A Dynamic Model of the Electricity Generation Market

by

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Abstract

This thesis proposes that the bidding process that occurs daily in the competitive short-run power market can be modeled as a dynamic system, or a dynamic game played by electricity generators. Such a game is a finitely repeated one of complete but imperfect information. In this thesis, a dynamic model representing a small short-run power market is formulated as a repeated game. Daily price competition provides sufficient information for the generators to estimate their bids. The next bids of each generator are proposed as functions of the previous and current bids. Results from the dynamic model show that the generators' bidding strategy affects the dynamics of the modeled power market. Different strategies yield different market clearing price patterns. Moreover, a step-supply function bid, in which an offer price relates to an offer quantity by a marginal-cost function, and the maximum available capacity of each generator, can cause inefficient prices in some scheduling periods and/or might result in inefficient dispatches in each scheduling day. In addition, depending on the bidding strategy that is uniformly applied to the model, although there is certainty of inelastic anticipated demand in the model, the repeated bidding processes tends to allow the generators to "learn" from the market so that they can tacitly collude to create demand deficiency. It is further suggested here that when demand deficiency unexpectedly occurs in peak-load periods, a real-time market (i.e., an hour-ahead market) might be needed so that supply always meets demand.

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Chapter 1

Introduction

This thesis offers a dynamic model of an electric power market under competition designed to capture market behavior that can be neglected in a static model . The dynamic model is focused on the market in a Poolco structure. The Poolco structure is one form of the power market under competition. A "Poolco market", a "power pool" or a "pool" refers to a short-run power market where its activities, such as power trading, etc., occur on a daily basis. In addition, electricity demand varies over time so that the market prices of the electricity vary over time (i.e., hourly or half-hourly). The customers pay for their power equal to the "spot" or "hourly" price. Therefore, a power pool is also known as a "spot market." Basically, in the power market, the characteristics of the pools are various. There are many power markets in the US that are forming. The UK power system is the first system that was successfully privatized or that was transformed from the government-regulated enterprise to the privately-owned business. Therefore, the characteristics of the UK pool are used to provide an overview of a pool.

The pool has been created to organize the scheduling, dispatch, and payment of power stations on a daily basis. Each generating set submits a multi-part bid, consisting of prices and technical information, and a computer algorithm (GOAL) then produces the schedule that minimizes the cost of meeting demand. The bid from the most expensive station scheduled for normal operation in each half hour is used to calculate the system marginal price, paid for every unit scheduled in that period. The pool price tends to be volatile, moving with the level of demand, which depends on the weather, and with the level of capacity available, which is affected by random faults. To provide greater certainty, most electricity sales in the pool are covered by contracts for differences [13, 20].

In summary, a simplified power pool consists of the Independent System Operator (ISO), power producers and customers. The ISO schedules the power producers to generate their power such that supply meets anticipated demand. The power producers (generators) bid their supply functions into the power pool on a daily basis. The customers (loads) buy power from the pool. Therefore, power trading in the power pool repeats daily. Consequently, the static model representing the short-run power market at steady-state might not be sufficiently general to analyze certain market behavior, such as daily price competition among the generators resulting from the repetitive bidding process.

Instead, a short-run power market process could be formulated as a dynamic process in which the next state of the system is a function of the previous and current states. Hence, a dynamic model that can capture the market behavior such as daily price competition, is introduced in this thesis as a new analytical tool to study the dynamics of the power market.

1.1 Background of the Power Market Model

In previous research [13, 15, 18, 19], the power market under the Poolco structure is usually modeled by using a static model, such as the standard oligopoly model. The standard oligopoly model represents the competitive market with a limited number of producers. Typically, there are two producers and one customer. The customer or demand is represented by a demand function. Two producers are often assumed to be symmetric or identical. For instance, they have the same operating-cost functions. Two symmetric producers in one market are known as a "duopoly". In a short-run competitive market, producers sell their output such that they maximize their profits. Assuming that, in a perfectly competitive market, market prices are given, the optimal solution to profit maximizing problem is that the producers produce output at the market price equal to their marginal costs. The marginal cost refers to the cost of producing one additional unit of output. When market price equals to marginal cost pricing is efficient.

The UK power market changed from the regulated to the deregulated market at the same time as it is privatized in 1990. The short-run UK market has a Poolco structure. Two large generators, National Power and PowerGen, dominate the market. Therefore, the UK power pool was modeled as a symmetric duopoly [13]. Two generators submit the symmetric marginal-cost supply functions into the power pool to bid for being scheduled to operate the next day.

The equilibrium in the supply schedules (the optimal supply function of the generator) resulting from the competition in the UK power pool is calculated by using the technique called Supply Function Equilibrium (SFE). The SFE technique is used to find an equilibrium (i.e., supply meets demand) in the market modeled by the standard oligopoly model. This technique assumes that demand is price-elastic i.e., that the customers vary their consumption in response to the market prices. The market price is not given as in the perfectly competitive market model. The solutions from the SFE technique are the producers' supply functions. The outcome of the UK power-pool model shows that the UK market is imperfect because the duopoly has a very considerable market power which be exercised without collusion by offering a supply schedule that is considerably above marginal cost.

To extend the duopoly model to mimic the characteristics of the bidding process in the UK power pool, a sealed-bid multiple-unit auction model was proposed [15]. This model is based on the belief that the particular organization of the electricity spot market makes standard oligopoly models inadequate as an analytical tool. The sealed-bid multiple-unit auction refers to the method in which producers submit their bids (offer prices) to the auctioneer simultaneously. After all bids are submitted, the auctioneer opens the offer bids and determines according to the price order which producers are scheduled to supply demand.

In the sealed-bid multiple-unit auction model representing a UK power pool, there are two generators participating in the power pool and both have constant marginal-cost functions. They offer their bidding prices to the pool. The equilibrium of the market in this model is determined by applying the concept of the "price game."[1] Each generator has the capacity limit. According to the price game, two generators sell their power up to their capacity. When demand is more than the capacity of the smaller generator and less than the total capacity of two generators, the generators charge their customer equal to the marginal cost of the higher-cost generator. The results from this model suggest that inefficient pricing exists and that high-cost generators might bid lower offer prices than lower-cost sets and thus might be dispatched before these more efficient units (the lower-cost units).

Furthermore, experimental studies [19] have been performed by applying two bidding strategies: a sealed bid offer (SBO) and a uniform price double-auction mechanism (UPDA). For the SBO, all buyers (sellers) submit a single round of bids (offers) in a series of price-quantity steps chosen at their discretion. The bid schedule and the offer (supply) schedule are adjusted to satisfy the transmission constraints, etc., and then prices and allocations are computed so as to meet the specified discrete optimization conditions. In the UPDA, the same information conditions apply. At each t there are tentative allocations exactly as SBO mechanisms. At any $t < T$ (T the length of the bidding period), agents can alter their bids (offers), but bids and offers can only be improved [19].

The performance of the SBO and the UDPA, in terms of their impact on incentives affecting market efficiency, generator and wholesale buyer profitability, and delivery price, are compared and analyzed. The results from these experimental studies show that the SBO performs better than the UPDA in the nonconvex environment which assumes that the generators or sellers (consumers or buyers) have the multi-step-supply (step-demand) functions and that they also offer the multi-step-supply (step-demand) function bids into the pool. For instance, the SBO yields a significant increase in market efficiency over the UPDA. Moreover, that is, relative to the UPDA, the SBO redistributes surplus from sellers to buyers; while increasing efficiency.

[1] For more detail of the price game, see [7].

Recently, a strategic supply function was proposed as an option for the generators submitting their bids to the power pool in the duopoly model [18]. The generators in this model are allowed to submit not only the marginal-cost supply function to the ISO, but also the strategic supply function. The market equilibrium was evaluated by using the SFE. The result indicates that if a generator submits the strategic-supply function instead of the marginal-cost supply function, the market clearing price will be higher than the system marginal cost. The results also confirm that market power does exist in the power market. The more generators, the less market power each will hold and the lower the market clearing price will be.

However, in power markets, many dynamic phenomena occur. For instance, the bidding process in the power pool repeats each day. Moreover, electricity demand varies over time (i.e., hourly, daily, monthly etc.). Therefore, we propose that a dynamic model should be able to represent the power markets more accurately than a static model.

1.2 The Motivation

It is proposed in this thesis that *the competitive power market should consist of two subprocesses or submarkets. The first one occurs in the long-time horizon (i.e., a 3-month period). The second occurs in the short-time horizon (i.e., a daily period).* The subprocess occurring in the long-term horizon represents the market for quantity. More generally, quantity competition is really *a choice of scale that determines the firm's cost functions and thus determines price competition.* The choice of scale can be a capacity choice, but more general investment decisions are also allowable [7].

Therefore, in the long-term power market, the generator plans for the available capacity which he will supply to the pool within a given period. After the generators commit their available capacity, they will not operate at more than their committed capacity. Similarly, the generator has one specific capacity constraint within one long-term period. After the generators compete for the market share (quantity or capacity) in the long-run market, they will participate in the short-run market.

The short-run market is a power pool in which the generators (consumers) bid their supply functions (demand functions) for scheduling (being served) on a daily basis. This market is also known as a *day-ahead market*. In this short-run subprocess, the generators compete with prices. The price competition happens one day before the physical dispatch. The ISO accepts the bids such that the transaction can be completed and the physical power can be actually transfered. The generators are informed about anticipated demand each hour (or half-hour) of the following day, and of the current (today) market clearing prices by the ISO when the current schedule is announced.

Hence, the generators have sufficient information about anticipated demand and the market prices from the ISO. Assuming that the generator is rational, it is reasonable to expect that the generator is able to learn from the daily market. Thus, the learning process should affect the generators' behavior which impacts the dynamics of the market. This is the basic rationale the short-run market as a dynamic system instead of a static model. In addition, in the real market, the generators are able to vary their bids depending on their bidding strategies. They might be restricted to bid at their (real) marginal cost, but they might bid strategically as well. Moreover, in general power markets, there are more than two generators with different technologies participating in the market. Therefore, the standard oligopoly model or the duopoly model, in which two symmetric generators offer their symmetric marginal-cost supply functions to the ISO, are no longer suitable to represent those markets.

Consequently, this thesis provides **one approach to formulate a short-run power market by using a dynamic model**. The proposed model is limited to the short-run subprocess. The long-run subprocess is complicated because of the long-term uncertainties. For instance, entry or the addition generators will play an important role in the long-run time-scale. Moreover, demand can vary considerably over several years. The long-run subprocess is an open research question. The dynamic model, representing the short-run sunprocess, can be considered as either a dynamic system or a *dynamic game*, in which the generators are players. Instead of applying the standard oligopoly model, a sealed-bid multiple unit auction model is proposed to represent the daily pool's activities. A learning process of each generator is also allowed in the new model. The proposed model consists of three asymmetric generators. However, in this model, demand is assumed that load has no significantly price-elasticity. Similarly, the demand-side bidding is omitted for simplicity of modeling. The proposed model should be able to depict the dynamics of the market such as daily price competition among the generators.

1.3 A Dynamic Game

Generally speaking, a short-run power market is a repetitive process. For instance, the generators, participating in the market, repeat bids to the pool each day. Demand in the short-run period varies periodically on weekly basis. Based on this, it seems reasonable to assume that the generators engaging in daily trading learn from the repetitive market process.

In an electricity market, the generator, as a firm, generates and sells power for profit. Although the generators sell their power at their marginal costs, there is no guarantee that the generators can sell their power at all periods, because the anticipated demand varies with time and the costs of generating power from each generator is different. Thus in this proposed model, the generators should learn the market so they can maximize their estimated profits based on market behavior and the available information, including previous day and current market prices, and next-day anticipated demand.

Game theory is a way of modeling and analyzing situations in which each player's optimal decisions depend on its belief or expectation about the actions of its opponents. A game in which the generators are players, is a so-called *daily bidding game* or a *bidding game*. The theory of dynamic games is applicable to this proposed model. Under perfect competition, each generator constructs a bid individually, therefore, there is no explicit collusion among the generators. Each generator's bidding strategy is assumed to be symmetric. Consequently, the bidding game is a noncooperative game.[2]

While the generators are informed of the previous day and current market clearing prices and next-day anticipated demand by the ISO, the generators do not know cost functions of other generators. Moreover, in the short-run market, the generators submit their bids to the ISO simultaneously, and this bidding game is played repeatedly every day over a long period of time. The characteristics of the bidding process are generalized to a *finitely repeated game of complete but imperfect information.* The information is complete because, in each bidding game, the generator knows exactly which "move" it could choose. But the fact that next bids by the other competitors, which are unknown, are simultaneously submitted to the ISO, makes each bidder's information imperfect.

A bidding strategy applied to the bidding game seems to have a ve a significant impact on the market clearing price. Different bidding strategies yield different market clearing price patterns. In addition, results from the dynamic model also show that the three-generator market is imperfect. For instance, the market clearing price is higher than the system marginal cost. Both market power issues (i.e., the tacit collusion among the generators to set the market price above the system marginal cost) and the efficiency in pricing and dispatch are important factors in determining whether the competition will tend to increase benefits to both generators and customers participating in the market. Therefore, like in the static model, market power is an issue. Efficiency in pricing and dispatch are also included in the analyses.

1.4 Thesis Outline

This thesis is divided into seven chapters. In Chapter 2, an overview of game theory is presented. In Chapter 3, model formulation is presented. Several assumptions that characterize the model are outlined. The details of the model are described. An example model that consists of three generators with asymmetric operating-cost functions, plus the ISO is presented. Also, a mathematical formulation of the dynamic

[2]For more detail of noncooperative game, see [8, 12].

model, in which the offer price of each generator is a state-variable, is derived. In Chapter 4, the three-generator model is analyzed in more detail. Engineering features of the dynamic system and the economic aspects of a dynamic game in concept are compared to highlight some similarities. In addition, because the short-run power market will be treated as a dynamic game or a so-called bidding game, the strategies applied to the bidding game are explained. In Chapter 5, results from the model's simulation are presented. The discussion is focused on market power issues and efficiency in pricing and dispatch. The conclusion is presented in Chapter 6. In Chapter 7, further research directions, especially in the long-run market modeling, are suggested.

Chapter 2

An Overview of Game Theory

This chapter provides an overview of "noncooperative game theory," especially, the concept of a *repeated game of complete but imperfect information*, which will be applied to formulate a dynamic model of the electricity generation market or a bidding game. The overview mainly concerns with game theory in an analytical perspective.

Noncooperative game theory is a way of modeling and analyzing situations in which each player's optimal decisions depend on its beliefs or expectations about the moves of its opponents. The distinguishing aspect of theory is its insistence that players should not hold arbitrary beliefs about the moves of their opponents. Instead, each player should try to predict its opponents' play, using its knowledge of the rules of the game and the assumption that its opponents are themselves rational, and are thus trying to make their own predictions and to maximize their own payoffs. In this thesis, only details of a game of complete information will be provided.[1] Each player's payoff function is commom knowledge among all players.

[1] For more information about games of incomplete information see [12].

2.1 Noncooperative Game

Game theory is divided into two general branches: cooperative and noncooperative. Cooperative games are not included in this thesis.[2] (The market is modeled such that no explicit collusion is allowed. Hence, the concept of noncooperative game is applicable to the model.) In noncooperative game theory, the unit of analysis is the individual participants in the game who is focused on doing as well for itself as possible subject to clearly defined rules and possibilities.

In addition, there are two forms of games characterized by the moves of players: *static* and *dynamic* games. In a static game, the players simultaneously choose their actions or each player's choice of actions is independent of the moves of their opponents. The players receive their payoffs (the outcome resulting from their moves) which depend on the combination of actions chosen. The dynamic game, on the other hand, is the game of sequential moves and refers to the game in which the players can observe and respond to their opponents' actions.

Moreover, each game is also divided into the game of *complete information* and *incomplete information*. In the game of complete information, each player's payoff function is common knowledge among all players. In games of incomplete information, in contrast, at least one person is uncertain about another player's payoff function. Nonetheless, games of incomplete information are not considered here.

Repeated games are one type of dynamic games. These games will be played repeatedly in either finite or infinite time, depending on the characteristics of the games. The dynamic game will be discussed later in more detail.

[2]For more detail of the cooperative game see *Game Theory with Application to Economics*, Friedman (1986).

2.2 Basic Notions of Noncooperative Game Theory

2.2.1 Game Representation

In general, there are two (almost) equivalent ways of formulating a game. The first one is an extensive form and the other is a normal form. Both forms are described below.

An Extensive Form *or* a Game Tree

An extensive-form representation of a game specifies:

1. The order of the each player's play.
2. The choices available to a player whenever it is its turn to move.
3. The information a player has at each of these turns.
4. The payoffs to each player as a function of the move selected.
5. The probability distributions for moves by "Nature."[3]

A Normal Form

A normal-form or a strategic-form representation is defined by condensing the details of the tree structure into three elements:

1. The players in the game.
2. The strategies available to each player.
3. The payoff received by each player for each combination of strategies that could be chosen by players.

[3] "Nature" means that some elements are random in the game (see [12]).

Note that, for dynamic games, extensive-form representation is often more convenient because the sequences of moves are shown explicitly. However any dynamic game can be represented in the normal form. On the other hand, a static game can also be described in the extensive form as well. Roughly speaking, the extensive form provides an "extended" description of a game while the normal (strategic) form provides a "reduced" summary of a game.

2.2.2 Strategies

Strategies are the choices that each player could make. A strategy for player i is a map that specifies each of its information sets, namely a probability distribution over the actions that are feasible for that set.

A Pure Strategy

A behavioral pure strategy specifies a single action at each information set or an action made with probability 1 (or 100% certainty).

A Mixed Strategy

A mixed strategy is a probability distribution over the pure strategies. It means that players randomly choose the available moves. If the player chooses one move with 100% certainty, its move is a pure strategy.

2.2.3 Payoffs

Payoffs indicate the utility (benefit) that each player receives if a particular combination of strategies is chosen. As a result, the payoffs for mixed strategies are the expected value of the corresponding pure-strategy payoffs.

2.2.4 Solution Techniques

To analyze or to predict what will happen in the games that are modeled, two solution techniques are generally used: dominance argument and equilibrium analysis.

Dominance

The general technique used is called *iterated elimination of strictly dominated strategies.* One strategy of a game is called a strictly dominated strategy when the players in that game receive payoffs from playing that strategy less than playing other available strategies. This process derives from the concept that rational players do not play strictly dominated strategies because there is no belief of each player that the other players will choose those strategies such that they would receive optimal payoffs.

Equilibrium Analysis

In some types of games fairly strong predictions about what the players will do can be figured out if the dominance criterion are iteratively applied to the game. But in many games, there are no strictly dominated strategies to be eliminated. Since all the strategies survive iterated elimination of strictly dominated strategies, the process produces no prediction whatsoever about the play of the game. The other stronger solution concept is a **Nash equilibrium**, which produces much tighter predictions in many broad classes of games. **A Nash equilibrium** is a strategy selection such that no player can gain by playing differently, given strategies of its opponents. Similarly speaking, it is a strategy of each player, such that no player has an incentive (in terms of improving its own payoff) to deviate from its part of strategy array. Let us define

- I the set of players.
- S_i player i's strategy space S.
- π^i player i's payoff function.

Hence, (I, S, π) completely describes a normal-form game.

Definition: *Strategy selection s^* is a pure-strategy* **Nash equilibrium** *of the game (I, S, π), if for all players i in I and all feasible s_i in S_i*

$$\pi^i(s^*) \geq \pi^i(s_i, s^*_{-i})$$

Thus, if the theory offers the strategy (s^*) as the solution but this strategy is not a Nash equilibrium, then at least one player will have an incentive to deviate from the theory prediction.

Theorem (Nash): *Every finite n–player normal form game has a mixed-strategy equilibrium.*

Let us consider this classic example:[4] **The Prisoner's Dilemma** in Table 2.1. In

Table 2.1: The Prisoner's Dilemma

	Cooperate	Defect
Cooperate	(3,3)	(0,4)
Defect	(4,0)	(1,1)

the original story, Row and Column were two prisoners who jointly participated in a crime. They could cooperate with each other and refuse to give evidence or they could defect and implicate the other.

A unique Nash equilibrium of this game is when both prisoners play "Defect."

2.3 Dynamic Games of Complete Information

Dynamic games of complete information are games in which players observe the previous stages of the games before choosing the actions in the next stage. The payoff of each player is *commom knowledge* among all players. These games can be subdivided into two categories: the game of *perfect information* and the game of *imperfect information.* Perfect information refers to a game where each player making the move knows the full history of the play of the game thus far. On the other hand, in a game of imperfect information, there exists information that the players do not know; for instance, the players move simultaneously during any stage game, therefore, the information of each player at that stage is not common knowledge.

[4]For more detail see such as [12, 14].

2.3.1 Games of Perfect Information

These games are described as follows:

1. The moves occur in sequences.

2. All previous moves are observed by players before the next move is chosen.

3. The players' payoffs from each feasible combination of moves are common knowledge.

2.3.2 Games of Imperfect Information

Unlike the game of perfect information, the simultaneous moves are allowed within each stage of the dynamic game of complete but imperfect information.

Because the dynamic game is related to sequential moves, a question arises: "what beliefs should players have about the way their current play will effect their opponents' future decisions?" Before clarifying the question, it is important to note that the central issue in all dynamic games is *credibility* because the payoffs for all players in these games will be different if the players admit any threats by other players and they play the game for very long time. Explanations about how credibility impacts payoffs in dynamic games can be found below.

In addition, the dynamic game also concerns how long the game is played. The finite game is the game that players play and stop in some limited periods. While the infinite game is played infinitely, both games might yield different outcomes, although the players play the same game (constituent game) in each stage.

2.3.3 Strategies of Dynamic Games

Definition: *A* **strategy** *for a player is a complete plan of action–it specifies a feasible action for the player in every contingency in which the player might be called on to act.*

2.3.4 Backward Induction

Backward induction is one solution concept to identify a Nash equilibrium that does not rely on such non-credible threats defined as the threats that the threatener would not want to carry out, but will not have to carry out if the threat is believed. The solution to the game can be found by working backwards through the game tree. The method is simple: go to the end of the game and work backwards, one move at a time. This process has been carried out until the initial node is reached and finally yields a Nash-equilibrium outcome.

Note that this method can be used in either sequential-move or simultaneous-move games because both types of games can be represented by an extensive-form representation. In some games, there are several Nash equilibria, some of which rely on non-credible threats or promises. The backward-induction solution to a game is always a Nash equilibrium that does not rely on non-credible threats. Backward induction can be applied in any finite game of complete information. However, in dynamic games with simultaneous moves or an infinite horizon, this method cannot be directly applied.

2.3.5 Subgame Perfection

Definition: (Selten 1965) *A Nash equilibrium is subgame-perfect if players' strategies constitute a Nash equilibrium in every subgame.*[5]

From the above definition, subgame-perfect equilibrium strategies must yield a Nash equilibrium not just in the original game, but in every one of its proper subgames. In a multi-period game, the beginning of each period marks the beginning of a new subgame. Thus, for these games, subgame-perfection can be rephrased as simply requiring that the strategies yield a Nash equilibrium from the start of each period. More generally, in any game of perfect information, subgame perfection yields the same answer as backward induction. In finite-period simultaneous-move games (or the games of imperfect information), subgame-perfection performs backward induc-

[5]Quoted from [12].

tion period by period. In the last period, the strategies must yield a Nash equilibrium, given the history of that game (which indicates how the players played in the game in the previous stages).

2.4 Repeated Games

The repeated game is one type of dynamic game. It is a game where players face the same constituent game in each of finitely or infinitely many periods. Both types of games will yield different Nash-equilibrium outcomes under certain rules, although the players play the same constituent game; therefore, those two types of repeated games, which are finitely and infinitely repeated games, will be analyzed separately.

2.4.1 Finitely Repeated Games

In the finitely repeated game, the following proposition is held.

Proposition: *If the stage game G has a unique Nash equilibrium, then for any finite T, the repeated game $G(T)$ has a subgame-perfect outcome: the Nash equilibrium of G is played in every stage.*[6]

The payoffs of the games are the sum from the payoff in each stage of the game. However, the credible threats or promises about future behavior can influence current behavior.[7] If there are multiple Nash equilibria of stage game G then there may be subgame-perfect outcomes of the repeated game $G(T)$ in which, for any $t < T$ the outcome of stage t is not a Nash equilibrium of G.

Suppose that the Prisoner's Dilemma (Table 2.1) is played repeatedly a fixed number of times. Let us consider one move before the last. According to the matrix-game shown in Table 2.1, even if the players agree to play "Cooperate", in the next move both players will want to play "Defect". Hence there is no advantage to cooperate on the next to the last move as long as both players believe that the other will play

[6] quoted from [12].

[7] For more detail and examples see [12].

"Defect" on the final move. There is no advantage to try to influence an opponent's future behavior by playing "Cooperate" on the penultimate move. The same logic of backward induction works for two moves before the end, and so on. Consequently, in repeated Prisoner's Dilemma with known number of repetitions, the Nash equilibrium ("Defect") is played in every round.

2.4.2 Infinitely Repeated Games

The infinitely repeated game is analyzed differently from the finite game because the sum of the payoffs from infinite sequences of state games does not provide a useful measure of a player's payoff in an infinitely repeated game (i.e., receiving payoff either 4 or 1 in each stage will yield the sum of payoffs equal to infinity ($1+1+\ldots+1+\ldots = 4+4+\ldots+4+\ldots = \infty$)). Moreover, even if the stage game has a unique Nash equilibrium, there may be subgame-perfect outcomes of the infinitely repeated game in which no outcome is a Nash equilibrium of that game. This comes from the proof of Friedman (1971)[8] who showed that any payoffs that are better for all players who play a Nash equilibrium of the constituent game are the outcome of a perfect equilibrium of the repeated game, if players are sufficiently patient. Under the proper circumstance, Chamberlin's intuition [8] (Chamberlin (1956)) can be partially formalized. Repetition can allow "cooperation" to be an equilibrium, but it does not eliminate the "uncooperative" static equilibrium and indeed can create new equilibria that are worse for all players than if the game had been played only once. To analyze this type of game in more detail, let us consider the following definitions:

Definition : **The discount factor** δ *measures the length of observation lag time between periods, as well as the players' impatience per unit time.*

The value of a dollar in the next period (i.e., month or year) is equal to today's value multiplied by a proper discount factor.

[8] Quoted from [8].

Definition : *Given the discount factor δ, the present value of infinite sequence of payoffs $\pi_1, \pi_2, \pi_3, \ldots$ is*

$$\pi_1 + \delta\pi_2 + \delta^2\pi_3 + \ldots = \sum_{t=1}^{\infty} \delta^{t-1}\pi_t$$

Hence, depending on the rules of the games, in the infinitely repeated game, a subgame-perfect outcome occurring in every stage might not be a static Nash equilibrium of every stage game. For instance, in the infinitely repeated Prisoner's Dilemma (Table 2.1), cooperation between players can occur in every stage of a subgame-perfect outcome of a infinitely repeated game, even though the only Nash equilibrium in the stage game is "Defect."

Consider the following strategy–*the punishment strategy*: "Cooperate on the current move unless the other player defected on the last move."

Let the $1/(1+r)$ be a discount factor δ.

The expected payoff from continuing to cooperate is $(3+3/r)$. On the other hand, if a player decided to play "Defect" on one move, it would get the expected payoff equal to $(4+1/r)$.

If $(3+3/r) > (4+1/r)$ or $(r < 2)$, this strategy forms a Nash equilibrium: if one plays the punishment strategy, the other party will also want to play it, and neither party can gain by unilaterally deviating from its choice. A famous result known as the *Folk Theorem* asserts precisely this: In the repeated Prisoner's Dilemma any payoff larger than the payoff received (if both parties consistently defect or play "Defect") can be supported as a Nash equilibrium.

2.4.3 State-space or Markov Equilibrium

In some games, the players maximize the present value of instantaneous flow payoffs, which may depend on state-variables as well as on the current actions. A *state-space* or *Markov equilibrium* is an equilibrium in state-space strategies that depends not on the complete specification of a past play, but also on the state (and, perhaps, on time.) A perfect state-space equilibrium must yield a state-space equilibrium for every initial state. Since the past's influence on current and future payoffs and opportunities is summarized in the state, if a player's opponents use state-space strategies, it could not gain by conditioning its play on other aspects of the history of the play.

Chapter 3

Model formulation

The model presented in this thesis is based on the simplified version of the UK power pool. The simple market consists of the ISO, the generators and the (aggregated) load. The only transaction allowed is the generators' bids to sell their bulk power to the pool. No bilateral transactions are allowed. System constraints such as transmission constraints are not included; therefore, the power can be transfered in any amount. Very-long time-scales (i.e., more than 2 years) are not factored in. The threat of entry is also ignored. Hence, the profit maximization technique generally used to find equilibrium in the competitive market in the short-run time-scale is applicable [13].

In addition, for simplicity of the model, no must-serve and must-run units are taken into consideration. The maximum demand is always less than the total available supply; therefore, market equilibrium, in which supply meets demand at any market clearing price, always exists. Quantity competition in the long-run market is also omitted. The maximum available capacity, which is supposed to be a solution to the long-run market, is assumed to be given.

The following notations will be used in this thesis:

P	the (given) market price.
Π	the profit function.
Q	the actual power that the generator generates.
$C_i(Q)$	Generator No. i's operating cost of generating Q units of power.

$P_{mark}[K]$	the market clearing price at the kth hour of the Kth day.
$P_i[K]$	the offer price by Generator No. i of the Kth day.
$X_i[K]$	the offer power quantity by Generator No. i of the Kth day.

3.1 Application of Game Theory to the Short-run Power Market Model

In the electricity generation market under competition, we suggest that the competition for trading power should consist of two subprocesses. The first one is the longer-time competition (i.e., a 3-month period). In this period, the generators commit their available capacities to the ISO. Demand, in general, varies over time. The generators respond to that variation (elasticity of demand) and decide how much of their generation should be available to the market. The other subprocess is in the shorter period (i.e., within three months) or the faster time-scale. After the available capacity is committed, the genertor must respond to the daily price variations within that period.

Consequently, within one certain short-run period (i.e. within three months), the generator plays a bidding game under one limited-capacity constraint. In the next period, due to changing demand, the generator changes its available capacity as well. (The generator might be able to create a deficiency in supply such that it can "game" the market.) Therefore, in each individual short-run period the generators play different price-competition games. Such a new game will start when the new short-run period starts.

Consider the process of offering a bid by each generator participating in a power pool. Each generator faces this repeating process every day. Hence, the daily bidding by each generator can be formulated as a repeated game. The reasons why the bidding process will be modeled as a repeated game are the following:

1. The bidding process, which occurs every single day, is considered to be one stage game. The generators "play" the *same stage game* daily.

2. The generators should have some common criteria to determine their offer bids that are rational and practical. As a result, each generator should play the same game under the same rules or strategies.[1]

3. The daily bid is defined as a short-run competition, assuming that there is no entry (new generators) and all the players (the generators) will not encounter the long-term effects (i.e. entry, technology changes and etc.).

4. The effects of transmission congestion or network constraints are not included in the proposed model; therefore, all the transactions are implemented and there is only a single price for the whole market (no nodal prices).[2]

In each day, the generators submit their bids to the ISO. The ISO schedules the generators in price merit order (in order of prices) and announces the market-clearing prices before the next bid starts. The set of market clearing prices for each hour (or half-hour) becomes common knowledge among the generators. The generators normally observe these prices and estimate their next bids. On the next day, the generators simultaneously submit their bids to the ISO again. This process happens repeatedly so it is simply interpreted as a finitely repeated game of complete but imperfect information. Paritcularly, in this thesis, the daily bidding process is called a *daily bidding game* or a *bidding game.*

3.2 General Assumptions

In the framework of this thesis, the dynamic model is formulated under the following assumptions.

[1]Nonetheless, the generators might also play the same game but under diverse strategies as well. However, the game with a nonuniform strategy is not considered in this thesis.

[2]For more detail about the uniform price or nodal prices in an electric power market, see [13].

3.2.1 The Generator

Operating Costs

Generators have quadratic operating-cost functions that are increasing and convex. The cost functions are asymmetric so one function is assigned to one generator. Note that we assume that the generators have no fixed costs.

$$C_i(Q) = a_i Q^2 + b_i Q \qquad \text{where} \quad a_i, b_i > 0 \tag{3.1}$$

In the competitive market, the generator behaves as a *price taker*.[3] The marginal-cost function is calculated by setting the generator's marginal operating cost equal to the (given) market price [2]. In the short-run market, in this thesis, a generator maximizes its profit by assuming that it is a competitive firm participating in a competitive power market. Therefore the marginal cost function is derived as:

$$\max_Q \ \Pi(Q) = \max_Q \ (P \times Q - C(Q))$$

$$P = \frac{\partial C(Q)}{\partial Q} \qquad \text{or} \qquad P = \Lambda \tag{3.2}$$

$$P = \Lambda = 2a_i Q + b_i \tag{3.3}$$

The marginal-cost function P is an affine function of Q and Λ is known as a "system lambda."

Available Capacity

Available capacity is supposed to be the outcome of the long-run market, in which the generators compete for quantities. However since this thesis examines only the short-run market, the maximum available capacity of each generator is a known information.

[3]For more detail about the competitive market and a price taker, see Appendix A.

The total available capacity is assigned to the generators such that the maximum demand is covered if all generators are fully scheduled.

3.2.2 Anticipated Demand

The day-ahead market is a market in which the activities such as proposing the bids and scheduling, occur one day before the actual dispatch occurs. The generator learns from the ISO the anticipated demand of the next day. In this thesis, the anticipated demand is assumed to be *inelastic*[4] and *deterministic.*

Demand is time-varying on an hourly basis, and demand on weekdays and weekends is also different. However, it is periodic on a weekly basis, and the maximum demand usually occurs during weekdays. The characteristics of the anticipated demand used in this thesis is shown in Figure 3-1; each day is divided into twenty-four hours. During each hour, demand is constant. In this thesis, the maximum anticipated demand is equal to 72.5 (power) units and the minimum anticipated demand is equal to 30.4 units.[5]

3.2.3 Bids

The generators submit their bids consisting of price and quantity ($P_i[K]$,$X_i[K]$). Although the anticipated demand varies with time, the generator proposes just one price for a one-day schedule. The generators are obliged to set their offer prices ($P_i[K]$) relating to their offer quantities ($X_i[K]$) by their marginal-cost functions. Hence,

$$P_i[K] = \frac{\partial C_i(X_i[K])}{\partial X_i[K]} \tag{3.4}$$

[4]von der Fehr and Harbord [15] state (in footnote 2, p. 532) that "Green and Newbery also assume downward sloping demand curves, whereas completely inelastic demand would seem to be more appropriate for the UK industry."

[5]Note that the total assigned capacities always exceed the maximum anticipated demand or are more than 72.5 units.

$$X_i[K] = \frac{P_i[K] - b_i}{2a_i} \tag{3.5}$$

Note that the offer quantity determines the maximum power that the generator will be willing to generate at that offer price or

$$Q \leq X_i[K] \tag{3.6}$$

The $P_i[K]$ is maximum when

$$P_{i,max}[K] = \frac{\partial C_i(X_i[K])}{\partial X_i[K]} |_{X_{i,max}} \tag{3.7}$$

Similarly, the generator actually offers a *single step-supply function* bid in which the maximum power supplied relates to the offer price by the generator's marginal-cost function. An example of the offer bid is shown in Figure 3-2. This figure shows that the generator submits the bid such that it will not operate more than the offer capacity ($X_i[K]$) at the offer price ($P_i[K]$).

3.2.4 The ISO

The ISO schedules the generators so that the anticipated demand is always met. The ISO sets the market clearing price for each hour equal to the price proposed by the marginal operating generator in that hour. The generators are scheduled to operate in "*the price merit order.*" In this model, the generator submits a single-step supply function bid; therefore, there is one price from each generator. The ISO arranges the offer prices of the generators in price-order. The generator offering the lowest price is scheduled to operate first. Thus the offer price of the generator with the highest price scheduled in each period, is announced as the market-clearing price. In addition, in some periods, the generators might offer the bids such that the demand might not be covered by the total offer quantities. The ISO will uniformly reschedule all generators' supply by using the *Economic Dispatch (ED)* technique so demand is met, and set the market clearing price in that period equal to the maximum price of the offer prices and the prices calculated from ED.[6]

3.2.5 The Strategy

The market is formulated as one dynamic game. From a game theory perspective, the players participating in any game, play the game according to the previously specified strategies. *A strategy for a player is a complete plan of action—it specifies a feasible action for the player in every contingency in which the player might be called to act.* In this bidding game, the generators adopt the given decision-making strategy uniformly. Therefore, the generator will 'move' based on the same strategy.

3.2.6 Common Knowledge

Common knowledge is defined here as the information that is shared among the generators. Similarly, the generators will have the same information, if the information is common knowledge. In this bidding game, common knowledge includes:

1. The anticipated demand for scheduling for the following day.

2. The previous and current market clearing price each hour.

However, as mentioned previously, the operating cost functions of individual generators is not common knowledge. The amount of power purchased from each generator is also not public. Note that the maximum available capacity of each generator should be common knowledge, but it is not included here.[7]

[6]See Appendix A for more detail about the ED. In fact, other criterion could be applied instead of the ED, if anticipated demand is not covered by the offer supply. For instance, it is possible to let the generators offer two price bids so that if demand is not met by the first price, there is room for the second-price market. But for simplicity, the ED solution is used. However, the ED will provide an optimal solution.

[7]The generators are assumed to have their "secret" long-term contracts (for the commited capacity) with the ISO.

3.3 The Three-Generator Market Model

The dynamic model formulated in this thesis is the extended version of the static power market model. The static model which appears in previous research [13, 15] mainly consists of two generators or a duopoly. The static model assumes that operating cost functions of two generators are symmetric. In this thesis, however, one more generator is added and the operating cost functions of the generators are asymmetric. The additional generators in the model represent the general power markets more precisely because, in general, there are more than two generators participating in the market. In the real market, there are many generators with different technologies. Hence, the model consisting of at least three generators with the asymmetric cost functions depicts the real market more accurately than a duopoly with the symmetric operating cost functions.

Because anticipated demand is inelastic and deterministic, price variations will have no effect on the customers' behavior. Therefore, the load has no significant role in this model. Moreover, there is no bidding process for the demand side. Thus demand has no dynamics.

As a result, in the simplified power market including the generators, the ISO and the load, only the generator section forms the dynamics of this market. Hence the 3-generator model is reduced to two sections: the generators and the ISO. In the generator section consisting the generators participating in the market, each generator determines its next bid based on the available information (i.e., common knowledge and the operating cost function). The ISO section performs the ISO's tasks which include equating supply and anticipated demand, and setting the market clearing price. The ISO announces the market clearing price in each period together with the anticipated demand of the next day. However the amount of power that each generator must produce is not public information so it is also not common knowledge.

3.3.1 The Generator Section

The generator section represents the generators' behavior. In this thesis, each generator will offer only a single-step supply function bid. The generator's job is then to evaluate the next bid, such that the next day will yield the maximum profits. The generator determines its next bid by using all available information. The dynamic game is played by each generator individually, but it is assumed that each generator has the same decision-making strategy. The bidding game is a finitely repeated game of complete but imperfect information; therefore, if there existed a unique Nash equilibrium in each bidding game, the generator would play the Nash equilibrium at every stage.

There are three generators with asymmetric operating cost functions in this model. The operating cost function of each generator is shown below :

Generator No. 1	$C_1(X)$	$= X^2 + X$
Generator No. 2	$C_2(X)$	$= 2X^2 + 0.5X$
Generator No. 3	$C_3(X)$	$= 0.5X^2 + 2X$

The marginal cost function of each generator is:

Generator No. 1	$MC_1(X)$	$= 2X + 1$
Generator No. 2	$MC_2(X)$	$= 4X + 0.5$
Generator No. 3	$MC_3(X)$	$= X + 2$

(Note that the operating cost function and the marginal-cost function are the functions of X instead of Q, because they refer to the functions that the generators use to formulate their bids.)

In this thesis, the marginal operating cost is an affine function of X, because the operating cost function of each generator is quadratic. The marginal cost function is derived by using the efficient pricing rule in which the marginal cost is set equal to the (given) market price [2]. The marginal-cost function of each generator is shown in Figure 3-3.

Definition 1 An expensive generator *is a generator whose marginal cost is higher than others for the same amount of power* (X) *generated over normal generation* *ranges (i.e., $X > 1$). Similarly, that the most expensive generator's marginal-cost function lies above others' marginal-cost functions (along the (X) axis).* (For instance, from Figure 3-3, Generator No. 2 is the most expensive generator and Generator No. 3 is the cheapest generator.)

3.3.2 The ISO Section

The ISO section, on the other hand, represents the ISO's task, which is to equate the offer supply and the anticipated demand. Because the demand is inelastic, the ISO predicts the demand which has no relation to the variation in the market-clearing price. Hence, there are no dynamics in this section. The ISO is an operator who organises the power pool until the transaction is completed. The ISO arranges the offer step-supply functions provided by the generators in the price merit order (i.e., the lowest to the highest cost), schedules each generator until the anticipated demand is met and sets the market clearing price equal to the offer price by the marginal operating generator in each period. The market clearing prices become common knowledge as well as the anticipated next day demand. The special task for the ISO in this model, when the anticipated demand is not covered by the total offer supply, is to reschedule all generators' supply equally by using the solution calculated from the ED so the demand at that time is met.

3.4 Model Formulation

In order to formulate the bidding game, some specific definitions are key to understanding the model set-up. Depending on the demand level, 24 hours can be divided into three periods: peak-load, mid-level and off-peak.[8] Further assume that in off-peak periods at least one generator can supply the entire demand. On the other hand, during peak-load periods, all generators must activate their operations. Let us introduce:

[8]This is similar to what von der Fehr and Harbord propose in their model [15].

Definition 2 The peak-load generator *is a generator that creates the highest market clearing price.*[9]

Definition 3 The off-peak or base-load generator *is the generator that is scheduled in every period, 24 hours a day.*

Definition 4 The mid-level generator *is the generator that is neither peak-load nor off-peak.*

Follow the assumption that the operating cost $C(Q)$ of each generator is an increasing and convex function. The profit function is defined by:

$$\Pi(Q) = R(Q) - C(Q) \tag{3.8}$$

where $\Pi(Q)$ the profit.

$R(Q)$ the revenue $= P \times Q$.

$C(Q)$ the quadratic operating cost function.

Proposition 1 *The profit function defined in (3.8) is an increasing and concave function.*

Proof of Proposition 1

$$\begin{aligned}
\text{Suppose} \quad Q_2 &> Q_1 \\
P_2 &= \frac{\partial C(Q_2)}{\partial Q_2} \\
P_1 &= \frac{\partial C(Q_1)}{\partial Q_1} \\
P_2 &> P_1 \quad (C(Q) \text{ is an convex and increasing function.}) \\
\text{Let} \quad Q &= \lambda Q_1 + (1-\lambda)Q_2
\end{aligned}$$

where $0 \leq \lambda \leq 1$

$$\begin{aligned}
\text{Let} \quad \Pi'(Q) &= P_1 \times Q - C(Q) \\
&= P_1 \times (\lambda Q_1 + (1-\lambda)Q_2) - C(\lambda Q_1 + (1-\lambda)Q_2)
\end{aligned}$$

[9] This is for the case when all generators submit their bids such that the anticipated demand is less than the total offer supply.

Because $C(Q)$ is convex and increasing, $C(Q)$ can be rewritten in terms of λ as

$$C(\lambda Q_1 + (1-\lambda)Q_2) \leq \lambda C(Q_1) + (1-\lambda)C(Q_2)$$

$$\text{Hence,} \quad \Pi'(Q) = (P_1 \times (\lambda Q_1 + (1-\lambda)Q_2) - C(\lambda Q_1 + (1-\lambda)Q_2))$$

$$P_1 \times (\lambda Q_1 + (1-\lambda)Q_2) - C(\lambda Q_1 + (1-\lambda)Q_2)$$

$$\geq P_1 \times (\lambda Q_1 + (1-\lambda)Q_2) - (\lambda C(Q_1) + (1-\lambda)C(Q_2))$$

$$\text{Then} \quad \Pi'(Q) \geq \lambda(P_1 \times Q_1 - C(Q_1)) + (1-\lambda)(P_1 \times Q_2 - C(Q_2)) \qquad Q.E.D.$$

Note that when $P_1 < P$, then the proof also holds for all $P \geq P_1$. In addition, the operating cost function is strictly convex and increasing, thus the equality can be eliminated in the inequality. Consequently, the profit function is also strictly concave and increasing.

Note also that the profit function is an increasing function in P if Q is held constant. Let

$$\begin{aligned} \Pi_1 &= P_1 \times Q - C(Q) \\ \Pi_2 &= P_2 \times Q - C(Q) \\ \text{if} \quad \Pi_1 > \Pi_2 &\Leftrightarrow P_1 > P_2 \end{aligned}$$

Next, if we consider only the particular case in which:

1. one scheduling day is divided to three equal periods that are peak-load, mid-level and off-peak periods.

2. the total offer supply is larger than the anticipated demand.

3. one generator supplies demand during off-peak periods and it is known as a base-load generator.

4. two generators, a base-load generator and another generator, supply demand during mid-level periods.

5. all generators participating in the market supply demand during peak-load periods.

6. $\bar{Pl}_{op} \geq (\bar{Pl}_{ml} - \bar{Pl}_{op}) \geq (Pl_{pl} - \bar{Pl}_{ml})$ where $\bar{Pl}_{op}$ is average load during off-peak periods, $\bar{Pl}_{ml}$ is average load during mid-level periods, and Pl_{pl} is peak load (the maximum load of the day). (Note that we use Pl_{pl} instead of $\bar{Pl}_{pl}$ because of (5) that all generators supply load during peak periods, the maximum load is also met.)

the following proposition will hold.

Proposition 2 *Each day, the total profits of the generator are maximized when the generator is scheduled as a base-load generator under a single-offer bid.*

Proof of Proposition 2 Let us define total profits as:

$$T\Pi = \sum_k \Pi_k = \sum_k (P_{mark} \times Q_k - C(Q_k))$$

and

Q_k	the scheduled power at the kth period.
$B_k = 1$ when $Q_k > 0$	
$B_k = 0$ when $Q_k = 0$ $(Q_k \geq 0)$	
$\{op\}$	a set of hours where load is in the off-peak periods.
$\{ml\}$	a set of hours where load is in the mid-level periods.
$\{pl\}$	a set of hours where load is in the peak periods.
$\hat{P}_{mark}$	the market clearing price at the kth peak period.
$\bar{P}_{mark}$	the market clearing price at the kth mid-level period.
$\tilde{P}_{mark}$	the market clearing price at the kth off-peak period.

Rewrite the total profit function in terms of B_k :

$$T\Pi = \sum_k B_k (P_{mark} \times Q_k - C(Q_k))$$

$$T\Pi = \sum_{k \in \{op\}} B_k (\hat{P}_{mark} \times Q_k - C(Q_k)) + \sum_{k \in \{ml\}} B_k (\bar{P}_{mark} \times Q_k - C(Q_k))$$

$$+\sum_{k\in\{pl\}} B_k(\tilde{P}_{mark} \times Q_k - C(Q_k))$$

Let us consider the following cases:

1. If the generator is a base-load type, it will receive total profits equal to

$$T\Pi_{base} = \sum_{k\in\{op\},\{ml\},\{pl\}} B_k(P_{mark} \times Q_k - C(Q_k))$$

2. If the generator is a mid-level type, it will receive total profits equal to

$$T\Pi_{mid} = \sum_{k\in\{ml\},\{pl\}} B_k(P_{mark} \times Q_k - C(Q_k))$$

3. If the generator is a peak-load type, it will receive total profits equal to

$$T\Pi_{peak} = \sum_{k\in\{pl\}} B_k(P_{mark} \times Q_k - C(Q_k))$$

In the power pool, the generator is paid $\{P_{mark}|P_{mark} \in \{\hat{P}_{mark}, \bar{P}_{mark} \text{ or } \tilde{P}_{mark}\}\}$ in any kth period if it is scheduled. For each generator, this inequality is always true.

$$T\Pi_{base} \geq T\Pi_{mid} \geq T\Pi_{peak}$$

The equality signs hold only when all generators propose the equal prices and are dispatched at the same quantity. The equality signs can be deleted and making the inequality strictly hold. Consequently, a generator's total profit is maximized when the generator operates as a base-load generator. *Q.E.D.*

3.4.1 A Profit Maximizer

Within the long-run time horizon, after the generator commits to supply the power for not more than a specified level, the generator participates in the short-run market. In the short-run market, the generator places its bid or offers its price and quantity into the pool. At this stage, the generator determines its offer bid so that its profit is always maximized.[10]

If the generator is a profit maximizer, it will maximize its profit with respect to the estimated market price every day (or in every game). Hence

$$\max_P \ \Pi(P) = \max_P \ (P_{mark} \times Q(P) - C(Q(P)))$$

$$\text{subject to} : X - Q \geq 0$$
$$X_{max} - X \geq 0$$

Formulating Lagrange's equation:

$$L(P) = P_{mark} \times Q(P) - C(Q(P)) + \mu(X - Q) + \beta(X_{max} - X)$$

$$\max_P \Pi(P) \rightarrow \frac{\partial L(P)}{\partial P} = 0, \frac{\partial L(\mu)}{\partial \mu} = 0, \frac{\partial L(\beta)}{\partial \beta} = 0$$

$$\frac{\partial L(P)}{\partial P} = 0 \ \rightarrow \ P_{mark}\frac{\partial Q}{\partial P} - \frac{\partial C(Q(P))}{\partial Q}\frac{\partial Q}{\partial P} + \mu(\frac{\partial X}{\partial P} - \frac{\partial Q}{\partial P}) - \beta\frac{\partial X}{\partial P} = 0 \quad (3.9)$$
$$\frac{\partial L(\mu)}{\partial \mu} = 0 \ \rightarrow \ X - Q = 0 \quad (3.10)$$
$$\frac{\partial L(\beta)}{\partial \beta} = 0 \ \rightarrow \ X_{max} - X = 0 \quad (3.11)$$

From Equation (3.4):

$$\frac{\partial C(X)}{\partial X} = P_k \quad (3.12)$$

$$\frac{\partial C(Q(P))}{\partial Q} = \frac{\partial C(X)}{\partial X}\frac{\partial X}{\partial Q} = P_k\frac{\partial X}{\partial Q} \quad (3.13)$$

Rewrite Equation (3.9):

$$P_k\frac{\partial X}{\partial Q} = (P_{mark}\frac{\partial Q}{\partial P} + \mu(\frac{\partial X}{\partial P} - \frac{\partial Q}{\partial P}) - \beta(\frac{\partial X}{\partial P}))/(\frac{\partial Q}{\partial P}) \quad (3.14)$$

[10]This comes from the concept of "long-run competition in capacity and short-run competition in price."

Note that the offer price is the function of the offer quantity; hence, P_k is equal to $\partial C(X)/\partial X$, and

$$(\frac{\partial X}{\partial P})/(\frac{\partial Q}{\partial P}) = \frac{\partial X}{\partial Q} = \delta$$

Let $P_k \to P_k[K]$, $P_{mark} \to P_{mark}[K]$, $Q \to Q_k[K]$, $\mu \to \mu_k[K]$, $\beta \to \beta_k[K]$, $\delta \to \delta_k[K]$

For the Kth day, rewrite Equation (3.14) in term of $[K]$ as:

$$P_k[K] = \frac{P_{mark}[K]}{\delta_k[K]} - \mu_k[K]\frac{(1-\delta_k[K])}{\delta_k[K]} - \beta_k[K]\delta_k[K] \tag{3.15}$$

For the $K+1$th day, rewrite Equation (3.15) in term of $[K+1]$ as:

$$P_k[K+1] = \frac{P_{mark}[K+1]}{\delta_k[K+1]} - \mu_k[K+1]\frac{(1-\delta_k[K+1])}{\delta_k[K+1]} - \beta_k[K+1]\delta_k[K+1] \tag{3.16}$$

Equation (3.16) - (3.15) $\to$

$$\begin{aligned} P_k[K+1] - P_k[K] = (&\frac{P_{mark}[K+1]}{\delta_k[K+1]} - \mu_k[K+1]\frac{(1-\delta_k[K+1])}{\delta_k[K+1]} \\ -\beta_k[K+1]\delta_k[K+1]) - (&\frac{P_{mark}[K]}{\delta_k[K]} - \mu_k[K]\frac{(1-\delta_k[K])}{\delta_k[K]} - \beta_k[K]\delta_k[K]) \end{aligned} \tag{3.17}$$

$$\begin{aligned} P_k[K+1] - P_k[K] = &\frac{P_{mark}[K+1]}{\delta_k[K+1]} - \frac{P_{mark}[K]}{\delta_k[K]} + (\mu_k[K]\frac{(1-\delta_k[K])}{\delta_k[K]} \\ &-\mu_k[K+1]\frac{(1-\delta_k[K+1])}{\delta_k[K+1]}) + (\beta_k[K]\delta_k[K] - \beta_k[K+1]\delta_k[K+1]) \end{aligned} \tag{3.18}$$

$$\begin{aligned} P_k[K+1] = P_k[K] + &\frac{P_{mark}[K+1]}{\delta_k[K+1]} - \frac{P_{mark}[K]}{\delta_k[K]} + (\mu_k[K]\frac{(1-\delta_k[K])}{\delta_k[K]} \\ &-\mu_k[K+1]\frac{(1-\delta_k[K+1]}{\delta_k[K+1])}) + (\beta_k[K]\delta_k[K] - \beta_k[K+1]\delta_k[K+1]) \end{aligned} \tag{3.19}$$

Let

$$\frac{P_{mark}[K+1]}{\delta_k[K+1]} - \frac{P_{mark}[K]}{\delta_k[K]} + (\mu_k[K]\frac{(1-\delta_k[K])}{\delta_k[K]} - \mu_k[K+1]\frac{(1-\delta_k[K+1])}{\delta_k[K+1]})$$
$$+(\beta_k[K]\delta_k[K] - \beta_k[K+1]\delta_k[K+1]) = B_k[K]U_k[K] \quad (3.20)$$

Substitute Equation (3.20) into (3.19):

$$P_k[K+1] = P_k[K] + B_k[K]\,U_k[K] \quad (3.21)$$

3.5 A Finitely Repeated Game of Complete but Imperfect Information

In this thesis, the bidding process in the short-run power market or the bidding game is treated as a finitely repeated game of complete but imperfect information. The explanation can be found below.

Let us begin with a dynamic system in Equation (3.21), which is defined by:

$$P_k[K+1] = P_k[K] + B_k[K]U_k[K]$$

This discrete-time system can be interpreted as a dynamic game, or more specifically, as a repeated game. Consider this system as a bidding game played by the generators participating in the power market as the following:

1. $[K]$ refers to the Kth day.
2. P_k refers to the market clearing price, which is a state-variable of this system.
3. B_k is an action of the generator in each day.
4. U_k is one positive number.

Generally, the action shows how the generator responds to previous-day and current market clearing prices, and it is determined by $B_k[K]$. $B_k[K]$ is derived from the so-called strategies of the bidding game, which are described later in the next chapter.

Based on Drew Fudenberg and Jean Tirole's *Game Theory* (1989), a model of a repeated game is defined as the following:

> The game, which is repeated, is called the *stage game*. Assume that the stage game is a finite I-player simultaneous-move game with finite action spaces A_i and stage-game pay-off functions $g_i : A \rightarrow \Re$, where $A = X_{i \in \mathcal{F}} A_i$. Let $\mathcal{A}_i$ be the space of probability distributions over A_i.
>
> Let $a^t \equiv (a_1^t, \ldots, a_I^t)$ be the actions that are played in period t. Suppose that the game begins in period 0, with the null history h^0. For $t \geq 1$, let $h^t = (a^0, a^1, \ldots, a^{t-1})$ be the realized choices of actions at all periods before t, and let $H^t = (A)^t$ be the space of all possible period-t histories.
>
> Since all players observe h_t, a pure strategy s_i for player i in the repeated game is a sequence of maps s_i^t—one for each period t– that map possible period-t histories $h^t \in H_t$ to actions $a_\imath \in A_i$.

The similarities between the bidding-game model and the repeated-game model are explained as the following: In the bidding game, the generators participating in the pool are players playing the bidding game every day. The game in which the generators offer supply-function bids simultaneously to the ISO is considered a stage game. This stage game consists of 3 players. Each stage refers to a one-day period so it is represented by $[K]$ instead of t. The action spaces A_i are comparable to $B_k[K]$ where $B_k[K] \in \{-1, 0, 1\}$. The action a_i is just a choice of $\{-1, 0, 1\}$. Thus a_i^t is equivalent to the set consisting of "-1", "0" or/and "1".

The payoff in the bidding game is not evaluated explicitly. It shows whether the generator might either earn more, less or unchanged profits. The actual profits (payoff) of each generator is determined after the scheduling process ends or the generator knows how much power it is going to produce in each period and it is able to calculate its profits. Therefore the profits are not common knowledge.

Strategies assigned to the bidding game are the methods to determine $B_k[K]$, thus this is comparable to the strategy s_i which is a sequence of maps s_i^t that map possible period-t histories $h^t \in H_t$ to actions $a_i \in A_i$. The strategy in the bidding game is uniform for every player. It maps past plays $\{P_k[K] \mid K \in \{0, \ldots, K-1\}$ and other common knowledge; anticipated demand, to the action space $B_k[K]$.

As a result, the above comparison leads to the simple conclusion that the discrete–time dynamic system defined in Equation (3.21), is just one repeated game. According to the fact that the dynamic model of the electricity generation market is divided into two submarkets the short-run market and the long-run market–the bidding game which represents the short-run market occuring within one long-run horizon is played with the limited number of games; therefore, the bidding game is a *finitely* repeated bidding game.

There is another question as to whether this game includes complete and imperfect information. After the operation schedule is executed for one-day period by the ISO, the ISO announces the market clearing price of each hour. This game should be considered a complete-information game because the generators have information to determine their bids. In this game the previous and current market clearing prices are particularly used as information in determining their bids. The generators have the information about the market clearing prices from the ISO so their information needed should be complete.

Next, let us consider the common knowledge of the generators: the generators are informed of the anticipated demand for the next scheduling day and the market clearing prices from the previous and current trades. However, the payoffs, which are supposed to be the generators' profits, are not announced. Thus, the payoff is not well-defined. The generator will know only its profit after the schedule is finished. This implies that the information that the generators should have might not be complete.

However, in this thesis, the bidding strategies have been derived without using the information about the generators' profits. Therefore, this information can be ignored. It plays no role in determining the "move" therefore it is not necessary to include this in common knowledge. Hence, the conclusion is that each generator has *complete information* to determine its move or to find $B_k[K]$ in each stage game.

On the other hand, the simultaneous move or the fact that the bid from each generator is submitted simultaneously, implies *imperfect information.* Consequently, the bidding game represented by Equation (3.21) is generalised to a *finitely repeated game of complete but imperfect information.*

The subgame of each generator starts each day. The total payoff of each generator at the current stage is the sum of its payoffs from the first stage to the latest one. As described previously, because the profit function of each generator is concave and increasing, there exists an optimal solution which is unique to the profit-maximization problem. This means that if the generator wants to maximize its profit in each stage game, it will play a unique Nash equilibrium. Moreover, because this game is finite, therefore, the generator will play the Nash equilibrium in every stage game. In other words, this repeated game has a subgame-perfect outcome or the Nash equilibrium, which is, for instance, the solution from profit-maximization, and it is played by the generator in every stage of the game.

Similar to the price competition in the short-run period, to commit how much available capacity in the long-run horizon should be provided to the ISO is also a repeated process. This game, therefore, can be formulated similarly to the game–the price competition–in the faster time-scale. However, the competition in the longer time is for capacity, not price. Given the elasticity of demand in the long-run time-scale, the Cournot quantity of each generator can be determined.[11]

[11] In fact, the market for power is still imperfect, therefore, the Cournot technique is used to find the solution to quantity competition. For instance, the UK power market is considered as a duopoly market, not a competitive one [16].

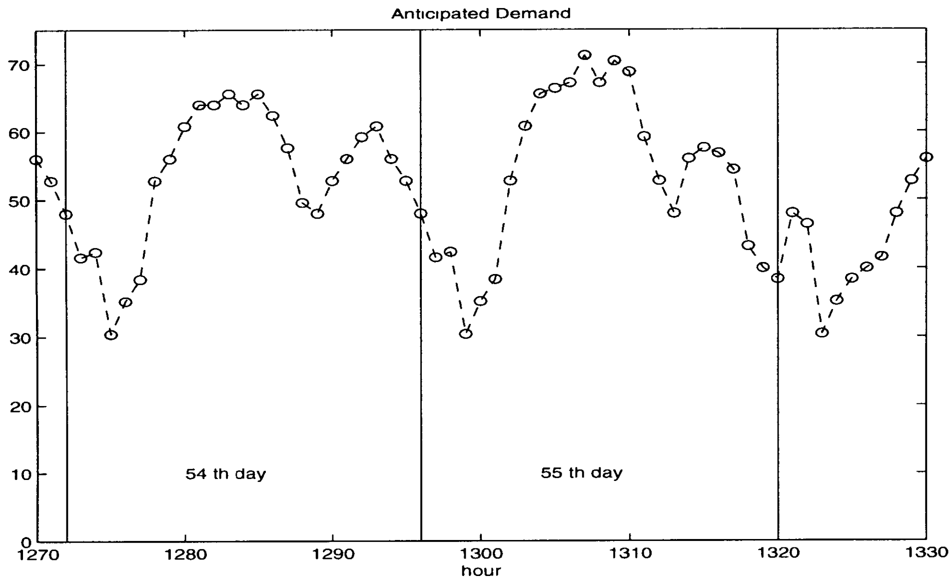

Figure 3-1: Anticipated Demand.

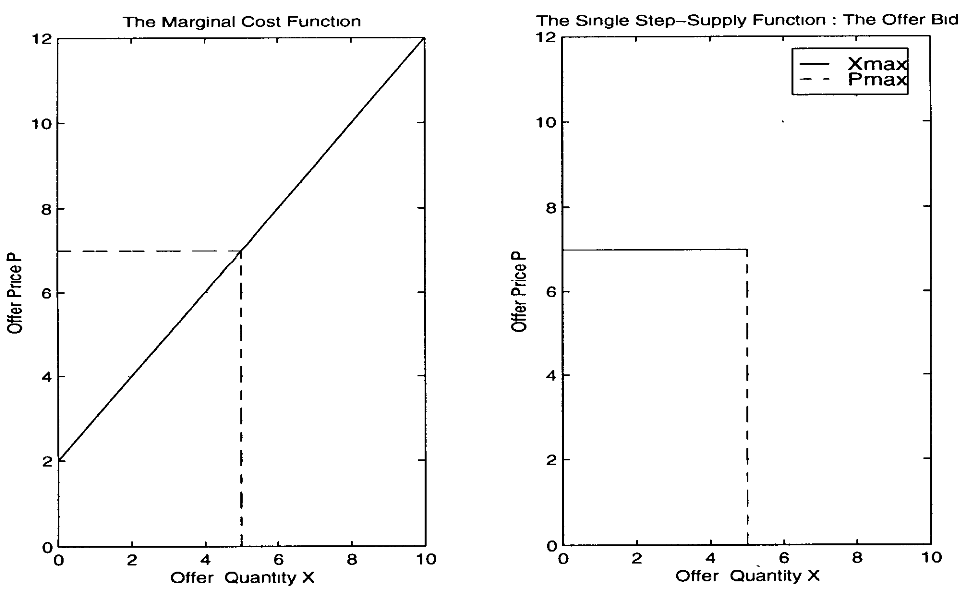

Figure 3-2: Generator's Offer Bid.

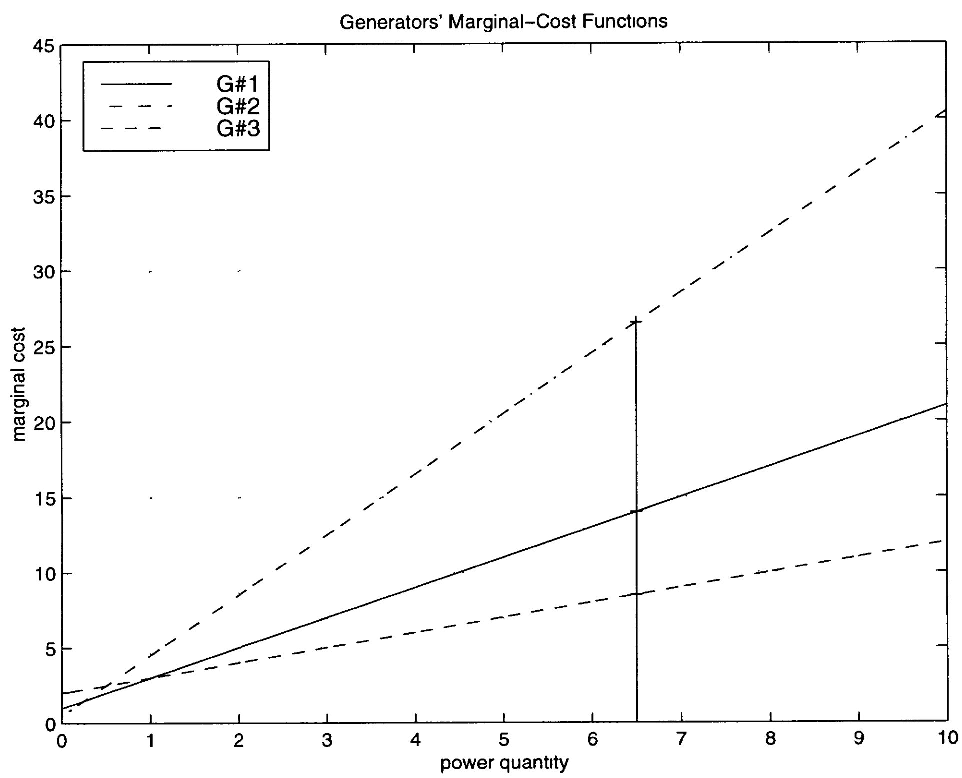

Figure 3-3: Generator's Marginal-Cost Functions.

Chapter 4

Three-Generator Model Analysis

This chapter describes how a dynamic game is formed. The generators' learning process is characterized in the context of strategies. Two strategies are presented: Estimated Profit Maximization and Competition to be a Base-load Generator. Strategy plays a role in the dynamic system shown in Equation (3.21) because strategy is used as a specific criterion to update $B_k[K]$ in each stage ($[K]$). Equation (3.21) depicts the dynamic behavior of each generator as a rational profit-maximizer. The dynamics of this dynamic system will illustrate how the generators respond to variation in the market.

4.1 A Dynamic System and a Dynamic Game

To characterize the power market as a dynamic system, the discrete state-space model derived in Equation (3.21) is introduced as a model formulation. This discrete system has P_k as a state variable. P_k or more specifically $P_{k,i}$ refers to the generator's offer price. $\{BU\}$ is interpreted as a control signal. The model in Equation (3.21) is interpreted as though the generator, maximizing its profits, wants to receive the maximum profit whenever it participates in the market, and that its next offer price will be the function of the current price and the "market-control" signal.

Let us consider the generator section as one system consisting of many subsystems equal to the number of the generators participating in the market. Because P_k is the offer price of each generator in the bidding game, it represents one subsystem. Therefore, the generation section is also known as the so-called *decentralised system*, which refers to the system whose subsystems are not coupled (directly related). For instance, if this generator section is formed in the usual dynamic discrete-system form:

$$x[K+1] = a[K]x[K] + b[K]u[K]$$

with $x[K]$ state-variables (in this thesis $x[K] \equiv (P_{1,k}[K], P_{2,k}[K], P_{3,k}[K])$) and $a[K]$ a system matrix, $b[K]$ a input matrix and $u[K]$ input signals. In the generator-section system, the system matrix $a[K]$ is a diagonal matrix in which all diagonal elements are equal to one.

Let us consider Equation (3.21) as a closed-loop system with full-state feedback control. $\{BU\}$ is now the function of the previous state variables and it is interpreted as the "market-control" signal that is the function of the previous state (previous offer price) and other information, such as the market clearing prices and the generator's maximum available capacity.

Similarly, the dynamic system represented by Equation (3.21) is also considered as a dynamic game. Either k or $[K]$ represents one stage game. If the game is repeated every hour, k represents the kth stage of the game. On the other hand, if the game is repeated every day, K represents the Kth stage of the game. The bidding game in this thesis is played repeatedly by the generators every day so K is used to represent the stage. In each stage, after the game ends, the players make their next moves in the next game based on knowledge received from the previous games. This is analogous to the bidding process in which the generators submit their next bids to the pool after the ISO announces the current schedule and the anticipated demand for the next day.

In addition, in each dynamic game, the strategy, which is a complete plan of actions which determines the "moves" of the players, is uniformly applied to the generators. In the bidding game, both common knowledge and strategy yield the "move" of the generator that can be interpreted as a feedback signal that can be used to determine the next bid, $P_k[K+1]$. A credible and rational strategy will, therefore, affect the outcome of the game as well as the feedback signal in the dynamic system. Consequently, the dynamics of the market model may depend significantly on the strategy applied to the model.

4.2 Strategies

A strategy is a complete plan of actions. As mentioned previously, the strategy will affect the generator's behavior. The strategy of any generator is generally not unique. A Nash equilibrium of any game depends on the strategy space of that game. The strategy space is a set of "moves" that the players can choose to act in the game. When the strategy space is defined, a Nash equilibrium is determined.[1] In the bidding game, after the strategy space and a Nash equilibrium are verified, the generator will be able to make "move" in that game because it will play the Nash equilibrium in every stage game.

In this model, two slightly different strategies are presented; each strategy is described in more detail below. The first strategy is to maximize estimated total profits the next day so it is called *Estimated Profit Maximization.* The second strategy is to compete for being a base-load generator among the generators so it is called *Competition to be a Base-load Generator.* The generator which adopts the second strategy for determining its bid does not directly depend on its estimated total profits. However, according to Proposition 2, this strategy implies that if the generator were a base-load generator, its profits would be maximized.

4.2.1 A Nash Equilibrium

Let us consider Equation (3.21) in the context of the bidding game in more detail. $B_k[K]\,U_k[K]$ is defined as the function of all common knowledge available for each generator:

[1] In the n-player normal-form game, if a number of players and their strategy space are finite, then there exists at least one Nash equilibrium, possibly involving mixed strategies. See [12].

- $B_k[K]$ is a set of specific numbers, $B_k[K] \in \{-1, 0, 1\}$. $B_k[K]$ is interpreted as a *strategic variable.* Similarly, when $U_k[K]$ is a (given) positive number, the next bid will differ from the current one based on whether the value of $B_k[K]$ is "-1", "0" or "1". Therefore, $B_k[K]$ is a set of *moves* corresponding to the assigned strategy space of each generator in one stage of the game. This is further explained as:

    1. $B_k[K] = -1$ means that the generator decreases its price the next day.
    2. $B_k[K] = 0$ means that the generator does not change its price the next day.
    3. $B_k[K] = 1$ means that the generator increases its price the next day.

- $U_k[K]$ is a positive constant and cannot be varied over periods of time (i.e., at least within a long-run horizon). Similarly, within a long-run horizon in which a short-run market, known as a day-ahead market, occurs every day, $U_k[K]$ cannot change. In this model, $U_k[K]$ *is set equal 5% of the maximum marginal cost* ($P_{i,max}$). Hence, $U_k[K]$ is determined from the maximum available capacity and the marginal-cost function. Therefore, each generator has an individual $U_k[K]$. If $B_k[K]$ is equal to 1, $U_k[K]$ is an incremental price. Note that 5% of the maximum marginal cost is assumed for this model. However, if this number is changed to another number, the results might change. This number might be considered as one *strategic variable* as well. In this thesis, it is set at 5% of the generator's maximum marginal cost (in one short-run period).

Particularly, in the three-generator model, each generator is either a base-load, a mid-level or a peak-load generator. To compute the next bid, the generator competes with other generators scheduled in the next higher rank and/or the next lower rank. Each generator wants to be better-off, for instance, according to Proposition 2, if the generator becomes a base-load generator instead of the mid-level generator, its profit will increase. On the other hand, the generator does not want to be worse-off in the next bid , for instance, if it loses the position scheduled as a base-load generator and becomes a mid-level generator instead, its profit will decrease. Consequently, this bidding game is simply generalized to a *two-player game,*[2] in which one generator and its rival (either the generator one rank higher or lower) are the players.

Two strategies to determine $B_k[K]$ (the generator's move) are presented in this thesis. A *Nash equilibrium* for this stage game required that the generator and its rivals determine their moves such that their moves result in the optimal solution (the estimated profits). Note that the strategy of the game is not unique. If the strategy space includes a unique Nash equilibrium, the best response of each player to the finitely repeated game is to play the Nash equilibrium in every stage of the game. In this model, payoffs of the generators, which are the estimated profits, are not calculated explicitly. They show whether the generator's estimated total profits either "increase", "decrease" or "unchange".

4.3 Strategy Formulation

Defining the strategy in this bidding game is analogous to defining how one can evaluate $B_k[K]$. In this thesis, there are two methods to determine $B_k[K]$ or two strategies that generators can employ to play the bidding game. Both strategies are based on the standard scenario when the generators' total offer supply is equal to or exceeds anticipated demand in any period. Consequently, in this scenario, there are at most three market clearing prices in each day because there are three generators with asymmetrical operating-cost functions, and the generators are obliged to bid a single step-supply function. *When the generator plays the two-player bidding game to determine the next bid, the generator assumes that the next offer price of its opponent does not change. Similarly, the generator assumes that its opponent's next offer price might change but the generator does not know.* Each strategy is described separately below. Both strategies actually aim for the same goal, that is the generators want to maximize their profits.

Note that the generators play the bidding game today in order to determine their

[2]For more detail of the two-player game, see [12, 14].

offer bids tomorrow. Similarly, the generators evaluate their $B_k[K]$ so that they can determine their $P_{i,k}[K+1]$.

4.3.1 Estimated Profit Maximization

In Estimated Profit Maximization strategy, the generator maximizes its estimated total profits in the next scheduling day by evaluating its next offer price that it might gain the maximum profit the next day. The generator will make its "move" such that its total profits might be increased or at least remain unchanged.

The generator determines whether it is scheduled as a base-load, mid-level or peak-load generator so it knows who its opponent is (i.e, if the generator is a current base-load generator, its opponent is the current mid-level generator.). In any case, the generator must employ specific decision-making processes. The decision-making processes for the generator being scheduled as a base-load, mid-level or peak-load generator and the special case are described below.

The Base-load Generator

If the generator is being scheduled as a base-load generator, it will want to be scheduled as a base-load generator the next day as well. This concept is directly derived from Proposition 2, which states that each day that the generator is scheduled as a base-load generator will yield the maximum profit to the generator. If the generator increases its bidding price and it remains a base-load generator the next day, the generator will increase its next offer price. The generator might be scheduled to generate the same power quantity as the previous day, because, according to Proposition 1, the profit function is an increasing function in $P_k[K]$. Its profits will increase, although it might be scheduled to produce the same quantity as the previous day. If this is the case, operating as a base-load generator,

- the generator must check whether it is operating at its maximum committed capacity. If the generator is, it will propose the same bid the next day. Similarly, the generator will assign "0" to its $B_k[K]$ because, according to Proposition 1 and 2, that the generator is offering its maximum price and that the generator is scheduling as a base-load generator, it already gains the maximum profit. Therefore,

$$P_k[K+1] = P_k[K]$$

On the other hand, if the generator does not offer its maximum capacity bid,

- the generator must compare its current offer price to the current mid-level market clearing price. If the generator increases its price, and the increased price exceeds the current mid-level market clearing price, it should not alter its offer bidding price for the next day, because the generator might lose its chance to become a base-load generator the next day. Similarly, the generator will assign "0" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K]$$

- the generator must compare its current offer price to the current mid-level market clearing price. If the generator increases its offer bidding price, and the increased price does not exceed the current mid-level market clearing price, the generator should increase its price so that its profit might increase (from Proposition 1). Thus the generator will assign "+1" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K] + U_k[K]$$

The Mid-level Generator

The mid-level generator's task is more complicated than a base-load generator's task, because the mid-level generator must consider both whether it can become a base-load generator and whether it might lose the position as a mid-level generator and become a peak-load generator instead. Therefore, the mid-level generator must play two games before deciding on its next "move". To determine the next move,

- the generator must check whether its current profits are more than the profits that it would have made if it were scheduled as a base-load generator. If its current profit (as a mid-level generator) is more than from that of a base-load generator, the generator should attempt to become a mid-level generator the next day. As a result, the following two options are available:

  1. If the generator increases its price and this increased price is less than the previous peak-load market clearing price, it can increase its offer price and still be scheduled as a mid-level generator. In addition, if its offer price is not the maximum value, the generator can increase its profits by selling power at the higher price. Hence, the generator will increase its offer price the next day or the generator will assign "+1" to its $B_k[K]$. Therefore,

     $$P_k[K+1] = P_k[K] + U_k[K]$$

     On the other hand, if the generator offers the maximum bidding price, it cannot increase its offer price in its next bid. Similarly, the generator will assign "0" to its $B_k[K]$. Therefore,

     $$P_k[K+1] = P_k[K]$$

  2. If the generator increases its price and the increased price is not less than the previous peak-load market clearing price, it can lose its position of being scheduled as a mid-level generator in the next period to the other generator. Hence, the generator will decrease its offer price in the next period or the generator will assign "-1" to its $B_k[K]$. Therefore,

     $$P_k[K+1] = P_k[K] - U_k[K]$$

- If the generator's current profit is less than the profit that it would have made if it were scheduled as a base-load generator, the generator would be eligible to be scheduled as a base-load generator during the next period by reducing its offer price. Hence, the generator will make more profits by decreasing its offer price or the generator will assign "-1" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K] - U_k[K]$$

The Peak-load Generator

Being scheduled as a peak-load generator produces the least profits for the generator for one offer bid. Therefore, to make greater profits, the generator should offer a bid such that it might be scheduled to operate more as a mid-level generator or it will receive the higher market price. This leads us to the following scenarios:

- The generator must compare its current profits as a peak-load generator with the profit that it could have made if it were scheduled as a mid-level generator. If its current profit is more than the estimated profits, the generator must consider two options:

    1. If the generator is not offering its maximum available quantity, according to Proposition 1, it will make more profits the next day by selling its power at the higher price for the current amount. Hence, the generator will increase its offer price in the next bid or the generator will assign "+1" to its $B_k[K]$. Therefore,

    $$P_k[K+1] = P_k[K] + U_k[K]$$

    2. If the generator is offering its maximum value, it cannot offer more due to the assumption in Section 3.2.3. Hence, the generator will not change its offer bid in the next day or the generator will assign "0" to its $B_k[K]$. Therefore,

    $$P_k[K+1] = P_k[K]$$

- On the other hand, if its current profits as a peak-load generator are less than estimated profits as a mid-level generator, the generator must reduce its price instead, so that it is eligible to be scheduled as a mid-level generator the next day. Hence, the generator will decrease its offer price or the generator will assign "-1" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K] - U_k[K]$$

The Special Case

A special case is added because in some sets of the assigned maximum available capacities, the generator might not be scheduled for operating at all. Hence, in this case, the only way to adjust its bid so that it might be scheduled, is to reduce its bidding price or to assign "-1" to $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K] - U_k[K]$$

4.3.2 Competition to be a Base-load Generator

In the Competition to be a Base-load Generator strategy, according to Proposition 2, the generator will maximize its profits if it is scheduled to operate in each day as a base-load generator. In addition, being scheduled as a mid-level generator is better than being scheduled as a peak-load generator. This strategy implies that the generator will undercut its rivals so that its offer price is less than its rivals so the generator will be scheduled to operate every hour. In each bid the generator is allowed to change its price not more than $U_k[K]$. The decision-making of each generator is analyzed in the same fashion as in the first strategy. Each case (base-load, mid-level and peak-load) is explained separately below.

Using this strategy, instead of estimating the next-day profits, the generator compares its current offer price to the current market clearing prices and decides whether it should either increase, decrease or not vary its next bid. The proposed decision-making processes are explained below.

The Base-load Generator

Based on Proposition 2, the generator is more profitable if it is scheduled as a base-load generator in every period. Hence, if the generator is operating as a base-load generator in the current period,

- the generator must check whether it is offering its maximum committed capacity and price. If the generator is, it will propose the next bid equal to the current bid, because its profits are already maximized. Similarly, the generator will assign "0" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K]$$

On the other hand, if the generator is not offering the maximum available capacity,

- the generator must compare its current offer price to the mid-level market clearing price. If the generator increases its next bidding price and the increased price does not exceed the current mid-level market price, the generator must increase its bid price to gain more profits. Also the generator will assign "+1" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K] + U_k[K]$$

- the generator must compare its current offer price to the mid-level market clearing price. If the generator decreases its next offer price and the decreased price is equal to or less than the mid-level price, the estimated profit can be maximized because the generator can operate as a mid-level generator in the next day. Hence the generator will decrease its next offer price or the generator will assign "-1" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K] - U_k[K]$$

The Mid-level Generator

The current mid-level generator must play two games before deciding on its next move. The mid-level generator will either undercut the base-load generator so it can become a base-load generator or the generator can maximize its profit as a mid-level generator. In the next move,

- the generator must compare its current offer price to the current off-peak market clearing price. If the generator decreases its price and the decreased price is less than the current off-peak price, it will decrease its price. Because the generator can be a base-load generator in the next period and this implies that it can make more profits than the current ones. Hence, the generator will assign "-1" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K] - U_k[K]$$

On the other hand, if the decreased price is not less than the current off-peak price, the generator may not make more profit by undercutting the current base-load generator to become a base-load generator. Therefore,

- if the generator increases its price and the increased price does not exceed the current peak-load price, it can make more profit from selling power as a mid-level generator at the price higher than its current price the next day. Hence,

  1. if the generator is offering its maximum committed capacity, it cannot increase its offer price due to the assumption in Section 3.2.3. Thus the generator will assign "0" to its $B_k[K]$. Therefore,

  $$P_k[K+1] = P_k[K]$$

  2. if the generator is not offering its maximum price, it can increase its price so that it can make more profits in the next day. Hence the generator will

assign "+1" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K] + U_k[K]$$

- If the increased price exceeds the current peak-load price, the generator will not increase its next offer price because it might be scheduled as a peak-load generator instead and its profit would definitely decrease. Hence, the generator will assign "0" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K]$$

The Peak-load Generator

In the next day, the peak-load generator will want to either sell its power at the price higher than its current price, or become a mid-level generator. However, according to Proposition 2, being scheduled as a mid-level generator, the generator makes more profits than as a peak-load generator, the generator will generally undercut the mid-level generator such that it might become a mid-level generator in the next day. The generator will choose its next move base on the following scenarios:

- If the generator decreases its price and its decreased price is more than the current mid-level price, it might not be scheduled as a base-load generator the next day. Thus,

  1. If the generator is not offering its maximum committed capacity, selling power as a peak-load generator at the increased price will yield more profits to the generator than the current price. Hence, the generator will increase its price or the generator will assign "+1" to its $B_k[K]$. Therefore,

  $$P_k[K+1] = P_k[K] + U_k[K]$$

  2. If the generator is offering its maximum committed capacity, according to the assumption in Section 3.2.3, it cannot increase price. The generator

will assign "0" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K]$$

- If the generator decreases its next offer price and the decreased price is not more than the current mid-level price, its profits in the next day can be maximized because the generator can operate as a mid-level generator the next day. Hence the generator will decrease its next offer price or the generator will assign "-1" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K] - U_k[K]$$

The Special Case

Like the first strategy, if the generator is not scheduled to operate at all in any day, one possible way for the generator so that it can make some profits the next day, is to reduce its offer price. The generator will assign "-1" to its $B_k[K]$. Therefore,

$$P_k[K+1] = P_k[K] - U_k[K]$$

4.4 The Marginally Stable Market Model

Let us consider the dynamic market model in Equation (3.21). It can be analyzed as a discrete open-loop dynamic system. This system represents the dynamics of one generator. Hence the overall system representing the market, which contains three generators in this thesis, consists of all open-loop eigenvalues (λ_{Ol}) equal to "1". So the open-loop system is marginally stable. However, $B_k[K]\,U_k[K]$ can be considered the full-state feedback signal evaluated using $P_k[K]$, $P_{mark}[K]$ and $C_i(Q)$ as well. In this model, $B_k[K]$ is a set of {-1, 0, 1} and it is determined from $P_k[K], P_{mark}[K]$ and $C_i(Q)$. Therefore, $B_k[K]$ refers to a function of $P_k[K]$, while $P_{mark}[K]$ and $C_i(Q)$ are treated as reference input-signals to set $B_k[K]$ equal to the assigned values.

$U_k[K]$ is analogous to the control gain. One positive number equal to $0.05 \times P_{imax}$ (5% of the maximum operating cost evaluated at the maximum available capacity) is specified as $U_k[K]$. Consequently, the maximum available capacity will have impact on determining the $U_k[K]$. All generators will be scheduled during the peak periods; therefore, they will generally offer positive bidding prices and also quantities. Besides, the maximum available capacity is assigned such that $U_k[K]$ is always less than 1.

On the other hand, if the system represented in Equation (3.21) is considered as a closed-loop system, the eigenvalues (λ_{Cl}) will change by a small value at each stage. The bidding system is also a closed-loop system. This closed-loop bidding game has a feedback signal, which is common knowledge. The common knowledge in this game consists of the market clearing price and anticipated demand.

Based on the above set-up, without loss of generality, the closed-loop system represented in Equation (3.21) yields that ($|\lambda_{Cl} - \lambda_{Ol}| < 1$); similarly, $|\lambda_{Ol}[K] - \lambda_{Ol}[K+1]| < 1$. The eigenvalues ($\lambda_{Cl}$) of the closed-loop system are

1. $\lambda_{Cl} = 1$ when $B_k[K] = 0$.

2. $\lambda_{Cl} > 1$ when $B_k[K] = 1$. or

3. $\lambda_{Cl} < 1$ when $B_k[K] = -1$ and $|\lambda_{Cl}| < 1$

Hence the closed-loop system is time-varying. The eigenvalues change when k and K change. Therefore, the initial condition used to initiate the simulation is also considered one factor that might influence the outcome.

Let us consider the following possible cases:

- The overall closed-loop system will be asymptotically stable only when all generators reduce their prices the next day. The absolute value of the closed-loop eigenvalues will be less than 1.

- The overall closed-loop system will be marginally stable when the generators maintain their bids at the same level in every period. This happens when the system reaches its steady-state in which the price competition ends. In this model, the generator is not allowed to offer more than $X_{i,max}$ and $P_{i,max}$. The maximum possible offer bid will be $(P_{i,max}, X_{i,max})$. Hence price competition will reach steady-state at least when all generators offer their maximum bids.

- The overall closed-loop system will be unstable when the generators do not change their behavior uniformly. Therefore, the set of eigenvalues of the system consists of at least one λ_{Cl} greater than 1.

In this thesis, the dynamic game representing the bidding process in the three-generator market is marginally stable; therefore, the initial condition will affect the system because of the time-varying property of the system. Different initial conditions create different trajectories of $P_k[K]$. Consequently, the initial conditions are varied while the load characteristics and the maximum available capacity remain unchanged, to observe how the initial conditions influence the market price.

Chapter 5

Results and Discussion

The results of this three-generator market model are simulated based on the assumptions in Chapter 3 and the two given strategies: Estimated Profit Maximization and Competition to be a Base-load Generator. Each strategy is uniformly applied to all generators to observe the dynamics of the market. The maximum available capacity and the initial condition used to start the simulation, which might affect the market price, are also studied. The generators choose the scale of operation (or the maximum available capacity) in the long-run horizon before they make any decisions about their offer prices in the short-run horizon. We assume that the generators have already decided on how much of their maximum capacity will be available in one long-run horizon. Within that long-run period, the generators do not operate beyond their committed capacities. Consequently, in the simulation, the available capacity is treated as one variable. This variable is varied to observe how it affects the market price under the same load and initial conditions.

In addition, the initial condition used to initiate the simulation is also considered an important factor that might influence the outcome. If the system in Equation (3.21) is treated as a discrete full-state feedback system, it is time-varying and might not be stable. Hence, the initial condition might have an impact on the outcome which is the market clearing price pattern. The initial condition is varied while the maximum available capacity of the generator is kept unchanged to observe how it influences the results.

Finally, to study the outcome from the three-generator dynamic model in terms of efficiency in pricing and also dispatch, the solution from the ED is used as a reference.[1] The efficiency in pricing is analyzed by comparing the simulation results with the ED solution. Note that the simulation results indicate the price competition of the short-run market in the steady-state.

Although the solution in steady-state is considered, this is not the same solution from the static model. In this dynamic model, each generator *individually "calculates (estimates)"* its supply to the pool based on the given information, which is not included the operating-cost function of each generator. The generators learn the market daily, therefore, they (optimally) adjust their behavior according to the market. On the other hand, in the static model, the solution is calculated from the assumption that all generators have all information including the other generators' operating-cost functions. The solution to the static model is centralized. The steady-state solutions, which are obtained from different approaches in both static and dynamic models, are, therefore, different.

In this thesis, the activities during the 1270th and 1330th hours are presented. During this period, the response from the dynamic model (i.e., the market price) is in the steady-state. Note that the set of maximum available capacities or the previously committed capacities in one long-run period, and initial conditions (I_i) of three generators are represented by $[X_{1,max}; X_{2,max}; X_{3,max}]$ and $[I_1; I_2; I_3]$ respectively. The analyses are explained in the following sections.

5.1 The Strategies

The two different strategies applied to the model result in different market clearing price patterns, although the simulation is carried out in the same environment (i.e., the load characteristic, the maximum available capacity of each generator and the initial condition).

[1]For more detail about how the ED solution is used as a reference, see Appendix A.

5.1.1 Estimated Profit Maximization

The Estimated Profit Maximization strategy is known as the "first" strategy. Some examples of the simulation results, when the first strategy is applied to the model, are shown in Figure 5-1 and Figure 5-2.

1. Figure 5-1 shows the simulation result when the maximum available capacities are equal to [25;20;35] and the initial conditions are equal to [20;10;30]. In this case, Generator No. 1 has the largest available capacity while Generator No. 2 has the smallest available capacity. The result indicates that Generator No. 1 is a mid-level generator, Generator No. 2 is a peak-load generator and Generator No. 3 is a base-load generator. The generators offer the bidding prices equal to their maximum marginal costs ($P_{i,max}$) ($2 \times (25) + 1 = 51$ for Generator No. 1, $4 \times (20) + 0.5 = 80.5$ for Generator No. 2, and $(35) + 2 = 37$ for Generator No. 3). The simulation also shows that the market clearing prices are above the ED price for every hour.

2. Figure 5-2 shows the simulation result when the maximum available capacities are equal to [30;15;30] and the initial conditions are equal to [20;15;20]. In this case, Generators No. 1 and No. 3 have the same available capacities while Generator No. 2 has the smallest capacity. The result shows that Generator No. 2 is a peak-load generator and Generators No. 1 and No. 3 are base-load generators.[2] However, Generator No. 3 does not create the market price. There are two prices within one scheduling day (24 hours). Generator No. 2 offers the maximum price $P_{2,max}$ ($4 \times (15) + 0.5 = 60.5$). Generator No. 1 offers a price that is lower than Generator No. 2's, but this price is not its maximum price. Although, Generator No. 3 offers its maximum price $P_{3,max}$, its maximum capacity is less than the minimum load (30.4 units) so Generator No. 3 offers no market price. In addition, the market clearing prices are higher than the ED price in every hour.

[2] According to Definition 2, Generator No. 3 operates in every period, so it is a base-load generator.

This result indicates that Generator No. 1 undercuts Generator No. 2 so that Generator No. 1 operates as a mid-level generator and it does not increase its price further because it might become a peak-load generator instead. On the other hand, because $U_{k,2}[K] = 0.05 \times (4 \times (15) + 0.5) = 3.025$, Generator No. 2 cannot undercut Generator No. 1 so Generator No. 2 offers its bid such that its profit is maximized as a peak-load generator.

The simulation suggests that the generators compete for scheduling by undercutting their rivals; but, if they cannot be scheduled in the better rank, they will increase their prices such that they continue to operate in the same rank. (As described in Proposition 1, the profit function is an increasing function in the market price (P); consequently, selling power in any rank at the highest possible price yields the maximum profit to the generator.)

In addition, in some scenarios (not presented here), the results show that the market clearing prices from the bidding process are higher than the ED prices in most periods. There are few scenarios in which the total supply is less than the anticipated demand, such that the ISO has to reschedule all generators and set the market clearing prices equal to the ED prices.

5.1.2 Competition to be a Base-load Generator

The Competition to be a Base-load Generator strategy is known as the "second" strategy. Some examples of the simulation results, when the second strategy is applied to the model, are shown in Figure 5-3 and Figure 5-4.

1. Figure 5-3 shows the simulation results when the maximum available capacities are equal to [25;20;35] and the initial conditions are equal to [20;10;30]. In this case which is similar to case (1) in Section 5.1.1, Generator No. 3 has the largest capacity. On some days, it is scheduled as a base-load generator but on other days it is scheduled as a mid-level generator. Like Generator No. 1, Generator No. 2 is scheduled as either a base-load or a mid-level generator; however, Generator No. 2 does not offer its maximum price when it is either

a base-load or a mid-level generator. Generator No. 1 is always scheduled as a peak-load generator and does not offer its maximum price every day. In addition, there is supply deficiency during some peak-load periods which make the supply does not meet peak-load demand, therefore, the ISO reschedules the generators and sets the market clearing prices. There are more than three values of market clearing prices. The outcome can be attributed to the fact that, on some days of the game, Generator No. 2, as a mid-level generator, undercuts Generator No. 3 and becomes a base-load generator. Generator No. 1, as a peak-load generator, on the other hand, does not undercut Generator No. 2, but increases the offer price. The supply deficiency, occurring in other days, causes Generator No. 1 to misinterpret the market. The generator increases its price so that it operates as a peak-load generator instead of undercutting Generator No. 2. In most periods, especially during the off-peak periods, the market prices are higher than the ED prices.

2. Figure 5-4 shows the simulation result when the maximum available capacities are equal to [30;15;30] and the initial conditions are equal to [20;15;20]. In this case which is similar to case (2) in Section 5.1.1, both Generator No. 1 and No. 3 have the largest capacities and Generator No. 2 has the smallest capacity. Generator No. 1 operates as a base-load generator on some days and as a peak-load generator on other days. Like Generator No. 1, Generator No. 2 is scheduled as a peak-load generator and as a base-load generator. Both Generator No. 1 and No. 2 do not offer their maximum prices. Generator No. 3, on the other hand, offers its maximum price $P_{1,max}$ $((30)+2 = 32)$ and it operates as a mid-level generator. Besides, there exists supply deficiency almost every day. The ISO has to reschedule the generators so that the demand is met, and set the market clearing prices equal to the ED prices. Therefore there are more than three values of the market clearing prices. Moreover, Generator No. 1 and No. 2 undercut each other such that they alternately operate as a base-load generator and a peak-load generator. In few periods, especially in off-peak

periods, the market prices are higher than the ED prices.

The simulation suggests that the generators mainly compete by undercutting their rivals; but this price competition creates a deficiency in supply which can occur very often. The resulting prices tend to be lower than the first strategy in the low-demand periods; however, during the high-demand periods supply deficiency causes the market clearing prices to be equal to the ED prices which vary from the offer prices. (The ISO reschedules the generators and sets the market clearing prices.) Because there are more than three market clearing prices within one day, the generators might receive the "wrong" signal from the offer prices of their opponents and thus, the generator might not undercut its rival such as Generator No. 1 as detailed in Figure 5-3.

The price competition under the two strategies creates a very interesting outcome. The different strategies applied to the model yield different market clearing price patterns; for example, the previous results in Figures 5-1 and 5-3, and Figures 5-2 and 5-4. Obviously, not only do different strategies have an impact on the market price, but the total maximum available capacities, the maximum available capacity of each generator, and the initial conditions also impact market clearing prices.

The common outcome of the two strategies is that the market clearing prices are usually higher than the ED prices. Particularly, in applying the first strategy to determine their bids, the generators offer the prices so that the resulting market prices are always higher than the competitive level (the ED price) in every period. This outcome implies that there is inefficiency in the generators' pricing. The market price is higher than the competitive price. Note that, in cases where supply exceeds demand, the single step-supply function bid for a one-day schedule restricts the market price to some specific values. This usually causes the market price, especially during the low-demand periods, to be higher than the prices from the ED. The result from this model tends to confirm that the inefficiency in pricing, at least in off-peak periods, is a predictable bottom line.

Besides, in some scenarios, the generators undercut their rivals and submit the bids such that the total supply does not meet the anticipated demand, especially during peak-load periods. This outcome suggests that a sealed-bid auction in which a generator offers a single-step supply function, might cause a deficiency in supply and might not be enough to make the transaction complete. In this thesis, the ISO is allowed to reschedule the generators by using the ED solutions such that the demand is always met. However, if the ISO were unable to reschedule (i.e., if the ISO had no information about each generator's marginal-cost function), the demand would not be satisfied.

In addition, the supply deficiency further implies that if the ISO did not be allowed to reschedule the generators in the real market, a second bid, which might be held in real-time, might be needed so that supply always meets demand. It also implies that the single step-supply function bid might be inadequate because, in this model, each generator offers only one bidding price. Therefore, the generators are likely to reduce their prices so that they are scheduled to operate. The multi-step-supply function bid might be a better choice. However, in this model the generator is obliged to bid based on its affine marginal-cost function, hence, the multi-step-supply function bid is inconsistent. *A* **real-time bid** *such as an* **hour-ahead market bid**, *tends to be an optimal method so that the inelastic anticipated demand is always met.* Such a market might lead to an increase in prices of power in the short-run market, especially during peak-load periods.

To clarify the concept that the real-time market might create higher market prices, let us consider, that each generator knows the current market clearing price in each hour and realizes that supply deficiency exists. The generator would be able to offer a price in the (assumed) real-time market that is at least equal to the maximum offer price from the day-ahead market. Each generator must offer a bid based on its marginal-operating-cost function so it cannot lower its price below the price offered in the day-ahead market to sell more power in a real-time market.

$$P_{i,k}[K_k] = \arg \max_{i} \{P_{i,k}[K]\}$$

where $P_{i,k}[K_k]$ is the offer price of Generator No. i in the (assumed) real-time market at the kth period (hour) of the Kth day.

Therefore, the price of electricity from the real-time market tends to be higher than the price of the day-ahead market because the generator with the lowest bidding price from the day-ahead market can offer its real-time market bidding price equal to the highest offer price from the day-ahead market. The conclusion drawn at this point is that although there are no (real) generation-capacity constraints in the system,[3] the generators tend to tacitly collude to create deficiency in supply (as a consequence of the repeated bidding processes) which might result in higher electricity prices.

The results, especially from the second strategy, further suggest that if there were enough generators in the market (more than the three generators in this model), the daily price competition and the learning process would let the generators undercut their rivals. Consequently, the market price might be driven down and converge closely with the competitive level, while demand would be met without creating deficiency in supply. For instance, during peak-load periods, although the generators undercut each other, there are still enough power available to supply the demand in peak-load periods. As a result, the electricity price should not be much higher than the first-offer bidding price. Moreover, it is reasonable to conclude that not only uncertainty in load creates an hour-ahead market, but also a fewer number of generators participating in the pool also create such a market as well.

5.2 Maximum Available Capacity

Under the same load and initial conditions, different sets of maximum available capacities assigned to each generator yield different market clearing prices. In each strategy, the maximum available capacity turns out to be a significant factor affecting the market-clearing price pattern. A choice of scale determines the generator's supply functions and thus determines the conditions of price competition[4] [7]. Avail-

[3]In this model, the total available generation capacity always exceeds the maximum anticipated demand.

[4]In the bidding game, the maximum available capacity indicates the maximum limits of the offer prices.

able capacity affects the price pattern. For instance, if the generator has different maximum capacities, it will have different $U_{k,i}[K]$ and the maximum prices ($P_{i,max}$) which yield different offer prices to the generator. Under the same strategy and same load characteristic, the following cases are considered: excessive supply, sufficient supply and limited total capacity, which are detailed in the following sections.

5.2.1 Excessive Supply

The total available capacities are assigned to the generators such that they largely exceed the maximum anticipated demand. In this case, it is likely that the generators will increase their offer prices.[5] The results show that in some cases, the market clearing prices are much higher than the ED prices in most periods.[6] For instance, some examples are shown in Figure 5-5 and Figure 5-7 in which total capacities equal 90 units ($90 >> 72.5$).

Figure 5-5 indicates that, applying the first strategy, Generator No. 2 increases its price to the maximum price $P_{2,max}$ ($4 \times (25) + 0.5 = 100.5$) and operates as a peak-load generator. Generator No. 1 and No. 3 offer their prices such that they are always scheduled as base-load generators. The market clearing prices are higher than the ED prices in every period. Besides the market prices are higher than when there is less excessive supply. (Compare the results in Figures 5-5 to 5-2, in which the total supply is equal to 75 units.)

Figure 5-7 shows that when applying the second strategy, unlike the first strategy, each generator undercuts its rivals such that its price is set to one value. Generators No. 1 and No. 2 offer their price such that they are always scheduled as base-load generators. Generator No. 3 offers a higher price than Generators No. 1 and No. 2, and is scheduled as a peak-load generator every day of the game. The market prices are higher than the ED prices. Demand deficiency is unlikely to occur in this case.

In both cases, the available capacity of the cheapest generator (i.e., Generator

[5] The offer price is limited by the marginal-cost function and also available capacity.

[6] When available capacity is larger, the capacity constraint is binding at the higher demand level; as a result, the ED price is lower when there is little excessive available capacity.

No. 3 in this model) is the largest. Interestingly, the more expensive generator might be scheduled as a base-load generator instead. The results are shown in Figure 5-5 in which Generator No. 1, which is not the cheapest generator, is scheduled as a base-load generator, and in Figure 5-7, in which Generator No. 3, the cheapest generator, operates as a peak-load generator.

This outcome broadens the proposition presented by von der Fehr and Harbord [15]. They suggest that, given that the generators have constant marginal costs and offer step-supply function bids (or single constant-price bids), when demand is low (less than the maximum capacity of the cheapest plant), the market price reaches the maximum limit at the marginal cost of the more expensive generator. However the result from this model implies that *the repeated process allows any generator (even the expensive (highest-cost) one) to learn to undercut its rivals such that it might become a base-load generator.* Therefore, the generator with the highest cost might also serve demand as a base-load generator.

On the other hand, if the generator with the lowest cost (Generator No. 3) has a smaller capacity and the generator with the highest cost (Generator No. 2) has the largest capacity, the market price will be incredibly high because, for instance, in this model, Generator No. 2 will have the highest maximum capacity $X_{2,max}$ and price $P_{2,max}$ so its offer price is likely to be high. If total supply from Generators No. 1 and No. 3 (the cheaper generators) are small enough, Generator No. 2 might supply demand and set the market prices during both mid-level and peak-load periods. The results from the Estimated Profit Maximization strategy are shown in figure 5-6. In this figure, the total available capacities are also 90 units. The result illustrates that Generator No. 2 operates as a peak-load generator, setting the highest market prices. The market prices are considerably higher than the market prices in Figure 5-5, in which the total available capacities also equal to 90 units. Besides, the market prices are substantially higher than the ED prices.

In addition, compared to the first strategy, if the second strategy is applied to the model, a similar phenomenon happens. The results are depicted in Figure 5-8. The generators undercut their rivals, setting their prices at specific levels. Compared to the result in Figure 5-9, the market price is higher than when Generator No. 3 has the largest capacity. Supply deficiency occurs during peak-load periods on some days. Besides, in this case, Generator No. 3 operates as a base-load generator. In addition, the market prices are also higher than the ED in most periods, especially in off-peak periods.

When there is excessive demand in the model under the first strategy, that market clearing prices are higher than the ED price is likely to occur. On the other hand, under the second strategy, the generators undercut their rivals so the market clearing prices tend to be lower than the prices in the first strategy, especially in low-demand periods.

5.2.2 Sufficient Supply

The total available capacities are assigend to the generators such that they slightly exceed the maximum load. In this case, the results are explained by using the *price game* [7]. Because of capacity constraints (the maximum available capacity belongs to some region just above the maximum anticipated demand), the generators will sell their power up to their maximum available capacity. The results from the first strategy are shown in Figure 5-9 and Figure 5-10.

In Figure 5-9, all generators offer their maximum prices. Generators No. 1 and No. 3 operate as base-load generators. Generator No. 2 is scheduled as a peak-load generator. In this case, the total capacities are equal to 75 units. However, the total capacities of Generators No. 1 and No. 3 are less than the anticipated demand in most periods; therefore, Generator No. 2, as a peak-load generator, establishes the high market prices. Moreover, the results show that the market clearing prices are mainly higher than the ED prices.

In Figure 5-10, the generators sell their power at their maximum prices. Generator No. 1 operates as a mid-level generator, Generator No. 2 is scheduled as a peak-load generator and Generator No. 3 is a base-load generator. In this case, the total capacities are also 75 units. However, the market prices are lower than the previous case in every period. Generator No. 1 sets the market prices in some moderatedemand periods. Generator No. 3, as a base-load generator, sets the market prices when demand is very low as well.

According to the "price game", in the market where demand is less than the total capacity of all firms and more than the total capacity of the firms excepting the highest-marginal-cost firm, all firms with their capacity limits will maximize their profits by producing output to serve demand and charging prices equal to the marginal cost of producing their maximum amount. Therefore, the market price, resulting from that the firms charge their maximum marginal costs, is equal to the marginal cost of the highest marginal-cost firm. Similarly, in the Estimated Profit Maximization strategy, the generator maximizes its (estimated) profits in every period by offering its maximum price $P_{i,max}$. The results show in Figures 5-9 and 5-10. Less supply builds the generators confidence to offer the highest bid and be scheduled for operation. The market clearing price in each hour (in steady-state) is, therefore equal to the maximum marginal cost ($P_{i,max}$) of the marginal generator at that hour. Most likely in this scenario, all generators offer maximum bids.

On the other hand, the Competition to be a Base-load Generator strategy does not directly lead to profit maximization because the generators undercut their rivals so that they will have more market share each period without determining whether their estimated profits increase. The generators believe that if they operate as a peak-load generator, their profits will be maximized. The generators rarely offer a bid at the highest marginal cost because they undercut each other until they are satisfied their status as a peak-load, mid-level or base-load generator. Therefore, the generators' offer prices converge to some values rather than their maximum marginal costs ($P_{i,max}$). However there exist some cases in which the most expensive generator does not increase its price to its maximum marginal cost. In other cases, the generators undercut each other creating deficiency in supply and thus prompting the ISO to reschedule the the generators during demand deficiency period so that demand is always met.

5.2.3 Limited Total Capacity

In this scenario, the total capacity is one fixed value while the maximum available capacity of each generator is varied. The result is interesting in that the maximum available capacity of each generator affects the price pattern. Different available capacities of any generator yield different supply functions and thus, the set of possible offer bids. Furthermore, as mentioned in Section 5.2.1, the generator with the lowest marginal cost tends to have much more influence on the market price than the generator with the greater marginal cost. The examples are shown in Figures 5-2, 5-9 and 5-10 in which the total capacities are equal to 75 units, while the available capacity of the generator varies.

The common outcome from each scenario is that the market prices are usually higher than the ED prices. The market prices equal the ED prices only when the ISO reschedules the generators due to deficiency in supply. This outcome implies that there is inefficiency in pricing. The market price from bidding is above the competitive level. Besides the examples show that it is possible that the cheapest generator might not be a base-load generator. This result confirms that inefficiency in dispatch exists as well.

The simulation indicates that the available-capacity committed in one long-run period plays a significant role in the short-run price competition within that period. The decision-making in the long-run market affects the short-run market. The inefficient planning might cause the generator to lose its market share in the short-run market (i.e. the lowest-marginal-cost generator is not scheduled as a base-load generator.). Inefficient pricing tends to be the robust outcome from this dynamic model. Besides, this dynamic model suggests that the generator with the highest marginal cost may bid at a lower offer price than the cheaper generators and thus be dispatched before these more efficient units.

5.3 Initial Conditions

The simulation shows that when the initial conditions are changed the outcome of the dynamic model also changes. This outcome is shown in Figure 5-11 and Figure 5-12. In both cases, the second strategy is applied to the model and the maximum available capacities are equal to [25;15;35]. In the first case (Figure 5-11), the initial conditions are equal to [20;15;25]. In the second case (Figure 5-12), the initial conditions are equal to [15;15;30]. The results indicate that the different initial-conditions have an impact on the price pattern. Although, the same maximum capacity and strategy are applied to the model, while the initial conditions are varied, and the market prices are also varied. Consequently, the initial condition also affects market prices as well as the maximum available capacity. This might be due to the fact that after the market starts in any long-run period, the first estimation of each generator (in the first-day market) can affect the generator's behavior in the following days.

The bidding game, represented by Equation (3.21), in steady-state is marginally stable. This can be explained as the following. Because the generator is not allowed to offer the price into the power pool more than its marginal cost evaluated at the maximum available capacity ($P_{i,max}(X_{i,max})$), the generator can increase the bidding price until the maximum limit is reached, and then it will stop changing its bidding price. The market moves toward stead-state, in which $B_k[K]$ is equal to "0". Considering this bidding game as a discrete dynamic system, the system at steady-state is then marginally stable (the eigenvalues of the integrated system equal to "1".). On the other hand, the generator might undercut its rivals such that it is satisfied at some steady-state prices. In this case, $B_k[K]$ is also equal to "0", which implies that the steady-state system is also marginally stable. As a result, the previously-specified initial condition turns out to be an important condition on this model.

Note that because the generator is obliged to limit the bidding price to no higher than $P_{i,max}(X_{i,max})$, the offer prices of the generator will ultimately converge to one finite value. In some cases, if the generators are not restricted to offer bids that are not higher than their maximum marginal costs, they might be able to increase their prices infinitely. This is analogous to the fact that the bidding game is marginally stable. In stead-state, depending on the set of initial conditions and the maximum available capacity, the system might be unstable. For instance, in Figures 5-9 and 5-10, the peak-load generator, Generator No. 2, can increase its offer price as high as possible, if it is not restricted to offer a price not more than the marginal cost of its maximum available capacity $P_{i,max}$ ($P_{2,max}$ equals to $4 \times (35) + 0.5 = 140.5$ in Figure 5-9 and equals to $4 \times (15) + 0.5 = 60.5$ in Figure 5-10.).

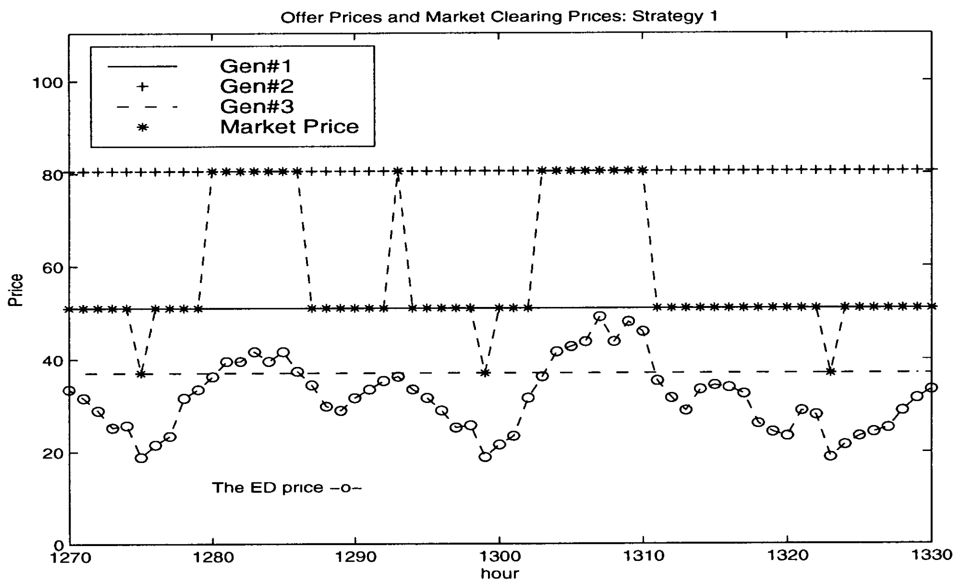

Figure 5-1: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the first strategy): Maximum Available Capacities = [25;20;35] and Initial Conditions = [20;10;30].

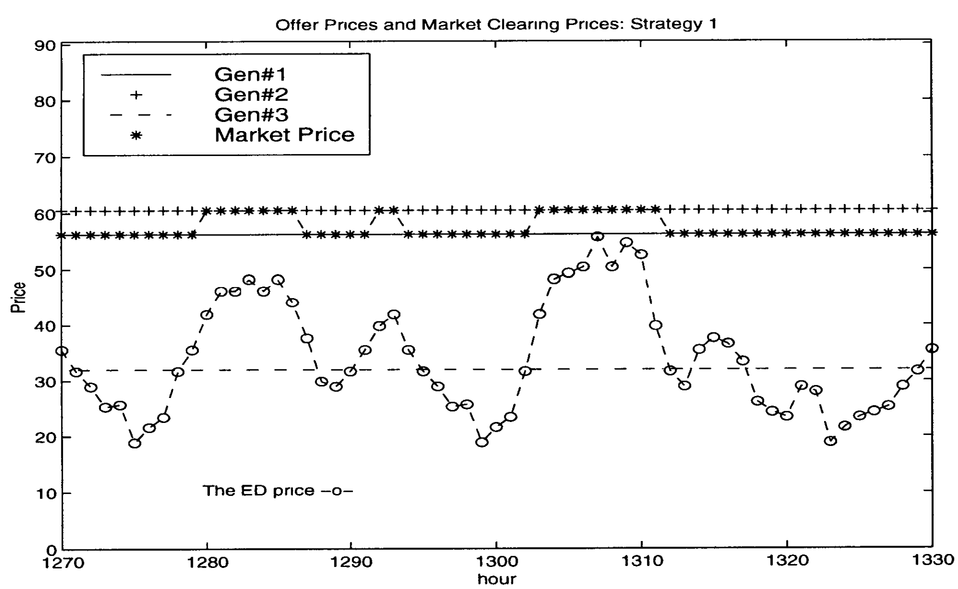

Figure 5-2: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the first strategy): Maximum Available Capacities = [30;15;30] and Initial Conditions = [20;15;20].

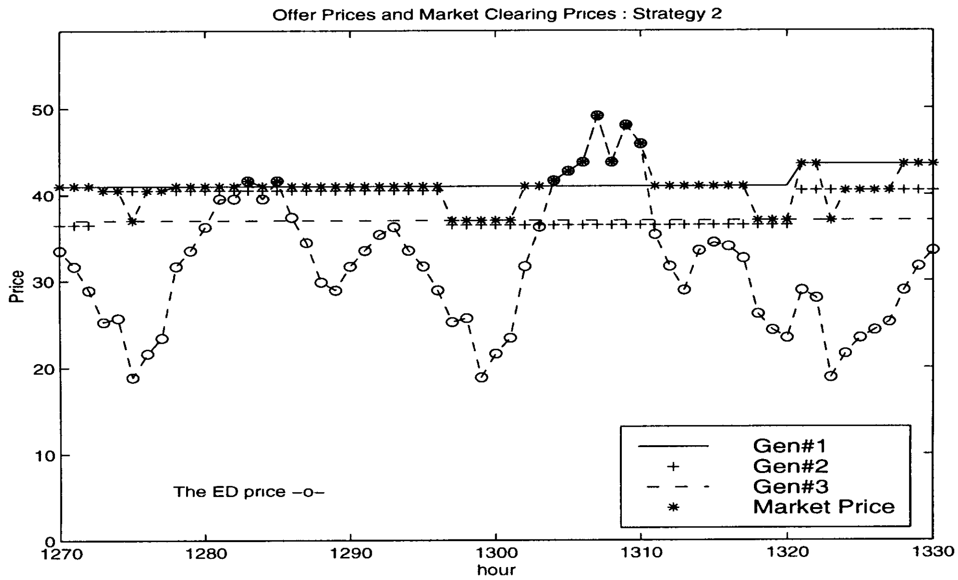

Figure 5-3: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the second strategy): Maximum Available Capacities = [25;20;35] and Initial Conditions = [20;10;30].

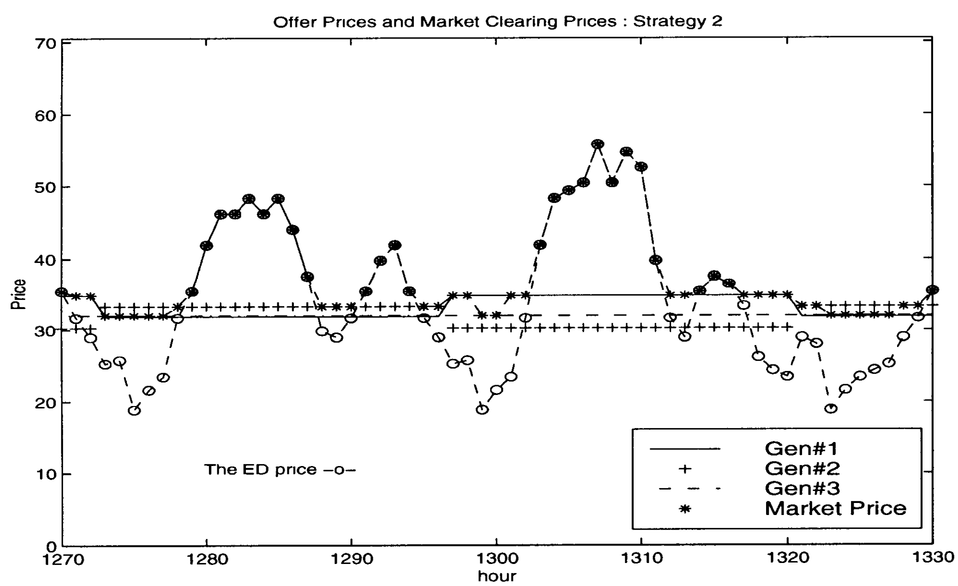

Figure 5-4: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the second strategy): Maximum Available Capacities = [30;15;30] and Initial Conditions = [20;15;20].

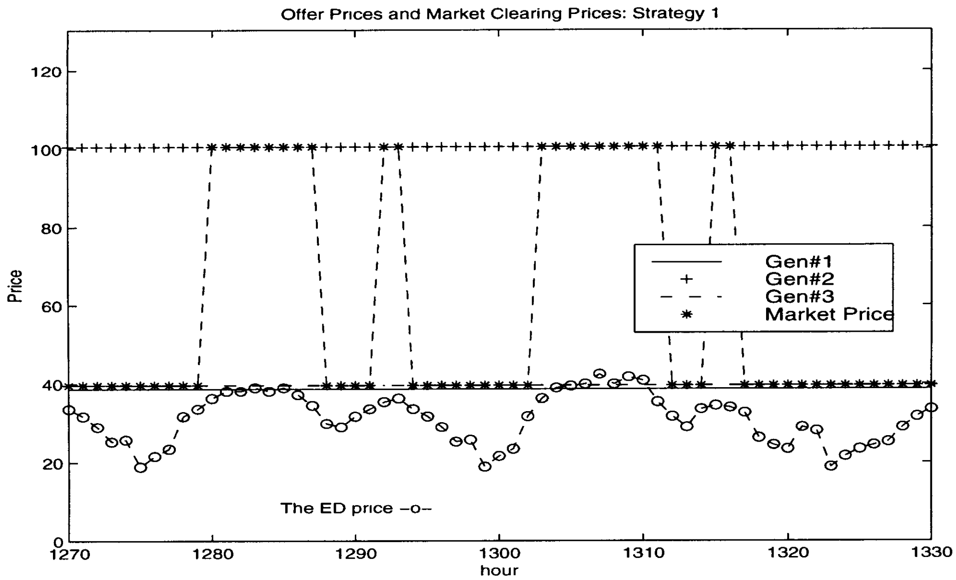

Figure 5-5: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the first strategy): Maximum Available Capacities = [25;25;40] and Initial Conditions = [15;15;25].

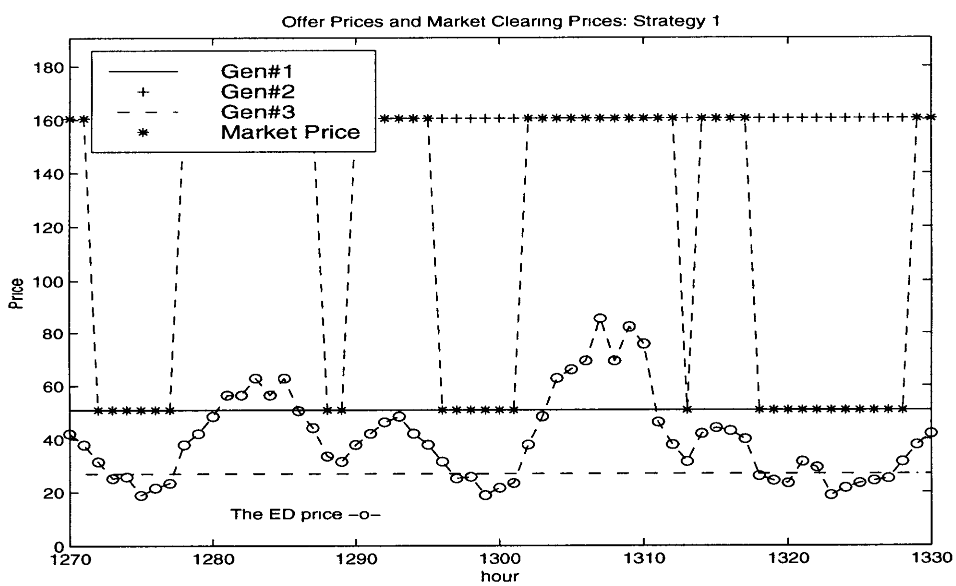

Figure 5-6: Generators's Offer Prices and Resulting Market Clearing Prices (Applying the first strategy): Maximum Available Capacities = [25;40;25] and Initial Conditions = [15;15;25].

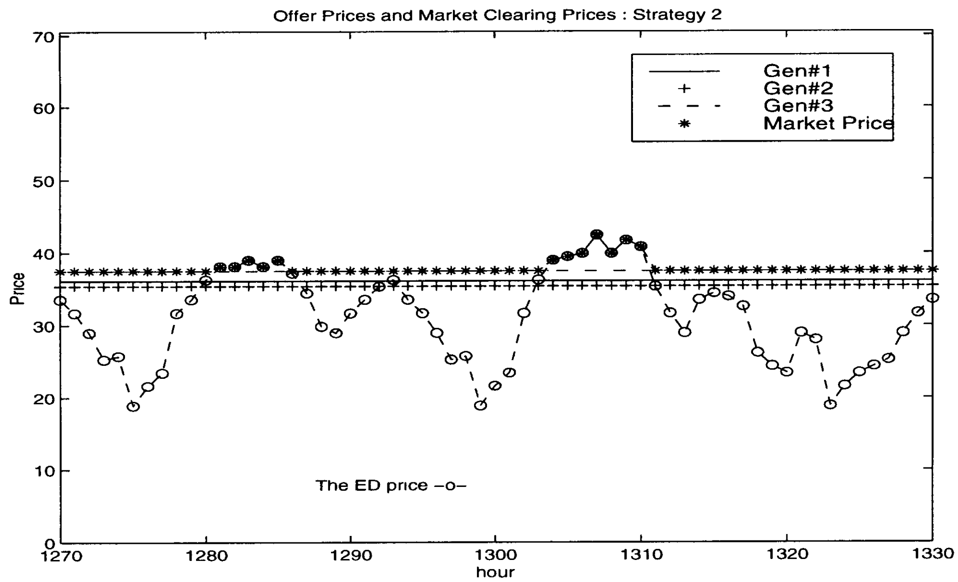

Figure 5-7: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the second strategy): Maximum Available Capacities = [25;25;40] and Initial Conditions = [15;15;25].

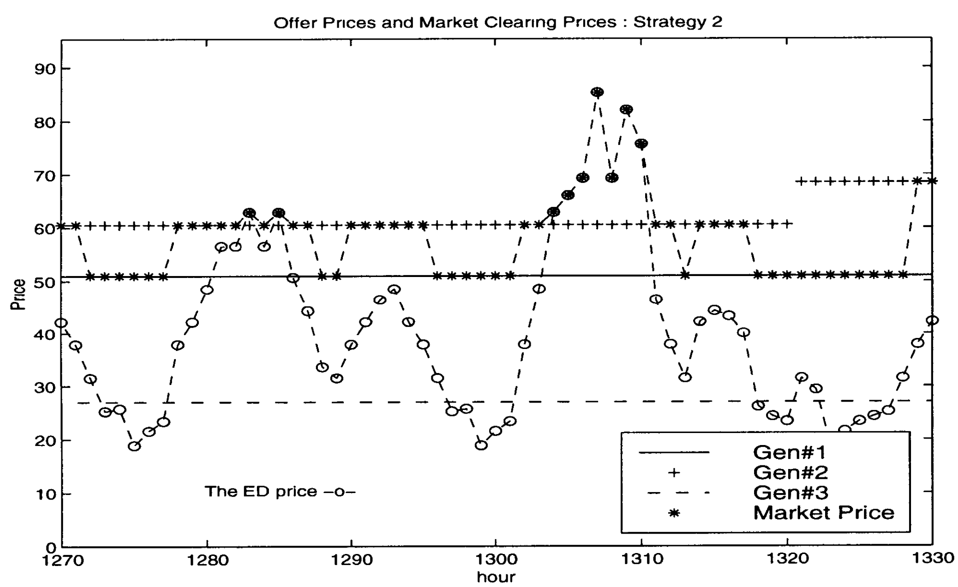

Figure 5-8: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the second strategy): Maximum Available Capacities = [25;40;25] and Initial Conditions = [15;15;30].

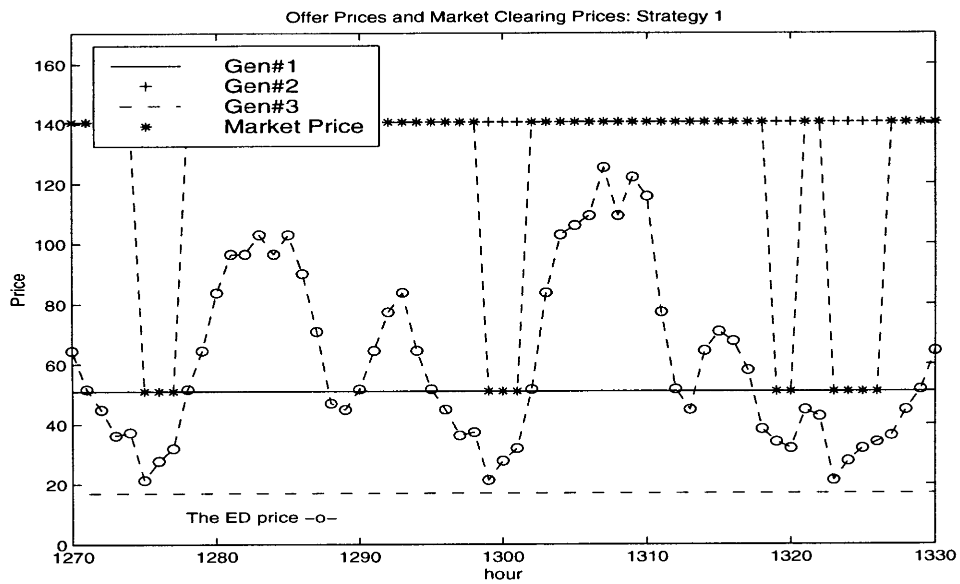

Figure 5-9: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the first strategy): Maximum Available Capacities = [25;35;15] and Initial Conditions = [15;10;10].

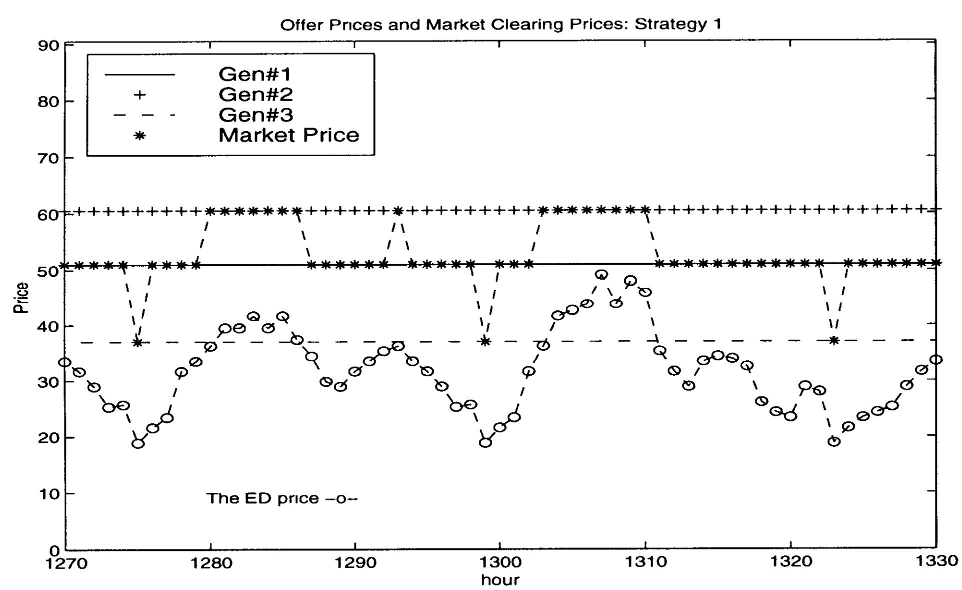

Figure 5-10: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the first strategy): Maximum Available Capacities = [25;15;35] and Initial Conditions = [15;10;10].

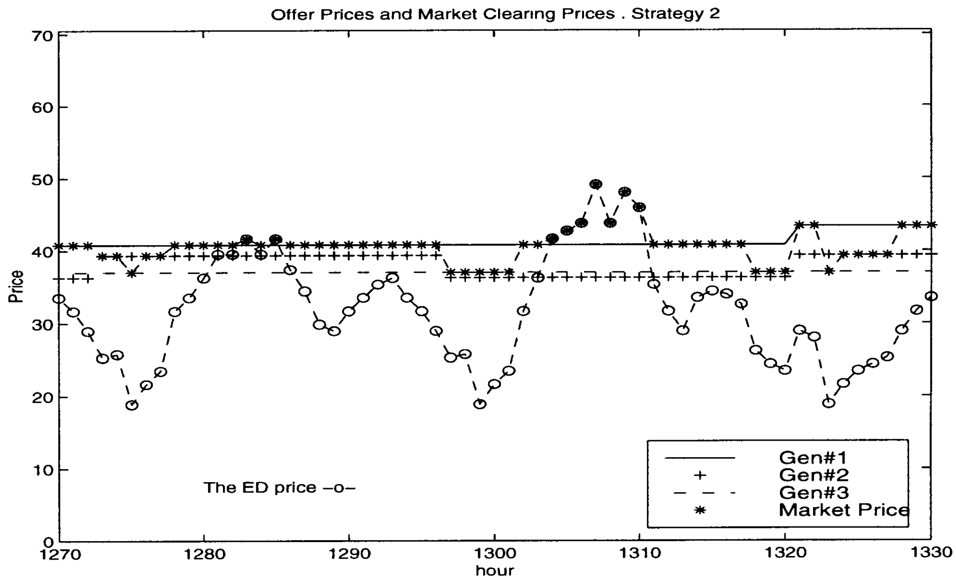

Figure 5-11: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the second strategy): Maximum Available Capacities = [25;15;35] and Initial Conditions = [20;15;15].

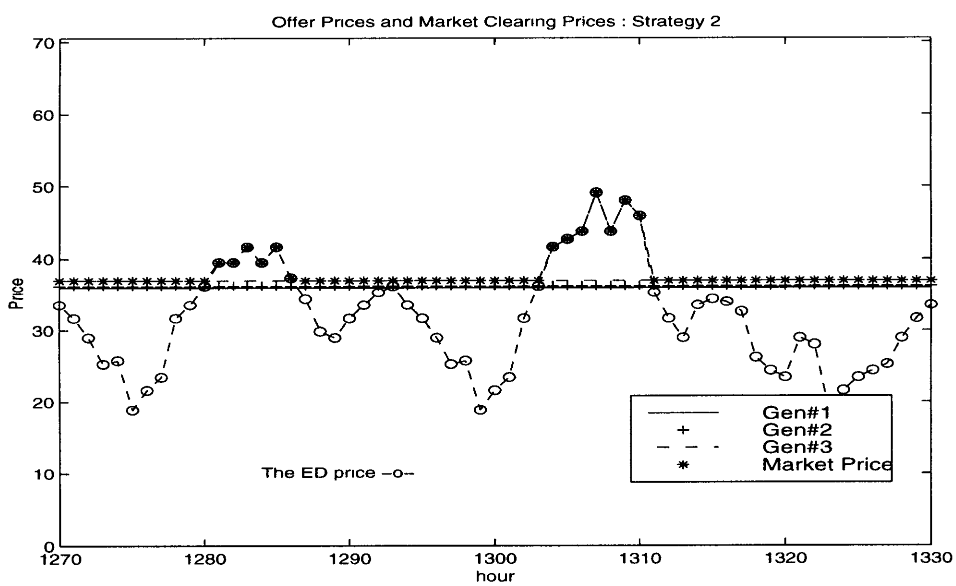

Figure 5-12: Generators' Offer Prices and Resulting Market Clearing Prices (Applying the second strategy): Maximum Available Capacities = [25;15;35] and Initial Conditions = [15;15;30].

Chapter 6

Conclusion

We suggest in this thesis that power markets should be divided into two subprocesses or submarkets: short-run and long-run subprocesses. The long-run subprocess relates to the quantity competition or the maximum committed capacity of each generator. The long-run market will be analyzed by using the Cournot analysis. However, this subprocess is not included in this thesis, and therefore, the maximum available capacity is assumed to be given.

The other subprocess is the competition of the generators in term of price in the short-run horizon. The proposed model in this thesis concerns with this subprocess. The short-run power market under competition, in which the trading of bulk power occurs every day in the form of bidding, was modeled by applying dynamic game theory because of the belief that a static model, such as the standard oligopoly model or a sealed-bid mutiple-unit auction model representing the pool, is not significantly efficient to capture the dynamic behavior of the market resulting from the daily bidding process. This dynamic game, played by the generators, is the daily bidding game which is considered a repeated game of complete but imperfect information. Also, this game is analogous to a discrete-dynamic system in which the offer price of each generator is a state-variable.

Because the short-run power market is considered a dynamic game, the generators can learn the market day by day. They evaluate their next offer bids by following the previously specified strategies which are the Estimated Profit Maximization and

Competition to be a Base-load Generator strategies, and using common knowledge which are the previous-day market clearing prices, the anticipated demand and their marginal-cost functions. The dynamic behavior of the generators responding to the market depends on the strategy, the maximum available capacities of the generators and the initial conditions assigned to the model.

Two strategies are proposed in this model. One might argue that many assumptions limit the credibility of the outcome. A uniform strategy imposed on all generators does not sound reasonable in the real world, because the rational generators should have their own strategies to construct their bids. However, this model is intended to demonstrate that the repeated process actually plays a significant role in the market. Some phenomena, such as price undercutting among the generators that creates supply deficiencies and increases in prices above the real supply level, cannot be captured in the static model.

The generators are obliged to propose their single step-supply function bids such that the price relates to the power quantity by the marginal-cost function. However the market price is always higher than the competitive level. In addition, the dynamic model is formulated in the short-run time-scale, and therefore, the threats of entry are not valid. There are a limited number of generators in the model (three generators in this model) and the generators have limited generation capacity. Hence, these allow the generators to vary (especially increase) their offer prices as much as they can so that their profits are always maximized.

In addition, the generators indirectly create the supply deficiency mainly resulting when the Competition to be a Base-load Generator strategy is applied to the model. It sounds reasonable to conclude that the learning process tends to allow the generator to tacitly collude in the market. Consequently, in both strategies, the results are that the generators implicitly manipulate the market by using common knowledge and available information. As a result, there is room for another market which can be thought of as a real-time market, a second-bid auction or an hour-ahead market. Such a market is aimed for that inelastic demand is always fully served.

It is suggested, especially from the Competition to be a Base-load Generator strategy, that if there were enough generators participating in the power pool (i.e., more than three generators in this model), the learning process from the daily price competition would allow the generators to undercut their competitors, and might finally drive the market price close to the competitive level without creating demand deficiency.

Besides, the previously-committed maximum available capacity considerably impacts the market clearing price in this model. This implies that the scale of operation determined in the long-run horizon plays a significant role in the short-run market. The different maximum available capacities determine the different limits of the generator's supply curve and its incremental price, which eventually brings about the different sets of market clearing prices. The inefficient dispatch is also one consequence of varying the generator's maximum capacity.

Moreover, the market equilibrium of this dynamic model, which represents the repetition of the price competition in the short-run power market, is marginally stable. Therefore the initial condition used to initiate the model affects the resulting market clearing price. This implies that, within one long-run horizon, the first offer bid, when the market starts, plays an important role in the short-run market. Different initial conditions can yield different market clearing prices.

In summary, the results from this dynamic model agree with the results from the static-setting models in that inefficient pricing turns out to be robust to the alternative form of modeling because, from this dynamic model, although the generators are obliged to offer their bids based on their marginal costs, market clearing prices are higher than the competitive prices . Besides, the results from the dynamic model further confirms that the generator with high marginal cost may bid at the lower offer price than the generators with low marginal costs, and thus be dispatched before these more efficient units [15]. In addition, from the results of the dynamic model, depending on the strategy that the generators use to determine their bids, the considerably high market clearing prices can occur, although, there is no supply deficiency. The results from the dynamic model, therefore, strongly confirm that the generators can learn from the market and tacilty collude to create the high market prices, consequently, they are very likely to confirm that the Poolco market, where each generator is obliged to offer a single-step supply function, is imperfect and the market power that is defined as price above marginal cost [7] does exists.

Chapter 7

Further Research

In this thesis, only the short-run power market is considered. In fact, this simplified model includes the significant characteristics of the market such as generators with asymmetric operating-cost functions, time-varying demand, step-supply function bids. Moreover, the market each day is modeled as a sealed-bid multi-unit auction that chracterizes market activities in a power pool more correctly than the standard oligopoly model. However, in this model, other assumptions are also specific. For instance, anticipated demand is inelastic and deterministic. The offer price and quantity are related to the marginal-cost function. Only the single step-supply function bid is allowed and the generators have the uniform strategy. These factors will impact the results. The outcomes will be different when these assumptions are varied.

To model the market in terms of its dynamics more precisely, the dynamic programming should be applied to the model so that real-time information, such as the response to the price variation of the load, can be taken into consideration. The demand should not be inelastic. Like the generators, the power consumers should be able to learn from the market and respond to the fluctuations in market prices. Moreover, in the real market, there exists uncertainty in demand. The stochastic process representing the demand side should be introduced into the model.

In addition, the generator should not be restricted to the symmetric strategy because each generator should have different criteria to offer its bids. Therefore, the non-uniform strategies should be applied to the model. The generators might bid strategically or they might cooperate with each other as well. These possible conditions, which will affect the market prices, should be considered.

Generally, electricity demand changes with time either in a short period, such as an hour, or in a longer period, such as a month. On the long-time horizon, the demand characteristic varies considerably. Demand is higher in the summer than in the winter; for example many air-conditioners are turned on simultaneously on the hot summer days. In different long-run periods (i.e., summer or winter), demand should be represented by different characteristics or different demand functions. This long-run market, as proposed in this thesis, is the market for quantities or capacities. Maximum available capacities will not be given as in the short-run model because they will be derived from the long-run market model. Such a model for the long-run market should show how the generators respond to the time-varying demand. Hence, the outcome from the long-run model (i.e., the generators' capacities) will be used to determine the supply function in the short-run model.

Further research, consequently, is needed to formulate both the short-run market and the long-run market more accurately. The best model will include significant details of the real market, such as demand variation over time (during both short-run and long-run periods), price-elasticity of demand and generators' asymmetric strategies for determining their bids. The generators (customers) should be able to bid various supply (demand) functions, such as multi-step-supply (demand) and the linear-supply (demand) functions. Moreover, more strategies for determining bids of each generator should be introduced. In addition, dynamic programming seems to be the better method for modeling the dynamics of the bidding processes, and for capturing the customers' behavior, such as their responses to the market prices, especially, in real time.

Therefore the dynamic model for the short-run market and the long-run market will be very useful as an analytical tool to predict market behavior in the short term, such as in a 3-month period. Future electricity prices, either in daily or monthly periods, might be forecasted in advance by using available information, such as the available generators and the anticipated-load characteristics. The model will simulate the possible sscenarios; for instance, how some low-cost generators which are not available to operate due to maintenance, will impact the electricity price at that time. Moreover, how gaming by some generators in the market, such as declaring their supply shortage, will affect the other generators' pricing. In addition, if customers response to variation in market prices in real-time, whether generators will tacitly collude to create demand deficiency as occuring in the short-run model in this thesis.

Appendix A

Competitive Equilibrium and Economic Dispatch

This appendix provides an overview of a "competitive" market focusing especially on a "competitive equilibrium" and the Economic Dispatch (ED). In fact, deregulation in a power market is aimed to create perfect competition, resulting in competitive prices for the generators and customers. Therefore, in this thesis, competitive equilibrium is calculated as a reference. The solution (i.e., market clearing prices) of competitive equilibrium is compared to the solution of the equilibrium from the bidding game.

In the power market, the ED is seen as an optimal and efficient solution to the dispatching problem. Generally, there exists constraints due to the limited capacity of the generators. Besides, each generator has the different operating cost. The optimal solution to the dispatch problem is to maximize the total welfare of the system (the generators and customers). In essence, the generator is seen as a price taker, maximizing its profits or minimizing its costs. Therefore, the ED solution is similar to the competitive outcome. This will be analysed below.

A.1 Competitive Equilibrium

Varian (1992) decribes the competitive market in the following:

> A competitive firm is one that takes the market price of output as being given and outside of its control. In the competitive market each firm takes the price as being independent of its own actions, although it is the actions of all firms taken together that determine the market price.

Hence, in the perfectly competitive power market, the market participants, which are the generators, will take the market price of the electric power as a given. In the this dynamic model, the market price will be determined on an hourly basis. Within each hour, the market price is uniform. The price, basically, can be set at any level. However, if the generator sets its price higher than others, no one will purchase its power.

An **equilibrium price** is one where the demand equals supply. To find the equilibruim price, let $x_i(p)$ be the demand function of individual i for $i = 1, \dots, m$, and $y_i(p)$ be the supply function of Firm i th in the industry with m firms. Hence, an equilibrium price is simply a solution to the equation:

$$\sum_{i=1}^{n} x_i(p) = \sum_{j=1}^{m} y_j(p)$$

competitive equilibrium is, therefore, the price at which supply equals demand. In addition, the competitive equilibrium level of output will maximize the total surplus (or the total welfare).

A.2 Economic Dispatch

Economic Dispatch (ED) is a method used to calculate operating schedules for the generators participating in the regulated power market, such that total operating costs in the system are minimized, while (anticipated) demand is satisfied. All the constraints that will affect the dispatch are included, thus the solution from the ED will reflect an impact of the system constraints on the electricity prices. The solution to the ED problem is optimal because the social welfare or the total surplus, consisting of generators' surplus and the load's surplus, is maximized. Let us define

$P_{mark}[K]$	the market clearing price at the kth period of the Kth day.
P	the market price.
$Q_{i,k}[K]$	the power generated by Generator No. i at the kth period of the Kth day.
$C_i(Q)$	the cost of producing Q units of power by Generator No. i.
$MC_i(Q)$	marginal-cost function of Generator No. i.
$X_{i,max}$	the maximum available capacity of Generator No. i, committed to the ISO.
$P_{i,max}$	the maximum offer price of Generator No. i.
$Pl_k[K]$	anticipated demand at the kth period of the Kth day.

In the regulated market, the standard method to find an optimal equilibrium is to maximize the net social benefit. Traditional measures of net social benefits (welfare) employed in evaluating public utility policies[1] is that:

$$\mathcal{W} = \mathcal{TR} + \mathcal{S} - \mathcal{TC} \tag{A.1}$$

where $\mathcal{W}$ = net social benefit (welfare), $\mathcal{TR}$ = total revenue, $\mathcal{S}$ = consumer surplus and $\mathcal{TC}$ = total cost. The generators's surplus ($\mathcal{GS}$) is defined as

$$\mathcal{GS} = \mathcal{TR} - \mathcal{TC}$$

The generator's surplus is the difference between total revenue and total cost. While the load's surplus is defined as

$$\mathcal{S} = \mathcal{U} - \mathcal{TR}$$

where $\mathcal{U}$ is the load's utility function. $\mathcal{TR}$ is interpreted as the total payment by the load. The load's surplus is the difference between the load's utility and its expense. Therefore, from Equation (A.1), the net social benefit is the sum of the generators' surplus and the load's surplus. In the case of a single product such as electric power,

[1]Electric power in the regulated world is one of public utilities.

Equation (A.1) is reduced to:

$$\mathcal{W} = \int_0^Q P(q)dq - \int_0^Q MC(q)dq \tag{A.2}$$

Where $P(q)$ and $MC(q)$ refer to (inverse) demand and supply curves respectively. The integration of $P(q)$ over q is the area under the demand curve, while the integration of $MC(q)$ over q is the area under the supply curve; hence, the difference between the two areas is simply the total social benefit ($\mathcal{W}$). The optimal market equilibrium is determined by:

$$\max \mathcal{W} = \max \int_0^Q P(q)dq - \int_0^Q MC(q)dq \tag{A.3}$$

$$\frac{\partial \mathcal{W}}{\partial Q} = P(Q) - MC(Q) = 0 \tag{A.4}$$

In the three-generator model, the ED problem is formulated as one where the generators have asymmetrical operating cost functions. Their operating cost functions are:

$$\begin{array}{lrcl} \text{Generator No. 1} & C_1(Q) & = & Q^2 + Q \\ \text{Generator No. 2} & C_2(Q) & = & 2Q^2 + 0.5Q \\ \text{Generator No. 3} & C_3(Q) & = & 0.5Q^2 + 2Q \end{array}$$

The supply curves of the generators are determined from the assumptions that, in a perfectly competitive market, the generators participating in the power pool behave like price takers. Assume that the market price is given. With no constraints, the generators' profit will be maximized when the marginal-operating cost is set equal to the market price in any period. This is also considered the rule of efficient pricing. Hence

$$P = MC(Q) = \frac{\partial C(Q)}{\partial Q} \tag{A.5}$$

This is also interpreted as short-run marginal costs. The generators' short-run supply functions are :

$$
\begin{aligned}
&\text{Generator No.1} \qquad & MC_1(Q) &= 2Q + 1 \\
&\text{Generator No.2} \qquad & MC_2(Q) &= 4Q + 0.5 \\
&\text{Generator No.3} \qquad & MC_3(Q) &= Q + 2
\end{aligned}
$$

Only one utility function represents the (aggregated) inelastic anticipated load.

To modify the standard process to fit the dynamic model, system constraints are added to Equation (A.3). The first constraint is that demand meets supply. It has been assumed perviously that there were no constraints according to transmission congestion; hence, only the capacity constraints from the generators are added to the first constraint. The generator commits to the ISO, not operating more than the capacity limit[2] within one long-run period; hence, the committed maximum capacity is the generator's capacity constraint in the short-run period. Thus, Equation (A.3) is reformulated for finding the optimal operation in each period (either k or K or both) as:

$$\max \mathcal{W} = \max \quad P_{mark}[K] \times Pl_k[K] - \sum_{i=1}^{3} C_i(Q_{i,k}[K]) \tag{A.6}$$

$$
\begin{aligned}
\text{subject to} \quad & \sum_{i=1}^{3} Q_{i,k}[K] = Pl_k[K], \quad \forall \ \{k, K\} \\
& Q_{i,k}[K] \leq X_{i,max}, \quad \forall \ \{i, k, K\}
\end{aligned}
$$

The next step is to form the Lagrangian L:

$$
\begin{aligned}
L = P_{mark}[K] \times Pl_k[K] - \sum_{i=1}^{3} C_i(Q_i) + \lambda_k[K]\Big(\sum_{i=1}^{3} Q_{i,k}[K] - Pl_k[K]\Big) \\
+ \sum_{i=1}^{3} \mu_{i,k}[K](X_{i,max} - Q_{i,k}[K])
\end{aligned} \tag{A.7}
$$

[2]This is the Cournot quantity evaluated in the long-run market. However, the short-run model assumes that this quantity is a given.

The solution to the maximizing problem in Equation (A.6) is also the solution to Equation (A.7). Both k and K are dropped in the following equations because the same method is used to find the ED solution in any period.

$$\frac{\partial L}{\partial Q_i} = 0 \rightarrow \lambda = MC_i(Q_i) + \mu_i, \quad \forall \ \{i, k, K\} \tag{A.8}$$

$$\frac{\partial L}{\partial Pl} = 0 \rightarrow P_{mar} = \lambda \tag{A.9}$$

$$\frac{\partial L}{\partial \lambda} = 0 \rightarrow \sum_{i=1}^{3} Q_i = Pl, \quad \forall \ \{i, k, K\} \tag{A.10}$$

$$\frac{\partial L}{\partial \mu_i} = 0 \rightarrow X_{i,max} - Q_i = 0, \quad \forall \ \{i, k, K\} \tag{A.11}$$

Where Equation (A.11) is equivalent to

$$\mu_i(X_{i,max} - Q_i) = 0, \quad \forall \ \{i, k, K\} \tag{A.12}$$

Hence, when "Pl" is known, there are seven unknown variables (i.e., Q_i, λ and μ_i, $\forall \ \{i\}$) and there are seven linearly independent equations[3] (i.e., 3-equation (Equation (A.8)), 1-equation (Equation (A.10)) and 3-equation (Equation (A.12))), therefore, there is a unique solution to the problem stated in Equation (A.6). The solution to Equation (A.9) indicates that

$$\text{The ED price} \quad = P_{mar} = \lambda = MC_i(Q_i) + \mu_i, \quad \forall \ \{i, k, K\} \tag{A.13}$$

and

$$\text{Generator No. } i \text{ will be scheduled as the ED quantity equal to } Q_i \tag{A.14}$$

[3]The marginal-cost function of each generator is an affine function of Q.

The solution to the problem in Equation (A.6) is analyzed as the following :

- When demand is low, all the generators are scheduled at less than their full capacities. Hence, $\mu = 0$ for all i. Therefore, the ED price will be equal to

$$P_{mar} = MC_1(Q_1) = MC_2(Q_2) = MC_3(Q_3)$$
$$\text{and } \; Q_i < X_{i,max}, \quad \sum_{i=1}^{3} Q_i = Pl \quad \forall \; \{i, k, K\} \tag{A.15}$$

- When the demand is high, at least one generator is operating at its maximum available capacity. In this case, Generator No. j operating at its maximum limit, its $\mu_j \neq 0$. The ED price will be equal to

$$P_{mar} = MC_i(Q_i) = MC_j(X_{j,max}) + \mu_j \quad \text{and } \; Q_i < X_{i,max}$$
$$\sum_{i=1}^{3} Q_i = Pl, \quad Q_j = X_{j,max}; \quad \forall \; \{i(i \neq j), k, K\} \tag{A.16}$$

  Note that if two generators operate at their maximum available capacities, the market price will be the marginal cost of the last generator operating at less than its available capacity.

Both cases are efficient pricing (resulting in optimal equilibrium) because in both cases, Equation (A.5) still holds. This is only a sufficient condition because efficiency requires only that the last unit of output is consumed by the load be priced at marginal cost. Therefore, intramarginal units purchased by the load need not be priced at marginal cost.

Therefore, in the power market, **competitive equilibrium is the same as the solution to the ED problem**. Consequently, the price calculated from the ED can be used as the reference of optimal and efficient pricing from the (perfectly) competitive market because competiton in the power market is aimed for that power will be purchased at efficient prices. Hence, if the market price calculated from the competitive market model does not converge to the competitive price, it will be concluded that the competition will not tend to achieve the previous goal.

The price resulting from the bidding market is compared to the ED price. If the market price from the bidding process is higher, the market will not be perfectly competitive. The generators have market power because they are able to set the market price above the competitive level. The welfare is no longer maximized. With this comparison, how the bidding process impacts the market can be observed. Moreover, the degree to which each strategy influences the market price, which, deviates from the optimal level, can be observed as well.

Bibliography

[1] Varaiya, Pravin P. (1972). *Note on the Optimization Course, by P. P. Varaiya*, New York: Von Nostrand Reinhold CO.

[2] Jaskow, P. L. (1975). 'Applying Economic Principles to Public Utility Rate Structures: The Case of Electricity', *Studies in Electric Utility Regulation*, edited by Charles J. Cicchetti and John L. Jurewitz, Cambridge MA: Ballinger Publishing Company.

[3] Šiljak, D. Dragoslav. (1978). *Large-Scale Dynamic Syetem: Stability and Structure*, New York: North-Holland.

[4] Crew, M. A. and Kleindorfer, P. R. (1986). *The Economics of Public Utility Regulation*, Cambridge MA: The MIT Press.

[5] Davidson, Carl and Deneckere, Raymond. (1986). 'Long-run Competition in Capacity, Short-run Competition in Price, and the Cournot Model', *Rand Journal of Economics* 17(3) : 404-415.

[6] Friedland, Bernard. (1986). *Control System Design: An Introduction to State-Space Methods*, McGrawHill.

[7] Tirole, Jean. (1988). *The Theory of Industrial Organization*, Cambridge MA: The MIT Press.

[8] Fudenberg, Drew and Tirole, Jean. (1989). 'Noncooperative Game Theory for Industrial Organization : An Introduction and Overview.' *Handbook of Industrial*

Organization Volume I, edited by R. Schmalensee and R. D. Willig, Elsevier Science Publisher B.V. : 259-327.

[9] Klemperer, P. D. and Meyer, M. A. (1989). 'Supply Function Equilibria in Oligopoly under Uncertainty', *Econometrica* 57(6): 1243-1277.

[10] Kreps, David M. (1990). *Game Theory and Economic Modelling.*, Oxford : Clarendon Press.

[11] Fundenberg, Drew and Tirole, Jean. (1991). *Game Theory*, Cambridge, MA : The MIT Press.

[12] Gibbons, Robert. (1992). *Game Theory For Applied Economists*, Princeton, NJ: Princeton University Press.

[13] Green, Richard J. and Newbery, David M. (1992). 'Competition in the British Electricity Spot Market', *Journal of Political Economy* 100(5): 929-953.

[14] Varian, Hal. (1992). *Microeconomics Analysis*, New York: Norton.

[15] von der Fehr, N. M. and Harbord, David. (1993). 'Spot Market Competition in the UK Electricity Industry', *Economic Journal* 103(5): 531-546.

[16] Green, Richard J. (1994). 'Britain's Unregulated Electricity Pool', *From Regulation to Competition: New Frontiers in Electricity Markets*, edited by Michael, A. Einhorn., Boston MA: Kluwer Academic Publisher.

[17] Gibbons, Robert. (1996). 'An Introduction to Applicable Game Theory' *Technical Working Papers:199*, National Bureau of Economic Research, INC.

[18] Rudkevich, A., Duckworth, M. and Rosen, R. (1997). 'Modeling Electricity Pricing in a Deregulated Generation Industry: The Potential for Oligopoly Pricing in a Poolco', Boston, MA: Tellus Institute.

[19] Denton, M. J., Rassenti, S. J. and Smith, V. L. (1998). 'Spot Market Mechanism Design and Competitivity Issues in Electric Power', 31st Annual Hawaii International Conference on System Sciences.

[20] Green, Richard J. (1998).'The Political Economic of The Pool', *Power Systems Restructuring: Its Value in a Changing Industry*, edited by Marija Ilic, Francisco Galiana and Lester Fink, Boston, MA: Kluwer Academic Publisher.