secure the forward contracts for its output prior to incurring the capital costs, from 1.49 to 0.92.

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