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ESTIMATING THE DYNAMIC EFFECTS OF MARKETING  
COMMUNICATIONS EXPENDITURES\*

*Bruce* David B. Montgomery \*\* and *John* Alvin J. Silk \*\*\*

558-71

September, 1971

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ABSTRACT

This paper is concerned with the problem of measuring market response to a "communications mix" -- the various means which a firm employs to transmit sales messages to potential buyers. Distributed lag models are applied to time series data for an ethical drug to estimate the short-run and long-run effects on market share of expenditures made for journal advertising, direct mail advertising and samples and literature. Important differences were found among the communications variables with respect to the magnitude and over-time pattern of effect each had on market share. The managerial implications of the findings are discussed.



TABLE OF CONTENTS

|   | Page |
|---|------|
| 1. INTRODUCTION.....                              | 4    |
| 2. PROBLEM BACKGROUND AND DATA DESCRIPTION.....   | 6    |
| 2.1 The Communications Mix for Ethical Drugs..... | 6    |
| 2.2 Variable Definitions and Measures.....        | 8    |
| 3. MODEL FORMULATION.....                         | 10   |
| 4. EMPIRICAL RESULTS.....                         | 16   |
| 4.1 Direct Distributed Lag Estimation.....        | 17   |
| 4.2 Modified Koyck Estimation.....                | 22   |
| 4.3 Pattern of Effects.....                       | 27   |
| 4.4 Magnitude of Effects.....                     | 29   |
| 5. DISCUSSION AND SUMMARY.....                    | 30   |



## 1. INTRODUCTION

One of the problem areas in marketing of great practical and theoretical significance about which much remains to be learned is the nature of market response to a firm's "marketing mix" [24]. Extensions and elaborations of economic theory have been proposed which lead to normative decision rules for obtaining an optimal mix of price, advertising, and product quality given certain general assumptions about sales response functions [38, Chapters 1 and 2]. Explicit response functions of a complex nature have been formulated by Kotler [23] and Urban [44] in building models for marketing mix decisions in the case of new products. In applying these structures, the parameter-setting was done judgementally.

Estimates of the effects on sales or market share of various combinations of the basic mix variables of advertising or promotional expenditures, distribution, price, and product quality obtained through statistical analyses of historical data have been reported in several published studies [e.g. 19, 26, 48]. In a similar vein, Frank and Massy [13] investigated how price, deals, and retail advertising affect market share and Urban [45] examined response to price and shelf-facings. To date, however, very little empirical work appears to have been done on an important subset of the marketing mix, the "communications mix".<sup>1</sup> By communications mix we mean that set of marketing activities by which a firm transmits product information and persuasive messages to a target market. In this study we examine the response of a particular market to a communications mix consisting of the following elements: product



samples, media advertising, and direct mail advertising. More specifically, the research discussed here is concerned with the following questions:

1. What is the magnitude of response to the various elements within the communications mix? How do they compare with one another?
2. What is the pattern of effects for each mix variable over time? Does the greatest response occur in the period of expenditure or does response build up for a few periods and then damp out? How do the short-run effects relate to the long-run effects? How do the mix elements compare to each other in these terms?

The rise of marketing information systems has created new demands for model-based analyses of historical data to answer questions of the type listed above. Montgomery and Urban [34] have discussed the importance of recognizing that a marketing decision-information system consists of four interdependent components: a model bank, a data bank, a measurement-statistics bank, and a communications capability. Given a set of decision-oriented models that address themselves to particular problems, attention centers on the estimation of key response parameters. Judgement, analysis of historical data, and special purpose studies are the basic sources of measurements for models. Information systems evolve over time and in such a context these three methods become stages in an iterative, continuing measurement process rather than ends in themselves. The usefulness of normative marketing models that rely partly or solely on subjective estimates of response parameters has been demonstrated in several problem areas including pricing [9, 10, 16], advertising [18, 27, 28] and sales force management [29, 33]. Experience indicates that almost invariably the users, if not the developers, of such models soon begin to seek ways of evaluating





and/or improving their judgemental inputs with data-based measurements. At this point, the capacity of the data and measurement-statistics bank to provide the historical records, model structures, estimation procedures etc. which are needed for a particular analysis becomes critical. Under favorable circumstances, an analysis of existing data may yield results which become inputs to a larger planning model or they might be used to revise prior judgemental estimates. Armstrong's work [1] in estimating sales potential in foreign markets provides an illustration of the latter type of application. In other situations the results may be less directly applicable but still valuable such as in pin-pointing the need for, and in giving direction to, designing an experiment or some other more refined method of data collection and measurement. Within the framework of a marketing information system then, the role of model-based analyses of historical data is that of a tool which can help decision-makers to learn systematically from past experience. The types of models and analyses reported here are intended to serve such purposes.

The remaining sections of the paper are organized as follows:

- (2) Problem Background and Data Description, (3) Model Formulation
- (4) Empirical Results, and (5) Discussion and Summary.

## 2. PROBLEM BACKGROUND AND DATA DESCRIPTION

### 2.1 The Communications Mix for Ethical Drugs

The empirical setting for this study is the market for an established ethical drug. An ethical drug is one which can only be sold to persons



possessing a prescription written by a licensed physician. The ethical drug market represents a particularly favorable setting in which to study communications mix effects. First of all, compared to many industries the ethical drug field is rather data-rich in that most firms have extensive data bases generated by both commercial sources and their own in-house marketing research activities. Secondly, the communications mix is a prime competitive tool in these markets. Other elements in the marketing mix such as distribution and price are generally considered to be of lesser importance. Ethical drugs are typically widely distributed and attempts to measure response to a communications mix will not ordinarily be confounded by changes in availability. While price is very important in the institutional market for ethical drugs, physicians in private practice tend to be much less sensitive to price differentials. This study is concerned only with prescriptions written by physicians in private practice and price competition was absent in the particular market investigated.

The communications mix for an ethical drug consists of advertisements in medical magazines and journals, promotional material mailed directly to practicing physicians, and sales calls made on doctors -- referred to in the trade as "detailing". The ethical drug salesman (or detailman) frequently leaves product samples and literature with the doctor when he visits him. There is a substantial body of empirical evidence from survey research which indicates that physicians use these commercial channels of information about drugs and view them as legitimate sources of such information [5].



## 2.2 Variable Definitions and Measures

Sales and market share are the principal alternatives that suggest themselves for consideration as a measure of market response to a communications mix. In the case of ethical drugs, there is a two-fold advantage in using market share (MS). First, obtaining a favorable share of the new prescriptions written by physicians in any time period is the basic objective of a firm's communications mix. For established drugs, the total number of new prescriptions written for a given class of drugs depends primarily upon the incidence of need for a particular kind of treatment which, in turn, is determined by exogenous factors. Hence, industry promotion ordinarily can have very little effect on the level of total sales for an established drug category. This would not, of course, hold for a product which opened up an entire new class of drug therapy.

A second reason for using market share is that environmental factors which affect the absolute level of sales but which exert the same basic influence on all competing products need not be included in the response function. In contrast, raw sales figures are subject to seasonal and cyclical fluctuations due to variations in external conditions such as the incidence of disease in the population. Such problems can often be avoided by using a relative sales measure like market share. (A graphical analysis of the present data indicated such was the case here). These considerations led us to employ market share of new prescriptions as the dependent variable in our communications mix response models.<sup>2</sup> The figures analyzed were derived from audits of pharmacy records.



One problem which can arise in attempting to use market share as a measure of response is that of determining which are and which are not competing brands and products [49]. Intuition and convention can be quite misleading. As Stefflre [42] has discussed in a different context, it is often no simple matter to delineate the set of competing products and/or brands which constitute the "market". Fortunately, the situation in the ethical drug industry is generally more straightforward. There, market share may be computed with reference to a particular therapeutic class of drugs -- i.e., relative to a class of drugs which are designed to achieve a certain type of therapeutic action. While a certain amount of inter-class competition does occur in some instances, by and large competition is an intra-class phenomenon.

The communications mix variables analyzed here are journal advertising (JA), direct mail advertising (DM), and samples and literature (SL). The units in which these variables were expressed are the dollar amounts expended for each mix element utilized in a given month. Fixed creative and production costs were not included in the expenditure figures used here. Hence, variations in the expenditure figures approximate variations in the quantity of communication transmitted to physicians via the media which the mix components represent. Journal advertising expenditures reflect the space costs for advertisements placed in medical publications. Expenditures for direct mail cover printing and mailing costs. Samples and literature represent expenditures on those materials which detailmen leave with doctors.

During the data period studied, a major competitive product was forced to withdraw from the market as a result of action taken by gov-





environmental regulatory agency. Thus, a method of treating this occurrence was required. We conjectured that the withdrawal of the competitor was more likely to affect the level of the market share of the drug under study rather than the responsiveness of the market to the communications mix per se. Consequently, we include in the regression models a dummy variable (CO) which takes on a value of one for every month in which the competitor was present and zero in the subsequent months when he was absent. Clearly, a negative coefficient is anticipated for this variable in the models presented below.

### 3. MODEL FORMULATION

A distributed lag formulation was chosen as the basic model structure because it gives a dynamic, investment perspective to expenditures for marketing communications. Since the observation period is relatively short (a month), representation of carry-over or lagged effects was particularly important. Such effects could result from: (a) Brand Loyalty. If a communication appearing in month  $t$  attracts a physician who subsequently develops a preference for the drug, then the expenditure made for that communication in  $t$  will have contributed to sales realized in later periods. (b) Threshold Effects. Several exposures to communications on behalf of a drug may be required before a doctor is persuaded to prescribe it. Under such circumstances the eventual response is attributable to both current and previous communications expenditures. (c) Exposure and Use Opportunity. A lag may occur between the data at which a communication succeeds in convincing a doctor of the merits of a particular drug and the time when he has the opportunity



to prescribe that variety of drug therapy. As well, the occasions on which physicians are actually exposed to communications in a given month may be distributed over time. A supply of samples received today can be used over a period of months as the need arises. Similarly, some doctors may not get around to examining a medical journal until some time after it reaches them, while others may refer back to articles of interest in previous issues and in the process be re-exposed to the advertisements. All of these factors suggest the need for a model structure which incorporates lagged effects.

The basic distributed lag model we use is:

$$(1) \text{ LMS}(t) = a_0 + \sum_{i=0}^I a_{i+1} \text{ LJA}(t-i) + \sum_{j=0}^J b_{j+1} \text{ LSL}(t-j) \\ + \sum_{k=0}^K c_{k+1} \text{ LDM}(t-k) + e(t)$$

where:  $\text{LMS}(t)$  = log of market share for month  $t$

$\text{LJA}(t)$  = log of journal advertising expenditures in  $t$

$\text{LSL}(t)$  = log of expenditures on samples and literature on  $t$

$\text{LDM}(t)$  = log of direct mail expenditures in  $t$

$e(t)$  = residual in  $t$

Our use of the variables in log form implies an underlying multiplicative demand model. This enables us to interpret the coefficients in (1) as elasticities and further allows for mix or interaction effects of the communications variables on market share without undue burden on sample size and without magnifying multicollinearity problems.<sup>3</sup> Whenever a communication variable is zero for some period  $t$ , it is set



equal to \$1 for that t to avoid a log value of minus infinity. Since the communications variables average over \$1000 per period, the effect on the results should be negligible.

We refer to (1) as the "direct distributed lag model." Under the usual least squares assumptions on  $e(t)$ , least squares estimates of the coefficients of (1) will be best linear unbiased estimators. An important assumption is a lack of autocorrelation of the  $e(t)$ 's. The plausibility of this assumption may be tested using the Durbin-Watson d statistic and other sample indications. The latter is discussed in a later section of this paper. However, for the direct distributed lag model least squares estimates will remain unbiased even in the presence of autocorrelation of the  $e(t)$ 's. In such an event the estimators will not be fully efficient and in the usual case of positive autocorrelation, the least squares estimates of the sampling variances will tend to be biased downward [22, Chapter 7].

The direct distributed lag model has the advantage that it permits a different lag structure for each communication variable. Further, standard errors for sums of the elasticities for a given variable can be readily obtained for this model. We are interested in such sums in order to assess the intermediate and long-run effects of communications expenditures. On the other hand, it has a number of disadvantages such as: (a) there is no clear indications as to how many lags should be included for each communication variable (i.e., how large should I, J, and K be), (b) multicollinearity problems may arise when a communications variable is autocorrelated, (c) the available number of observations is reduced by the largest of I, J, or K thus reducing the



degrees of freedom for estimation, and (d) the number of parameters to estimate rapidly becomes large, further reducing degrees of freedom.

One solution to the problems of multicollinearity and degrees of freedom is to impose further structure on the basic lag model. The most popular method has been to impose an assumption that the effects of the exogenous variables will decline geometrically after some period. This then is a modified version of the distributed lag model proposed by Koyck [25] and widely applied in marketing demand studies [13], [26], [37], [41]. In our case we specify the model as:

$$(2) \quad \text{LMS}(t) = a_0 + a_1 \text{LJA}(t) + a_2 \text{LJA}(t-1) + a_3 \text{LJA}(t-2) + a_4 \sum_{i=0}^{\infty} \lambda^i \text{LJA}(t-3-i) \\ + b_1 \text{LSL}(t) + b_2 \text{LSL}(t-1) + b_3 \sum_{i=0}^{\infty} \lambda^i \text{LSL}(t-2-i) \\ + c_1 \text{LDM}(t) + c_2 \sum_{i=0}^{\infty} \lambda^i \text{LDM}(t-1-i) + e(t)$$

where:  $0 < \lambda < 1$ .

The following points should be noted concerning this model:

(a) The geometric decay in the effect of the communication variables may set in at different points in time for each variable. That is, the decay sets in at  $t-4$ ,  $t-3$ , and  $t-2$  for journal advertising, samples and literature, and direct mail, respectively. The decision as to when to specify the decay for each variable was made after examining empirical results.

(b) Once the geometric decay sets in, the same rate of decay  $(1-\lambda)$  is assumed to hold for all exogenous variables. This does not





imply that the effects of  $LJA(t-4)$ ,  $LSL(t-3)$ , and  $LDM(t-2)$  are identical (notice that these are the first terms exhibiting the geometric decay for each variable). This can be seen from (2) in that the coefficients for  $LVA(t-4)$ ,  $LSL(t-3)$ , and  $LDM(t-2)$  are  $\lambda a_4$ ,  $\lambda b_3$ , and  $\lambda c_2$ , respectively. Hence only the decay rate and not the magnitudes of the effects are assumed identical for all the communications variables.

(c) The decay terms form an infinite series and, hence, an arbitrary truncation of lagged terms is not required.

(d) Specific lags (e.g.,  $LVA(t-1)$ ,  $LJA(t-2)$ ,  $DM(t-1)$ , etc.) are included because there is reason to believe [20] that certain of the variables may have a greater effect after one or two periods than they do in the period during which the expenditure was made. Their inclusion provides an opportunity to examine the pattern of effects indicated by the data. In addition, the specific lags allow each variable to exhibit an individual decay rate up to the period in which the geometric decay sets in. By thus moving the common decay rate assumption back in time, the assumption becomes considerably less restrictive in our analysis since we would generally expect important differences to show up in the first few periods.

The basic model given in (2) may be transformed into a form which is readily estimated. First write (2) for  $LMS(t-1)$  (i.e., lagged one period), multiply both sides of the resulting equation through by  $\lambda$ , and subtract it from (2). This yields:



$$\begin{aligned}
 (3) \quad LMS(t) &= a_0(1-\lambda) + a_1LJA(t) + (a_2-\lambda a_1)LJA(t-1) + (a_3-\lambda a_2)LJA(t-2) \\
 &\quad + a (a_4-\lambda a_3)LJA(t-3) + b_1LSL(t) + (b_2-\lambda b_1)LSL(t-1) \\
 &\quad + (b_3-\lambda b_2)LSL(t-2) + c_1LDM(t) + (c_2-\lambda c_1)LDM(t-1) + \lambda LMS(t-1) \\
 &\quad + e(t) - \lambda e(t-1) \\
 &= \alpha_0 + \alpha_1LJA(t) + \alpha_2LJA(t-1) + \alpha_3LJA(t-2) + \alpha_4LJA(t-3) \\
 &\quad + \beta_1 LSL(t) + \beta_2LSL(t-1) + \beta_3LSL(t-2) + \gamma_1LDM(t) \\
 &\quad + \gamma_2LDM(t-1) + \lambda LMS(t-1) + U(t)
 \end{aligned}$$

where:  $U(t) = e(t) - \lambda e(t-1)$                       and:

$$\begin{aligned}
 (4) \quad a_0 &= \alpha_0/(1-\lambda) & b_1 &= \beta_1 \\
 a_1 &= \alpha_1 & b_2 &= \beta_2 + \lambda\beta_1 \\
 a_2 &= \alpha_2 + \lambda\alpha_1 & b_3 &= \beta_3 + \lambda\beta_2 + \lambda^2\beta_3 \\
 a_3 &= \alpha_3 + \lambda\alpha_2 + \lambda^2\alpha_1 & c_1 &= \gamma_1 \\
 a_4 &= \alpha_4 + \lambda\alpha_3 + \lambda^2\alpha_2 + \lambda^3\alpha_1 & c_2 &= \gamma_2 + \lambda\gamma_1
 \end{aligned}$$

The  $\alpha$ 's,  $\beta$ 's, and  $\gamma$ 's in (3) may be directly estimated using ordinary least squares. These estimates will be referred to as the "raw" coefficients. Our interest, however, centers in the  $a$ 's,  $b$ 's, and  $c$ 's which are the coefficients in the basic Koyck distributed lag form(2). Estimates of these coefficients (termed "adjusted" coefficients) may be derived from the raw coefficients by the equations given in (4). From



(4) we see that the raw coefficients provide direct estimates of the adjusted coefficients for current period (t) expenditures -- e.g.,  $a_1 = \alpha_1$ . However, the adjusted coefficients for each of the specific lag terms in (2) are non-linear functions of several of the raw coefficients. This makes it difficult to assess the standard errors of the adjusted coefficients for specific lag terms which in turn renders assessment of the standard errors of sums of the adjusted coefficients virtually impossible at the present state of the art.

Under the usual least squares assumptions (especially assuming that the  $U(t)$ 's are not autocorrelated), it is well known that least squares estimation of (3) will yield consistent estimates of the raw coefficients and thereby the estimates of the adjusted coefficients will be consistent.<sup>4</sup> There will, however, be a finite sample negative bias on the least squares estimate of  $\lambda$  in this case [21]. If the  $U(t)$  are not serially independent, then the least squares estimates of the raw coefficients are no longer consistent and a more complex estimation scheme should be employed [22]. The plausibility of assuming that the  $U(t)$  are serially independent is tested in the next section. There we also give consideration to several important econometric issues relating to the specification of the model and the direction of causal influence between the exogenous and the dependent variables.

#### 4. EMPIRICAL RESULTS

The data for this study consist of 54 monthly observations on the drug's market share of new prescriptions along with current and lagged expenditures for direct mail, samples and literature, and journal ad-



vertising. During the ninth month of our observations, a major competitive product was removed from the market. The presence of this competitor is represented by the dummy variable (CO) explained previously. The dummy variable was added to each of the estimating equations (1) and (3) in all of the results considered here.

In the remainder of this section we report the empirical results. The direct distributed lag estimates are discussed first along with certain econometric considerations. The modified Koyck results are presented next, including tests for autocorrelation and model specification. The third subsection analyzes the pattern of over-time effects for each of the communications variables. Finally, attention will be given to the comparison of the magnitude of the effects for the different communications variables.

#### 4.1 Direct Distributed Lag Estimation

Recall from (1) that the maximum direct lag for journal advertising, samples and literature, and direct mail is given by  $t-I$ ,  $t-J$ , and  $t-K$ , respectively. Since there is no theoretical basis for making a clear-cut prediction as to just what  $I$ ,  $J$ , and  $K$  should be, we present four sets of empirical results using the following scheme: a) estimate with  $I=3$ ,  $J=2$ ,  $K=1$ , b) increment each of  $I$ ,  $J$ , and  $K$  by one and re-estimate, and c) stop when the lag effects appears adequately estimated as judged on empirical grounds. The starting values for  $I$ ,  $J$ , and  $K$  seemed reasonable on the basis of earlier analysis using the modified Koyck model [32].

The direct distributed lag estimates are presented in the first four columns of Table 1. Notice that by the time we estimate  $(I, J, K) = (6, 5, 4)$  the lagged effects are virtually exhausted for all variables





as reflected by their magnitude and the insignificance of these terms in a test against zero (i.e., t statistics). Note also that the general pattern and magnitude of the effects for each variable are reasonably consistent across these four specifications. Further, the bulk of the estimates are statistically greater than zero as judged by the t statistics. Finally, the results all have the expected sign except for LDM(t-4), whose coefficient is only about 25% of its standard error. Both the unadjusted and adjusted coefficients of determination ( $R^2$  and  $R^{-2}$ ) are large and statistically significant given our degrees of freedom. The adjusted  $R^2$  is a nearly unbiased estimate of the population population proportion of the variation in the dependent variable associated with the linear function of the independent variables. For our sample size the bias must be less than .002 [31]. More will be said about the meaning of these results in the subsections on the patterns and magnitudes of effects. However, at this point consideration shall be given to the econometric issues of multicollinearity, autocorrelation of the independent variables, autocorrelation of the regression residuals in (1), and cross-lag correlations.

Multicollinearity analysis on the independent variables was performed using the methods suggested by Farrar and Glauber [12] and implemented on an interactive system by Gonzales and Montgomery [15]. The results for the smallest direct lag model are presented in Table 2 to illustrate the results.<sup>5</sup> Similar analyses were performed on all the remaining runs reported in this paper. The principle diagonal of the matrix in Table 2 presents the coefficient of determination ( $R^2$ ) variables as a linear function of all the other independent variables.



Below the principle diagonal we present the partial correlations between the pair of variables indicated by the row and column, the linear effects of all other variables in the analysis having been removed from each member of the pair. Above the principle diagonal are the t values having 44 degrees of freedom which correspond to the partial correlation coefficients. Below the table lie the F statistic having 9 and 44 degrees of freedom and which correspond to the multiple  $R^2$ 's given on the principle diagonal. The  $R$ 's and partial correlation results are distribution free, but use of the F and t statistics implies an assumption of multivariate normality for the set of independent variables. The results given in Table 2 indicate some multicollinearity for each variable except for direct mail which appears to be essentially free of the problem and SL(t-2) and JA(t) which are also not especially affected. While the multicollinearity problem becomes somewhat greater as we move toward the largest direct lag model, the multiple  $R^2$  for a communication variable never exceeded .47 and the locus of collinearity continued to be among the advertising variables and between the competitive dummy variable and samples and literature.

The autocorrelation of the independent variables is important in interpreting the results related to autocorrelation of the regression residuals. The simple correlations between LDM(t) and successively lagged values of LDM are, .05, -.19, .00, .02, so we see that direct mail is essentially not autocorrelated. Similarly, for samples and literature the correlation between LSL(t) and successively lagged values of LSL are -.04, .04, .12, .15, .01. Hence, samples and literature are not autocorrelated. However, the results for LJA are



.42, .10, .06, .01, .08, -.02 which indicate substantial positive first order autocorrelation for LJA. In essence this implies that LJA exhibits a somewhat smooth evolution over time.

A test for autocorrelation of the regression residuals,  $e(t)$ , in (1) may be made by applying the Durbin-Watson  $d$  statistic [30]. For our sample size of 54 and 10 to 19 independent variables (excluding the constant), all the  $d$  statistics given in Table 1 lie in the inconclusive region between  $d_L$  and  $d_U$  for a 5% significance test of the null hypothesis of no autocorrelation of the residuals in (1). Consequently, the Durbin-Watson test is inconclusive with respect to autocorrelation of the residuals. The Theil and Nagar [43] statistic which eliminates the inconclusive region cannot be used because both direct mail and samples and literature have essentially zero autocorrelation, thereby violating their assumptions. One rough test is to examine the estimate of the first order autocorrelation coefficient of the sample residuals, given as  $\hat{\rho}$  in Table 1.<sup>6</sup> For our direct distributed lag models these coefficients range from .34 for the smallest to .20 for the largest model. However, the empirical estimate of the autocorrelation of the  $e(t)$  is likely to be biased downward. We adjust for this bias by using Malinvaud's [30, p. 433] procedure under the condition of low autocorrelation of the independent variables. These results are presented as  $\hat{\rho}$  in Table 1. These results for the larger direct lag model are on the order of .3. We conclude that there probably is some modest amount of positive autocorrelation in our direct distributed lag models.

Even in the presence of autocorrelation of the  $e(t)$ 's our OLS estimates remain unbiased, although they may be somewhat less efficient



than GLS estimators. Since the autocorrelation of the independent variables is low and the autocorrelation of the residuals is relatively small, the OLS estimates of the standard errors of the coefficients will be very near the true standard errors. See Malinvaud [30, pp. 433-5]. Our overall conclusion is that our OLS results yield a good picture of the communications effects.

In using single equation models in this study we have assumed that market share is determined by the communications variables and not vice versa. There has been considerable discussion [e.g., 2, 4] of the need for simultaneous equation demand models given the problems of identification and estimation bias associated with the use of single equation models in situations where there are two-way relationships between sales and promotional variables.

The problem of a built-in interdependency between market share and communications expenditures arising out of the coincidence between the firm's planning cycle and the observation periods is less likely to occur when the latter are of short duration (e.g. months) since delays are involved in the reporting of market results to management. Furthermore, it takes time to implement policy changes once the decision to make them has been made and such lags are not the same for all the communications variables considered here.

One simple method that has been proposed for assessing causal priorities between pairs of variables is the "cross lag correlation" technique. Briefly, the approach rests on the following argument: If C determines E rather than vice versa, then the correlation  $r[C_{t-1}E_t]$  should be greater than the correlation,  $r[C_tE_{t-1}]$  where: r is the simple





correlation coefficient; C and E represent the variables designated cause and effect, respectively. The technique has been applied previously in both behavioral [6] and economic research [40].

We utilized the method here to check our implicit assumption about the causal ordering of market share and communications expenditures. Table 3 presents the simple pairwise product moment correlation coefficients for log market share and the logs of the various mix variables. If the dominant causal direction is from the communications variables to market share and not vice versa, then the correlation between  $LMS(t-1)$  and each variable should be smaller than the correlation between  $LMS(t)$  and each variable at  $t-1$ ,  $t-2$ ,  $t-3$ , ... . We see from Table 3 that this is indeed the case for all but two of the fifteen pairwise comparisons of the correlations. The reversals are for direct mail at  $t-3$  and  $t-4$ .

#### 4.2 Modified Koyck Estimation

The inclusion of the competitive dummy variable in (2) requires some comment. After going through the transformation to obtain the autoregressive form (3), the competitive term will be  $dCO(t) - \lambda dCO(t-1)$  where  $d$  is the coefficient of the competitive dummy variable in (2). Now since  $CO(t) = CO(t-1)$  except for  $t = 9$ , we would have nearly perfect collinearity if we tried to estimate (3) with both  $CO(t)$  and  $CO(t-1)$  in the equation.<sup>7</sup> Consequently, we must seek an estimable formulation. Since  $CO(t) = CO(t-1)$  at all but one data point, we estimated (3) with  $d(1-\lambda)CO(t)$  as a term. An estimate of  $d$  may then be obtained from the raw coefficient of  $CO(t)$  by simply dividing it by  $1-\lambda$ .

Both the raw coefficients and the adjusted coefficients are given



in the last two columns of Table 1. As in the direct estimation case, most of the raw coefficients are significant. The adjusted  $R^2$  of .89 is indicative of a good fit to the data. The adjusted coefficients are quite similar in pattern and magnitude to those obtained by direct estimation and all have the expected sign. The major exceptions in pattern and magnitude are for samples and literature at t-3, t-4, and t-5, where the Koyck geometric decay seems to understate the effects of this communication variable and for journal advertising at t-5 which exhibits an increase over t-4, which, of course, the geometric decline cannot represent unless specific lags up to t-5 are included. The latter would, of course, eliminate most of the advantages of a modified Koyck specification.

We turn now to some econometric considerations relating to the modified Koyck model. It is known that if the residuals,  $U(t)$ , in (3) are not autocorrelated, then OLS applied to (3) will yield consistent estimates of both the raw and the adjusted coefficients, although there will be a small sample bias in the estimate of  $\lambda$  [21]. However, if the  $U(t)$  are autocorrelated, the OLS estimates will be inconsistent. Since the estimating equation contains a lagged value of the dependent variable, the Durbin-Watson d statistic is inappropriate since it will be biased toward 2, the value indicating no autocorrelation [36]. Recently Durbin [8] introduced a new statistic which may be used to test for autocorrelated residuals in equations containing lagged values of the dependent variable. The statistic is given by

$$h = \beta \frac{T}{1-T \text{Var}(\lambda)}$$



where:

$\hat{\rho}$  = sample first order autocorrelation coefficients of the residuals,  $U(t)$ , ( $\hat{\rho} = 1-d/2$ ).

T = number of observations

$\hat{V}\hat{a}r(\lambda)$  = estimated variance of the coefficient of the dependent variable lagged one period.

Durbin has shown that  $h$  is asymptotically normally distributed with mean 0 and variance 1 under the null hypothesis that  $\rho$  is zero. For our model  $h = -.308$ , which is only about one third of its standard error and thereby is very consistent with the absence of autocorrelation in the residuals of the modified Koyck model.<sup>8</sup> The power of the test is asymptotically equivalent to the power of a likelihood ratio test.<sup>9</sup> Further, an indication that the residuals in the modified Koyck form are zero autocorrelated, or at least nearly so, may be found by comparing the  $\lambda$  of .348 estimated for the Koyck model with the adjusted  $\hat{\rho}$  of about .31 for the largest direct estimation models. The  $U(t)$ 's in (3) will not be autocorrelated if the  $e(t)$ 's in (2) are first order autocorrelated with autocorrelation coefficient  $\lambda$ . Since the largest direct lag model is an approximation of (2), we see that the condition is fulfilled within the sampling fluctuation of the estimate of  $\lambda$ .

We next present some specification checks of the modified Koyck model following several suggestions of Griliches [17] and Bass and Clarke [3]. We first suppose that the true relation is not a distributed lag model of the form of (2) but rather:

$$(5) \quad LMS(t) = a_0 + a_1LJA(t) + a_2LJA(t-1) + a_3LJA(t-2) + b_1LSL(t) \\ + b_2LSL(t-1) + c_1LDM(t) + dCO(t) + U_t$$



where:

$$(6) \quad U(t) = \rho U(t-1) + \eta_t$$

the regression residual in (5) is first order autocorrelated and where  $\eta_t$  satisfies the usual least squares assumptions. Griliches points out that if (5) and (6) are true but we estimate (3) we may well obtain significant and sensible coefficients and observe reduced serial correlation in the residuals even though the distributed lag model is wrong. A test for this potential problem can be made by substituting (6) into (5) and then expressing  $U(t-1)$  as the difference between  $LMS(t-1)$  and its corresponding regressors from (5) expressed at  $t-1$ .

This yields:

$$(7) \quad LMS(t) = a_0(1-\rho) + a_1LJA(t) + (a_2-\rho a_1)LJA(t-1) + (a_3-\rho a_2)LJA(t-2) \\ - \rho a_3LJA(t-3) + b_1LSL(t) + (b_2-\rho b_1)LSL(t-1) \\ - \rho b_2LSL(t-2) + c_1LDM(t) - \rho c_1LDM(t) + d(1-\rho)CO(t) \\ + \rho LMS(t-1) + \eta_t$$

where we again treat the competitive dummy variable as discussed above, If (5) and (6) are true, when we estimate (7) we should have estimated coefficients for  $LJA(t-3)$ ,  $LSL(t-2)$ , and  $LDM(t-1)$  approximately equal  $-\rho a_3$ ,  $-\rho b_2$ , and  $-\rho c_1$ , respectively, when we estimate without constraining these coefficients to these values. We have already presented such unconstrained estimation of (7) as our raw Koyck results in Table 1. If (5) and (6) were true rather than the distributed lag model, the coefficient estimates for  $LJA(t-3)$ ,  $LSL(t-2)$ , and  $LSL(t-1)$  should be  $-.009$ ,  $-.010$ , and  $-.001$ , respectively. However, the estimated values





of these coefficients in Table 1 are all positive and in the cases of LJA(t-3) and LDM(t-1) are more than two and one half times their standard errors. The LSL(t-2) result is about one and one half times its standard error. From this it would appear that our Koyck results are not spurious.

Further specification tests were made against higher order lag models -- i.e., models containing LMS(t-2) and both LMS(t-2) and LMS(t-3). The procedure follows Bass and Clarke [3]. We first examine the parameter estimates for these additional lagged values of the dependent variable and we then apply the Scheffe test [39] for the significance of the contribution of these terms when added to our basic "modified Koyck model" (3). The coefficient of LMS(t-2) was .134 with a standard error of .112. Hence the coefficient value was scarcely greater than its standard error. When we also added LMS(t-3) to the equation its coefficient value was -.090 with a standard error of .112. In both of these cases the modified Koyck model of (3) therefore appears more plausible than a higher order lag scheme.

The Scheffe test provides an F statistic for testing the contribution to regression from adding variables to the set of regressors.<sup>10</sup> If the observed F statistic for the additional variables exceeds the critical level, the additional variables are taken to provide a significant contribution to the regression. When we added LMS(t-2) to our basic Koyck model (3), the F value for its contribution was 1.43 which is far below the critical value which is  $F(.05, 1, 42) = 4.07$ . Hence, LMS(t-2) does not add significantly to the regression. When we tested the joint contribution of LMS(t-2) and LMS(t-3) to (3),



the F value was 1.06, which again is less than the critical  $F(.05, 2, 41) = 3.23$ . Hence adding both these variables to the basic Koyck model does not significantly enhance the fit of the model.

We conclude from these results that the first order Koyck lag is more plausible than a second or third order model. But what about the contribution of  $LMS(t-1)$ ? When this variable is added to the smallest direct lag model to make it a modified Koyck model, the F value for  $LMS(t-1)$  is 13.7 which is much larger than  $F(.01, 1, 43) = 7.27$ . Hence, the modified Koyck model dominates the smallest direct lag model.

#### 4.3 Pattern of Effects

The over-time pattern of effects for each communication variable may be examined in Table 1. The direct mail effect is initially miniscule, peaks at  $t-1$ , and is essentially zero by  $t-4$ . Similarly the samples and literature effects are small for period  $t$ , peak at  $t-1$ , remain relatively high through  $t-3$ , and are essentially zero by  $t-5$ . Hence, both direct mail and samples and literature exhibit lag functions which peak at other than the current period. In contrast, journal advertising effects exhibit a high initial effect, followed by a long trail off extending to  $t-6$  with modest peaking at  $t-3$  and  $t-5$ . The seemingly peculiar pattern for journal advertising is probably the result of the high collinearity between successive advertising terms. See Table 2. An indication of this is the small t ratios in Table 1 for many of the advertising terms. This collinearity does not create a problem for estimates of sums of the elasticities (coefficients).

An interesting summary form in which to examine the pattern of



effects of the communication variables is to determine the quarterly and long-run elasticity for each variable. The quarterly elasticity for any variable is simply the sum of its elasticity values at  $t$ ,  $t-1$ , and  $t-2$ . For example, the quarterly elasticity for journal advertising is  $a_1 + a_2 + a_3$ . Similarly, the long-run elasticity is the sum of all its coefficients. Since we use the largest direct lag estimates in our computations, we see that the long-run elasticity for journal advertising is  $\sum_{i=1}^7 a_i$ . The results are presented in Table 4, where we also give the corresponding results for the Koyck estimates.<sup>12</sup> Note that the short-run elasticities are given directly as  $a_1$ ,  $b_1$ ,  $c_1$ .

Since the quarterly and long-run elasticities are linear combinations of random variables, we may estimate their standard errors by utilizing the estimated variance -- covariance matrix of the elasticities in Table 1, which we readily obtain from regression analysis. These results are reported in Table 4 for the direct estimates.<sup>13</sup> With the exception of the short-run elasticity for direct mail, the estimates are all large relative to their standard errors. Note in particular that the precision of estimation problem for journal advertising coefficients which arose due to collinearity problems have been eliminated in the quarterly and long-run results.

The short-run, quarterly, and long-run elasticities form an interesting pattern for each variable. Direct mail exhibits little initial response, while virtually the entire response occurs within a quarter of an expenditure. This result is consistent with other company studies in which the receipt of reply cards from direct mail promotions tends to follow the pattern exhibited by the short-run, quarterly, and



long-run elasticities estimated for direct mail. The bulk of the reply cards are typically returned between four and thirteen weeks of a mailing but a small number will continue to trickle in for some time. Such a flow corresponds to the elasticities estimated here.

Samples and literature also have a low initial response, while the response within a quarter of an expenditure is roughly 70% of the estimated total response. This latter result is consistent with other company studies which indicate that physicians tend to use samples within a few months of receiving them and that they discard any old supplies.

In contrast, the response to journal advertising is very large during the month in which the expenditure was made. While the quarterly elasticity is substantially greater than the short-run elasticity in absolute terms, the short-run elasticity provides a substantial proportion of the total quarterly elasticity. However, the long-run elasticity is more than double the short-run elasticity and is very large in absolute terms. This indicates a substantial long-term effect on market share from journal advertising expenditures.

#### 4.4 Magnitude of Effects

The short-run, quarterly, and long-run elasticity results given in Table 4 indicate substantial response to journal advertising, smaller but still significant response to samples and literature, and extremely small response to direct mail. Yet, the average monthly expenditures made for each communication variable was:

|                        |         |
|------------------------|---------|
| Journal Advertising    | \$120.9 |
| Samples and Literature | \$135.5 |
| Direct Mail            | \$163.0 |





where the magnitude of the expenditures has been coded to preserve the confidential nature of the data. It appears that the company had been allocating its communications expenditures in inverse relation to the actual market response! Although further modeling would be required to link these results for market share to sales and profitability, the clear implication of these results is that the company has tended to underinvest in journal advertising and overinvest in direct mail. In any case, the results strongly suggest that a re-examination of the firm's allocation and budgeting procedures is in order.

To assure that the conclusion based on the elasticity differences are on firm statistical ground, we calculated the standard errors of these differences. The results are presented in Table 5. The differences in elasticities indeed exceed that expected from sampling fluctuations by a goodly margin in all cases except for the short-run elasticity difference between samples and literature and direct mail. Recall however, that for both of these variables, the short-run elasticities are very small.

## 5. DISCUSSION AND SUMMARY

Dynamic estimation models of market share response to direct mail, journal advertising, and samples and literature indicated substantial differences between these communication variables in terms of both the magnitude and over-time pattern of their effects. These results caution against attempts to analyze response to an aggregate communication variable for a pharmaceutical product, even though such a practice



might be tempting in order to conserve degrees of freedom. A variety of econometric checks on the models indicated that there is reason to have confidence in the results.

But what is the managerial significance of these results? Recall that the estimates indicate that management had historically allocated resources to their three communication vehicles in inverse relation to their effectiveness considered as estimated in the above models. Although cost considerations would have to be considered explicitly to arrive at specific allocation decision, these results do suggest the need for a major reallocation of expenditures among these three communication vehicles. In the absence of a formal measurement program to help managers learn systematically from their past actions, it is perhaps not surprising to find that an apparent misallocation occurred. Reply cards returned by doctors were generally utilized in connection with direct mail promotions so such activities were accompanied by a clear, observable response. Hence, the heavy use of direct mail is probably related to the rapid and detectable nature of response to this communication medium. The fact that the product managers were themselves all former detailmen suggests that their own past field experience had convinced them that substantial expenditures for samples and literature were justified. In contrast, there was little to indicate the product managers just how the market was affected by responding to journal advertising. The uncertainty with respect to payoff likely contributed to the relative underutilization of journal advertising. Measurement approaches such as the one taken in this paper can provide a methodology whereby managers may begin to learn from their previous



experience by analyzing the effects of their past actions. Such measurement programs should help to improve the quality of judgments which managers bring to bear in judgmentally parameterized models such as DETAILER [33] and ADBUDG [28].

Finally, having seen these historical results, the company is now in a better position to design a direct market experiment. The magnitudes of effects provide clues as to how large an experimental change would have to be for each communication variable in order to observe any effects in a market experiment. Further, the over-time pattern of effects provides information as to the required duration of an experiment (e.g., how many months) or the proper design to use (e.g., a switch-over design in order not to bias the results against a variable such as advertising which appears to have a considerable long-run payout.

In summary then, this work indicates that analysis of historical relationships using dynamic measurement models can help marketing managers in at least three distinct ways: a) to learn about the nature of communications response systematically from their past experience and thereby sharpen their future judgements in this area; b) to identify problem areas where a re-direction of previous policies with respect to the allocation of communication effort would be profitable; and c) to design proper market experiments that will yield more refined knowledge about response to the firm's marketing communications mix.



FOOTNOTES

- 1 One exception is Nakanishi's study [35] of consumer response to various forms of promotion and advertising for new products.
- 2 Sales arising from new prescriptions should be distinguished from those which occur when an existing prescription is refilled. Refills or repeat sales on a prescription filled previously were excluded from the data utilized here.
- 3 To be sure, interaction or mix effects could be incorporated in a model which is linear in the variables by introducing a large number of interaction terms. This could greatly expand the set of independent variables which, in turn, might gravely strain the available data base. It could also contribute to reduced precision in estimating the coefficients by introducing additional multicollinearity.
- 4 From (4) we see that the adjusted coefficients are continuous functions of the raw coefficients. Consequently, we can invoke Slutsky's Theorem which states: "If  $\hat{\theta}$  is a consistent estimator of  $\theta$ , and if  $\psi = g(\theta)$  is a continuous function of  $\theta$  then  $\hat{\psi} = g(\hat{\theta})$  is a consistent estimator of  $\psi$ " [14, p. 118].
- 5 Space constraints precludes their inclusion here. Copies of the remaining collinearity analyses are available from the authors upon request.
- 6 Note that  $\hat{\rho} \cong 1 - d/2$  [14].
- 7 In addition we'd also have to use restricted least squares since the coefficient of  $C0(t-1)$  is  $d\lambda$ .
- 8 It should be noted that sampling fluctuations could lead to  $T \hat{V} \hat{r} > 1$ , in which case  $h$  cannot be computed. Durbin [ 8 ] offers an alternative, but more computationally burdensome approach which may be used in such cases.
- 9 The finite sample properties and the associated power of the  $h$  statistic are under study now by Montgomery and van den Abeele using Monte Carlo methods [45]. Preliminary results lead us to have considerable confidence that the above result is reasonable.
- 10 Let  $\Omega$  denote the assumptions associated with a regression model having  $r$  independent variables and  $\omega$  denote the set of assumptions associated





with a nested regression model such that  $\omega$  differs from  $\Omega$  only by the fact that under  $\omega$ ,  $q$  of the variables under  $\Omega$  have zero regression coefficients. The test of  $H_0: \omega$  vs.  $H_1: \Omega$  is  $F = \frac{(n-r)(S_\omega - S_\Omega)}{q S_\Omega}$  where  $n$  is the number of observations and  $S_\omega$  is the residual sum of squares under  $\omega$  and  $S_\Omega$  is the residual sum of squares under  $\Omega$ . Then we reject  $H_0$  in favor of  $H_1$  if  $F > F_{\alpha, q, n-r}$ .

- 11 The competitive term was appended to (3) in all of these results.
- 12 For the Koyck estimates, the long-run elasticities are  $a_1 + a_2 + a_3 + a_4/(1-\lambda)$ ,  $b_1 + b_2 + b_3/(1-\lambda)$ , and  $c_1 + c_2/(1-\lambda)$  for journal advertising, samples and literature, and direct mail, respectively.
- 13 The Koyck estimates of all the long-run elasticities and the quarterly elasticity for DM involve non-linear functions of the elasticities from Table 1, rendering estimation of the standard errors problematical at the current state of the art.



TABLE I  
Distributed Lag Estimates<sup>1</sup>

| Variable       | Direct Estimates <sup>2</sup> |                   |                   |                   | Koyck Estimates <sup>3</sup> |          |
|----------------|-------------------------------|-------------------|-------------------|-------------------|------------------------------|----------|
|                | I, J, K=3, 2, 1               | I, J, K=4, 3, 2   | I, J, K=5, 4, 3   | I, J, K=6, 5, 4   | Raw                          | Adjusted |
| a <sub>0</sub> | -3.78<br>(-11.64)             | -4.30<br>(-12.64) | -4.67<br>(-13.31) | -4.69<br>(-12.07) | -2.75<br>(-6.83)             | -4.22    |
| CO             | -.458<br>(-7.82)              | -.383<br>(-6.65)  | -.346<br>(-6.12)  | -.341<br>(-5.70)  | -.28<br>(-3.95)              | -.423    |
| LDM(t)         | .001<br>(.22)                 | .003<br>(.93)     | .002<br>(.55)     | .002<br>(.62)     | .002<br>(.71)                | .002     |
| LDM(t-1)       | .012<br>(3.11)                | .008<br>(2.33)    | .010<br>(2.74)    | .010<br>(2.56)    | .010<br>(2.86)               | .001     |
| LDM(t-2)       |                               | .008<br>(2.30)    | .006<br>(1.67)    | .006<br>(1.62)    |                              | .004     |
| LDM(t-3)       |                               |                   | .001<br>(.38)     | .001<br>(.21)     |                              | .001     |
| LDM(t-4)       |                               |                   |                   | -.001<br>(-.24)   |                              | .000     |
| LSL(t)         | .013<br>(1.60)                | .015<br>(2.10)    | .013<br>(1.94)    | .013<br>(1.76)    | .015<br>(2.17)               | .015     |
| LSL(t-1)       | .028<br>(3.60)                | .032<br>(4.43)    | .033<br>(4.73)    | .032<br>(4.26)    | .025<br>(3.68)               | .030     |
| LSL(t-2)       | .021<br>(2.93)                | .026<br>(3.88)    | .030<br>(4.47)    | .029<br>(4.23)    | .010<br>(1.51)               | .020     |
| LSL(t-3)       |                               | .018<br>(2.94)    | .022<br>(3.64)    | .023<br>(3.40)    |                              | .007     |
| LSL(t-4)       |                               |                   | .008<br>(1.46)    | .009<br>(1.44)    |                              | .002     |
| LSL(t-5)       |                               |                   |                   | .002<br>(.48)     |                              | .000     |
| LJA(t)         | .139<br>(5.17)                | .150<br>(6.17)    | .146<br>(6.37)    | .146<br>(5.99)    | .157<br>(6.48)               | .157     |
| LJA(t-1)       | .008<br>(.27)                 | .005<br>(.21)     | .014<br>(.53)     | .014<br>(.52)     | -.053<br>(-1.71)             | .002     |
| LJA(t-2)       | .021<br>(.72)                 | .024<br>(.91)     | .024<br>(.97)     | .027<br>(1.02)    | .026<br>(1.00)               | .026     |
| LJA(t-3)       | .091<br>(3.15)                | .085<br>(3.12)    | .079<br>(2.97)    | .079<br>(2.85)    | .068<br>(2.61)               | .077     |
| LJA(t-4)       |                               | .040<br>(1.59)    | .024<br>(.94)     | .020<br>(.70)     |                              | .027     |
| LJA(t-5)       |                               |                   | .054<br>(2.22)    | .054<br>(1.97)    |                              | .009     |
| LJA(t-6)       |                               |                   |                   | .005<br>(.18)     |                              | .003     |
| LMS(t-1)       |                               |                   |                   |                   | .348<br>(3.66)               |          |
| R <sup>2</sup> | .8862                         | .9185             | .9326             | .9333             | .9137                        |          |
| $\bar{R}^2$    | .860                          | .892              | .904              | .896              | .891                         |          |



TABLE I (CON'T)

| Variable    | Direct Estimate <sup>2</sup> |             |             |             | Koyck Estimates <sup>3</sup> |          |
|-------------|------------------------------|-------------|-------------|-------------|------------------------------|----------|
|             | I,J,K=3,2,1                  | I,J,K=4,3,2 | I,J,K=5,4,3 | I,J,K=6,5,4 | Raw                          | Adjusted |
| $d^4$       | 1.32                         | 1.52        | 1.54        | 1.60        |                              | -.308    |
| $\hat{p}^5$ | .34                          | .24         | .23         | .20         |                              | -.03     |
| $\bar{p}^6$ | .42                          | .32         | .33         | .31         |                              |          |



Footnotes to Table 1

- 1 The sample size in all cases is 54 monthly observations. The number in parentheses below each estimate is the corresponding t ratio having 54-k degrees of freedom where k = number of independent variables in that equation including the intercept.
- 2 I, J, K represent the number of lagged terms included in the model for journal advertising, samples and literature, and direct mail, respectively. See (1).
- 3 Adjusted Koyck coefficients beyond the values for the last variable included in the Koyck regression were computed using the geometric decline with  $\lambda = 0.348$ , the estimated value of  $\lambda$ .
- 4 The Durbin-Watson d statistic is not appropriate for the Koyck model. The value in this row for the Koyck model is the Durbin h statistic which is appropriate for autoregressive models.
- 5  $\hat{\rho} = 1-d/2$  and is an estimate of the first order autocorrelation coefficient of the observed regression residuals.
- 6  $\bar{\hat{\rho}} = \hat{\rho} (1 + \frac{m}{T-m})$  where m = the number of independent variables (excluding the intercept) and T = the number of observations. This adjusts for the downward bias in the empirical correlogram when the independent variables have irregular evolutions (i.e., low autocorrelation) as in the present case. See Malinvaud [30, p. 433].





TABLE II

## Multicollinearity Analysis\*

|          | LSL(t)     | LSL(t-1)   | LSL(t-2)   | LDM(t)     | LDM(t-1)   | LJA(t)     | LJA(t-1)   | LJA(t-2)   | LJA(t-3)   | CO         |
|----------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| LSL(t)   | <u>.35</u> | -1.99      | -.87       | .15        | .79        | -1.15      | 1.25       | -1.22      | -2.23      | -3.79      |
| LSL(t-1) | -.29       | <u>.31</u> | -.97       | -.03       | -.29       | .05        | -.61       | .53        | -2.24      | -3.80      |
| LSL(t-2) | -.13       | -.14       | <u>.19</u> | 1.55       | -.61       | -.97       | .91        | -1.08      | -.01       | -2.27      |
| LDM(t)   | .02        | -.01       | .23        | <u>.09</u> | .39        | -.55       | -.30       | .42        | .66        | .36        |
| LDM(t-1) | .12        | -.04       | -.09       | .06        | <u>.08</u> | .67        | -1.22      | .04        | .56        | -.56       |
| LJA(t)   | -.17       | .01        | -.14       | -.08       | .01        | <u>.25</u> | 3.01       | -.37       | 0.31       | -1.13      |
| LJA(t-1) | .19        | -.09       | .14        | -.04       | -.18       | .41        | <u>.38</u> | 2.84       | .21        | .01        |
| LJA(t-2) | -.18       | .08        | -.16       | .06        | .01        | -.06       | .39        | <u>.38</u> | 2.25       | -1.02      |
| LJA(t-3) | -.32       | -.32       | .00        | .10        | .08        | -.05       | .03        | .32        | <u>.33</u> | -1.75      |
| CO       | -.50       | -.50       | -.32       | .05        | -.08       | -.17       | .00        | -.15       | -.25       | <u>.48</u> |
| F(9,44)  | 2.64       | 2.18       | 1.13       | 0.49       | 0.42       | 1.62       | 3.03       | 2.94       | 2.45       | 4.52       |

\*The entries on the principle diagonal (underlined in the table) are the  $R^2$ 's for that variable as a linear function of all the other variables in the table. The F(9,44) at the bottom of each column corresponds to this  $R^2$ . The critical values of F at the .05 and .01 levels are 2.10 and 2.84, respectively. The entries below the principal diagonal are the partial correlations between the row and column variables. The entries above the principle diagonal are the t statistics having 44 degrees of freedom which are associated with the partial correlation coefficients. The critical t values for a two-tailed test at the .05 and .01 levels are 1.96 and 2.32, respectively.



TABLE III

Cross Lag Correlations

| Communication Variable | Correlation of LMS(t-1) with Communication Variable in t | Correlation of LMS(t) with Communication Variable in: |     |      |      |     |     |
|------------------------|--|---|-----|------|------|-----|-----|
|                        |  | t-1   | t-2 | t-3  | t-4  | t-5 | t-6 |
| LDM                    | .01  | .25   | .16 | -.04 | -.13 |     |     |
| LSL                    | .19  | .42   | .28 | .33  | .25  | .19 |     |
| LJA                    | .17  | .48   | .31 | .29  | .28  | .18 | .20 |



TABLE IV  
Short Run, Quarterly, and Long Run Elasticities

|                   | <u>DM</u> |       |       | <u>SL</u> |       |       | <u>JA</u> |      |      |
|-------------------|-----------|-------|-------|-----------|-------|-------|-----------|------|------|
|                   | SR        | Q     | LR    | SR        | Q     | LR    | SR        | Q    | LR   |
| Direct Estimation | .0020     | .0180 | .0180 | .0130     | .0740 | .1080 | .146      | .187 | .365 |
| Standard Error    | .0037     | .0065 | .0094 | .0072     | .0150 | .0190 | .024      | .031 | .057 |
| Koyck Estimation  | .0020     | .0170 | .0190 | .0150     | .0650 | .0760 | .157      | .185 | .303 |
| Standard Error    | .0035     | *     |       | .0070     | *     |       | .031      | *    |      |

Key: SR = Short Run  
Q = Quarterly  
LR = Long Run

\*Not applicable since the Koyck model does not provide an estimate of these standard errors.



TABLE V  
Short Run, Quarterly, and Long Run

|                              | <u>JA-DM</u> |      |      | <u>JA-SL</u> |      |      | <u>SL-DM</u> |      |      |
|------------------------------|--------------|------|------|--------------|------|------|--------------|------|------|
|                              | SR           | Q    | LR   | SR           | Q    | LR   | SR           | Q    | LR   |
| Elasticity Difference        | .144         | .167 | .347 | .133         | .113 | .257 | .011         | .056 | .090 |
| Standard Error of Difference | .024         | .046 | .057 | .022         | .033 | .052 | .008         | .014 | .020 |





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