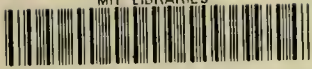


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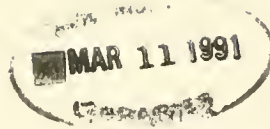


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An Empirical Investigation of Asset Pricing  
with Temporally Dependent Preference Specifications

by  
John Heaton

WP. #3245-91-EFA

February 1991

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AN EMPIRICAL INVESTIGATION OF ASSET PRICING  
WITH TEMPORALLY DEPENDENT PREFERENCE SPECIFICATIONS

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February 1991

Comments Invited

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ABSTRACT

In this paper, I empirically investigate a representative consumer model in which the consumer is assumed to have temporally dependent preferences of several forms. Comparing the model's implications for asset prices to those of a time-additive model, I find substantial evidence for the local substitutability of consumption with habit formation occurring over longer periods of time. Neither local substitution nor habit formation alone is found to be significant, so that the interaction between the two effects is quite important. Although the model provides a better explanation of the behavior of asset prices, the model is still statistically rejected. A potential explanation for this rejection is that with local substitution the Hansen and Jagannathan (1990, 1991) bounds on the moments of the marginal rate of substitution, can be fit only with extreme curvature in the period utility function.

Keywords: Asset Pricing, Local Substitution, Habit Persistence, Temporal Aggregation

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## I. INTRODUCTION

In this paper, I empirically investigate a representative consumer model in which the consumer is assumed to have temporally dependent preferences of several forms. Comparing the model's implications for asset prices to those of a time-additive model, I find substantial evidence in favor of a preference structure under which consumption at nearby dates is substitutable (called *local substitution*) and where habit over consumption levels develops slowly. Neither local substitution nor habit persistence alone is found to be significant, so that the interaction between the two effects is quite important. Although the model provides a better explanation of observed asset prices, the model is still statistically rejected. A potential explanation for this rejection is that the local substitution that is needed to fit many of the moments of asset prices causes difficulties on other dimensions. In particular, to fit estimated Hansen and Jagannathan (1991) bounds on the moments of the marginal rate of substitution, the local substitutability of consumption can be maintained only if extreme curvature in the period utility function is allowed.

The possibility that temporal dependencies in preferences could help explain some of the poor performance of asset pricing models has been suggested by a number of authors. For example, Constantinides (1990) argued that by specifying a preference structure where consumption is complementary over time, the equity premium puzzle of Mehra and Prescott (1985) could be solved<sup>1</sup>. In an empirical investigation of the Euler equations implied by this model, Ferson and Constantinides (1990) found evidence in favor of the model, using quarterly and annual data<sup>2</sup>.

In the model analyzed by Constantinides (1990), habit persistence was

used to make consumption complementary over time. One problem with this type of specification is that it violates the notion that consumption at dates *local* to time  $t$  should be relatively substitutable for consumption at time  $t$ . Huang and Kreps (1987), and Hindy and Huang (1989) argued in favor of making consumption locally substitutable. In a continuous-time environment, Hindy and Huang (1989) showed that when consumption is locally substitutable, asset prices will be "smooth" if there is no new arrival of information. This occurs even if the consumption endowment is very erratic. This seems to correspond quite well with our intuition about the behavior of prices over short periods of time. In fact, using monthly consumption and asset market data, Dunn and Singleton (1986), Eichenbaum and Hansen (1989), Gallant and Tauchen (1989) and Ogaki (1988), all found evidence in favor of consumption being *locally* substitutable.

*Temporal aggregation* is one potential explanation for the fact that studies using monthly data find evidence for local substitution whereas using quarterly data Ferson and Constantinides (1990) found evidence for consumption being complementary over time. In studies that use aggregate consumption data, temporal aggregation is an important problem because observed consumption, whether it be at monthly, quarterly or annual frequencies, is an average of consumption expenditures over a period of time. In a study of consumption dynamics, Heaton (1990) showed that temporal aggregation and temporal dependencies in preferences can interact in an important way. For example, if consumption is locally substitutable, as the length of the time averaging increases (for example going from monthly to quarterly data), it is possible for the data to suggest that preferences are time additive or even that there is habit formation. Of course, a different explanation is that consumption is locally

substitutable, but habit develops much more slowly<sup>3</sup>. In this case, it is possible that when we examine low frequency data, the habit effects will dominate.

A goal of this paper is to try to sort out the importance of all of these different effects. In studying these issues I investigated a model in which consumption is *locally substitutable*. I modeled this by assuming that the consumption good is durable. Habit was modeled as developing over the flow of services from the consumption good and, as a result, habit over consumption itself develops much more slowly. I was careful to take account of the fact that observed consumption is time averaged.

I found that habit persistence substantially improves the fit of stock and bond returns *only if local substitution is also present*. The estimated parameters of the model imply that consumption is relatively substitutable over a period of about a year, with habit effects dominating beyond that point. However, imposing the requirement that consumption be locally substitutable implies that a high degree of curvature in the period utility function is needed to fit Hansen and Jagannathan (1990, 1991) bounds on the moments of the marginal rate of substitution. This difficulty may explain why the model is still statistically rejected by the data.

The rest of the paper is organized as follows. In section 2, I outline the preference structures that are examined throughout the rest of the paper. In section 3, I present the asset pricing environment. In section 4, I use a diagnostic proposed by Hansen and Jagannathan (1990) to examine whether the model of section 3 fits some aspects of the asset market when local substitution in consumption is maintained. In section 5 I present the results of formal estimation of the entire asset pricing model. Section 6 concludes the paper.

## 2. PREFERENCES

In this section, I outline the class of preferences that will be used throughout the rest of the paper. The preferences of the representative consumer are assumed to display simple forms of temporal dependence. To model the temporal dependence in preferences, assume that the preferences of the representative consumer are time additive over a potentially fictitious good called *services*. Let services at time  $t$  be given by  $s(t)$  and  $s \equiv \{s(t): t=0,1,2, \dots\}$ . Preferences over  $s$  are then assumed to be given by the utility function:

$$(2.1) \quad U(s) = E\left\{\sum_{t=0}^{\infty} \beta^t u(s(t))\right\}.$$

Assume further that the period utility function is characterized by constant relative risk aversion, so that  $U(s)$  is given by:

$$(2.2) \quad U(s) = E\left\{\sum_{t=0}^{\infty} \beta^t \frac{s(t)^{1-\gamma} - 1}{1-\gamma}\right\}, \quad \gamma > 0.$$

The constant relative risk aversion parameterization of  $u(\cdot)$  is convenient in this investigation because it makes it possible to use simple transformations of the data that induce stationarity.

To induce temporal dependencies in the consumer's preferences over *consumption goods*, services are assumed to be linked to current and past consumption via:

$$(2.3) \quad s(t) = \sum_{j=0}^{\infty} a(j)c(t-j).$$



I use several different parametric examples of the function  $\{a(j)\}$  in later sections:

1. *Durable goods subject to exponential depreciation.*

Suppose that the consumption good is durable and depreciates exponentially. In this case let  $a(j) = \Delta^j$  where  $0 < \Delta < 1$  and  $s(t)$  is given by<sup>4</sup>:

$$(2.4) \quad s(t) = \sum_{j=0}^{\infty} \Delta^j c(t-j).$$

Note that this specification makes the consumption good substitutable over time. This specification of durability was also used, for example, by Dunn and Singleton (1986), Eichenbaum and Hansen (1990) and Ogaki (1988). In sections 3, 4 and 5 I assume that the time interval between decisions taken by the consumer is very short. This allows me to account for the potential of temporal aggregation. When the interval between decisions is very short, it seems reasonable that consumption at adjacent dates will be very substitutable. In fact, the continuous-time limit of this preference structure satisfies the continuity restriction of Huang and Kreps (1987) and Hindy and Huang (1989). This continuity restriction captures the notion that consumption at relatively adjacent dates should be substitutable.

2. *Habit Persistence.*

Suppose that the consumer cares about the level of consumption today relative to an average of past consumption so that the consumer develops an acceptable level of consumption over time. *Habit persistence* of this form

has been studied, for example, by Pollack (1970), Ryder and Heal (1973), Sundaresan (1989), Constantinides (1990) and Detemple and Zapataro (1990). Following Constantinides (1990) I model habit persistence by assuming that  $s(t)$  is given by:

$$(2.5) \quad s(t) = c(t) - \alpha(1-\theta)\sum_{j=0}^{\infty}\theta^j c(t-1-j), \quad 0 < \alpha < 1, \quad 0 < \theta < 1.$$

Note that the term  $(1-\theta)\sum_{j=0}^{\infty}\theta^j c(t-1-j)$  is a weighted average of past consumption, and  $\alpha$  gives the proportion of this average that is compared to current consumption. The process  $x(t) \equiv (1-\theta)\sum_{j=0}^{\infty}\theta^j c(t-1-j)$  is referred to as the *habit stock*. The introduction of habit persistence effects makes consumption complementary over time.

### 3. Habit Persistence with local durability

With short decision intervals, the preference specification given by (2.2) and (2.5) violates the notion that consumers can readily substitute consumption at adjacent dates. In fact, Hindy and Huang (1989) showed that preferences with habit persistence do not satisfy their continuity restrictions. Instead it seems reasonable that habit may develop more slowly and that the consumer is willing to substitute consumption local to time  $t$ , for  $c(t)$ . One way to model this is to assume that the good is durable and that the consumer develops habit over the flow of services from the durable. In other words:

$$(2.6) \quad s(t) = \hat{s}(t) - \alpha(1-\theta)\sum_{j=0}^{\infty}\theta^j \hat{s}(t-1-j), \quad 0 < \alpha < 1, \quad 0 < \theta < 1,$$

where

$$(2.7) \quad \hat{s}(t) = \sum_{j=0}^{\infty}\Delta^j c(t-j), \quad 0 < \Delta < 1.$$



As I demonstrate in Sections 4 and 5, it is possible for this parameterization to capture local substitution with long-run habit persistence. For example, if  $\Delta < \Theta$ , at longer lags the effect of past consumption on preferences due to the durability of the good will die out and the habit persistence effects will dominate. In this sense, the durability would be local.

In the next section I discuss the asset pricing implications of this type of model.

### 3. ASSET PRICING MODEL

Suppose now that the representative consumer of section 2 is faced with a random payoff of  $y(t+1)$  units of consumption at time  $t$ . Then the time  $t$  price of this payoff is given by:

$$(3.1) \quad p(t) = E \left\{ \beta \frac{muc(t+1)}{muc(t)} y(t+1) \mid \mathcal{F}(t) \right\}$$

where  $muc(t)$  is the marginal utility of consumption at time  $t$ . For the preference structure given by (2.2) and (2.3), the marginal utility of consumption at time  $t$  is given by:

$$(3.2) \quad muc(t) = E \left\{ \sum_{\tau=0}^{\infty} \beta^{\tau} a(\tau) s(t+\tau)^{-\gamma} \mid \mathcal{F}(t) \right\}.$$

Because of the presence of the conditional expectation in (3.2), testing the Euler equation given in (3.1) is somewhat difficult. However if

observed consumption is used to proxy for consumption in the model, and the good has a finitely lived impact on services, then it is possible to use the restrictions embodied in (3.1) and (3.2) directly. Examples of this approach can be found, for example, in Dunn and Singleton (1986), Eichenbaum and Hansen (1989), and Ferson and Constantinides (1990)<sup>5</sup>. Gallant and Tauchen (1989) follow a different approach and use a nonparametric expansion of the utility function to estimate the degree of dependence in preferences.

However, Heaton (1990) presents evidence that temporal aggregation can have important effects upon the estimation of the temporal dependencies in preferences. The temporal aggregation problem occurs in this context because consumers make decisions at intervals much finer than the observed frequency of the data and the observed consumption data consists of averages of consumption over a period of time. This issue is important within the context of models with temporally dependent preferences because time-averaging consumption induces positive autocorrelation in consumption growth rates. Without accounting for temporal aggregation, this "smoothness" in consumption could spuriously be interpreted as being induced by habit persistence (see Heaton (1990)). Grossman, Melino and Shiller (1987), Hall (1988), and Hansen and Singleton (1990) also show that it is important to account for temporal aggregation within the context of models with time additive preferences<sup>6</sup>.

Given this evidence, I will assume that the interval between decisions is very short: one fourth of a month (i.e. a week). To handle the complications created by this assumption, a more complete specification of the economic environment is needed than in Euler equations investigations (as in Ferson and Constantinides (1990), for example). However, these additional assumptions create an advantage.

*Keeping muc(t) nonnegative*

The marginal utility of consumption in models with temporally dependent preferences involves forward looking terms as in (3.2). In Euler equation investigations where a complete specification of the data generation mechanism is not necessary, it is not possible to verify that the marginal utility of consumption is well behaved. This is a potential problem in models with habit persistence because the marginal utility of consumption at time  $t$  involves expectations of future marginal utilities of services with *negative weights*. Without proper restriction on the preference parameters the estimated utility function could be *decreasing* in consumption, at the observed consumption levels.

For example suppose that services at time  $t$  are given by:

$$(3.3) \quad s(t) = c(t) - \alpha c(t-1), \quad \alpha > 0 .$$

Then the marginal utility of consumption at time  $t$  is:

$$(3.4) \quad muc(t) = s(t)^{-\gamma} - \alpha \beta E\{s(t+1)^{-\gamma} | \mathcal{F}(t)\} .$$

Suppose further that the decision interval of the consumer is a quarter so that we can use observed quarterly expenditures on consumption to evaluate the model. Under the assumption that  $c(t)/c(t-1)$  is stationary, the process:

$$(3.5) \quad muc^*(t) \equiv muc(t)/c(t)^{-\gamma}$$

is stationary. Notice that  $muc(t)$  is nonnegative if and only if  $muc^*(t)$  is nonnegative since  $c(t) \geq 0$ .

To illustrate the potential problems, suppose that  $\gamma=1$ ,  $\beta=0.95^{1/4}$  and  $\alpha = 0.9$ . Setting  $\alpha = 0.9$  is admittedly extreme, as it was chosen to dramatically illustrate the potential problems. However, Ferson and Constantinides (1990) often obtain parameter estimates for  $\alpha$  that are of this order of magnitude.

Using these parameter values the process:

$$(3.6) \quad \tilde{muc}(t) \equiv [s(t)/c(t)]^{-\gamma} - \alpha\beta [s(t+1)/c(t)]^{-\gamma}$$

is plotted in figure 3.1. The consumption data consists of quarterly observations of per capita, seasonally-adjusted aggregate expenditures on nondurables from 1950 through 1989, taken from CITIBASE<sup>7</sup>. Notice that  $\tilde{muc}(t)$  is highly erratic and is negative quite often. For the entire sample  $s(t)$  is positive so that the negative values of  $\tilde{muc}(t)$  are due to the forward looking term in (3.6). Because of the negative values of  $\tilde{muc}(t)$ , the chosen parameter values may not be consistent with a well defined utility function if observed consumption corresponds to the optimal consumption choice made by the representative consumer when consumption is freely disposable. However, the object of interest is  $muc^*(t) = E\{\tilde{muc}(t); \mathcal{F}(t)\}$ .

As a proxy for the conditional expectations needed to evaluate  $muc^*(t)$ , consider:

$$(3.7) \quad \hat{muc}(t) \equiv [s(t)/c(t)]^{-\gamma} - \alpha\beta Proj\{ [s(t+1)/c(t)]^{-\gamma} ; H(t) \}$$

where  $Proj$  is the projection operator.  $H(t)$  is taken to be the linear space generated by a constant,  $\{c(t-j)/c(t-1-j)\}_{j=0}^2$ ,  $\{R^f(t-j)\}_{j=0}^2$  and  $\{R^{vw}(t-j)\}_{j=0}^2$ , where  $R^f(t)$  is the real return on a 3 month treasury bill and  $R^{vw}(t)$  is the quarterly real return from the CRSP value-weighted portfolio. Nominal returns were converted to real returns using the quarterly implicit price deflator for nondurables. The parameters governing  $Proj\{[s(t+1)/c(t)]^{-\gamma} : H(t)\}$  were estimated using ordinary least squares using the entire sample period of the data.

Figure 3.2 plots the resulting realized values of  $\hat{muc}(t)$ . Notice that several values of  $\hat{muc}(t)$  are negative. To the extent that the estimated linear projection is proxying for the true conditional expectation, these chosen parameters cannot be consistent with the data and should not be part of the admissible parameter space. Although there are relatively few observations of the marginal utility process that are negative, the negative observations are potentially important as can be seen in the plot of the realized values of the marginal rate of substitution as given in figure 3.3. Notice that there are several outliers that are very negative. These results suggest that it may be important to restrict the estimation of these models such that the implied utility function is nondecreasing in consumption. Only with a complete description of the economic environment (as is done in this paper) is it possible to determine whether the imposition of the restriction  $muc(t) \geq 0$  is important or not.

*The Economic Environment: Endowments and Asset Payoffs*

To model the economic environment more completely, assume that the consumer is embedded in an endowment economy with some exogenously given law of motion for endowments. In particular suppose that consumption, the durable stock, the habit stock and the asset payoffs at time  $t$  are elements of a markov process  $\{X(t)\}_{t=0}^{\infty}$ . The time  $t$  price [denoted  $p(t)$ ] of a sequence of payoffs  $\{d(\tau)\}_{\tau=t+1}^{\infty}$  is then characterized by:

$$(3.8) \quad p(t) = E \left\{ \beta \frac{muc(t+1)}{muc(t)} [p(t+1) + d(t+1)] \mid X(t) \right\} .$$

Given a law of motion for  $\{X(t)\}$ , the price of the asset at time  $t$  as a function of  $X(t)$  can be found by finding a fixed point to (3.8) as in Lucas (1978).

Although it would be possible to use this framework to price a large number of assets, it is of interest to first see if the model is capable of pricing a few "aggregate" returns. As a result, I will consider pricing the dividend stream embedded in the CRSP value weighted return, and a real discount bond<sup>8</sup>. Assume that the law of motion for the logarithm of real consumption and dividend growth rates is given by a two dimensional Gaussian process  $\{Y(t)\}_{t=0}^{\infty}$ .

Monthly observations of the logarithm of the real dividend growth rate are highly seasonal. To model this seasonal, I assumed that the seasonal component of dividends can be modeled by using seasonal dummies and a seasonal autoregressive polynomial. In particular, I assumed that consumption and dividends evolve according to:



$$(3.9) \quad Y(t) = \begin{bmatrix} 1 & 0 \\ 0 & \alpha_s(L) \end{bmatrix}^{-1} \Gamma(L)\varepsilon(t) + \begin{bmatrix} 0 \\ \mu^s \end{bmatrix} D(t) + \mu$$

where:

$$Y(t) \equiv \begin{Bmatrix} \log[c(t)/c(t-1)] \\ \log[d(t)/d(t-1)] \end{Bmatrix},$$

$\Gamma(L)$  is a 2 by 2 matrix of polynomials in the lag operator,  $\alpha_s(L)$  is a (seasonal) polynomial in the lag operator,  $\{\varepsilon(t)\}_{t=0}^{\infty}$  is a 2 by 1 sequence of iid Gaussian shocks ( $E(\varepsilon(t)\varepsilon(t)') = I$ ),  $D(t)$  is a sequence of seasonal dummies,  $\mu^s = [\mu_1^s \mu_2^s \cdots \mu_{48}^s]$ , and  $\mu$  is a vector of means<sup>9</sup>.

The observed data consists of averages of consumption and dividends over different periods<sup>10</sup>. The observation interval that I used was monthly since this is the finest observation interval for aggregate consumption data. Dividend data is available at very high frequencies. However, I used the data at monthly frequencies to avoid having to model finer details like the number of weeks in a month.

To estimate (3.9) I used a simulated method of moments estimator as discussed by Ingram and Lee (1987) and Duffie and Singleton (1989). Although the model could be fit using Maximum Likelihood techniques, the simulation method is more convenient to use in this context because fitting the asset pricing implications of the model requires simulation as well. The data consists of observations of  $\{c^a(t), d^a(t)\}$  for  $t=0, 4, 8, \dots$  (remember I am assuming a weekly model) where, for example:

$$(3.10) \quad c^a(t) \equiv \sum_{\tau=0}^3 c(t+\tau).$$

The simulation estimator could be implemented by first simulating a path for consumption and dividend levels using (3.9), then averaging and computing the moments of the simulated ratios which would be "matched" to the data. Application of this technique resulted in very severe numerical instabilities. This occurred because a slight misspecification of the growth properties of consumption (for example) resulted in very large values of consumption levels along the simulated path. To avoid this problem, I followed Grossman Melino and Shiller (1987), Hall (1988) and Hansen and Singleton (1990) and assumed that geometric averages are reasonable approximations to arithmetic averages. In other words I assumed, for example, that  $\sum_{\tau=0}^3 \{ \log(c(t+\tau) - \log(c(t-4+\tau)) \} \approx \log(c^a(t)) - \log(c^a(t-1))$ .

The observed data is of the form:

$$(3.11) \quad Y^a(t) \equiv \begin{bmatrix} \log(c^a(t)) - \log(c^a(t-4)) \\ \log(d^a(t)) - \log(d^a(t-4)) \end{bmatrix}$$

for  $t = 0, 4, 8, \dots$ . The first set of moments I fit were the twelve seasonal means of log dividend growth, and the mean of consumption growth. Because all of the seasonal dummies in the weekly model of (3.9) cannot be identified, I assumed that only the dummy for the first week of each month is non zero. Under this assumption, the seasonal dummies and the mean  $\mu$  were estimated directly without using simulation. The estimates of these parameters are given in table 3.1. The standard errors were estimated using the method of Newey and West (1987), with two years of lags, to correct for serial correlation in the error term of the regression.



I parameterized  $\Gamma(L)$  by setting

$$\Gamma(L) = \begin{bmatrix} \alpha_1(L) & 0 \\ 0 & \alpha_2(L) \end{bmatrix}^{-1} B(L)$$

where  $B(L)$  is a 2 by 2 matrix of first order polynomials in the lag operator and  $\alpha_1(L)$  and  $\alpha_2(L)$  are polynomials of order 2.  $\alpha_s(L)$  was parameterized by setting  $\alpha_s(L) = 1 - \alpha_s L^s$  where  $s$  was 48. These parameters were fit using the simulated method of moments estimator described above using 1344 observations in the simulated sample (approximately four times the observed sample size). Let  $\hat{Y}^a(t)$  denote the residual of  $Y^a(t)$  less estimated seasonal dummies and means. Similarly, let  $\hat{Y}(t)$  denote the residual of  $Y(t)$  less seasonal dummies and means. In fitting the parameters of  $\hat{Y}(t)$ , I used the following moments:  $E\{\hat{Y}^a(t)\hat{Y}^a(t)'\}$ ,  $E\{\hat{Y}^a(t)\hat{Y}^a(t-4)'\}$ ,  $E\{\hat{Y}^a(t)\hat{Y}^a(t-8)'\}$  and  $E\{\hat{Y}^a(t)\hat{Y}^a(t-12)\}$  along with the covariance of dividend growth with its values 12 and 13 months in the past. In estimating the weighting matrix used in the procedure, 1 year of lags in a Newey-West (1987) procedure were used.

A test of the over identifying restrictions yields a test statistic of 1.23 which implies a P-Value of 0.873. As a result, the model is doing a reasonable job of fitting the chosen moments.<sup>10</sup> Given this law of motion for the exogenous variables of the model, asset prices can now be calculated.

#### *Numerical Solution Method*

Even if the law of motion for consumption and dividend growth is taken as known, it is not possible to analytically calculate the marginal utility

of consumption or asset prices as a function of the state of the world at any given date. Instead, the approach taken in this paper is to calculate the marginal utility of consumption and asset prices numerically. There is a growing literature examining the properties of numerical solutions to dynamic economic models (see, for example, the group of papers in the January 1990 issue of the *Journal of Business and Economic Statistics*). The solution method that I chose was the method suggested by Judd (1990) which is based upon methods used to solve differential equations. I will provide a brief outline of the method I used.

Consider, for example, the marginal utility of consumption when the consumption good is subject to exponential depreciation. In this case, the marginal utility of consumption is given by:

$$(3.12) \quad muc(t) = E\left\{\sum_{\tau=0}^{\infty} \beta^{\tau} \Delta^{\tau} s(t)^{-\gamma} ; \mathcal{F}(t)\right\}.$$

Note that (3.12) is a solution to the difference equation:

$$(3.13) \quad muc(t) = s(t)^{-\gamma} + \beta \Delta E\{muc(t+1) ; \mathcal{F}(t)\}.$$

The difference equation (3.13) seems to be relatively simple to solve. However the problem is that observed consumption is growing over time which implies that the service flow from consumption is growing over time. To handle this problem, normalize (3.13) by  $c(t)^{-\gamma}$  to give:

$$(3.14) \quad muc^*(t) = s^*(t)^{-\gamma} + \beta \Delta E\{muc^*(t+1) [c(t+1)/c(t)]^{-\gamma} ; \mathcal{F}(t)\}$$

where  $muc^*(t) \equiv muc(t)/c(t)^{-\gamma}$ . A unique stationary solution to (3.14) exists

if  $-\gamma E\{\log[c(t+1)/c(t)]\} > \beta\Delta$  [see Cochrane (1989), for example].

In the model under consideration, information at time  $t$ ,  $\mathcal{F}(t)$ , is generated by a finite-dimensional state vector,  $X(t)$ , that includes the stock of durables, the habit stock, current and lagged values of consumption and dividend growth, and lagged values of the shock process  $\{\varepsilon(t)\}$ . The solution to (3.14) was approximated by a linear combination of finite-order polynomials in the state vector  $X(t)$ . Let  $\Gamma_n(X(t))$  denote a vector of polynomials of elements of  $X(t)$  of order less than or equal to  $n$ . Let  $\phi_n$  denote a vector of weights with the same dimension as  $\Gamma_n(X(t))$ . Then approximate  $muc^*(t)$  by:  $\tilde{muc}^*(t) \equiv \phi_n' \Gamma_n(X(t))$ . Of course  $\tilde{muc}^*(t)$  will not satisfy (3.14) exactly. Define a residual vector:

$$(3.15) \quad \zeta[\phi_n; X(t)] \equiv \tilde{muc}^*(t) - s^*(t)^\gamma - \Delta \beta E\{\tilde{muc}^*(t+1)[c(t+1)/c(t)]^{-\gamma} | \mathcal{F}(t)\}.$$

Then the parameters  $\phi_n$  are chosen such that  $\zeta[\phi_n; X(t)]$  is orthogonal to the polynomials in the state vector. That is choose  $\phi_n$  such that:

$$(3.16) \quad E\{\zeta[\phi_n; X(t)] \Gamma_n(X(t))\} = 0.$$

This method was applied repeatedly for the different examples of temporal dependencies considered in this paper. The method was also used to calculate asset prices. In the calculations presented in sections 4 and 5, I used second order polynomials to approximate the various functions<sup>11</sup>. To calculate the inner product given in (3.16) I created a times series with 1000 observations of  $X(t)$ . Sample averages were then used to approximate the expectation given in (3.16).

#### 4. HANSEN AND JAGANNATHAN DIAGNOSTICS

Before turning to the results of formal estimation and testing of the model of section 2, I will first present some simple implications of the model. In this section, I use a diagnostic developed by Hansen and Jagannathan (1990, 1991) which examines restrictions on moments of the marginal rate of substitution implied by asset market data. In Section 5 the results of estimation of the complete model are presented. The diagnostics presented in this section are useful before turning to these results because they aid in determining the effects on the asset pricing environment of the different parts of the preference structure. Also the diagnostics are useful as a way of determining whether the numerical solution algorithm discussed in Section 3 is performing reasonably well or not.

In examining whether the different forms of temporal dependencies can fit the bounds on the moments of the marginal rate of substitution implied by asset market data, it is important to examine whether the fit is due to habit persistence that is local in nature. As I argued in the sections 1 and 2, it seems reasonable to restrict the preference specifications so that consumption is locally substitutable. In fact, the parameter estimates found by Heaton (1990) imply that habit persistence fits the consumption data *only* if local substitution in consumption is present. A similar conclusion is reached in Section 5. The preference parameters estimates obtained by Heaton (1990) will be useful in guiding the analysis in this section.

Consider an asset that pays off  $y(t+4)$  units of consumption at time  $t+4$ . Then the price of the asset in units of consumption at time  $t$ ,  $q(t)$ , satisfies:

$$(4.1) \quad q(t) = E\{\beta^4[muc(t+4)/muc(t)]y(t+4)|\mathcal{F}(t)\}$$

where  $muc(t)$  is the marginal utility of consumption at time  $t$ . Write (4.1) as:

$$(4.2) \quad q(t) = E\{mrs(t+4)y(t+4)|\mathcal{F}(t)\}$$

where  $mrs(t+4) = \beta^4[muc(t+4)/muc(t)]$ .

Hansen and Jagannathan (1990, 1991) (hereafter H-J) show how to use observed asset prices and payoffs to estimate regions in which the mean and standard deviation for the process,  $mrs$ , must lie. They show that given a mean for the marginal rate of substitution, it is possible to estimate a lower bound for the standard deviation of the marginal rate of substitution. By changing the hypothetical mean for the marginal rate of substitution, a region for the moments of the marginal rate of substitution is created<sup>12</sup>. After creating this region, H-J propose to test different models by asking whether the moments of the marginal rate of substitution predicted by the model lie within the region. This is the diagnostic I use.

In estimating the H-J region, I used the one month treasury bill return and the monthly CRSP value weighted return constructed from daily returns (from 1962,7 to 1989,12)<sup>13</sup>. These two returns were multiplied by their



lagged values to create 6 asset payoffs and prices to use in estimating the region. Figure 4.1 gives a plot of the resulting H-J region. To test whether the region has content, I tested whether the minimum value of the function plotted in figure 4.1. is zero<sup>15</sup>. This resulted in a P-Value of essentially zero indicating that the region of figure 4.1 has empirical content.

I now present results of calculating estimated moments for *mrs* for the different models using the law of motion for the exogenous variables estimated in section 3.

#### *Moments of MRS with Time-Additive Preferences*

As a benchmark, first consider the moments of the *mrs* implied by a time-separable model, where the preferences of the representative consumer are given by:

$$(4.3) \quad E\{\sum_{t=0}^{\infty} \beta^t [c(t)^{1-\gamma} - 1] / (1-\gamma)\}.$$

Further suppose that we ignore the temporal aggregation issues and set  $c(t)$  equal to the observed monthly consumption of nondurables plus services (ie. setting the time interval to one month and assuming time-separable preferences). The estimated first and second moments of the marginal rate of substitution are plotted (the ■ 's) in figure 4.2, for values of  $\gamma$  starting from  $\gamma=1$  at the point  $E(mrs) \cong 1$  and  $std(mrs) \cong 0$ , to  $\gamma=12$  on the left in increments of 1.  $\beta$  was assumed to be 1. Also plotted in figure 4.2 is the H-J region of figure 4.1. Note that the estimated points for the moments of *mrs* do not come close to being in the region as  $\gamma$  is increased.

This was noted by Hansen and Jagannathan (1991). As  $\gamma$  increases, the standard deviation of the  $mrs$  is increased but  $E\{mrs\}$  is pushed very much out of line.

Suppose that we now shift the time interval back to a week, maintain the time-additive preference specification and use the estimated weekly law of motion of section 3 to estimate the moments of  $muc(t)$ . Plotted in figure 4.3 is the H-J region along with the estimated moments for the marginal rate of substitution for the weekly model (the O's) and the monthly model (the ■'s as before). The plot for the weekly model again starts at  $\gamma=1$  with  $std(mrs) \cong 0$  and  $E(mrs) \cong 1$ , and moves in increments of 1 to  $\gamma=20$  at the point  $E(mrs) \cong 0.994$  and  $std(mrs) \cong 0.063$ . Again  $\beta$  was set to 1. Note that the second moments estimated for the  $mrs$  do not increase as quickly as in the case of using time averaged data, but that  $E(mrs)$  performs much better as a function of  $\gamma$ . However, accounting for temporal aggregation alone does not seem to provide a greatly improved fit. This could explain why investigations that account solely for temporal aggregation problems are not completely successful [see, for example, Grossman, Melino and Shiller (1987)].

#### *Moments of MRS with Pure Habit Persistence*

Constantinides (1990) argued that habit persistence can help to explain the equity premium puzzle of Mehra and Prescott (1985). As a result, it is of interest to investigate habit persistence in this context. Figure 4.4 plots moments for the habit persistence model given by (2.2) and (2.5) with  $\Theta = 0.9$  and  $\alpha = 0.5$  and using the weekly consumption law of motion as outlined above. The value of  $\Theta$  chosen is consistent with the estimated

half-life for habit effects estimated by Heaton (1990).  $\beta$  was set to 1. The triangles in figure 4.4 give the moments with habit persistence. The circles are the same as those in figure 4.3 (except that  $\gamma$  ranges only from 1 to 10).  $\gamma$  ranges from 1 to 20 with  $std(mrs)$  rising as  $\gamma$  rises.

Notice that for any given level of  $\gamma$ ,  $std(mrs)$  is much larger than in the case of no habit effects. Hence with less curvature in the period utility function, the required standard deviation of the marginal rate of substitution could be more easily fit. This finding is similar to the results reported by Gallant, Hansen and Tauchen (1989) using a monthly model and a monthly law of motion for the exogenous variables of the model.

Figure 4.5 gives the weighting function  $a(j)$  of (2.3), implied by these parameter settings. Notice that this parameter setting puts very small negative weight on each of the past consumptions. For any value of  $\gamma$  the volatility of the marginal rate of substitution can be increased by raising the absolute sum of these weights. This sum is given by the parameter  $\alpha$ . Another way of increasing the volatility of the marginal rate of substitution is to increase the absolute value of  $a(j)$  for small values of  $j$ . This can be done by decreasing  $\theta$ . Figure 4.6 gives a plot of the moments of the marginal rate of substitution similar to figure 4.4 except that  $\theta$  is set to 0.5. Notice that for a given value of  $\gamma$  the standard deviation of the marginal rate of substitution is increased. Figure 4.7 gives a plot of the  $a(j)$  function implied by these parameter setting.

A difficulty with the result of figure 4.6 is that it that the improvement in the fit of the moments of the marginal rate of substitution is to a large extent due to *local* habit persistence.



Consider now the specification of preferences given by (2.2), (2.6) and (2.7) in which the good is assumed to be durable and the consumer develops habit over the flow of services from this good. Heaton (1990) shows that the observed consumption dynamics are consistent with this specification but *not with pure habit persistence*. The specification given by (2.2), (2.6) and (2.7) is also consistent with the notion that consumption is locally substitutable.

I first set  $\beta = 1$ ,  $\Delta = 0.7$ ,  $\Theta = 0.9$  and  $\alpha = 0.5$ . The values for  $\Delta$  and  $\Theta$  are consistent with the parameter estimates found by Heaton (1990). Figure 4.8 plots estimates of the moments of the marginal rate of substitution under this preference specification. The squares give plots for the case of habit persistence with local durability as  $\gamma$  ranges from 1 to 20. Also included in figure 4.8 is the similar plot for the case of pure habit persistence (the triangles). Notice that with the local durability, the estimated moments are very different than in the case of pure habit persistence. By making the good locally substitutable, the standard deviation of the marginal rate of substitution is *much lower*.

At issue is whether the result of figures 4.4 and figure 4.6 play off of *local* habit. To examine this issue consider what happens as  $\alpha$  is increased. Figure 4.9 gives a plot similar to figure 4.8 except that  $\alpha$  ranges from 0.5 to 0.8. Again for each value of  $\alpha$ ,  $\gamma$  varies from 1 to 20. Notice that as  $\alpha$  increases, the model with local durability comes much closer to fitting the H-J region. However very dramatic values of  $\gamma$  are still needed to come close to the region.

In figure 4.10, I plot the weights on past consumption in the mapping

from consumption to services implied by the habit persistence with local durability model with  $\alpha = 0.8$ . Notice that the weighting function implies that there is local substitution with long-run habit. As the local substitution is removed from the model, the model performs better. For example, figures 4.11 and figure 4.12 given plots similar to figure 4.9 and 4.10 except with  $\Delta=0.5$   $\Theta = 0.51$ . As figure 4.12 demonstrates under this parameterization the habit persistence effects dominate quite quickly. In this case much lower values of  $\gamma$  are needed to come close to the H-J region. In general it seems that large improvements in the fit of the model occur only when the local substitution is driven out in favor of local habit persistence.

## 5. ESTIMATION RESULTS.

In this section I report results of estimating and testing the models of section 2. The results of section 4 indicate that the model is likely to encounter difficulties because of the extreme parameter values that are needed in fitting the H-J regions. However the results of section 4 provide some evidence that the model solution algorithm is performing reasonable well since the plots of the moments of the *mrs* are well behaved as parameter values are changed.

In estimating and testing the asset pricing models of section 2, the estimated law of motion for consumption and dividends given in tables 3.1 and 3.2 was taken as given. Given this law of motion and the form of preferences, stock and bond prices, as a function of the state variable, were calculated using the numerical method described in Section 3. The stock price gives the price of the future dividend stream fitted in Section

3. The bond price is the price of a one month real discount bond.

The preference structures were fit using simulated method of moments<sup>15</sup>. I used observed moments of the vector  $r(t) \equiv [r^{vw}(t), r^f(t)]'$  where  $r^{vw}(t)$  is the monthly real return on the CRSP value weighted portfolio and  $r^f(t)$  is the monthly real T-bill rate<sup>16</sup>. The moments used<sup>17</sup> were  $E\{r(t)\}$ ,  $E\{[r(t) - \bar{r}][r(t) - \bar{r}]'\}$  and  $E\{[r(t) - \bar{r}][r(t-1) - \bar{r}]'\}$  where  $\bar{r} \equiv E\{r\}$ . A simulation that was three times the size of the observed sample of returns was used.

The complete model of local durability and habit persistence was estimated first with a weighting matrix that did not correct for the first-stage estimation error. In this initial round of estimation the parameters  $\gamma$  and  $\beta$  were driven to regions where the equilibrium in the model does not exist. To counter this problem, I fixed  $\beta$  at  $0.99^{1/48}$ . This value of  $\beta$  implies that an equilibrium exists for all values of  $\gamma > 0$ . Under this restriction, the rest of the parameters were estimated and the resulting estimates were used to estimate a weighting matrix that corrects for the estimation error in the law of motion for consumption and dividends. A second round of parameter estimation was done for each of the models. In each case the weighting matrix was fixed at its estimated value implied by the first-stage estimates of the complete model. This allows a simple comparison of the minimized criterion function across the different models.

Table 5.1 reports the results of estimating the complete model with local durability and habit persistence. Notice that the curvature parameter is estimated to be quite small and that the estimate of  $\alpha$  is quite large implying a large degree of habit persistence. The estimates of  $\Delta$  and  $\Theta$  are also quite large. The estimates imply that the half-life of the durable nature of consumption is 10.7 months and the half-life of the habit

persistence stock is 6.6 months. Figure 5.1 plots the function  $a(j)$ , implied by the parameter estimates of table 5.1. Remember that the function  $a(j)$  governs the form of the temporal dependencies in preferences [see (2.3)]. The shape of the function  $a(j)$  indicates that consumption is substitutable for 51 weeks (one year and one month) with the habit persistence effects dominating beyond 51 weeks<sup>18</sup>. The relatively long period of substitutability of the consumption goods is consistent with other estimates of the degree of durability in the components of aggregate nondurables [see Hayashi (1985), for example].

The estimated form of the time-nonseparabilities is consistent with the *a priori* restriction that consumption is locally substitutable. The shape of this underlying nonseparability also potentially explains the fact that studies using monthly data (e.g. Gallant and Tauchen (1989) and Eichenbaum and Hansen (1990)) find evidence for local substitution whereas studies that have used quarterly data find evidence for habit persistence (e.g. Ferson and Constantinides (1990)). Data averaged over longer periods of time could be picking up the presence of long-run habit persistence<sup>19</sup>.

Table 5.1 also reports standard errors for the estimated parameters of the model. The parameters governing the nonseparabilities are seemingly poorly estimated. The estimated standard errors rely upon a mean-value approximation that may be quite poor because of the restrictions on the parameters (e.g.  $\alpha \leq 1$ ). To overcome this problem, an assessment of the accuracy of the parameter estimates can be done by examining different hypothesis about the parameters directly.

A Chi-Square test of the over identifying restrictions ( $J_T$ ) yields a P-Value of 0.0009 so that the model is not very consistent with the data. It is perhaps not surprising that this simple asset pricing model is

rejected by the data. It is of greater interest to examine whether the addition of these parameters significantly improves the fit of the model.

Table 5.2 gives parameter estimates of the pure habit persistence model with the parameter  $\Delta$  set to zero. Under this restriction the criterion function deteriorates quite dramatically relative to that reported in table 5.1 for the complete model. The change in the criterion function (the  $J_T$  statistic) when this restriction is relaxed has a Chi-Square distribution with one degree of freedom<sup>20</sup>. The P-Value of the test that  $\Delta=0$  is .0005. Hence there is a great deal of evidence against the pure habit persistence model.

Table 5.3 presents parameter estimates of the time additive specification. Notice that the criterion function deteriorates very little from table 5.2 to table 5.3. Again a test of the time additive model relative to the model of habit persistence with local durability can be conducted by taking the difference in the minimized criterion functions values reported in tables 5.2 and 5.3. A problem with this test is that under the null of the time additive model ( $\alpha=0$ ) the parameter  $\Theta$  is not identified and hence the test has a nonstandard distribution. However a Chi-Square distribution with 1 degree of freedom can be used to calculate a lower bound on the P-Value of the test. This lower bound is 0.169 so that there is very little evidence against the simple time additive model relative to a model with habit persistence alone.

The change in the criterion function from the time additive case to the case of habit persistence with local durability is quite large which indicates that the addition of the two effects is important. Unfortunately because  $\Theta$  is not identified under the null that  $\Delta=0$  and  $\alpha=0$ , an exact probability value for the test cannot be easily derived.



Estimation of the pure durability model resulted in a parameter estimate for  $\Delta$  of essentially zero so that the presence long-run habit along with local substitution seems to be important. To further examine this issue, consider figure 5.2 which gives a plot of the criterion function as a function of the parameter  $\alpha$  where the criterion function is minimized over the parameters  $\gamma$ ,  $\theta$  and  $\Delta$ . As  $\alpha$  is driven toward zero, the criterion function deteriorates quite rapidly. For each fixed value of  $\alpha$  the difference between the criterion function value plotted in figure 5.2 and the value with  $\alpha$  unrestricted is asymptotically distributed as a Chi-Square random variable with one degree of freedom. Figure 5.3 gives a plot of the probability value of the resulting test that  $\alpha$  is the value given on the x-axis. There is quite dramatic evidence against values of  $\alpha$  smaller than about 0.7.

The function  $a(j)$ , under the restriction that  $\alpha = 0.7$ , is given in figure 5.4. Notice that the long-run habit evident in figure 5.1 has disappeared. As a result, the improvement in the fit of the model is due to the *simultaneous* presence of habit and local substitution effects. To help interpret this result, consider table 5.4 which reports values of the fitted moments as implied by the parameter estimates of the different models. Also reported in table 5.4 are the sample values of the moments. Both the habit persistence model and the complete model do a better job than the time-additive model in fitting the level of the risk free rate. However the complete model does a much better job of fitting the variance and first-order autocovariance of the risk-free rate. The large improvement in the criterion function with the complete model occurs because these moments are relatively well measured. The presence of local substitution seems to be needed to fit the observed smoothness in the risk-free rate, as measured

by its variance and first-order autocorrelation.

Although the habit persistence with local durability model does provide a better rationalization of some of the moments of stock and bond returns, the model is not a complete success. One potential explanation for this failure is provided by the results of Section 4 which indicate that local substitution makes it very difficult to fit the Hansen and Jagannathan (1990, 1991) bounds on the moments of the marginal rate of substitution. In fact the mean and standard deviation of the marginal rate of substitution implied by the estimates of table 5.1 are 0.999 and 0.0021 respectively. This point is well outside of the Hansen and Jagannathan region examined in Section 4.

## 7. CONCLUDING REMARKS

In this paper I modeled consumption as being durable and I assumed that the representative consumer develops habit over the flow of services from the durable consumption good. Compared with a simple time additive model I found that the model substantially improves the fit of the first few observed moments of stock and bond returns. This occurs only if local substitution and long-run habit are present *simultaneously*. Neither pure local substitution nor pure habit persistence significantly improves the fit of the model. A difficulty with the results is that the model is statistically rejected. A potential explanation for this rejection is that the local substitution that is needed to fit many of the moments of asset prices causes difficulties on other dimensions. In particular, to fit estimated Hansen and Jagannathan (1990, 1991) bounds on the moments of the

marginal rate of substitution, the local substitutability of consumption can be maintained only if extreme curvature in the period utility function is allowed.

There are several potentially important limitations to the investigation carried out in this paper. The first is the use of a single class of consumption goods: expenditures on nondurables and services. It would have been preferable to investigate a model with multiple goods and to examine durable goods along with nondurables and services as was done by Dunn and Singleton (1986) and Eichenbaum and Hansen (1990), for example. However this would have made it very difficult to undertake the numerical work necessary for the asset pricing calculations. Investigations with multiple consumption goods have not been entirely successful when no account is made for the presence of temporal aggregation in the data. As a result, it was important to first examine a model that is capable of dealing with temporal aggregation and time-nonseparabilities in a world of a single consumption good. An analysis of the model with multiple goods is left to future work.

A second issue not addressed in this paper is the effect of using seasonally adjusted consumption data. It could be that the process of seasonal adjustment hides some of the effects of temporal dependencies in preferences [see for example Heaton (1989)]. In an analysis of a linear "permanent income" model of consumption dynamics, Heaton (1990) found evidence for seasonal habit persistence [see also Osborn (1988)]. Also Ferson and Harvey (1990) find evidence for seasonal habit persistence in Euler equation investigations using seasonally unadjusted data. Adding a model of seasonality to the other effects considered in this paper would



also have dramatically complicated the calculations need for the analysis in this paper. An investigation of the model with seasonally unadjusted data is also left to future research.

## FOOTNOTES

<sup>1</sup>Abel (1990) also presents evidence that models in which consumption is complementary over time can fit the observed equity premium.

<sup>2</sup>Heaton (1990) and Ferson and Harvey (1990) find evidence for seasonal habit formation using seasonally unadjusted consumption data.

<sup>3</sup>Ferson and Constantinides (1990) argued that the use of quarterly and annual data helps them to capture habit persistence that develops slowly.

<sup>4</sup>The nonseparabilities are written as depending on the infinite past for notational convenience. To make this consistent with consumption only for  $t \geq 0$ , set consumption prior to time zero, to zero.

<sup>5</sup>Dunn and Singleton (1986) and Eichenbaum and Hansen (1990) examine models with multiple goods in which some of the goods may have an infinitely lived impact on services. However, the presence of a least one good with a finite life is necessary.

<sup>6</sup>Christiano, Eichenbaum and Marshall (1989) show that temporal aggregation can help to explain some anomalies in consumption data and income data. They do not apply the model to asset data however.

<sup>7</sup>In later sections of the paper I used monthly data as described in the data appendix. I used quarterly data in this section since this is the data set that tends to produce parameter estimates that imply habit persistence [see for Ferson and Constantinides (1990)].

<sup>8</sup>Mehra and Prescott (1985) proceeded by assuming that the aggregate stock return is the return to holding the aggregate endowment. Here, I broke the link between aggregate consumption and the object to be priced. This was also done by Tauchen (1986) and Cechetti, Lam and Mark (1990a, b), for example.

<sup>9</sup>The assumption that the logarithms of consumption and dividend growth follow a vector ARMA process with seasonal dummies is a simplifying assumption. Other models of dividends and consumption have been used. For example Cechetti, Lam and Mark (1990a, b) used a Markov switching model to model annual consumption and dividend growth. A different model such as a Markov switching model could be have been used in this investigation, however a very large state would be needed to capture the degree of correlation over time found in the monthly data. The ARMA specification seems to be the minimal state specification that captures a few of the important moments in consumption and dividend growth.

<sup>10</sup>A complete description of the data used in this paper can be found in the appendix.

<sup>11</sup>I tried several more simple models of consumption and dividends (for example with  $\alpha_1(L)$  and  $\alpha_2(L)$  constrained to be equal) that were rejected by the data when compared to the more general alternative reported in the paper.

<sup>12</sup>Due to the large dimension of the state variable  $X(t)$  it was not possible to experiment with higher order polynomials in the solution algorithm.

<sup>13</sup>Hansen and Jagannathan (1991) show how to create bounds for these moments both with and without the restriction that the marginal rate of substitution must be positive. In this paper I will consider those bounds without the restriction that the marginal rate of substitution be positive. For the securities considered in this paper, the added restriction that the marginal rate of substitution be positive does not seem to be very important.

<sup>14</sup>See the appendix for a more complete description of the asset return data used in this section.

<sup>15</sup>This test is described in detail in Hansen and Jagannathan (1990).

<sup>16</sup>A correction to the distribution of the test statistics and the parameter estimates was done to take account of the estimation error present in the estimates of the law of motion for consumption and dividends. See Newey (1986) and Hansen and Heaton (1991) for discussions of this procedure.

<sup>17</sup>In the model solution the price of a real discount bond was calculated. The comparison to the observed real T-bill rate is valid under the assumption that inflation risk is negligible.

<sup>18</sup>The estimation reported in this section was also carried out with a larger set of moment conditions in which second-order autocovariances were also used. The results using this second set were similar to the results reported in the paper. However some problems were encountered due to difficulties in estimating the very large weighting matrix needed for the estimation. As a result, the estimation results with the larger set of moments are not reported.

<sup>19</sup>Although the half-life of the habit stock is smaller than the half-life of the durable stock, the habit persistence effects are of a long-run nature when defined over consumption directly. This occurs because the stock of habit is created from the flow of services from the durable stock.

<sup>20</sup>I tried to assess this issue directly by simulating the economy that is implied by the point estimates of tables 3.1, 3.1 and 5.1. I then time-averaged the resulting data over periods corresponding to months and quarters and I applied the methods and models of Ferson and Constantinides (1990) to this data. Unfortunately this resulted in very strange parameter estimates of the models. This seemed to occur because the underlying model is very much at odds with the true data, as indicted by the test of the overidentifying restrictions of the model reported in table 5.1. As a result, the model is not capable of producing simulated data that closely resembles the observed data.

<sup>21</sup>See Eichenbaum, Hansen and Singleton (1988), for example.



## APPENDIX: DATA

Consumption: Real and nominal purchases of nondurables plus services from the U.S. National Income and Product Accounts. Seasonally adjusted. Monthly from 1959,1 to 1989,12. Taken from CITIBASE.

Dividends: Dividends from the value weighted return series of the New York Stock Exchange as constructed by CRSP. Monthly from 1959,1 to 1989,12. Converted to real dividends using the implicit price deflator for nondurables plus services.

Population: The consumption data was converted to a per capita basis using monthly U.S. population from 1959,1 to 1989,12. The data is published by the Bureau of the Census and was obtained from CITIBASE.

Treasury Bill Return: One month return on one month treasury bills from 1959,1 to 1989,12. From the CRSP bond file. Converted to real returns using the implicit price deflator for nondurables plus services.

Value weighted return: Value weighted returns from the NYSE were constructed from the CRSP daily value weighted return. The monthly value weighted return was not used directly since it assumes that all dividends are paid at the end of a month. Monthly from 1962,7 to 1989,12. This series was converted to real returns using the implicit price deflator for nondurables plus services.

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TABLE 3.1

ESTIMATES OF MEANS AND SEASONAL DUMMIES FOR WEEKLY LAW OF MOTION OF THE THE  
LOGARITHMS OF REAL CONSUMPTION AND REAL DIVIDEND GROWTH

$$X(t) \equiv \begin{Bmatrix} \log[c(t)/c(t-1)] \\ \log[d(t)/d(t-1)] \end{Bmatrix}$$

$$X(t) = \begin{bmatrix} 1 & 0 \\ 0 & \alpha_s(L) \end{bmatrix}^{-1} \Gamma(L)\varepsilon(t) + \begin{bmatrix} 0 \\ \mu'_s \end{bmatrix} D(t) + \mu$$

$$\mu'_s \equiv [\mu_s^1 \ \mu_s^2 \ \cdots \ \mu_s^{48}], \quad \mu' \equiv [\mu^1 \ 0]$$

Parameter	Estimate	Standard Error
$\mu^1$	0.00169	0.000214
$\mu_s^1$	1.205	0.054
$\mu_s^5$	-0.811	0.056
$\mu_s^9$	-0.511	0.054
$\mu_s^{13}$	1.388	0.070
$\mu_s^{17}$	-0.863	0.062
$\mu_s^{21}$	-0.443	0.057
$\mu_s^{25}$	1.280	0.080
$\mu_s^{29}$	-0.937	0.061
$\mu_s^{33}$	-0.082	0.057
$\mu_s^{37}$	1.084	0.094
$\mu_s^{41}$	-0.910	0.082
$\mu_s^{45}$	-0.360	0.042

TABLE 3.2

## SIMULATED METHOD OF MOMENTS ESTIMATES OF

$$\begin{bmatrix} \alpha_1(L) & 0 \\ 0 & \alpha_2(L) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \alpha_s(L) \end{bmatrix} \hat{X}(t) = B(L)\varepsilon(t)$$

$$\alpha_1(L) = 1 + \alpha_1^1 L + \alpha_1^2 L^2; \alpha_2(L) = 1 + \alpha_2^1 L + \alpha_2^2 L^2; \alpha_s(L) = 1 - \alpha_{48} L^{48}$$

$$B(L) = B_0 + B_1^1 L, \text{ where } B_0 = \begin{bmatrix} B_0^{11} & 0 \\ B_0^{21} & B_0^{22} \end{bmatrix}, B_1 = \begin{bmatrix} B_1^{11} & B_1^{12} \\ B_1^{21} & B_1^{22} \end{bmatrix}$$

Parameter	Parameter Estimate	Standard Error
$\alpha_1^1$	1.305	0.063
$\alpha_1^2$	0.575	0.027
$\alpha_2^2$	1.656	0.034
$\alpha_2^2$	0.922	0.206
$\alpha_{48}$	0.547	0.087
$B_0^{11}$	0.0089	0.0075
$B_0^{21}$	-0.0264	0.0083
$B_0^{22}$	0.0165	0.0104
$B_1^{11}$	-0.0076	0.0068
$B_1^{12}$	-0.0128	0.0027
$B_1^{21}$	0.0114	0.0081
$B_1^{22}$	-0.0213	0.0120

<sup>a</sup>Parameter estimates based upon 1344 simulated observations and the moments discussed in the text. One-year of lags was used in a Newey-West (1987) procedure to estimate the weighting matrix.



TABLE 5.1

## S.M.M. ESTIMATES OF COMPLETE MODEL

Using means, covariance and 1 autocovariance of stock and bond returns.

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Parameter	Parameter Estimates	Standard Errors
$\gamma$	0.356	0.995
$\alpha$	0.946	1.300
$\theta$	0.974	0.382
$\Delta$	0.984	0.390

$J_T$  20.794  
P-Value:  $8.86 (\times 10^{-4})$

TABLE 5.2

## S.M.M. ESTIMATES OF HABIT PERSISTENCE MODEL

Using means, covariance and 1 autocovariance of stock and bond returns.  
Weighting matrix from habit persistence with local durability model.

---



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Parameter	Parameter Estimates	Standard Errors
$\gamma$	0.0123	0.863
$\alpha$	0.874	4.573
$\theta$	0.970	0.611

$J_T$  32.816  
P-Value:  $1.14 (\times 10^{-5})$

TABLE 5.3

## S.M.M. ESTIMATES OF TIME ADDITIVE MODEL

Using means, covariance and 1 autocovariance of stock and bond returns.  
Weighting matrix from habit persistence with local durability model.

---

Parameter	Parameter Estimates	Standard Errors
$\gamma$	0.709	0.133

$J_T$  34.703

P-Value:  $3.028 (\times 10^{-5})$

TABLE 5.4

## SAMPLE AND FITTED MOMENTS

$$r(t) \equiv [ r^{vw}(t) \ r^f(t) ]', \quad \tilde{r}(t) \equiv r(t) - E\{r(t)\}$$

$$E\{r(t)\} \equiv \begin{Bmatrix} \mu_1 \\ \mu_2 \end{Bmatrix}, \quad E\{\tilde{r}(t)\tilde{r}(t)'\} \equiv \begin{Bmatrix} \sigma_0^{11} & \sigma_0^{12} \\ \sigma_0^{21} & \sigma_0^{22} \end{Bmatrix},$$

$$E\{\tilde{r}(t)\tilde{r}(t-1)'\} \equiv \begin{Bmatrix} \sigma_1^{11} & \sigma_1^{12} \\ \sigma_1^{21} & \sigma_1^{22} \end{Bmatrix},$$

---

Moment	Sample Value	Fitted		
		Complete Model	Habit Model	Time Additive
$\mu_1$	1.00541	1.00256	1.00225	1.00256
$\mu_2$	1.00094	1.00086	1.00083	1.00113
$\sigma_0^{11}$	2.019 ( $\times 10^{-3}$ )	1.019 ( $\times 10^{-3}$ )	6.116 ( $\times 10^{-4}$ )	5.843 ( $\times 10^{-3}$ )
$\sigma_0^{12}$	1.242 ( $\times 10^{-5}$ )	1.730 ( $\times 10^{-5}$ )	-3.177 ( $\times 10^{-6}$ )	-3.449 ( $\times 10^{-6}$ )
$\sigma_0^{22}$	8.492 ( $\times 10^{-6}$ )	8.955 ( $\times 10^{-6}$ )	3.977 ( $\times 10^{-6}$ )	3.669 ( $\times 10^{-6}$ )
$\sigma_1^{11}$	1.179 ( $\times 10^{-4}$ )	-2.515 ( $\times 10^{-4}$ )	-1.481 ( $\times 10^{-4}$ )	-1.404 ( $\times 10^{-4}$ )
$\sigma_1^{12}$	1.236 ( $\times 10^{-5}$ )	1.199 ( $\times 10^{-5}$ )	8.829 ( $\times 10^{-7}$ )	1.475 ( $\times 10^{-6}$ )
$\sigma_1^{21}$	1.387 ( $\times 10^{-5}$ )	5.980 ( $\times 10^{-6}$ )	3.884 ( $\times 10^{-6}$ )	3.425 ( $\times 10^{-6}$ )
$\sigma_1^{22}$	4.192 ( $\times 10^{-6}$ )	3.740 ( $\times 10^{-6}$ )	-9.116 ( $\times 10^{-8}$ )	-4.622 ( $\times 10^{-8}$ )

Figure 3.1

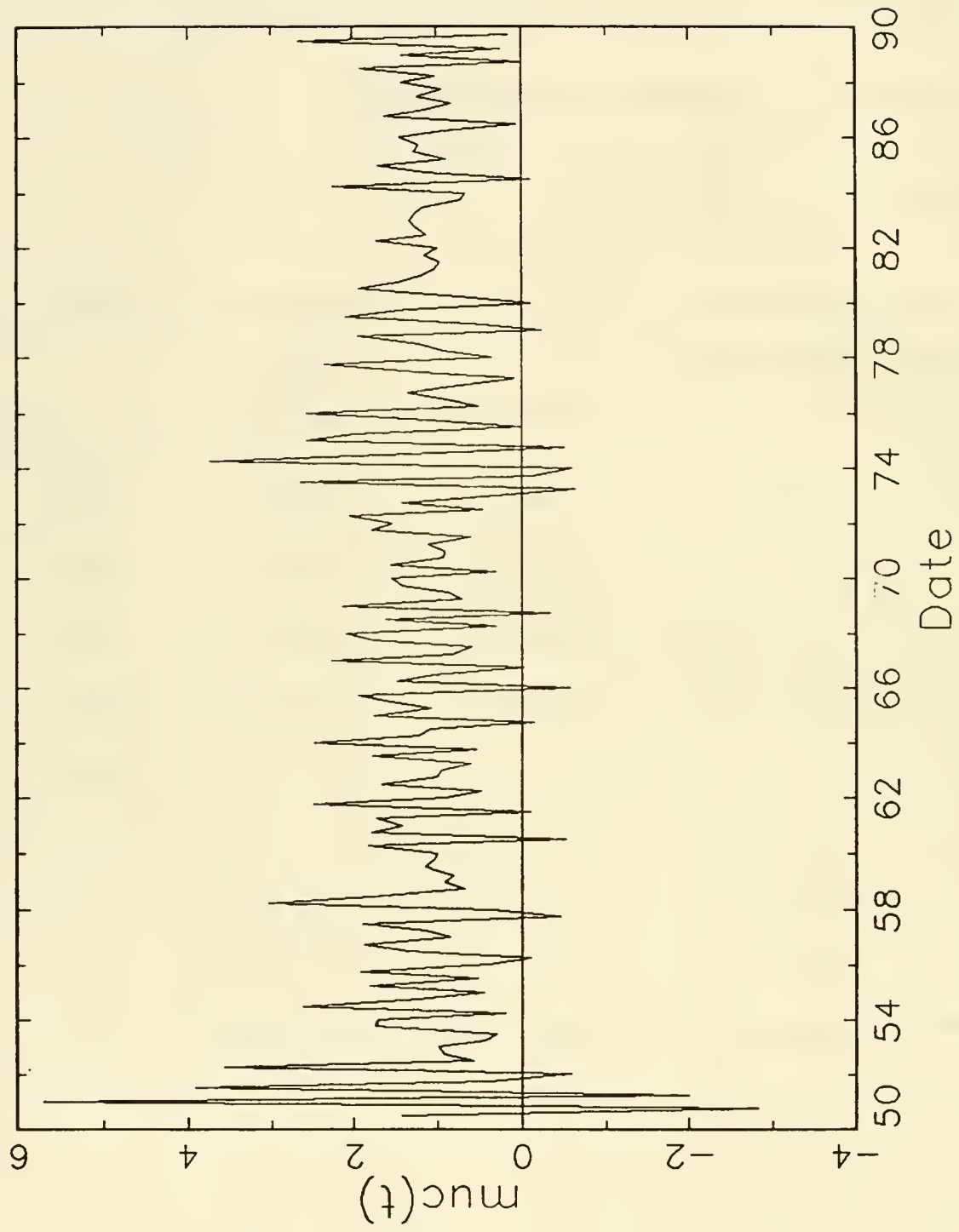


Figure 3.2

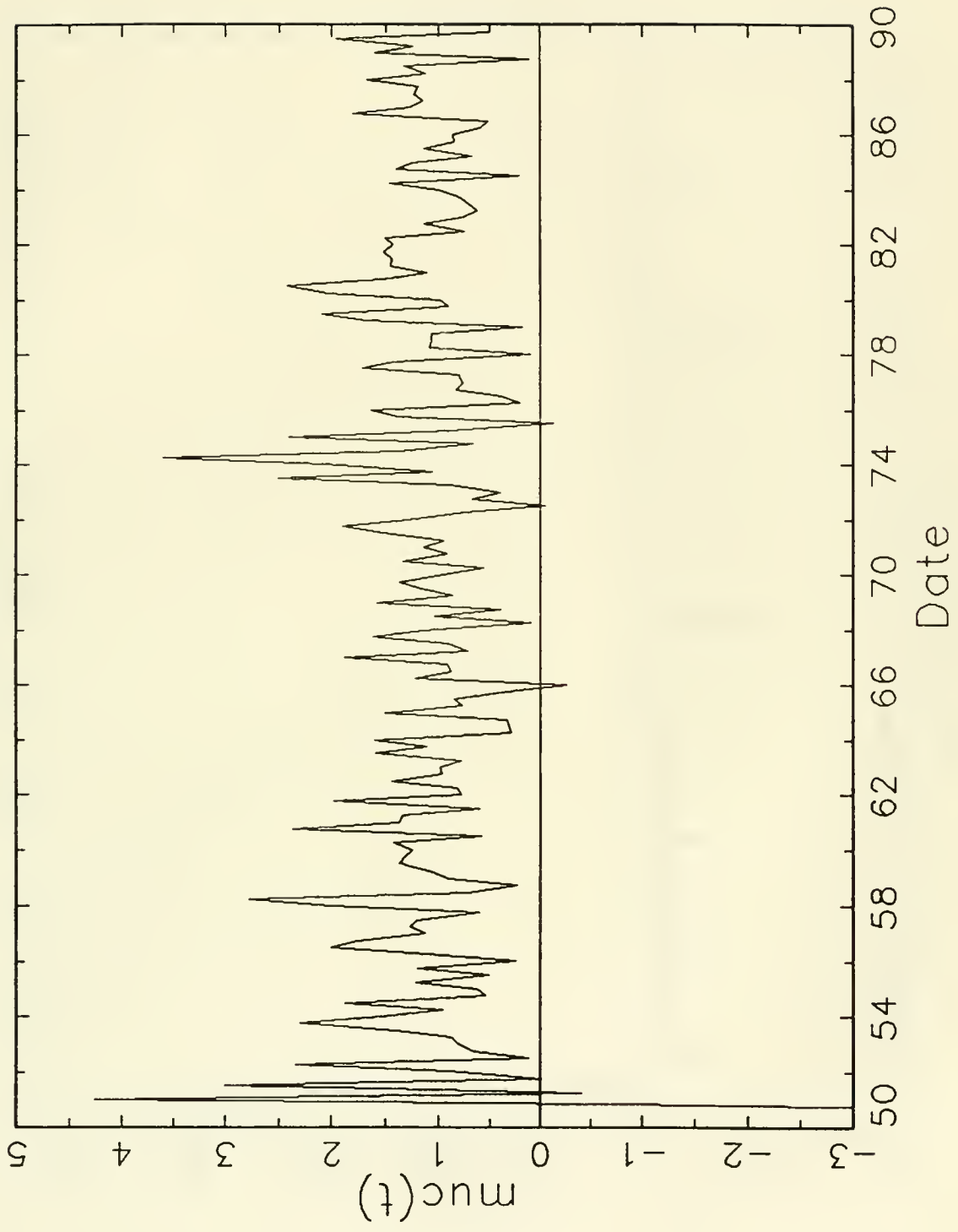


Figure 3.3

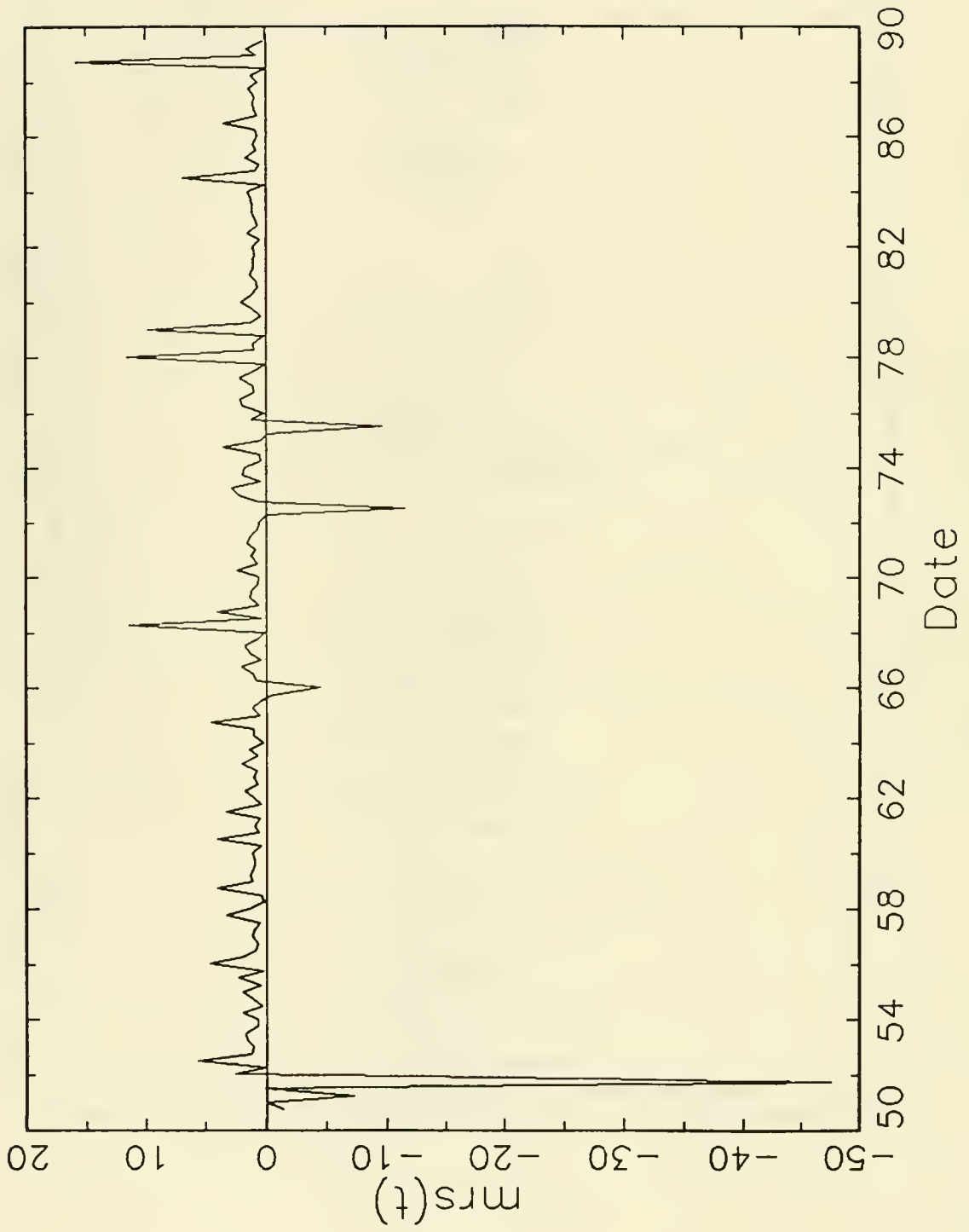




Figure 4.1

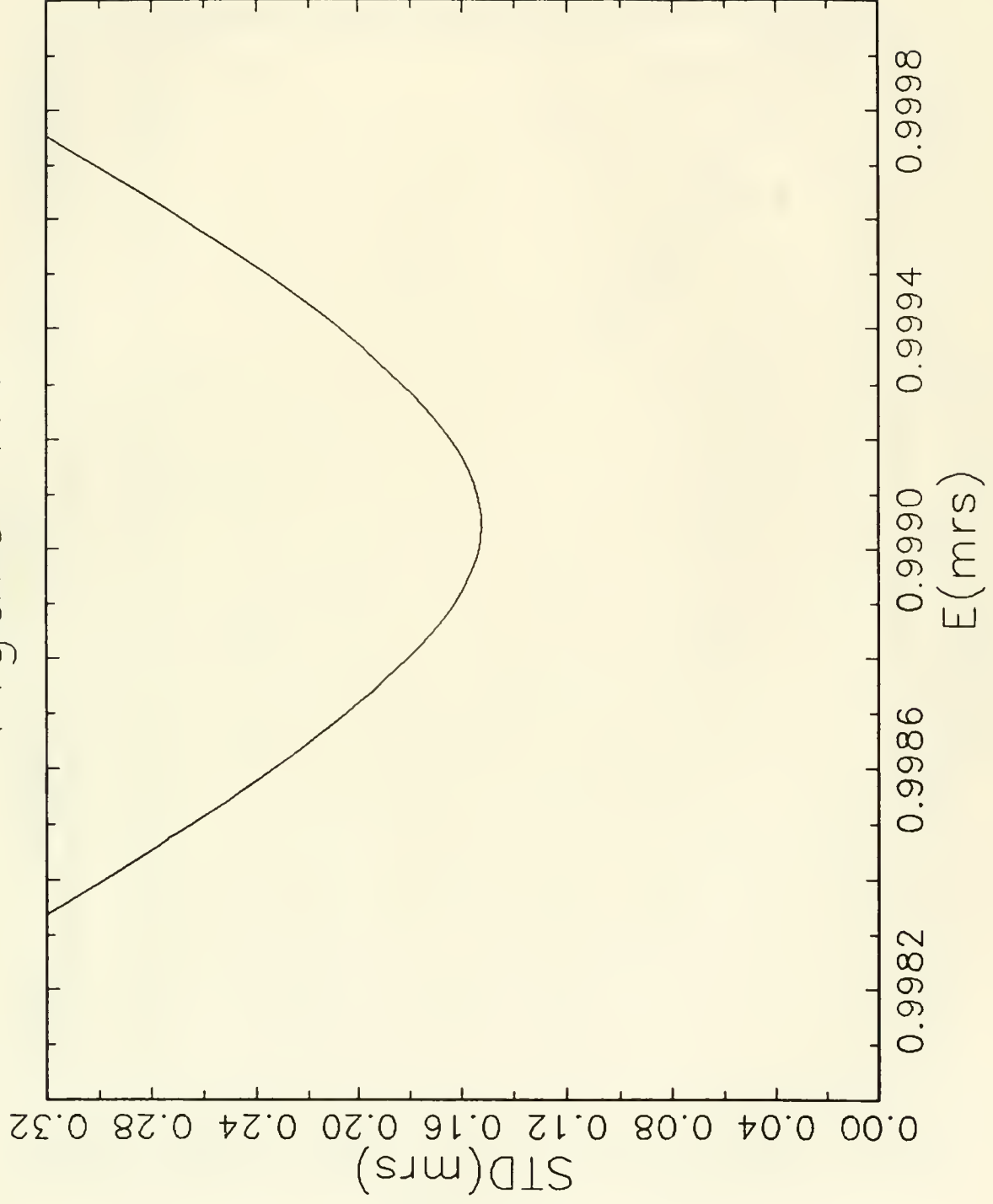


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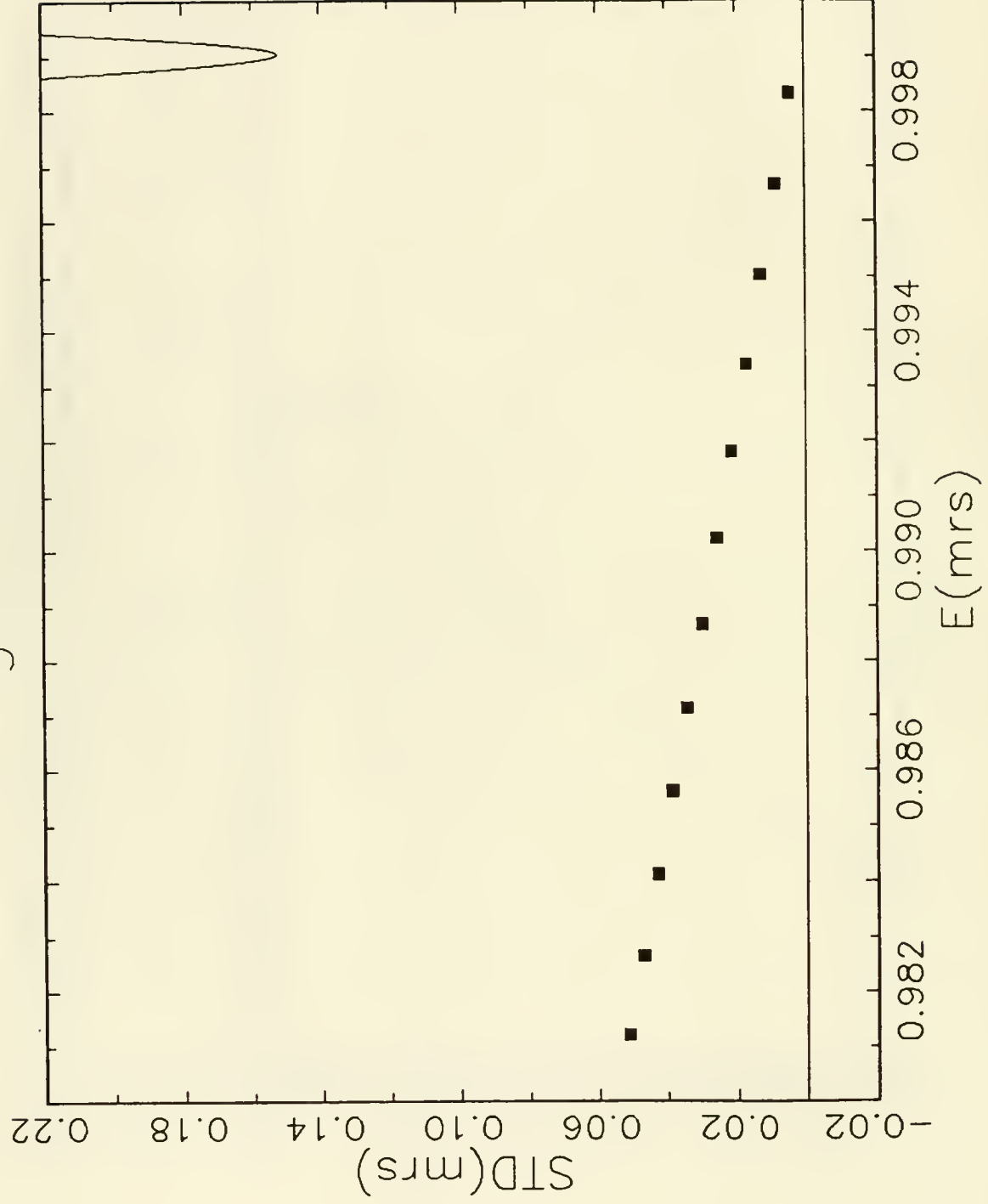


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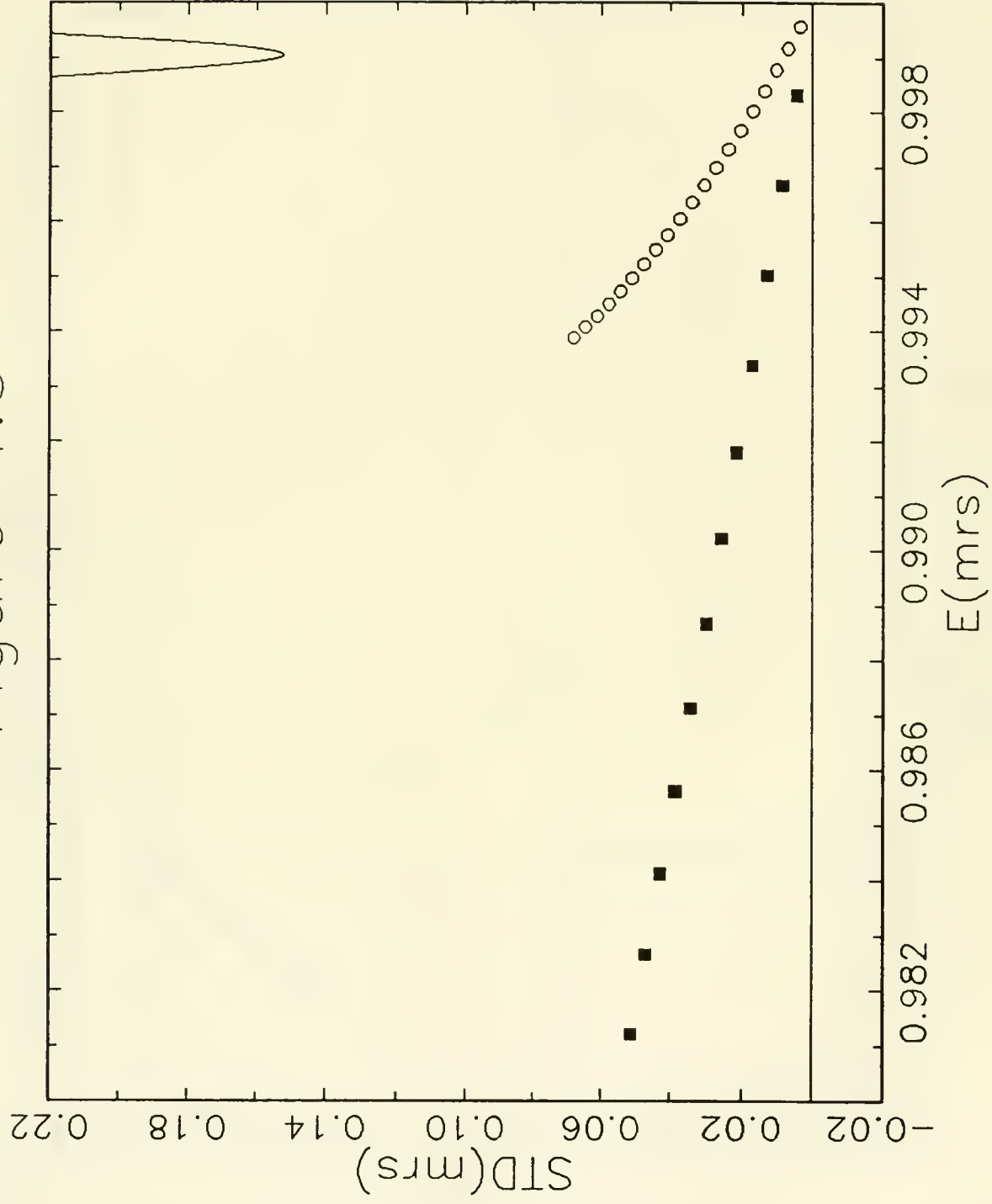


Figure 4.4

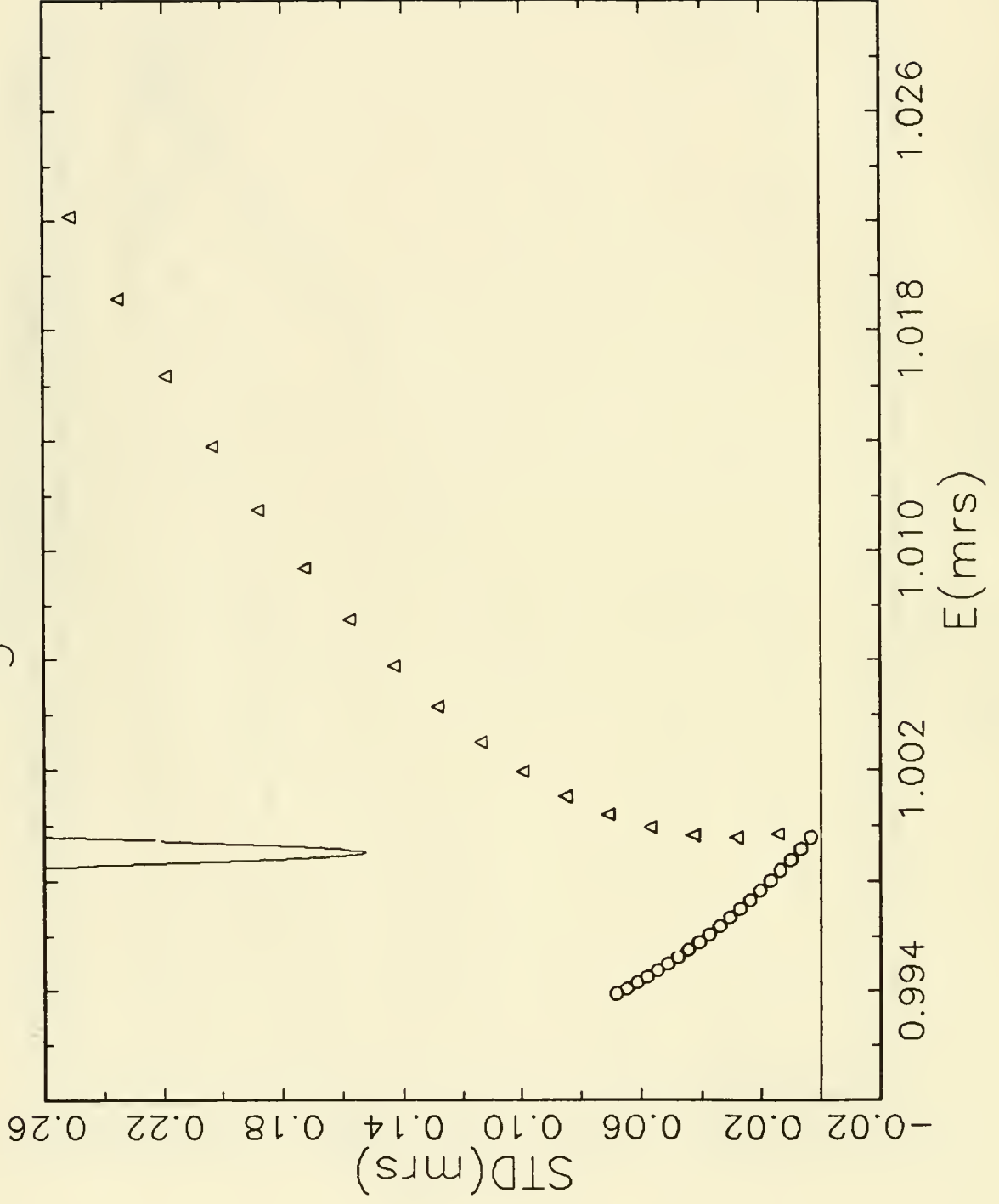


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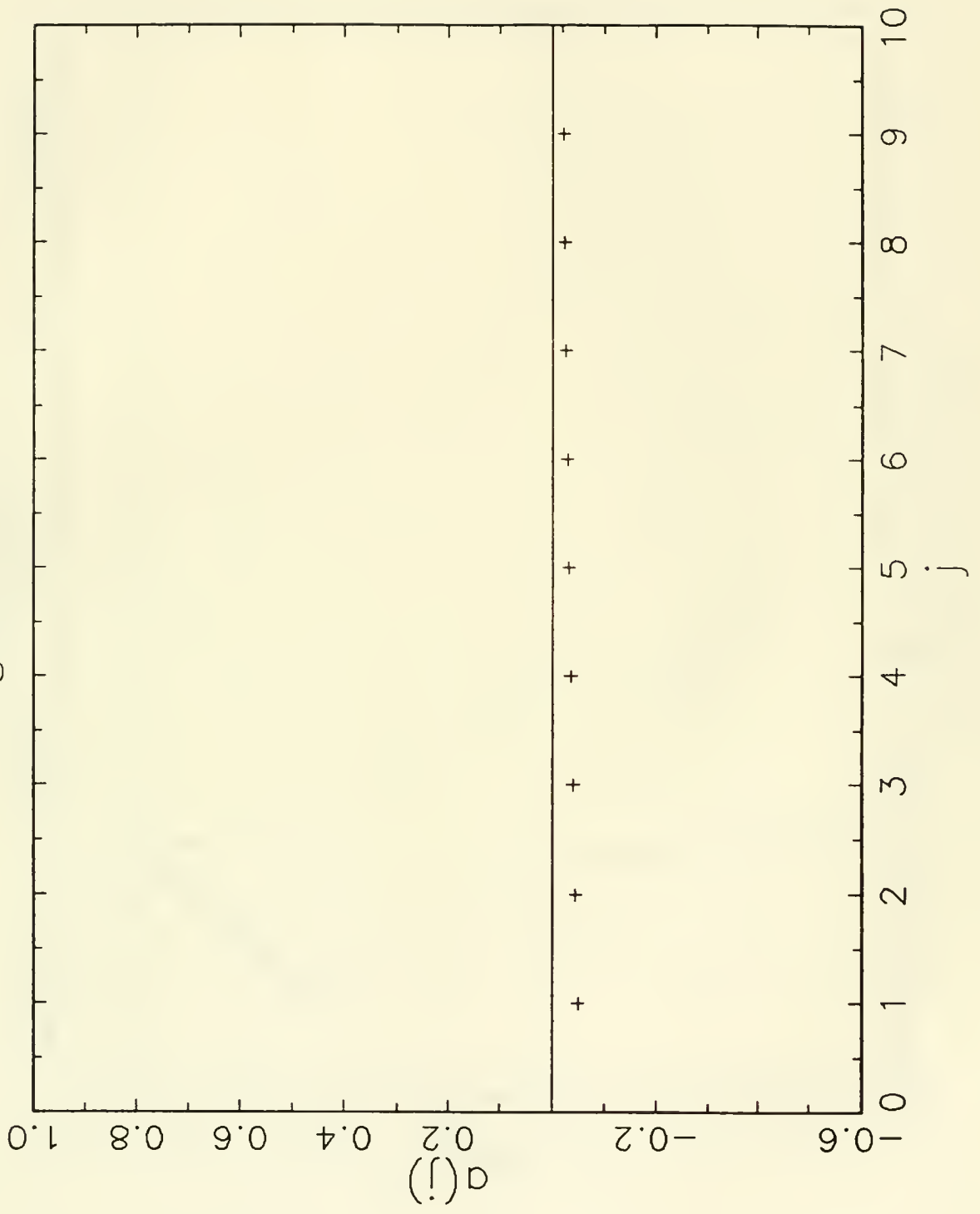


Figure 4.6

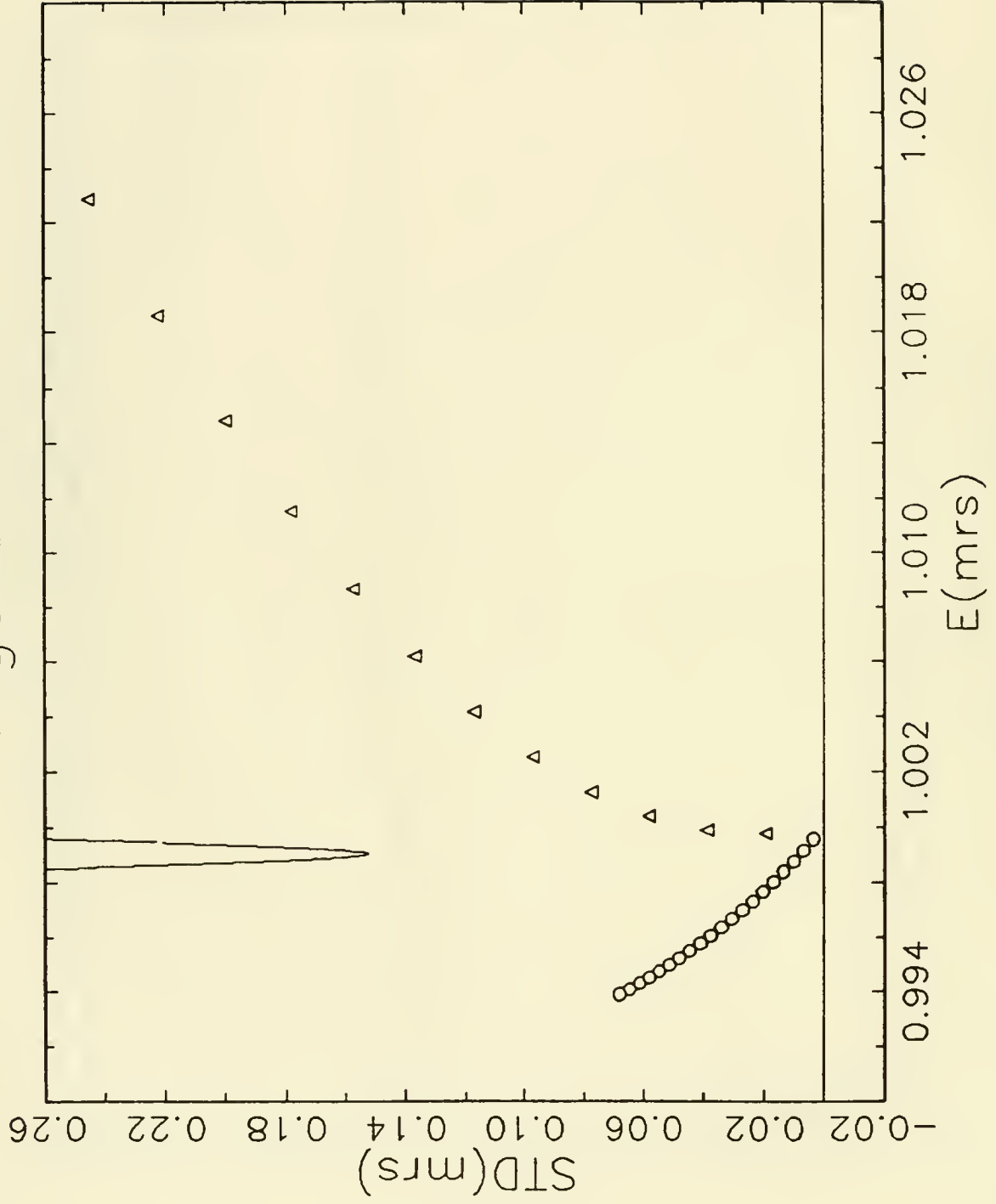




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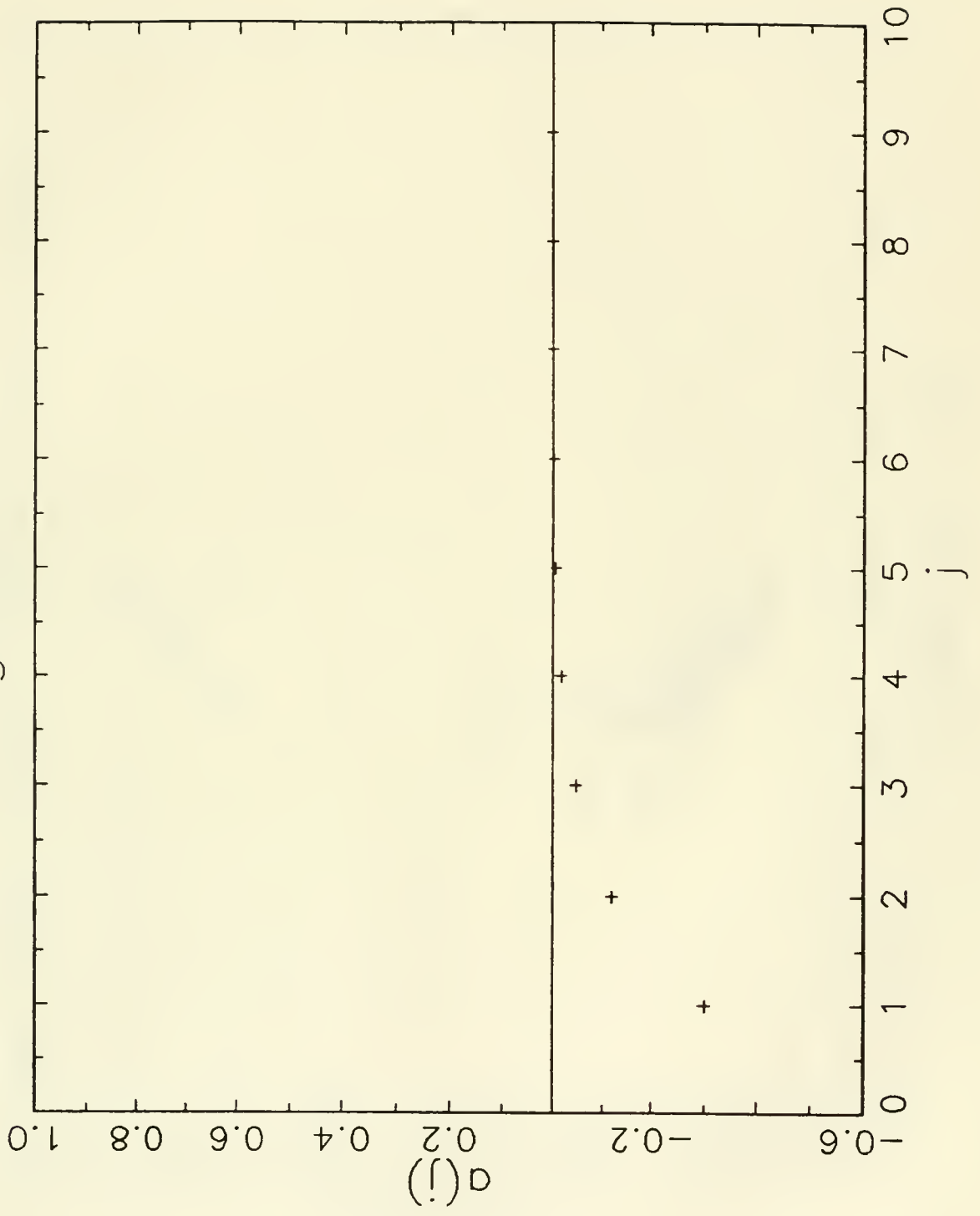


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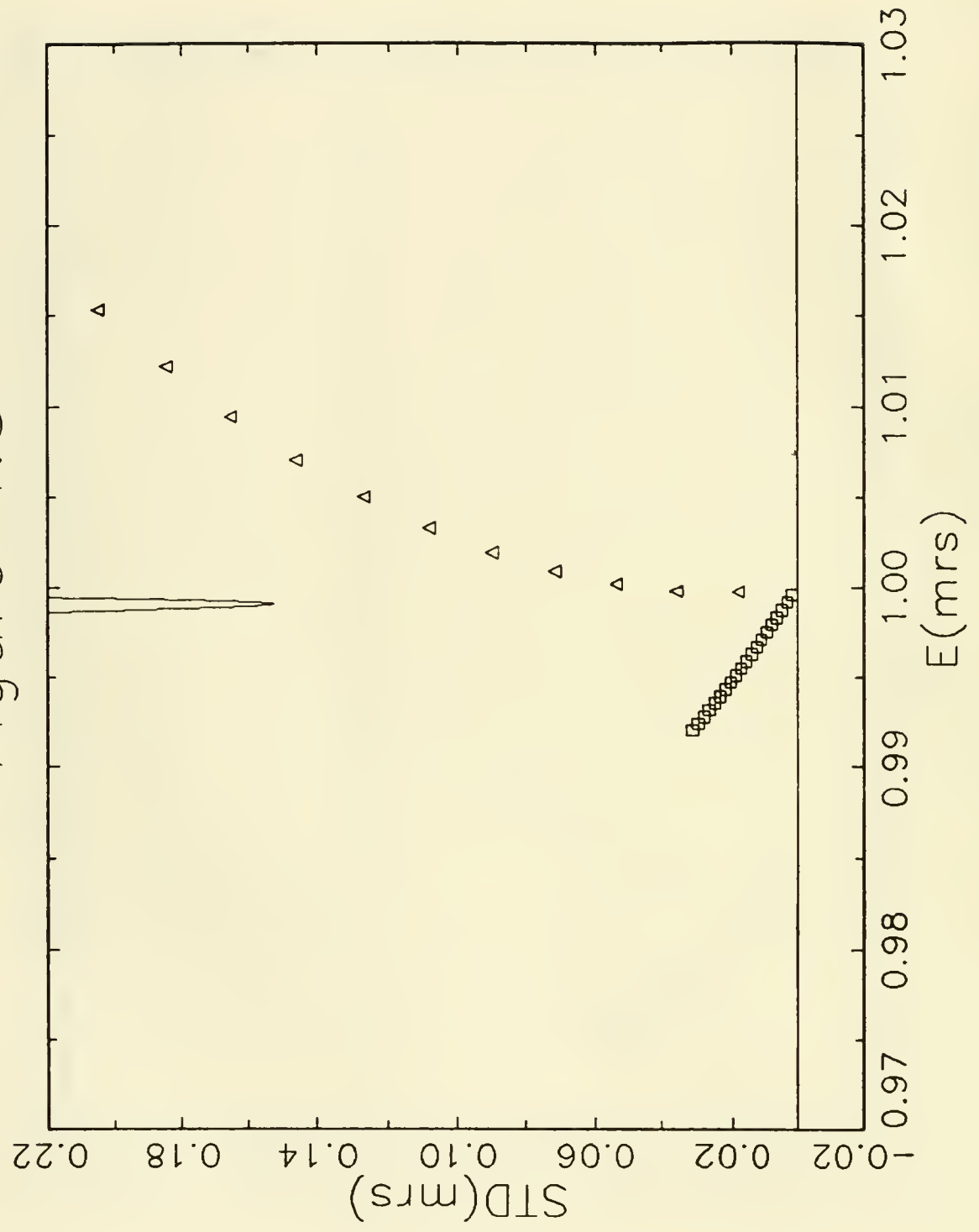


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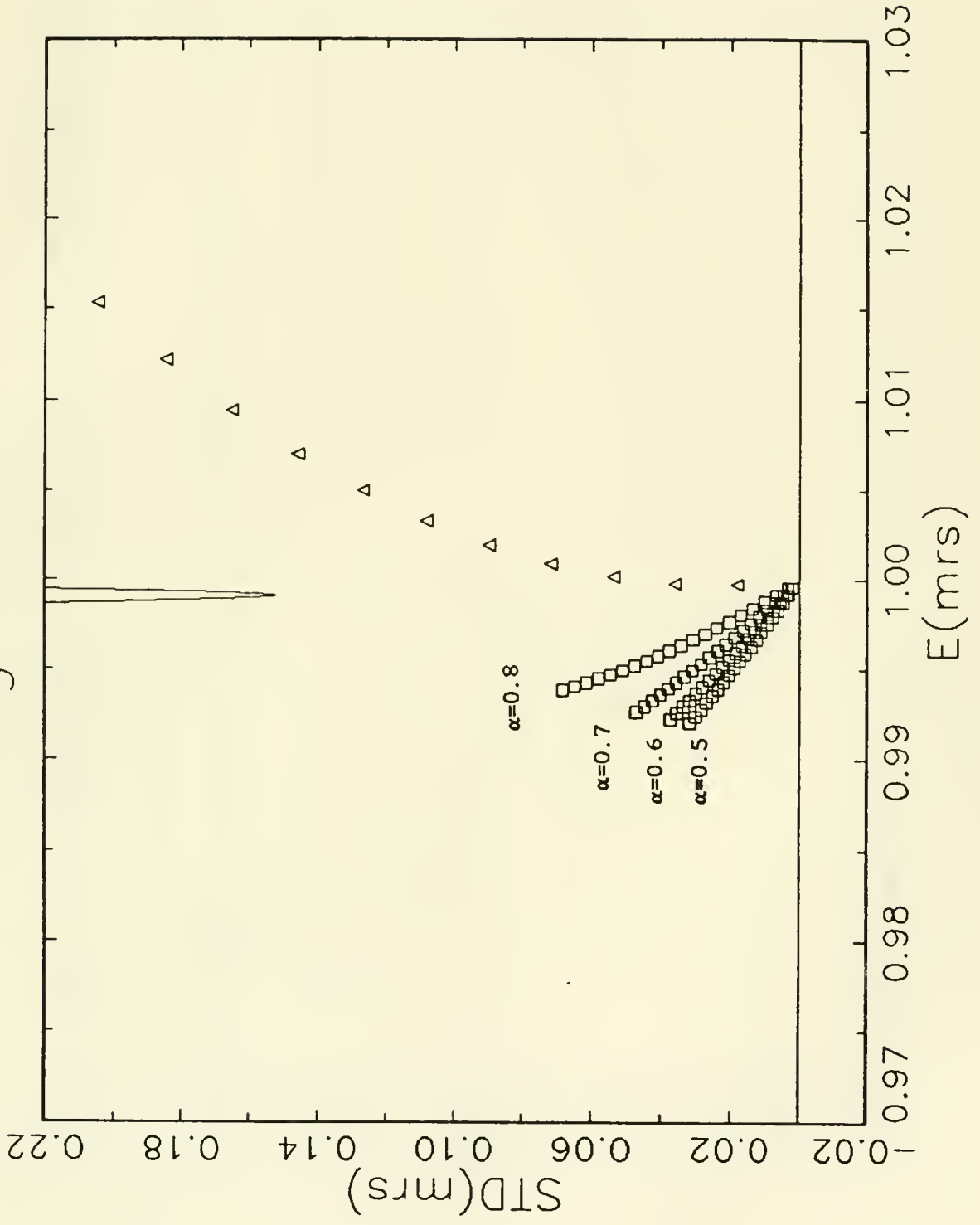


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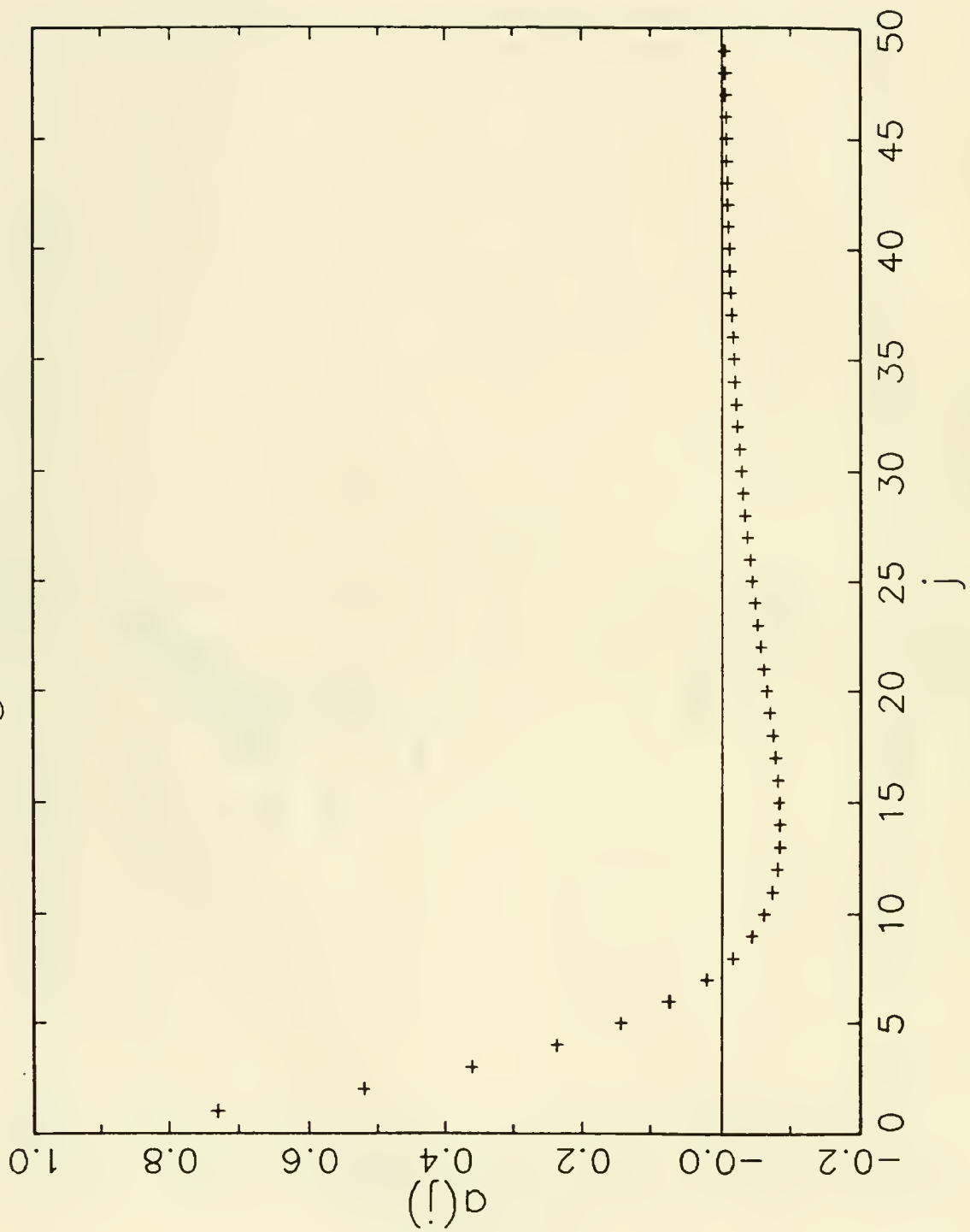


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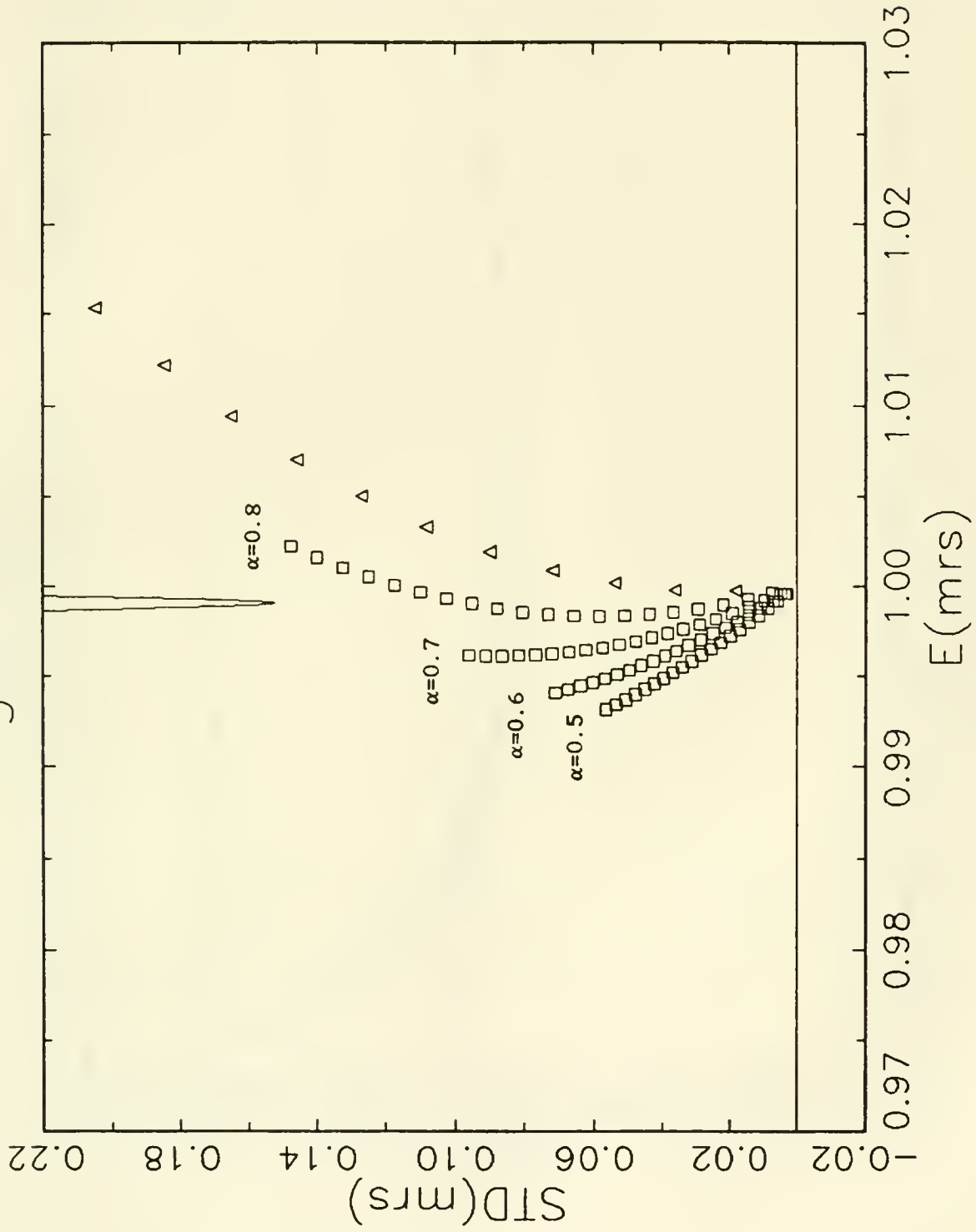


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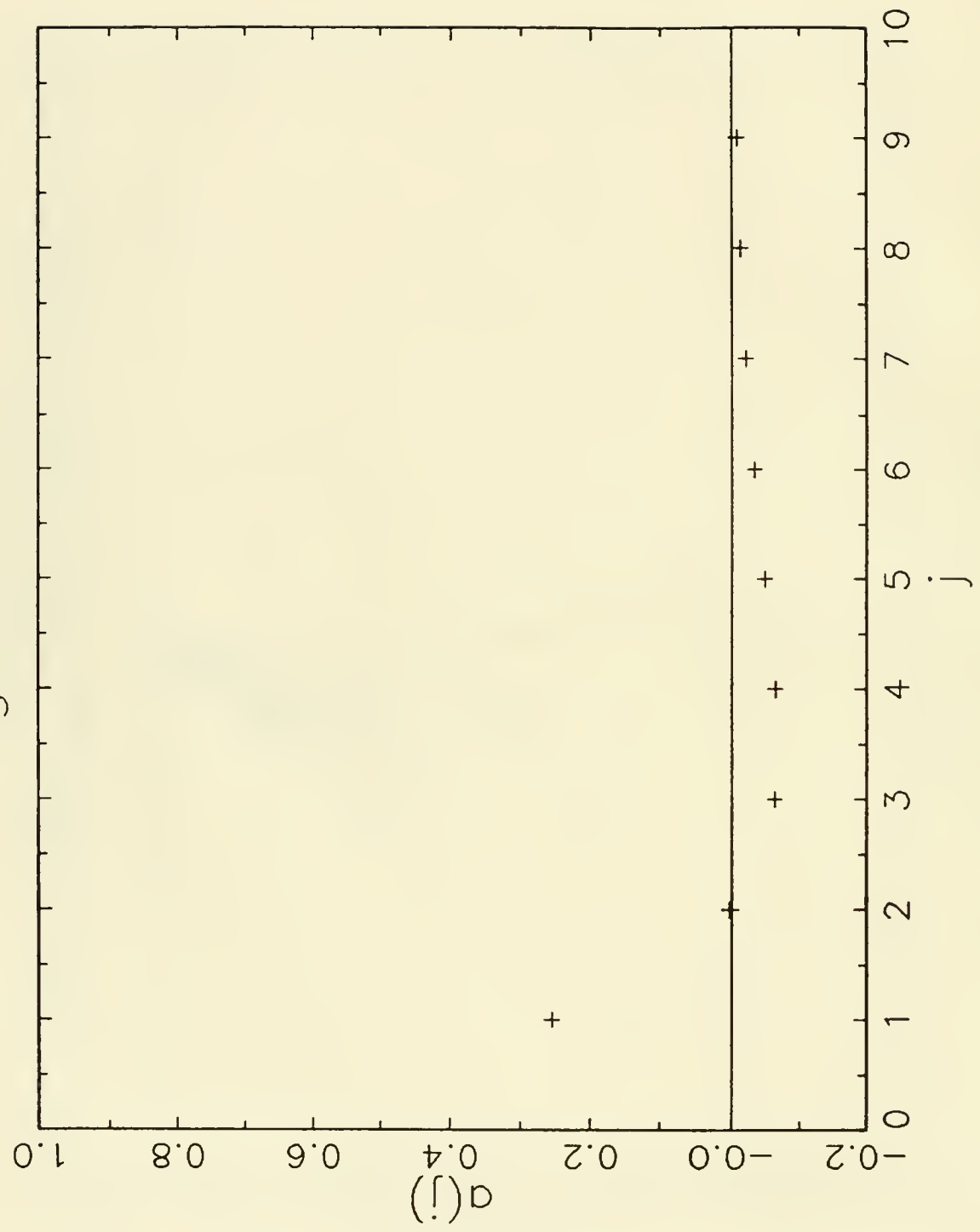


Figure 5.1

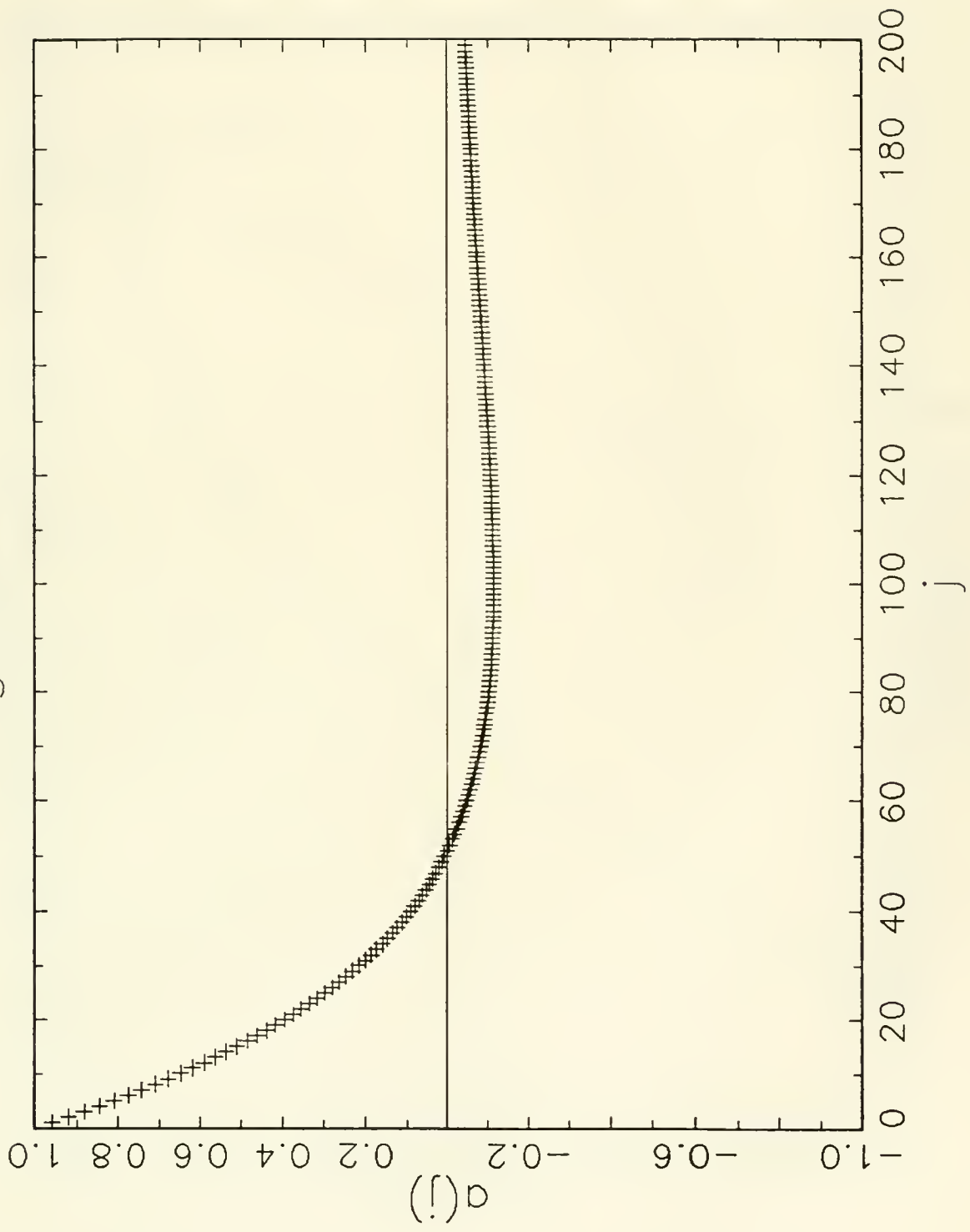




Figure 5.2

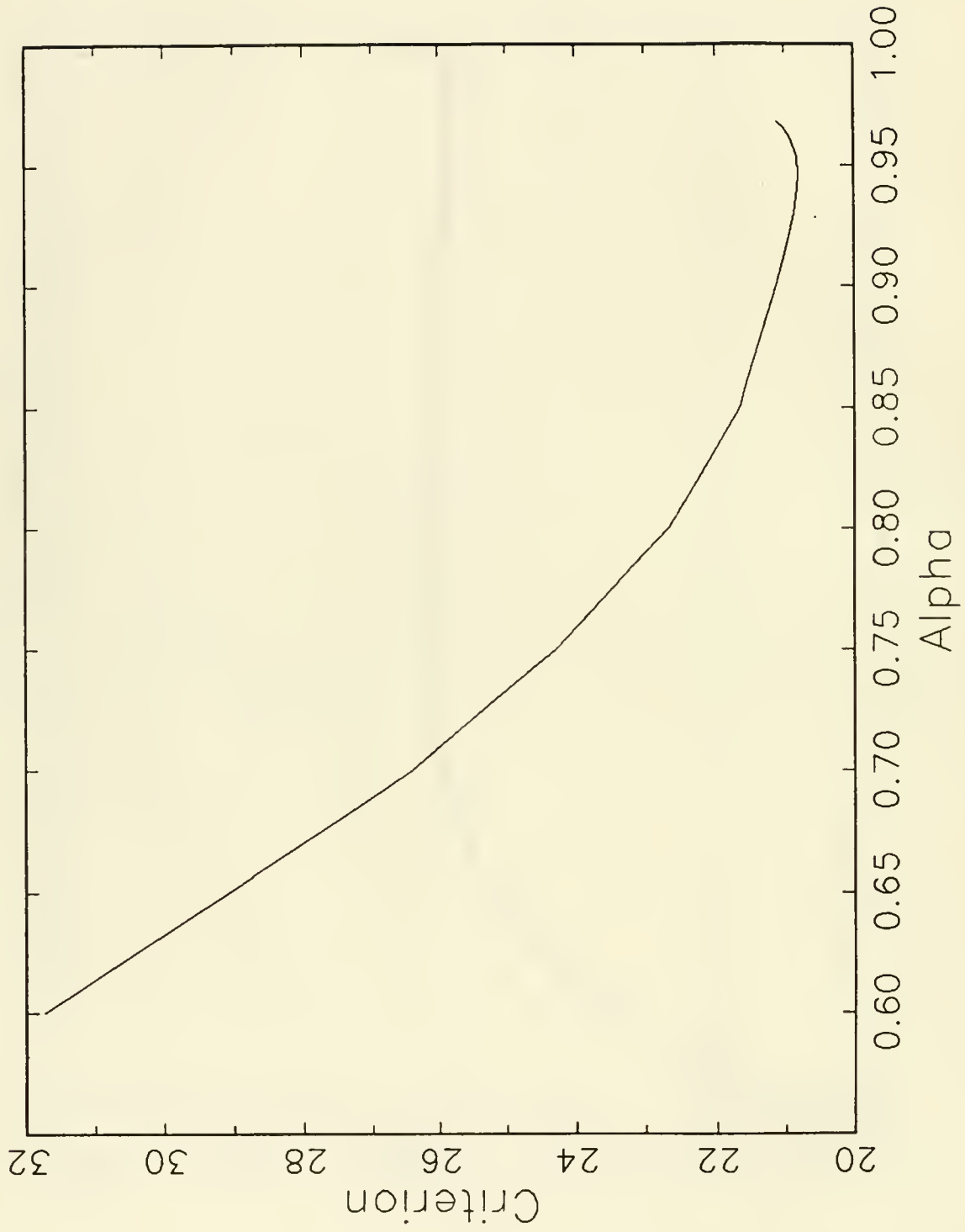


Figure 5.3

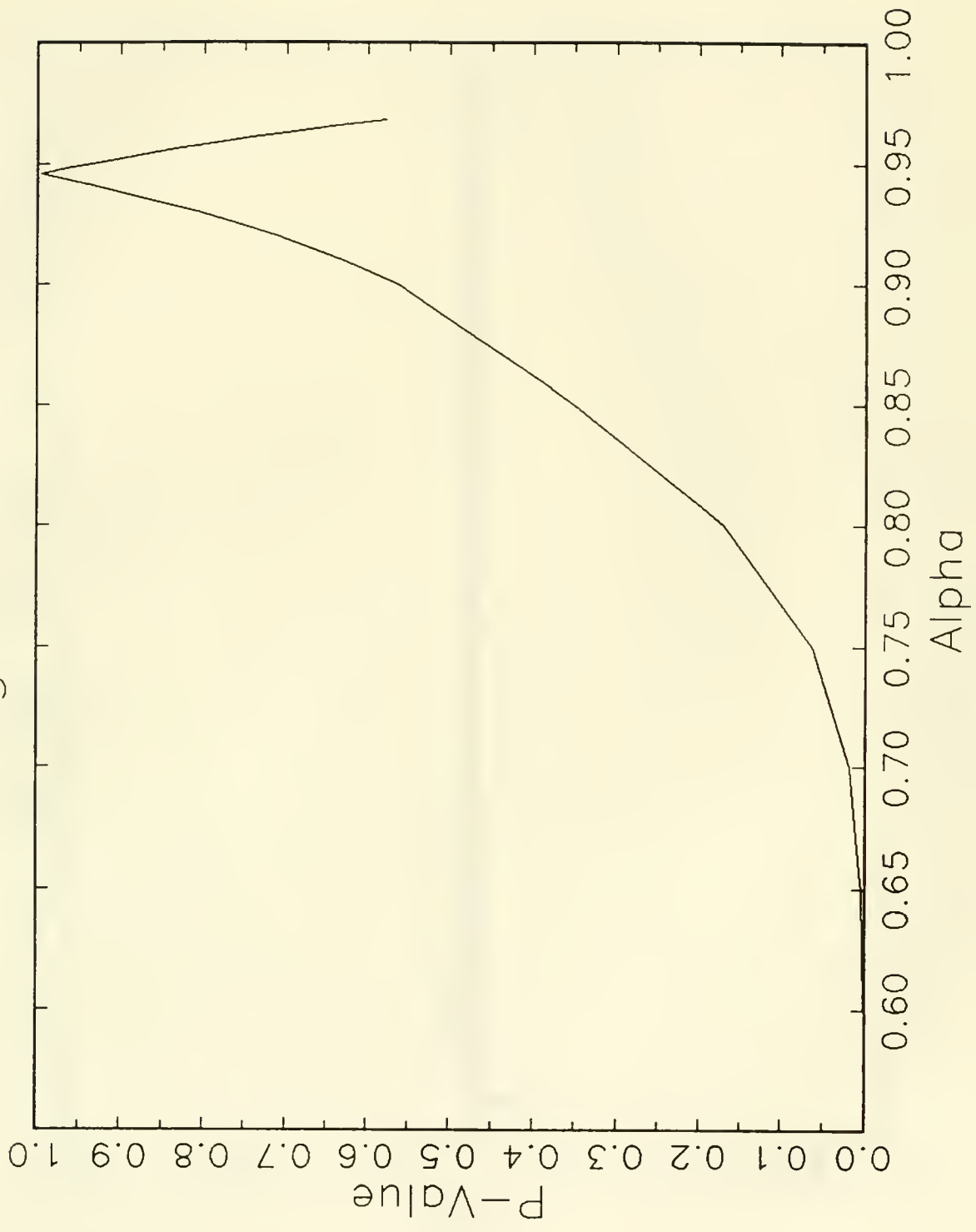
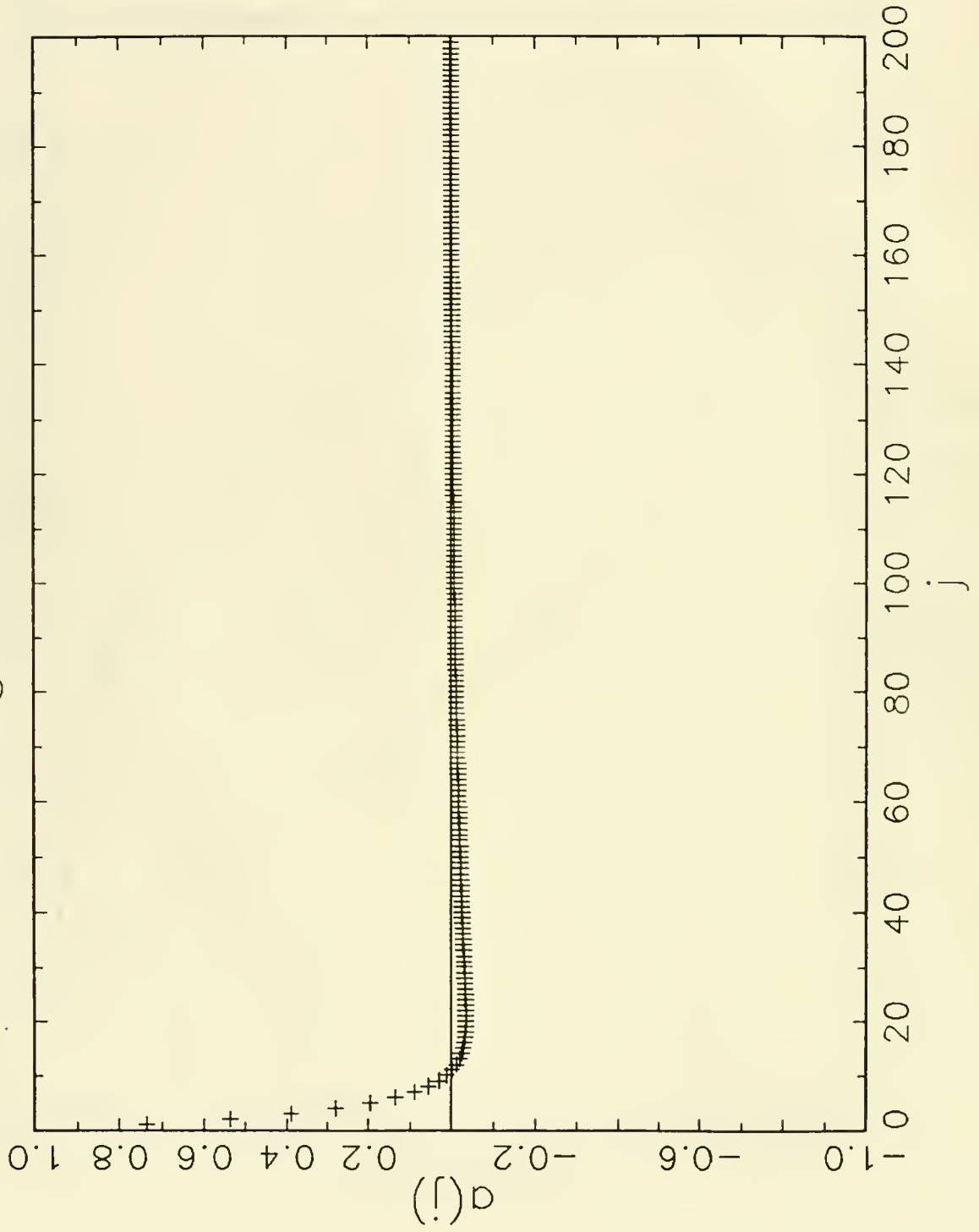


Figure 5.4









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