Monopolistic Insider Trading in a Stationary Market

by
Zhihua Qiao
Ph.D. Statistics
the University of Pennsylvania (2006)

Submitted to the Sloan School of Management
in partial fulfillment of the requirements for the degree of
Master of Science
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
February 2009

Author . . . .
Sloan School of Management
December 15, 2008

Certified by . . . .
Leonid Kogan
Nippon Telephone and Telegraph Professor of Management
Thesis Supervisor

Certified by . . . .
Jiang Wang
Mizuho Financial Group Professor of Finance
Thesis Supervisor

Accepted by . . . .
Ezra Zuckerman
Nanyang Technological University Professor
Head of the Ph.D. Program

Monopolistic Insider Trading in a Stationary Market

by

Zhihua Qiao

Submitted to the Sloan School of Management
on December 15, 2008, in partial fulfillment of the
requirements for the degree of
Master of Science

Abstract

This paper examines trading behavior of market participants and how quickly private information is revealed to the public, in a stationary financial market with asymmetric information. We establish reasonable assumptions, under which the market is not efficient in the strong form, in contrast to the Chau and Vayanos (2008) model. First, we assume that the insider bears a quadratic transaction cost. We find that the trading intensity of the insider is inversely related to transaction cost and that the market maker's uncertainty about private signals is positively related to transaction cost. As transaction cost approaches zero, the economy converges to that of the Chau and Vayanos (2008) model. Second, we assume that the insider can observe signals only discretely and at evenly spaced times, at a lower frequency than that at which trading takes place. The sparseness of signals induces insiders to trade patiently before the next signal comes in, as in the finite horizon model of Kyle (1985). Furthermore, the degree of market efficiency declines as signals arrive more sparsely. Finally, we assume that arrival times of private insider signals are random. In such case, the insider is less patient and trades more smoothly than with fixed arrival times. As a result, market prices incorporate private information more quickly.

Thesis Supervisor: Leonid Kogan
Title: Nippon Telephone and Telegraph Professor of Management

Thesis Supervisor: Jiang Wang
Title: Mizuho Financial Group Professor of Finance

Contents

List of Figures

Chapter 1

Introduction

Informed trading under asymmetric information has drawn the attention of numerous studies in the past few decades. Studies [1] have looked at such questions as: How are security prices formed and how quickly is the information about profitability incorporated into the prices? What is the role of market makers in the price discovery process? And will an insider trade slowly to control the cost of price impact or quickly to make a quick killing? These questions, which are part of the market microstructure research, are also related to market efficiency. The efficient market hypothesis formulated by Eugene Fama in 1970, suggests that, at any given time, all available information—public and private—is fully reflected in the stock prices. This type of efficiency is called strong-form efficiency. On the other hand, the semi-strong-form efficiency hypothesis states that only publicly available information, for example past prices, is incorporated into the market prices. While it is plausible to think that strong-form efficiency does not describe reality[2], it is important to understand what the conditions are for the market to closely resemble strong-form efficiency.

The first generation of the informed trading literature, which begins with Kyle (1985),

[1] for example, Kyle (1985), Back (1992), Back, Cao and Willard (2000).

[2] For example, Fama (1991)

examines strategic trading strategies and their price impact with competitive market makers. In the Kyle model of informed trading, a monopolistic insider strategically submits orders to a competitive market maker and some liquidity traders submit exogenous order quantities. The market maker can only observe the batch orders. In equilibrium, the insider patiently submits orders and thus gradually reveals his private information. The private information is fully revealed to the public only at the end of the trading session. Clearly, this model does not reveal strong-form efficiency

In the Kyle model, the insider receives a signal at the beginning of the trading session. This signal represents the final payoff of the risky asset at the end of the trading session. Chau and Vayanos (2008) (henceforth CV) conjectures that this is a critical assumption to induce the insider to trade slowly. CV studies the market efficiency in a stationary framework with infinite horizon. In CV, the financial market is similar to that in the Kyle model. The main difference is that the insider receives private information repeatedly. CV adopts the notion of Wang (1993) that the private information is a mean reverting stochastic process that determines the dividend growth rate of a risky stock. The insider's objective is the present value of expected future profits. It is shown that in a discrete time setting, the monopolistic insider reveals his information very quickly by placing a large order each period; as the market approaches continuous time, the insider's rate of order flow converges to infinity and the market maker's uncertainty about the insider's information converges to zero. This says that the market can be arbitrarily close to strong-form efficiency. However, the insider's profit does not converge to zero as the market approaches efficiency, which means that there is nontrivial return to the cost of information acquisition. This is in contrast with the usual postulation that the positive profits of the insider are inconsistent with the strong form of the efficient market model[3].

It is argued in CV that the strong-form efficiency outcome is due to the combination of impatience and stationarity rather than any peculiarity of their assumptions. More con-

[3]For example, see Rozeff and Zaman (1988)

cretely, these authors show that impatience is introduced if any of three factors is present: time discounting, publicly revealed information, and obsolescence of private information through mean-reversion in the firm's profitability. CV also argues that impatience alone cannot lead to the quick trading of the insider. It is the stationarity of the market, in addition to the insider's impatience that induces quick trading. Therefore, it may appear that strong-form efficiency is a robust result that holds under relatively weak model assumptions. In this paper, we argue that this is not the case. That is, some of the assumptions that are essential for strong form efficiency in CV may still be too strong to approximate reality. Some of the assumptions in the CV model that fall into this category are described as follows. First, the insider is risk neutral, so his signal precision is not taken into account as he optimizes his trading strategy. If the insider is risk averse, he will trade less aggressively and the degree of market efficiency goes down. Second, the market maker is risk neutral and hence does not charge any inventory cost. Given the risk aversion of the market maker, inventory cost may limit the insider's trading and acquisition of information by the market maker. Third, the insider is not subject to any transaction cost. It is intuitive that very high transaction cost will potentially prohibit the insider from trading quickly. Fourth, the insider is perfectly informed in the sense that he receives continuous private information with no time lag. This can be relaxed in several ways. For example, the insider can receive the signals at a lower frequency than the trading frequency. Another alternative is that the insider receives the signals at stochastic times. This dissertation examines the effect of these latter two assumptions.

First, we introduce a transaction cost in the quadratic form faced by the insider, that intuitively prevents the insider from trading quickly. Ordinarily, traders face three types of transaction cost: order processing cost, inventory cost and adverse selection cost. Here, transaction cost can be regarded as an order professing cost. Note that, strictly speaking, there should be no inventory cost if the market maker is assumed to be risk neutral. Adverse selection cost also exists in the economy, due to asymmetric information. We assume the transaction cost is quadratic with respect to the rate of the insider's trading, to gain tractability for the model. Also note that the transaction cost is on the total position of the insider, rather than a fixed cost per transaction. Nonetheless, this type of transaction cost has a strong deterrent effect on the insider. In the presence of the transaction cost, the insider trades slowly and the market is no longer strong-form efficient.

Next, we ask what is different if the insider receives private information at a lower frequency than the one at which the trading takes place. In CV, the assumption that the insider receives signals repeatedly is interpreted as an approximation of a proprietary-trading desk, which generates a flow of private information on a stock through superior research. There is no reason to believe that the agent always produces signals at a particular high frequency. It is also assumed in the CV paper, however, that the insider receives the signals at the same frequency as the one at which the trading takes place. Hence it is natural to relax the latter assumption with the assumption that the frequency of the private signal is lower relative to trading frequency. For maximal tractability, we study a continuous time model, i.e., the frequency of trading is infinitely high. Private signals are assumed to arrive at fixed and evenly spaced times. The infinite horizon market is still stationary, although the equilibrium has certain dynamics within each period between two consecutive signals. To focus on the effect of the frequency of the private signal, we let the transaction cost be very small. We show that during the interval between two consecutive signals, as time goes by the trading intensity of the insider increases, the price impact declines, and the insider's informational advantage declines as well. More importantly, we show that, as the frequency of the private signals of the insider decreases, the proportion of the private information that is incorporated into the market price, which can be regarded as a measure of market efficiency, also declines. In other words, the degree of market efficiency is lower than that in the CV model, which shows approximate strong-form efficiency.

Finally, we relax the assumption that the insider receives private information at deterministic and evenly spaced times. We study the alternative case, in which the arrival time of the next signal is random. With a prop-trading desk example, this says that the research department cannot guarantee that the new signal will be produced at a pre-specified time. Instead, once a signal is produced, the next signal can be produced at any subsequent time according to a probability distribution. To compare the two cases, we assume that the mean arrival time in the stochastic arrival case is roughly the same as the arrival time in the fixed arrival time case With the arrival time following a truncated exponential distribution, we show that the insider trades more smoothly. That is. while in the fixed arrival time case, the insider's trading intensity increases and shoots up immediately before the next signal arrives; in the stochastic arrival time case, he trades more aggressively right after receiving a new private signal, and less aggressively as time passes by, relative to the case in which the arrival time of the signals is fixed. This is intuitive because in the fixed arrival time case, the insider is very patient and waits until the moment immediately before the next signal to use up his private information on the last signal. By contrast, in the stochastic arrival time case, if he is too patient, his signal is likely to be wasted since the next signal can arrive at any time, thus making his last signal obsolete. On the other hand, the trading intensity of the insider increases less quickly in the later part of the trading period because he is not sure when exactly the next signal will arrive.

To make a stark comparison between the CV model and our model with transaction cost, we study a discrete time model in which the private signals of the insider arrive at the same times that the trading takes place. This model converges to the CV model when the transaction cost vanishes, and converges to a continuous time model with continuous trading and a continuous private signal process. The latter model can also be obtained as the limit in our primary model, as the frequency of the insider signals increases without bound. For this latter model with a continuous signal process, we obtain the comparative statics of the variables of interest to highlight the effects of trading costs, for example, liquidity and market maker's uncertainty, as the parameter corresponding to transaction cost changes. When transaction cost decreases, the insider trades more aggressively, the price impact increases, and the market maker's uncertainty about profitability decreases. In particular, when transaction cost diminishes, the equilibrium solution approaches strong-form efficiency.

The other assumptions we discussed earlier but do not study in this paper are the risk aversion of the insider and the market maker. Wang (1993) studies a rational expectation equilibrium model in a stationary infinite horizon economy, in which both the market maker and the insider are risk averse. As a result, the market maker charges traders for the inventory risks that the market maker bears. In that model, the insiders are assumed to be competitive, and the market maker and the insider submit demand schedules and price is determined by the market clearing condition. This equilibrium shows very different characteristics from those in CV(2008). In particular, no traders trade aggressively and there is no strong-form market efficiency. The insiders trade quickly on their information if they are risk neutral. Under the risk aversion assumption, however, they trade slowly and the private information is revealed to the market gradually.

Chau (1999) also studies dynamic trading and market making in a similar framework. The author's dynamic model involves the same market participants: a financial market with a strategic large trader, a market maker, and noise traders. However, a major difference is that in Chau (1999) the large trader and the market maker both face inventory costs, which constitute the major source of risk investigated.

This thesis is organized as follows. In the next chapter, we study the general environment of continuous time trading and discrete private signals. Our model is very comprehensive and nests many models as special cases. For example, if we take the limit when the time interval between private signals goes to zero, then the private signals occur continuously. This special case is discussed in Chapter 3. To make a clear comparison between our model and the Chau and Vayanos (2008) model, we study a discrete time model in Chapter 4, where trading times coincide with the times at which signals arrive. Chapter 5 extends the first chapter by introducing the more general assumption that the arrival times of private signals are stochastic.

Chapter 2

A General Framework with Discrete Private Signals

2.1 Introduction

It is argued in CV that the insider is impatient in a stationary financial market for three reasons: time discounting, public revelation of insider information through dividends and the obsolescence of insider information through mean reversion in the firm's profitability. Nonetheless, it is still quite puzzling to see such a prominent model prediction as the insider's quick trading. This dissertation seeks to contribute a deeper understanding of why insider trading volume is so large. In this section, I examine the continuous time model, in which the insider can receive only discrete signals. One simple conjecture is that, since the insider has less informational advantage, he will trade less aggressively. While this is true, I provide more detail on trading behavior between any two consecutive private signals. More importantly, my results indicate that risk neutrality is probably a more fundamental reason for the aggressive trading of the insider.

2.2 Model Setup

The model setup is similar to Chau and Voyanos (2008). There are two assets. The first is a riskless bond with exogenous constant return r. The second is a stock that pays a dividend at rate D_t, where D_t is a diffusion process with the following dynamics

$$dD_t = \nu\left(g_t - D_t\right)dt + \sigma_D dB^D. \tag{2.1}$$

It is mean reverting and its time varying mean g_t is itself an Ornstein-Ulenbeck (OU) process,

$$dg_t = \kappa\left(\bar{g} - g_t\right)dt + \sigma_g dB^g \tag{2.2}$$

For simplicity, I assume that the two Brownian motions B^D and B^g are independent. The dividend process and the dividend growth process, (2.1) and (2.2), are adopted from Wang (1993); these have become standard in modeling asymmetric information in an infinite horizon. As explained in Wang (1993), $\nu\left(g_t - D_t\right)$ is the expected growth rate of dividends. The state variable g_t can be interpreted as the true underlying profitability of the risky asset. I require that $\nu > 0$ so that the dividend process indeed depends on the underlying state variable g_t. When $\kappa = 0$, the process g_t is simply a Brownian motion. Since our aim is to investigate the implication of insider trading in a stationary financial market, I will focus on the case in which $\kappa > 0$.

The assumptions on the market participants are standard and follow Kyle (1985). There are three types of traders: a market maker, an insider and noise traders. The market maker is competitive and is risk neutral. The insider is risk neutral and behaves strategically. The market maker and the insider have the utility function

$$E\left[\int_t^\infty c_s^m e^{-r(s-t)} | \mathcal{F}_t^m\right]$$

where c_s^m denotes consumption at time s, and $\mathcal{F}_t^m$ denotes the information set at time t

of the market maker. It is helpful to notice that in the above utility function, since the discounting is the same as the riskless interest rate, agents are indifferent about the timing of consumption, and therefore value the consumption stream using the present value of expected cash flow. discounted at the riskless interest rate. Noise traders are exogenous and submit order flows as follows

$$dz_t = \sigma_u dB_t^u.$$

Let dx_t denote instantaneous order flow of the insider trader. The market maker observes only the aggregate order flow $dx_t + dz_t$.

For simplicity I assume that now the insider can observe signals only at evenly spaced time intervals, but that trading still takes place continuously. Denote by Δ the length of the time interval between signals of g_t. Clearly, one of the complications of the model is that the insider must estimate the true underlying short-run mean process g. We denote his filtering solution as $g^\imath$.

From the insider's perspective, the true value of the stock at time t is equal to the present value of expected dividends conditional on his information. This value is similar to the price set by the market maker, except that the expectation is conditional on the insider's information set.

$$v_t = E\left[\int_t^\infty e^{-r(s-t)} D_s ds | \mathcal{F}_t^\imath\right].$$

With the same calculation as for the market maker, the present value can be simplified as

$$P_t = A_0 D_t + A_1 g_t^\imath + A_2 \bar{g}.$$

The insider is also risk neutral, and has the object function

$$\max_{\theta_t} E\left[\int_{t_0}^\infty e^{-rt}\left[\left(v_t - p_t\right)\theta_t - c\theta_t^2\right] dt | \mathcal{F}_{t_0}^\imath\right]$$

where θ_t is the trading strategy. In the bracket of the integrand is the instantaneous net profit at future time t. The first part of the net profit is the payoff from trading a quantity θ_t at time t; the second part is a quadratic transaction cost[1]. The introduction of the transaction cost is the main difference between the current paper and CV (2008). This is motivated by the following economic intuition. The market price in the Kyle model is the batch order price; therefore, there is no bid-ask spread. The market price reflects only the market maker's inference based on batch orders, which includes the insider's trade. Trades move prices because the insider is better informed than the market at large. However, this kind of theoretical market price ignores two components in the actual transaction prices in the market, order processing costs and inventory costs. Since we assume that the market maker is risk neutral, then, strictly speaking under this assumption, there is no inventory cost. In this paper, transaction cost can be considered the order processing cost, which is assumed to be exogenous and has a quadratic form. On the other hand, if the market maker is not risk neutral, there is a cost that the market maker charges to compensate for bearing the inventory risk. In Wang (1993), the market maker is risk averse, so inventory costs are built in endogenously.

Since the market maker is competitive and thus makes zero profits, he sets price as the conditional expectation of the present value of expected future dividends of the stock

$$P_t = E\left[\int_t^{\infty} e^{-r(s-t)} D_s ds | \mathcal{F}_t^m\right]. \tag{2.3}$$

His information set $\mathcal{F}_t^m$ involves two stochastic processes: the dividend process D_t and the insider's trading strategy x_t. The conditional expectation can be brought into the integrand. By substituting in D and g we can show that the price is

$$P_t = A_0 D_t + A_1 E\left(g_t | F_t^m\right) + A_2 \bar{g}$$

where A_0, A_1 and A_2 are three positive constants depending on the parameters. The details

[1] Perhaps the cost should be $c\,|\theta| + \bar{c} \cdot 1_{[|\theta|>0]}$ This may be very difficult to solve though

are given in Appendix A. The resulting price is intuitive because the processes D and g are jointly Markov. The interpretation is that the price positively depends on the current level of dividends, and the levels of the short run mean g and the long run mean $\bar{g}$, in a simple linear way.

2.3 Equilibrium

Recall that the price set by the market maker is

$$P_t = A_0 D_t + A_1 g_t^i + A_2 \bar{g} \tag{2.4}$$

where g^m is the market maker's conditional expectation of the state variable g_t given his information set at time t. Similarly, the insider's valuation of the stock is given by the above formula, except that the mean process g_t is known by the insider.

$$v_t = A_0 D_t + A_1 g_t^i + A_2 \bar{g}.$$

As a result, the insider's objective can be expressed as

$$\max_{\theta_t} E\left[\int_{t_0}^{\infty} e^{-rt}\left[\left(g^i - g^m\right)\theta_t - A_1^{-1} c\theta_t^2\right] dt|\mathcal{F}_{t_0}^i\right]. \tag{2.5}$$

Below, we will let $\tilde{c} = A_1^{-1} c$ and abuse the notation to let c denote $\tilde{c}$. We consider linear equilibrium in which the agents' strategies are linear functions of the state variables. In particular, we assume that the insider has linear strategy of the following form

$$dx_t = \beta_t \left(g^i - g^m\right) dt. \tag{2.6}$$

This can be a candidate strategy only if g^m is observable by the insider. From the pricing equation (2.4), we see that the insider can infer a market maker's expectation g_t^m from observing the price. We also assume that the market maker sets the price according to equation (2.4) with the conditional expectation having the following dynamics

$$dg_t^m = \kappa \left(\bar{g} - g^m \right) dt + \lambda \left(t \right) \left[dx_t + dz_t \right] + \gamma \left(t \right) \left[\nu \left(D - g^m \right) dt + dD \right]. \tag{2.7}$$

The intuition of the above strategy by the market maker is as follows. The first term reflects mean reversion of the true underlying process g. The market maker further updates his belief on g with two pieces of incoming information, aggregate order flow, the second term, and dividend payout, the third term.

Definition 1 *A pair of linear strategies (x_t, p_t) satisfying (2.4), (2.7) and (2.6) is a Nash equilibrium if the following two conditions hold:*
1. Given the market maker's pricing rule (2.4) and (2.7), and insider chooses the optimal strategy $dx_t = \theta_t dt$ to maximize his expected future profit (2.5). Given the insider's strategy (2.6), the market maker sets prices equal to the conditional expectation of the present discounted value of the stock with equation (2.3).

To solve the Nash equilibrium, we need to solve the market maker's inference problem and the insider's optimization problem and then match their strategies.

2.4 Insider's Inference

At the beginning of each interval $[t, t + \Delta]$, the insider needs to solve the filtering problem. He observes g_t, and $\{D_s, t \leq s \leq t + \Delta\}$, and needs to find $E\left[g_s | \mathcal{F}_s\right]$.

Proposition 2 *The filtering has solution*

$$dg^{\imath}_s = \kappa\left(\bar{g} - g^{\imath}_s\right)dt + \frac{\Sigma\nu}{\sigma^2_D}\left[\nu\left(g_s - g^{\imath}_s\right)dt + \sigma_D dB^D\right]$$
$$= \kappa\left(\bar{g} - g^{\imath}_s\right)dt + \frac{\Sigma\nu}{\sigma_D}dB^{\imath D}_s$$
$$\frac{d\Sigma}{ds} = -2\kappa\Sigma(s) - \frac{\nu^2\Sigma(s)^2}{\sigma^2_D} + \sigma^2_g, \quad \Sigma(t) = 0$$

where $\tilde{B}^{\imath D}_s$ *is a standard Brownian Motion in* $[t, t+\Delta]$ *under insider's filtration.*

Proof. See appendix. ■

To make the behavior of Σ_s regular, we assume that σ_g and σ_D are sufficiently large.

2.5 Market Maker's Inference

Recall that the market maker sets the competitive price as $P_s = A_0 D_s + A_1 g^m_s + A_2\bar{g}$, where g^m_s is his estimation given his information framework to time s. Suppose the insider's strategy is $dx_s = \theta_s dt = \beta(s)\left(g^{\imath}_s - g^{\imath m}_s\right)dt$. Then the market maker observes (x_t, D_t) and updates his estimation on $\left(g_t, g^{\imath}_t\right)$, denoted by $\left(g^m_t, g^{\imath m}_t\right)$. Since the market maker knows that the insider receives a perfect signal at the beginning time t, the market maker would impose $E\left(g_t|\mathcal{F}^m_t\right) = E\left(g^{\imath}_t|\mathcal{F}^m_t\right)$. We denote the conditional variance of the market marker's filtering problem by $\Sigma(t) \equiv Var\left(\left(g_t, g^{\imath}_t\right)|\mathcal{F}^m_t\right)$, which is a 2 by 2 matrix. The (i, j) element of this matrix is denoted by $\Sigma_{\imath\jmath}$. To satisfy the stationarity condition, it is required that the variance $\Sigma(t)$, as a function of time, is the same on each interval $[l\Delta, (l+1)\Delta]$. In particular, let $t = l\Delta$ and we have

$$\Sigma_{11}(t+\Delta) - \Sigma_{11}(t) = \int_t^{t+\Delta} d\Sigma^m_{11}(s) = 0.$$

Now we state the results of the filtering problem for the market maker as follows.

Proposition 3 *The solution to the filtering satisfies $g^m(s) = g^{im}(s)$ and*

$$dg^m = \kappa(\bar{g} - g^m)\,dt + \left\{\Sigma_{11}^m(s)\,\nu^2\sigma_D^{-2}(g - g^m) + \Sigma_{12}^m(s)\,\beta_s^2\sigma_u^{-2}\left(g^i - g^{im}\right)\right\}dt$$
$$+ \Sigma_{11}^m(s)\,\nu\sigma_D^{-1}dB^D + \Sigma_{12}^m(s)\,\beta_s\sigma_u^{-1}dB^u$$

The conditional variances are given by

$$\begin{aligned}\frac{d\Sigma_{11}^m}{ds} &= -2\kappa\Sigma_{11}^m + \sigma_g^2 - \left(\Sigma_{11}^m\right)^2\nu^2\sigma_D^{-2} - \left(\Sigma_{12}^m\beta_s\right)^2\sigma_u^{-2}\\ \frac{d\Sigma_{12}^m}{ds} &= -2\kappa\Sigma_{12}^m - \Sigma\nu^2\sigma_D^{-2}\Sigma_{12}^m - \nu^2\sigma_D^{-2}\Sigma_{11}^m\Sigma_{12}^m - \Sigma_{22}^m\Sigma_{12}^m\beta_s^2\sigma_u^{-2}\\ \frac{d\Sigma_{22}^m}{ds} &= -2\left(\kappa + \Sigma\nu^2\sigma_D^{-2}\right)\Sigma_{22}^m - \left(\Sigma_{12}^m\right)^2\nu^2\sigma_D^{-2} - \left(\Sigma_{22}^m\beta_s\right)^2\sigma_u^{-2}\end{aligned} \qquad (2.8)$$

and they satisfy

$$\Sigma_{11}^m(s) = \Sigma_{12}^m(s) + \Sigma(s), \quad \Sigma_{12}^m(s) = \Sigma_{22}^m(s).$$

Proof. See appendix. ■

Therefore we can substitute the observable processes (x_t, z_t, D_t) into the above SDE of g^m, and the linear pricing rule expressed using the observable is

$$dg^m = \kappa(\bar{g} - g^m)\,ds - \gamma(s)\,\nu g^m ds + \lambda(s)\left[\theta_s ds + \sigma_u dB^u\right] + \gamma(s)\left[dD_s + \nu D_s ds\right]$$

where $\lambda(s) \equiv \Sigma_{12}^m(s)\,\beta_s\sigma_u^{-2}$ and $\gamma(s) \equiv \Sigma_{11}^m(s)\,\nu\sigma_D^{-2}$.

2.6 Insider's Optimization

According to the Nash equilibrium, we conjecture that the insider anticipates that the pricing rule of the market maker is

$$dg^m = \kappa(\bar{g} - g^m)\,ds - \gamma(s)\,\nu g^m ds + \lambda(s)\left[dx_t + dz_t\right] + \gamma(s)\left[dD_s + \nu D_s ds\right].$$

As before, the insider's objective function is the expected present value of the the total profit, which is equivalent to

$$\max_{\theta_t} E \int_0^{\infty} e^{-rt} \left[(g - \hat{g}) \theta_t - c\theta^2 \right] dt.$$

Now the insider receives discrete signals at times $\{l\Delta\}_{l\in Z}$, which makes his optimization problem mathematically more complex. We can approach the insider's dynamic programming problem as a discrete one with continuous control variable θ_s over each interval $[l\Delta, (l+1)\Delta]$. Formally, we define the insider's value function at time $l\Delta$ to be $V(l, g_l - g_l^m)$. The insider's optimization is a Markov control problem and the market maker's estimation error is the only state variable because it is Markov as

$$\left(g^{\imath}_{(l+1)\Delta} - g^m_{(l+1)\Delta} \right) = e^{\Lambda(l\Delta) - \Lambda((l+1)\Delta)} \left(g^{\imath}_{l\Delta} - g^m_{l\Delta} \right) \tag{2.9}$$
$$+ e^{-\Lambda((l+1)\Delta)} \int_0^{\Delta} e^{\Lambda(s)} \Lambda(s) \left\{ -\lambda(s) \left[\theta_s ds + \sigma_u dB^u \right] - \Sigma_{12}^m(s) \nu \sigma_D^{-1} d\tilde{B}_s^{\imath D} \right\}$$

where $\Lambda(\cdot)$ is defined by $(\log \Lambda(s))' = (\kappa + \gamma(s)\nu)$. This dynamic of the state variable is proved in the appendix. We explicitly write the time l in the value function simply to facilitate understanding the derivations. The market stationarity guarantees that the value function is time independent.

The discrete time dynamic programming problem with infinite horizon has the following Bellman equation

$$V(l, g_t - g_t^m)$$
$$= \sup_{\{\theta_s^{\imath}, t \le s \le t+\Delta\}} E_t^{\imath} \left\{ \int_t^{t+\Delta} e^{-r(s-t)} \left[\left(g_s^{\imath} - g_s^m \right) \theta_s - c\theta^2 \right] dt + e^{-r\Delta} V\left(l+1, g_{t+\Delta}, g^m_{t+\Delta} \right) \right\}$$

where the state variable evolves according to equation (2.9). The state variable is a discrete time stochastic process, observable to the insider at each discrete time $l\Delta$. Notice that at the discrete times $\{l\Delta\}_{l\in Z}$, the insider knows the true value of g, and thus $g^{\imath}_{l\Delta} = g_{l\Delta}$. As emphasized above, one special property about this dynamic programming problem is that the control variable is a continuous function over each period $[l\Delta,(l+1)\Delta]$ Therefore, the Bellman equation above cannot be solved by routine methods, such as taking derivatives to obtain first order conditions. Instead, we must solve it by considering the continuous time optimal control problem on the finite time interval $[l\Delta,(l+1)\Delta]$. In particular, let us consider the insider's short term (per period) objective function, given the value function $V(l, g_{l\Delta}-g_{l\Delta}^m)$ at the discrete times $\{l\Delta\}_{l\in Z}$

$$\sup_{\{\theta_s^\imath, t\leq s\leq t+\Delta\}} E_t^\imath\left\{\int_t^{t+\Delta} e^{-r(s-t)}\left[(g_s-g_s^m)\theta_s-c\theta^2\right]dt+e^{-r\Delta}V\left(l+1,g_{t+\Delta},g_{t+\Delta}^m\right)\right\}$$

$$d\left(g^\imath-g^m\right)=-\left[\kappa+\gamma(s)\nu\right]\left(g^\imath-g^m\right)dt-\lambda(s)\theta_s dt$$
$$-\lambda(s)\sigma_u dB^u+\left[\Sigma-\Sigma_{11}^m(s)\right]\nu\sigma_D^{-1}d\tilde{B}_s^{\imath D}.$$

This is the finite horizon stochastic control problem. Denote the value function by $J\left(s,g_s^\imath-g_s^m\right)$; then, the discrete time Bellman equation is just

$$V(l,g_{l\Delta}-g_{l\Delta}^m)=J\left(0,g_{l\Delta}^\imath-g_{l\Delta}^m\right). \tag{2.10}$$

We address this equation later. For the finite horizon per period problem, the terminal value is

$$J\left(\Delta,g_{(l+1)\Delta}^\imath-g_{(l+1)\Delta}^m\right)=e^{-r\Delta}E_{(l+1)\Delta}^\imath V\left(l+1,g_{(l+1)\Delta},g_{(l+1)\Delta}^m\right). \tag{2.11}$$

Therefore, the per period problem can be solved using the regular approach, the Hamilton-Jacobian-Bellman equation. To summarize, the insider's problem can be broken down into two pieces. The outside piece is a discrete time infinite horizon problem, with a continuous time control variable. The inside piece is a finite horizon continuous time stochastic control problem. The solution to the latter problem can be considered to play a similar role as the first order condition of an ordinary discrete time Bellman equation.

Proposition 4 *The finite horizon stochastic control problem has the following solution. The value function is given by*

$$J\left(s, g^{\imath} - g^{m}\right) = e^{-rs}\left[\alpha\left(s\right)\left(g^{\imath} - g^{m}\right)^{2} + \delta\left(s\right)\right]$$

where the functions $\alpha(\cdot)$ *and* $\delta(\cdot)$ *satisfy the ordinary differential equations*

$$0 = -r\alpha\left(s\right) + \alpha'\left(s\right) - 2\alpha\left(s\right)\left[\kappa + \Sigma_{11}^{m}\left(s\right)\nu^{2}\sigma_{D}^{-2}\right] + \frac{\left[1 - 2\alpha\left(s\right)\lambda\left(s\right)\right]^{2}}{4c} \tag{2.12}$$

$$0 = -r\delta\left(s\right) + \delta'\left(s\right) + \alpha\left(s\right)\left[\left(\lambda\left(s\right)\sigma_{u}\right)^{2} + \left[\Sigma - \Sigma_{11}^{m}\left(s\right)\right]^{2}\nu^{2}\sigma_{D}^{-2}\right]$$

The terminal value conditions for these equations are given below; they combine the terminal condition of the per period problem and the Bellman equation of the infinite horizon dynamic programming problem. The optimal control of the insider is given by

$$\theta_{s} = \frac{\left[1 - 2\alpha\left(s\right)\lambda\left(s\right)\right]}{2c}\left(g^{\imath} - g^{m}\right).$$

Proposition 5 *The value function of the infinite horizon problem is*

$$V\left(l, g_{l\Delta} - g_{l\Delta}^{m}\right) = \alpha\left(0\right)\left(g - g^{m}\right)^{2} + \delta\left(0\right)$$

where $\alpha(\cdot)$ *and* $\delta(\cdot)$ *satisfy the ordinary differential equations in the last proposition and the terminal value conditions*

$$\begin{aligned}\alpha\left(\Delta\right) &= \alpha\left(0\right)\\ \delta\left(\Delta\right) &= \alpha\left(0\right)\Sigma\left(\Delta\right) + \delta\left(0\right).\end{aligned}$$

The control variable is given in the last proposition.

2.7 Equilibrium Solution

In this subsection we fully solve the equilibrium. The insider receives the signals at discrete times $\{l\Delta\}_{l\in Z}$, which leads to the new properties of the model. During each interval $[l\Delta, (l+1)\Delta]$, the market maker and the insider have their time dependent strategies, involving deterministic functions $\lambda(t)$, $\gamma(t)$ and $\beta(t)$. The estimation uncertainty of the market maker $\Sigma(t)$, the functions $\alpha(t)$ and $\delta(t)$ in the insider's value function, are also time dependent. Since the insider's uncertainty has an exact relation with the market maker's uncertainty $\Sigma(s) = \Sigma_{11}^{m}(s) - \Sigma_{12}^{m}(s)$, we treat it as a separate function in the mathematical derivation of the solution. Because the market is of infinite horizon and is stationary, it is necessary that all strategies utilized by the participants and other related deterministic functions are the same during each time interval $[l\Delta, (l+1)\Delta]$. This is in the same spirit as a standard infinite horizon discrete dynamic programming problem.

To find the equilibrium solution, we combine the market maker's inference and the insider's inference and optimization, using the definition of Nash equilibrium. We have the following eight deterministic functions

$$\left(\Sigma_{11}^{m}(s), \Sigma_{12}^{m}(s), \Sigma_{22}^{m}(s), \lambda(s), \gamma(s), \beta(s), \alpha(s), \delta(s)\right)$$

to solve. We have eight equations and three terminal conditions for three differential equations. The number of constraints is just enough to identify the unknowns. It turns out that we can derive two equations involving only two functions $(\Sigma_{12}^{m}, \alpha(s))$ and thus solve them first. All other functions can be obtained consequently. Details are in the appendix C.

The equilibrium solutions can be visualized as in figure 2-1 and 2-2. In figure 2-1, the transaction cost coefficient is $c = 0.4$, while in figure 2-2, the coefficient is $c = 0.004$. The plots are very informative. We start by examining the case in which there is nearly no transaction cost, i.e. $c = 0.004$. First, the insider trades very patiently. In addition, the trading intensity of the insider explodes at the end of the trading interval. In this respect, the market within the interval between two consecutive signals of the insider has similar characteristics as that in the Kyle model. The factors behind this phenomenon are similar to the factors in the Kyle model. However, we observe several new and interesting points, as follows.

Second, it is helpful to examine the information asymmetry $\Sigma_{11}(s) - \Sigma^{i}(s)$ plotted in the top left panel. At the moment when each trading period starts, the market maker has the greatest information disadvantage since the insider knows the signal precisely. As the market maker learns from the aggregate order flow, which includes the insider's orders, information asymmetry is reduced gradually. At the end of the trading interval, the insider submits huge order flows and drives the difference between the uncertainties between the two players close to zero. With this result, it is not difficult to understand the market maker's and the insider's individual uncertainties. During each trading interval, the insider's uncertainty about the signal increases from zero, and how large it becomes depends on the length of the interval Δ. For example, if Δ is very large, the insider's uncertainty Σ increases until it finally stabilizes at the steady state variance of the true process g_t. By comparison, the market maker's uncertainty about the true signal g_t starts high, decreases, then increases to the extent that it becomes very close to the insider's uncertainty. In the end, the market maker's uncertainty reassumes its level at the beginning of the trading period, as required by the stationarity of the infinite horizon market. In the first part of the interval the market maker's uncertainty is dominated by the fact that his information asymmetry is decreasing. In the second part of the interval the market maker's uncertainty is mainly influenced by the fact that the insider's uncertainty is increasing.

Third, the price impact $\lambda(t)$ decreases during the trading period. This is in contrast with the Kyle model in which λ is a constant over the finite period. From the expression $\lambda(t) = \Sigma_{12}^{m}(s)\,\beta(s)\,\sigma_u^{-2}$, we see that there are two effects that determine the size of the price impact $\lambda(t)$. The first effect is the insider's relative information precision

$\Sigma_{12}^m(s) = \Sigma_{11}^m(s) - \Sigma(s)$. In our model, the insider knows the true mean g perfectly only at the beginning of the trading period. His uncertainty gradually increases, as depicted in the top-right panel. In the Nash equilibrium, the market maker anticipates this and therefore sets prices less responsively as time goes by. The second effect that influences λ is the proportion of the insider's orders relative to the order size of the noise traders $\beta(s)/\sigma_u^2$. This proportion increases over time within the interval and hence moves the price impact $\lambda(t)$ upward. The first effect dominates and consequently $\lambda(t)$ decreases over time. However, in the Kyle model there is only one signal received by the insider, the final payoff at the end of the trading period. The two effects defined above exactly offset each other in the Kyle model. Actually, it is the constant λ that achieves an equilibrium solution.

Finally, it is inspiring to notice the following seemingly contradictory properties in this market. On the one hand, the insider loses his information advantage gradually. On the other hand, at the end of the trading period, his trading intensity shoots up sharply. This is in accordance with the Kyle model but is surprising in our framework, since the insider can become ignorant of the true mean g himself but he still trades arbitrarily aggressively. Close to the end of the trading interval, the insider anticipates the next signal, and thus he will submit enormous orders to use up his information from the last signal, no matter how tiny is his information advantage. This demonstrates the fact that the risk neutrality assumption on the part of the insider is so strong that it produces implausible results.

When we turn to the chart for relatively greater transaction cost, i.e., $c = 0.4$, we find the expected results. The insider's trading intensity increases very slowly and does not jump up close to the end of the trading period. This is because the transaction cost deters him from doing so. As a consequence, information asymmetry deceases over time less rapidly and furthermore, the market maker's information uncertainty decreases to a lesser extent before rising.

We also compare the equilibria in two markets in which the insider receives signals at different frequencies, with everything else equal. The results of the case with higher frequency are plotted in figure 2-3. At high frequency, the insider's uncertainty only slightly increases before the next signal arrives. Since his estimation of the true mean is accurate, he trades more aggressively over the short period. This is turn explains why the market maker's uncertainly is small in absolute terms. In addition, each interval is shorter so the insider's trading intensity is smoother, explaining why the market maker's uncertainty is large relative to the insider's uncertainty. A reasonable measure of market efficiency is

$$\frac{\Sigma_{11}^{m}(s) - \Sigma^{\imath}(s)}{\Sigma^{*} - \Sigma^{\imath}(s)}$$

where the numerator is the excess of the conditional uncertainty of the market maker over that of the insider, and the denominator is the difference between the estimation uncertainty, assuming the dividend rate to be the only observable element of the filter, and the insider's conditional uncertainty. Equivalently, the denominator is the amount of private information that the insider holds, and the numerator is the difference between the precision of information between the market maker and the insider. In other words, the numerator is the portion of the private information that is not incorporated into the market price. This fraction is never greater than 1. In figure 2-3, this fraction is roughly between one-half and zero during the interval between the two consecutive signals. By contrast, in figure 2-2, this fraction is much larger, close to 1 most of the time. Therefore, we can conclude that with higher frequency of the insider's signals, the market is more efficient. The intuition is that when the insider receives signals more frequently, as explained above, his trading is smoother, and therefore the market maker can track his private information more closely.

Overall, when the insider receives signals only at discrete times, the market efficiency in strong form does not hold, even as the transaction cost is infinitesimal. The market is less efficient when the transaction cost is greater or when the insider receives signals less frequently.

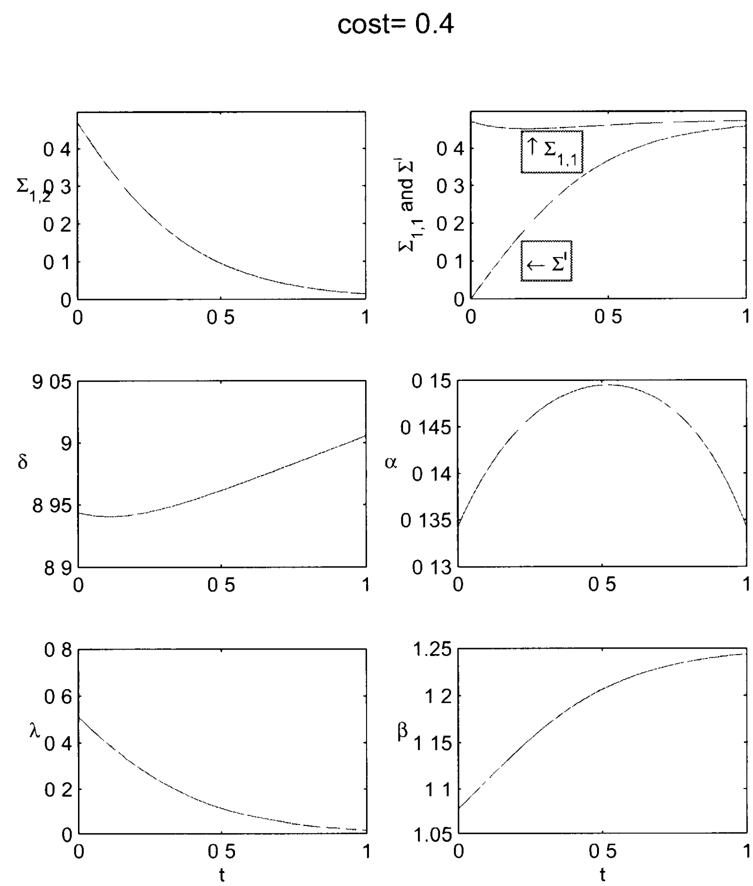

Figure 2-1: The insider receives signals at evenly spaced times. The coefficient of the transaction cost is 0.4.

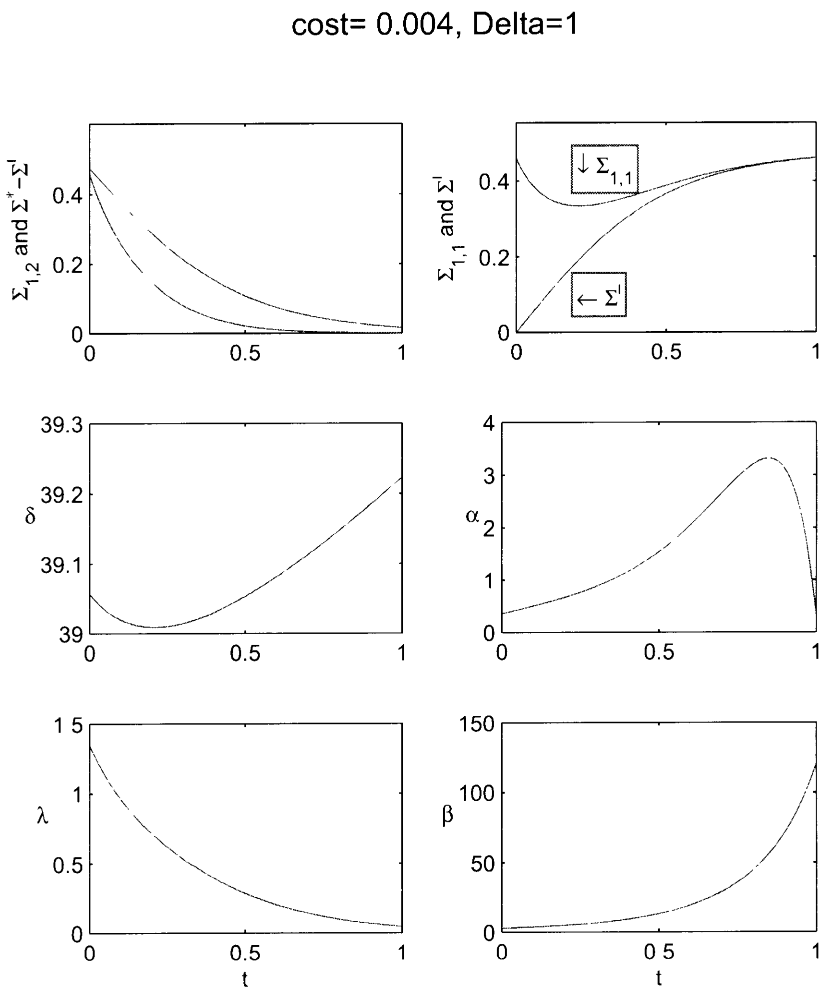

Figure 2-2: The insider receives signals at evenly spaced times. The coefficient of the transaction cost is 0.004.

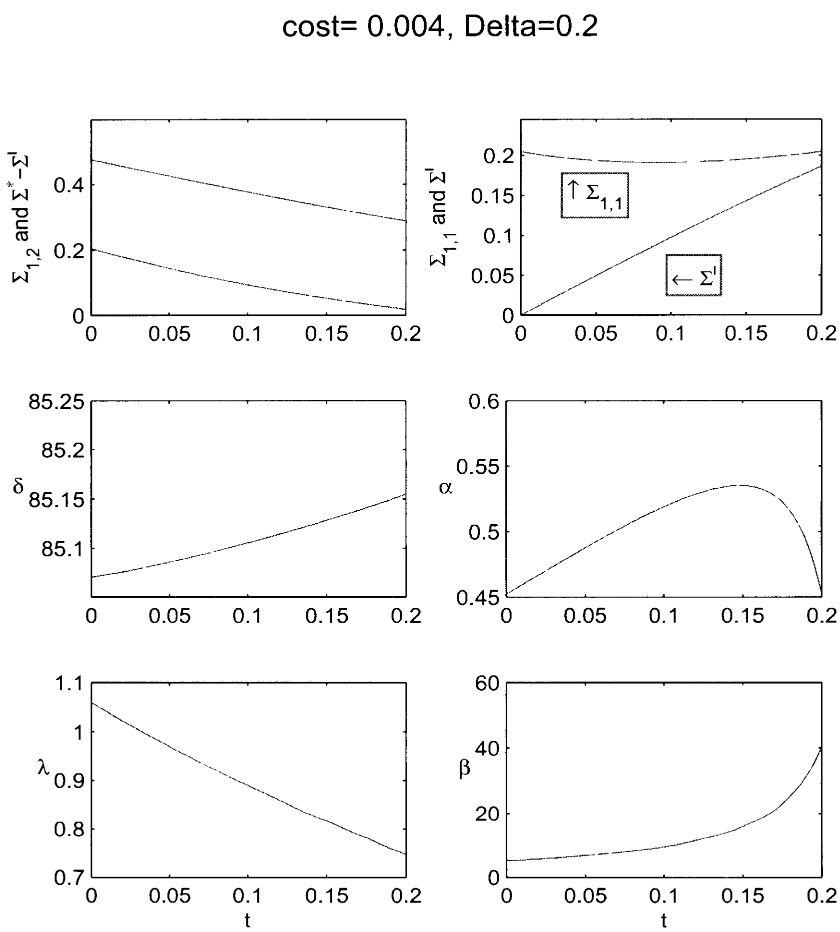

Figure 2-3: The insider observes signals at discrete times. The transaction cost is c=0.004. The interval between signals is fixed at $\Delta = 0.2$.

2.8 Conclusion

The results of Chau and Vayanos (2008), that a stationary market with a monopolistic insider is approximately efficient in the strong form, is quite interesting and surprising. However, these results do not seem to reflect real financial markets. In particular, it is not plausible that the insider would submit a huge order flow constantly over an infinite horizon. We find that in the CV model there are some seemingly moderate assumptions that actually deviate from reality. These assumptions are critical for the derivation of strong form efficiency in the CV model, and relaxing these assumptions can potentially reconcile the empirically doubtful conclusions of strong-form efficiency. We introduce two more realistic assumptions, namely that there exist transaction costs and that the frequency of signal arrivals and the frequency of trading are different.

We show that the presence of a transaction cost in the quadratic form faced by the insider will prevent the insider from trading quickly. Moreover, the market is no longer efficient in the strong form. In addition, we introduce the assumption that the insider receives private information at a lower frequency than the frequency at which trading takes place. The solution of the equilibrium shows that between two consecutive signals, there is an interesting pattern of trading intensity of the insider, price impact, and the insider's informational advantage. Furthermore, the degree of market efficiency is lower than the case in which the frequency of private signals equals the frequency of trading.

Chapter 3

Limiting Case with Continuous Private Signals

3.1 Model Setup

In this chapter, we study a limiting case where the length of time intervals between the insider's private signals goes to zero. The assumptions different from before will be highlighted.

As before, the market maker sets the price as the conditional expectation of the present value of expected future dividends of a stock

$$P_t = E\left[\int_t^{\infty} e^{-r(s-t)} D_s ds | \mathcal{F}_t^m\right]. \tag{3.1}$$

This can be simplified to be

$$P_t = A_0 D_t + A_1 \hat{g}_t + A_2 \bar{g} \tag{3.2}$$

where $\hat{g}$ is the market maker's conditional expectation of the state variable g_t, given his information set at time t.

Notice that the insider observes g perfectly, so we replace $E\left(g_t | F_t^m\right)$ in the market maker's valuation by g. From the insider's prospective, the present value of the stock can be simplified as

$$P_t = A_0 D_t + A_1 g_t + A_2 \bar{g}.$$

As a result, the insider's objective can be expressed as

$$\max_{\theta_t} E\left[\int_{t_0}^{\infty} e^{-rt}\left[(g-\hat{g})\,\theta_t - A_1^{-1} c\theta_t^2\right] dt|\mathcal{F}_{t_0}^{\imath}\right]^{1} \tag{3.3}$$

We consider linear equilibrium, in which the agents' strategies are linear functions of state variables. In particular, we assume that the insider has linear strategy of the following form [2]

$$dx_t = \beta_t\,(g-\hat{g})\,dt. \tag{3.4}$$

We also assume that the market maker sets the price according to equation (3.2) with the conditional expectation having the following dynamics

$$d\hat{g}_t = \kappa\,(\bar{g}-\hat{g})\,dt + \lambda\,(t)\,[dx_t + dz_t] + \gamma\,(t)\,[\nu\,(D-\hat{g})\,dt + dD]\,. \tag{3.5}$$

Definition 6 *A pair of linear strategies (x_t, p_t) satisfying (3.2), (3.5) and (3.4) is a Nash equilibrium if the following two conditions hold:*

1. Given the market maker's pricing rule (3.2) and (3.5), insider chooses the optimal strategy $dx_t = \theta_t dt$ to maximize his expected future profit (3.3). Given the insider's strategy (3.4), the market maker sets prices equal to the conditional expectation of the present discounted value of the stock with equation (3.1).

[1] In the following we will let $\tilde{c} = A_1^{-1}c$ and abuse the notation to let c denote $\tilde{c}$.

[2] This can be a candidate strategy only if $\hat{g}$ is observable by the insider. From the pricing equation (3.2), we observe that the insider can infer market maker's expectation $\hat{g}_t$ from observing the price.

3.2 Market maker's filtering problem

First we look at the market maker's inference problem. The market maker updates his belief about the underlying state variables based on the observed variables, the dividend process, and the aggregate order flow submitted by the insider and the noise traders. We follow the standard Kalman-Bucy [3] filtering technique to derive the solution to the inference problem.

Formally, the market maker observes the aggregate order flow and the dividend process (Y, D). Dividend evolves according to equation (2.1) and total order flow satisfies the following stochastic differential equation

$$dY_t = \beta_t (g_t - \hat{g}_t) dt + \sigma_u dB_t^u.$$

Equivalently, the processes $\left(\tilde{Y}, \tilde{D}\right)$, defined by the following SDE evolution

$$d \begin{bmatrix} \tilde{Y} \\ \tilde{D} \end{bmatrix} = \begin{bmatrix} \beta(t) \\ \nu \end{bmatrix} g dt + \begin{bmatrix} \sigma_u & 0 \\ 0 & \sigma_D \end{bmatrix} \begin{bmatrix} dZ^u \\ dB^D \end{bmatrix}$$

are observable to the market maker. The unobserved state variable is the dividend growth

$$dg_t = \kappa (\bar{g} - g_t) dt + \sigma_g dB^g.$$

Define $\hat{g}_t = E\left(g_t | \mathcal{F}_t^M\right)$ and $\Sigma_t = Var\left(g_t | \mathcal{F}_t^M\right)$.

Proposition 7 *Given the insider's strategy (3.4), the market maker will update the conditional expectation $\hat{g}$ and the conditional variance Σ of the state variable g by*

$$d\hat{g} = \kappa (\bar{g} - \hat{g}) dt + \lambda(t) [\theta dt + \sigma_u dZ^u] + \gamma(t) \left[\nu (g - \hat{g}) dt + \sigma_D dB^D\right] \tag{3.6}$$

[3] Lipster and Shiryaev (2001).

and

$$\Sigma'(t) = -2\kappa\Sigma(t) + \sigma_g^2 - \Sigma^2(t)\left(\frac{\beta^2(t)}{\sigma_u^2} + \frac{\nu^2}{\sigma_D^2}\right)$$

where $\lambda(t) = \Sigma(t)\beta_t/\sigma_u^2$ *and* $\gamma(t) = \Sigma_t\nu/\sigma_D^2$.

The filtering theory tells us that the estimate $\hat{g}$ is updated because g is expected to change, and because new information from observable processes is available. The first term reflects the part of $\hat{g}$ that is expected to change in the same manner as g. The new information from the observable processes is incorporated into the estimation, as there is a correlation between the state variable and the drift of the observable variables $\left(\tilde{Y}, \tilde{D}\right)$. The second term of the RHS of equation (3.6) reflects the correlation between g and $\tilde{Y}$, and the third term reflects the correlation between g and $\tilde{D}$.

Notice that the conditional variance evolves deterministically and satisfies an ordinary differential equation of the Reccati type, but in general there is no closed form solution of the differential equation. However, we can circumvent this problem if we are only interested in the steady state of the stationary financial market. The convergence of the filters to their steady state solution is proved, under mild conditions, in Anderson and Moore (1979). In the steady state, the uncertainty $\Sigma(t)$ is constant over time and therefore $\Sigma'(t) \equiv 0$. The insider's strategy is also constant; therefore, we obtain an algebraic equation of the two variables Σ and β

$$-2\kappa\Sigma + \sigma_g^2 - \Sigma^2\left(\frac{\beta^2}{\sigma_u^2} + \frac{\nu^2}{\sigma_D^2}\right) = 0. \tag{3.7}$$

Furthermore, when the market maker can only infer the process g_t based on observation of the dividend process, his conditional variance of g_t is greater than the case in which he observes both g_t and D_t. In the former case, the steady state conditional variance satisfies the quadratic equation (3.7) where the strategy β is replaced by zero, and the valid solution is given by

$$\Sigma^* = \frac{-\kappa + \sqrt{\kappa^2 + \sigma_g^2\nu^2/\sigma_D^2}}{\nu^2/\sigma_D^2}.$$

To facilitate the equilibrium analysis. we write the filtering solution (3.6) as a stochastic differential equation of the observable variables of the market maker

$$d\hat{g}_t = \kappa (\bar{g} - \hat{g})\, dt + \lambda [dx_t + dz_t] + \gamma [\nu (D - \hat{g})\, dt + dD]. \tag{3.8}$$

As explained above. the market maker updates his estimate of g using the information on the aggregate order flow and the dividend cash flow In the steady state of the stationary financial market. $\lambda(t)$ and $\gamma(t)$ are not time dependent.

3.3 Insider's optimization

Notice that the insider cannot observe the total order flow as the market maker does: therefore, he cannot imitate the market maker by working out the filtering problem to obtain the market maker's conditional expectation of the insider's private information. However, recall that the market maker sets the price as in equation (3.2). thus the insider can infer the market maker's conditional expectation of the insider's private information from the price process and the dividend cash flow.

In the Nash equilibrium, the insider anticipates a rational pricing rule of the market maker. In our model, due to perfect competition. the market maker sets price as the present value of the expected future dividend cash flow. conditional on his information set. By the assumed Markov property of the processes (D, g). the pricing rule is determined by equation (3.2). Therefore. the pricing rule essentially boils down to an updating scheme of short run mean g. We assume that the insider anticipates the market maker updates ĝ using equation (3.8), which is equivalent to

$$d\hat{g} = \kappa (\bar{g} - \hat{g})\, dt + \lambda \theta dt + \lambda (t)\, \sigma_u dZ^u + \gamma \left[\nu (g - \hat{g})\, dt + \sigma_D dB^D\right].$$

Then Insider's optimization is

$$\max_{\theta_t} E \int_0^\infty e^{-rt} \left[(g - \hat{g}) \theta_t - c\theta^2 \right] dt.$$

This is a standard stochastic control problem. The state variables are $(\hat{g}, g)$. However, a more careful look renders

$$\begin{aligned}
& d(g - \hat{g}) \\
& = \kappa (\hat{g} - g) dt + \sigma_g dB^g - \lambda(t) \theta dt - \lambda(t) \sigma_u dZ^u - \gamma(t) \left[\nu (g - \hat{g}) dt + \sigma_D dB^D \right] \\
& = \left[(\kappa - \nu\gamma_t)(\hat{g} - g) - \lambda(t) \theta \right] dt + \sigma_g dB^g - \lambda(t) \sigma_u dZ^u - \gamma_t \sigma_D dB^D.
\end{aligned}$$

This, in addition to the function form of the optimization objective, says that the estimation error of the market maker is the single state variable of the insider's stochastic control problem.

Let $J(t, g - \hat{g})$ denote the value function. Then the value function satisfies the HJB equation

$$\begin{aligned}
0 &= \sup_{\theta_t} J_t + J_g \left[(\kappa - \nu\gamma_t)(\hat{g} - g) - \lambda(t) \theta \right] + \frac{1}{2} J_{gg} \left[(\sigma_g)^2 + \lambda_t^2 \sigma_u^2 + \gamma_t^2 \sigma_D^2 \right] \\
&+ \left[(g - \hat{g}) \theta_t - c\theta^2 \right] e^{-rt}, \\
& s.t. \left[(\kappa - \nu\gamma_t)(\hat{g} - g) - \lambda(t) \theta \right] dt + \sigma_g dB^g - \lambda(t) \sigma_u dZ^u - \gamma_t \sigma_D dB^D \\
& \lim_{\tau \to \infty} E \left[J(\tau, g - \hat{g}) \,|\mathcal{F}_t^I \right] = 0.
\end{aligned}$$

Suppose the functions $\lambda(t)$ and $\gamma(t)$ are not time varying.

Proposition 8 *The HJB equation has a solution of the form*

$$J(t, g - \hat{g}) = e^{-rt} \alpha (g - \hat{g})^2 + e^{-rt} \delta$$

where α and δ satisfy the following equations

$$0 = -r\delta + \alpha\sigma_g^2 + \alpha\left(\lambda^2\sigma_u^2 + \gamma^2\sigma_D^2\right) \tag{3.9}$$

$$0 = -r\alpha - 2\alpha\left(\gamma\nu + \kappa\right) + \frac{1}{4c}\left(1 - 2\alpha\lambda\right)^2. \tag{3.10}$$

The optimal trading strategy is linear and given by

$$\theta = \frac{-2\alpha\lambda + 1}{2c}\left(g - \hat{g}\right). \tag{3.11}$$

Proof. See appendix. ■

The main conclusion is that given the market maker's pricing rule, the insider chooses a linear trading strategy. The value function is a quadratic function of the market maker's estimation error. It is important to notice that when deriving the insider's optimal solution, we have already incorporated the fact that in the steady state, the market maker's choices $\lambda(t)$ and $\gamma(t)$ are constants, which in turn implies that the insider's strategy depends on the state variable in a time invariant fashion, and so does the value function, except the time decaying e^{-rt}. The stationarity of the economy is the main difference between this paper (also CV (2008)) and Kyle (1985).

Comparison between our infinite horizon model of a stationary financial market and the finite horizon model is in order. In the Kyle (1985) model, trading occurs over a finite horizon $[0, T]$. At the beginning of the trading period, the insider learns about the final payoff $v \equiv v_T$ of the stock. There is no dividend payout of the stock. The noise trader is assumed, as before, to submit exogenous order flows $dz_t = \sigma_u dB_t^u$ independent of the insider. If the market maker's pricing rule is $dP_t = \lambda_t\left(dx_t + dz_t\right)$, then the risk-neutral insider has the value function

$$J\left(t, v - P\right) = \frac{1}{2\lambda}\left(v - P\right)^2 + \frac{1}{2}\lambda\sigma_u^2\left(T - t\right)$$

where the terminal condition is $J(T, v-P) = 0$. There are two major differences between the Kyle model and our model. First, in the Kyle model, the insider's trading strategy is not determined by his own optimization problem, since the HJB equation is linear in the insider's trading strategy θ_t. For the insider's optimal control problem to have a solution, the terms in HJB that do not relate to θ_t should add up to zero. This requires that the market maker sets $\lambda(t)$, the reciprocal of the market depth, to a constant. The terminal value condition of the value function implies $E(v|\mathcal{F}_T^m) = 0$. Therefore, the insider's strategy is determined in the following way. The insider chooses $\beta(t)$ such that the market maker, given $\beta(t)$, chooses a constant $\lambda(t)$ and infers the terminal payoff perfectly. i.e. $E(v|\mathcal{F}_T^m) = 0$. In the Kyle model the equilibrium solution strongly depends on the Nash equilibrium concept. This is quite different from the common property of optimization problems, that the solutions are usually given by first order conditions. In our model, the insider's strategy is determined by his optimal control problem, because with the quadratic transaction cost, the HJB equation is not linear in θ_t. Second, in the Kyle model, in principal the insider's trading intensity $\beta(t)$ and the market maker's liquidity choice $\lambda(t)$ are deterministic functions of time, since the financial market has a finite horizon. Although it turns out that $\lambda(t)$ is a constant, it is only a special property of the Nash equilibrium solution. On the other hand, in our model, since we have a stationary framework, any control variables must be constant over time. This places a strong requirement on the equilibrium solution. Later we will show that stationarity is indeed much more restrictive than it appears.

It is helpful to point out a caveat. The insider's strategy given by equation (3.11) may seem to implicate that insider's trading intensity $\beta = (1 - 2\alpha\lambda)/2c$ explodes as transaction cost diminishes. Although this result is true, the reasoning behind it goes beyond this equation alone. The variables α and λ are themselves affected by the parameter c. To analyze the comparative statics, we must solve the equilibrium, that is, to solve for the variables as functions of the parameters only.

3.4 Equilibrium solution

To solve the steady state equilibrium values of $(\Sigma, \lambda, \alpha, \delta, \gamma, \beta)$, we combine the solutions to the market maker's inference problem and the insider's optimization problem and obtain a system of equations. These can be simplified as follows.

Proposition 9 (λ, Σ) *are jointly determined by the following system of equations.*

$$\lambda^2 = \frac{1}{\sigma_u^2}\left(-2\kappa\Sigma + \sigma_g^2 - \frac{\nu^2}{\sigma_D^2}\Sigma^2\right),$$
$$0 = \Sigma\left(\Sigma - 2c\lambda\sigma_u^2\right)\left(r + 2\kappa + 2\Sigma\nu^2\sigma_D^{-2}\right) - 2c\lambda^3\sigma_u^4.$$

The remainder of the parameters $(\alpha, \delta, \gamma, \beta)$ can be solved subsequently.

$$\alpha = \frac{\Sigma - 2c\lambda\sigma_u^2}{2\lambda\Sigma},$$
$$\delta = \frac{2\alpha}{r}\left(\sigma_g^2 - \kappa\Sigma\right),$$
$$\gamma = \Sigma\nu\sigma_D^{-2},$$
$$\beta = \lambda\sigma_u^2\Sigma^{-1}.$$

The limiting behavior as the transaction cost diminishes is

$$\lim_{c\to 0}\lambda = \frac{\sigma_g}{\sigma_u},$$
$$\lim_{c\to 0}\Sigma = 0,$$
$$\lim_{c\to 0}\beta = \infty,$$
$$\lim_{c\to 0}\gamma = 0.$$

The results are easy to interpret. First, when the transaction cost is small, the solution is consistent with the discrete time setup in CV (2007), in the sense that there is approximate strong form market efficiency. The insider's rate of trading is huge and the market maker's uncertainty about the underlying true profitability g_t is negligible. However, market depth converges to a constant σ_u/σ_g and we show that the insider's profit converges to a positive constant. It is clear that there does not exist a Nash equilibrium in the continuous time model in the limit when there is no transaction cost. Second, our model explains more than CV about what is the outcome when the insider bears a transaction cost. There exists an equilibrium in which the insider's trading intensity is finite and the market maker cannot infer the true signal g_t perfectly from the aggregate orders and the dividend rate. In such case, the market is not strong form efficient. We can calculate comparative statics from the solution to the model. The parameters are plotted in Figure 1 as the transaction cost varies.

3.5 Conclusion

We take the limit in the general model as the frequency of the insider's private signals increasing to infinity, which means private signals are observed continuously. In this special case, every deterministic process that the market maker and the insider control becomes constant in equilibrium, rather than periodic as in Chapter 1. For example, $\Sigma(t)$, $\alpha(t)$, $\gamma(t)$ are constant over the infinite horizon. Major effects resulting from trading costs are highlighted in the simpler model. For example, we show that if there is a transaction cost in the quadratic form faced by the insider, it will prevent the insider from trading quickly. Moreover, the market is no longer efficient in the strong form.

Chapter 4

Discrete Trading and Comparison with Strong Form Efficiency

4.1 Discrete Trading Setup

In this section, we briefly describe the results for a discrete framework, in which trading takes place at fixed and evenly spaced times. The private signals of the insider are assumed to arrive exactly at these trading times. This framework extends that in the CV model by only bringing in a trading cost for the insider; therefore, the model here nests the CV model. The continuous time model in Chapter 2 can also be considered a limiting case, as the trading interval shrinks to zero, of the discrete time model in this chapter.

The assumptions are the same as in the above continuous time model, except that trading takes place at a set of discrete times $\{lh\}_{l \in Z}$. There is a quadratic transaction cost for the insider's trades, otherwise this economy is essentially the same as the one in CV (2007).

The dividend rate is mean reverting

$$D_l = D_{l-1} + \nu h \left(g_{l-1} - D_{l-1} \right) + \varepsilon_{D,l}$$

with the short run mean process g_l itself reverting to a constant $\bar{g}$ according to

$$g_l = g_{l-1} + \kappa h \left(\bar{g} - g_{l-1}\right) + \varepsilon_{g,l}.$$

The parameters ν and κ determine the reversion rate of these processes. The errors $\varepsilon_{D,l}$ and $\varepsilon_{g,l}$ are both *i.i.d.* and independent of each other, and normally distributed with mean zero and variances $\sigma_D^2 h$ and $\sigma_g^2 h$, respectively. The market maker is competitive and sets the price as the expected present value of future dividend streams conditioned on his information set

$$P_l = E\left[\sum D_{l'} h e^{-r(l'-l)h} | \mathcal{F}_l^m\right] = A_0 D_l + A_l E\left(g_l | \mathcal{F}_l^m\right) + A_2 \bar{g}$$

where the second equality is derived as above by substituting into the processes $(D_{l'}, g_{l'})$; this essentially results from the joint Markov property of the dividend rate and the true underlying mean. Noise traders' orders at time lh are $\varepsilon_{u,l}$, with mean zero and variance $\sigma_u^2 h$.

We consider only linear strategies for the market maker and the insider. The market maker's inference is conjectured to evolve according to

$$\hat{g}_l = (1 - \kappa h)\, \hat{g}_{l-1} + \kappa h \bar{g} + \lambda_D \left(D_l - (1 - \nu h)\, D_{l-1} - \nu h \hat{g}_{l-1}\right) + \lambda_x \left(x_l + u_l\right).$$

We also conjecture the insider's trading strategy to be

$$x_l = \beta \left(g_{l-1} - \hat{g}_{l-1}\right).$$

The insider is risk neutral and bears transaction cost cx_l^2/h for trading at time lh. Thus, his objective function is

$$\max_{x_l} E\left\{\sum_{l'=l}^{\infty}\left[x_l\left(g_l-\hat{g}_l\right)-c\frac{x_l^2}{h}\right]e^{-r(l'-l)h}|\mathcal{F}_l^i\right\}.$$

The equilibrium can be solved in the usual fashion. We first solve the market maker's inference problem given the insider's strategy $\beta\left(g_{l-1}-\hat{g}_{l-1}\right)$, and define his conditional variance of the state variable to be $\Sigma_g \equiv Var\left(g_l|\mathcal{F}_l^m\right)$. Then given the market maker's estimation $\hat{g}$ and the pricing rule, we solve the insider's optimization problem, obtaining a quadratic value function $B\left(g-\hat{g}_{l-1}\right)^2+C$ in our case, and optimal trading intensity β. Finally, we combine the solutions of the two players. There is no simple closed form solution to the equilibrium. The equilibrium is characterized by a system of equations, which are presented in the appendix. However, the limiting behavior when the trading interval vanishes can be expressed easily as in the following proposition.

Proposition 10 *The equilibrium is determined by system of equations, as follows*

$$\begin{aligned}
\beta &= \frac{\left[1-\left(\kappa+\nu\lambda_D\right)h\right]\left(1-2\lambda_x e^{-rh}B\right)}{2\left(\lambda_x+ch^{-1}-\lambda_x^2 e^{-rh}B\right)},\\
B &= \left[1-\left(\kappa+\nu\lambda_D\right)h\right]^2\frac{1+4e^{-rh}Bch^{-1}}{4\left(\lambda_x+ch^{-1}-\lambda_x^2e^{-rh}B\right)},\\
C &= \frac{1}{1-e^{-rh}}e^{-rh}B\left(\lambda_D^2\sigma_D^2+\lambda_x^2\sigma_u^2+\sigma_g^2\right)h,\\
\lambda_D &= \frac{\left(1-\kappa h\right)\Sigma_g\nu\sigma_u^2h}{\Sigma_g\left(\beta^2\sigma_D^2+\nu^2\sigma_u^2h^2\right)+\sigma_D^2\sigma_u^2h},\\
\lambda_x &= \frac{\left(1-\kappa h\right)\beta\Sigma_g\sigma_D^2}{\Sigma_g\left(\beta^2\sigma_D^2+\nu^2\sigma_u^2h^2\right)+\sigma_D^2\sigma_u^2h},\\
\Sigma_g &= \frac{\left(1-\kappa h\right)^2\Sigma_g\sigma_D^2\sigma_u^2h}{\Sigma_g\left(\beta^2\sigma_D^2+\nu^2\sigma_u^2h^2\right)+\sigma_D^2\sigma_u^2h}+\sigma_g^2h.
\end{aligned}$$

When $h\to 0$, the equilibrium converges to the equilibrium solution to the continuous time *model as presented in proposition 9*

$$\lim_{h\to 0}\left(\Sigma_g, \lambda_x, B, C, \lambda_D, \frac{\beta}{h}\right) = (\Sigma, \lambda, \alpha, \delta, \gamma, \beta).$$

Proof. See appendix. ■

4.2 Comparison with Strong-form Market Efficiency

CV (2008) computes the equilibrium in a discrete time setting with infinite horizon. Our discrete time model nests their model, since we add a transaction cost. As the transaction cost approaches zero, we can derive the equilibrium results of the CV model. For convenience, we list the main results from CV (2008) below.

Proposition 11 *When the insider is trading, the asymptotic behavior of the equilibrium is characterized by*

$$\begin{aligned}
\lim_{h\to 0}\frac{\lambda_D}{\sqrt{h}} &= \frac{\sigma_g^2\nu}{\sigma_D^2\sqrt{r+2\kappa}},\\
\lim_{h\to 0}\lambda_x &= \frac{\sigma_g}{\sigma_u},\\
\lim_{h\to 0}\frac{\Sigma_g}{\sqrt{h}} &= \frac{\sigma_g^2}{\sqrt{r+2\kappa}},\\
\lim_{h\to 0}\frac{\beta}{\sqrt{h}} &= \frac{\sigma_u\sqrt{r+2\kappa}}{\sigma_g},\\
\lim_{h\to 0}B &= \frac{\sigma_u}{2\sigma_g},\\
\lim_{h\to 0}C &= \frac{\sigma_g\sigma_u}{r}.
\end{aligned}$$

What, as CV (2007) claims, is most striking is that when calendar time converges to zero, the size of insider's trading β is $\sqrt{h}$, which is larger than order h. Therefore, the rate of order size β/h converges to infinity. In addition, the insider's trading reveals more and more information, as seen from the fact that Σ_g is also of order $\sqrt{h}$, which means that the market maker's uncertainty about the insider's private information converges to zero as the time interval converges to zero. Another important variable is the insider's trading volume, $x_l = \beta\,(g_{l-1} - \hat{g}_{l-1})$, which is of order $h^{3/4}$. Thus the volume generated by the insider within a fixed time interval is of order $h^{-1/4}$, which converges to infinity when h goes to zero. The marker maker's estimation is also interesting. The updating of his belief about g depends on total order flow and dividend rate through λ_x and λ_D, respectively. As the trading interval approaches zero, the price impact of the dividend rate converges to zero. This is intuitive, because the market maker's update needs not rely as much on dividend rate when the insider places a huge, informative order flow. It may be surprising that the price impact of the aggregate order flow converges to a finite number, since total order flow is more informative than the Kyle model. This is explained in CV (2008) that the market maker faces less uncertainty and the information has smaller effect on the prices.

As we are interested only in model implications for the near continuous trading case, our discrete time model stands in notable contrast with the CV model. First, the CV model does not converge to an equilibrium model in continuous time, since the equilibrium solution diverges as h approaches zero. Our discrete time model converges to a continuous time model when the trading interval approaches zero, as seen in proposition 11. In our discrete time model, so long as the transaction cost is not zero, the insider's rate of trading does not explode when h approaches zero. In particular, the rate of trading of the insider, measured by parameter β, is of order h, as opposed to order $\sqrt{h}$ in the CV model. Another difference is the insider's profit margin $g_l - \hat{g}_l$. This also converges to zero in the CV model but does not do so in our model. The magnitude of the profit margin not only determines the insider's profit, but also has an effect on the insider's trading volume. It is natural to see that the insider's profit has a nonzero limit. Combining the properties of β and $(g - \hat{g})$, we observe that the volume of the insider's trading $x_l = \beta\,(g_{l-1} - \hat{g}_{l-1})$ is also h, which means that the rate of trading volume with respect to time between trading converges to a finite number. This is a crucial difference from the CV model. Consequently, the market maker cannot infer insider information perfectly. In other words, the market maker's uncertainty Σ_g about the true mean g converges to a positive number. Therefore the continuous time limit of the financial market is not strong form efficient any more, so long as the transaction cost exists.

CV explains in detail that the main difference between their model and the Kyle model is the stationarity of the market. Here, stationarity refers mainly to the fact that the insider repeatedly receives private signals, the true underlying short run mean g of the dividend rate. It is this kind of stationarity that drives the insider to be much less patient than the insider in the Kyle model. In our model, however, the insider is also impatient. The additional quadratic transaction cost prohibits him from trading aggressively.

Our model nests the CV model. When the transaction cost approaches zero, our discrete time model converges to their model. On the other hand, as the time interval h approaches zero, our discrete time model converges to the continuous time model in chapter 3 while the CV model does not converge, as trading frequency increases without bound. The relationship between the three models is described in diagram 4-1. The solutions to the CV model, however, quantitatively have a limiting property when $h \to 0$, although in the limit, the CV model has no solution. The solutions for our continuous time model on the right hand corner of diagram 4-1 exhibit the same limiting characteristics as trading cost vanishes, i.e., $c \to 0$.

This last convergence between the right two models in diagram 4-1 can be seen in Figure 4-2, which shows what happens to the solution of our continuous time equilibrium when trading costs vary. It is clear that when there is almost no transaction cost, the solution has the same properties as the solution to the CV model in the near continuous trading case. For example, the trading intensity of the insider can be arbitrarily large and the uncertainty Σ of the market maker can be arbitrarily close to zero for sufficiently small transaction costs c. The comparative statics when c varies are straightforward. Since transaction cost is the

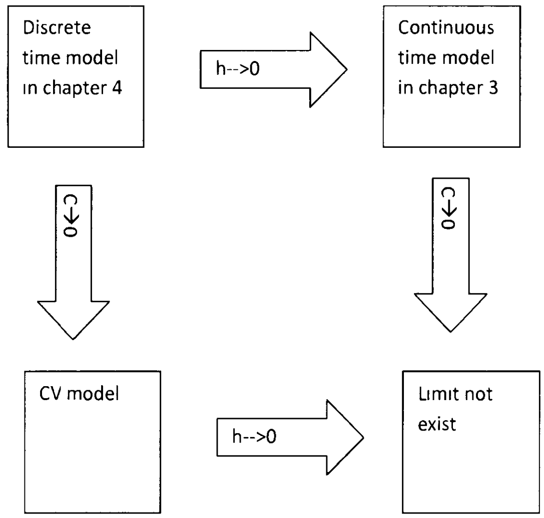

Figure 4-1: relation between the models

only factor keeping the impatient insider away from aggressive trading, the insider's trading intensity increases as transaction cost decreases. Moreover, as transaction cost decreases, price impact increases, and equivalently, market depth decreases. The reason for this is intuitive. As c decreases, the trading volume of the insider $\beta(g - \hat{g})$ increases. The market maker anticipates this in equilibrium, therefore he believes that a larger proportion of the aggregate order flow comes from the insider traders. As a result, the market maker moves prices more responsively to the total order flow. This means that market depth is shallower when transaction cost is lower. Finally when transaction cost is lower, the higher trading volume of the insider carries more information; therefore, the market maker's uncertainty about the signal Σ is less.

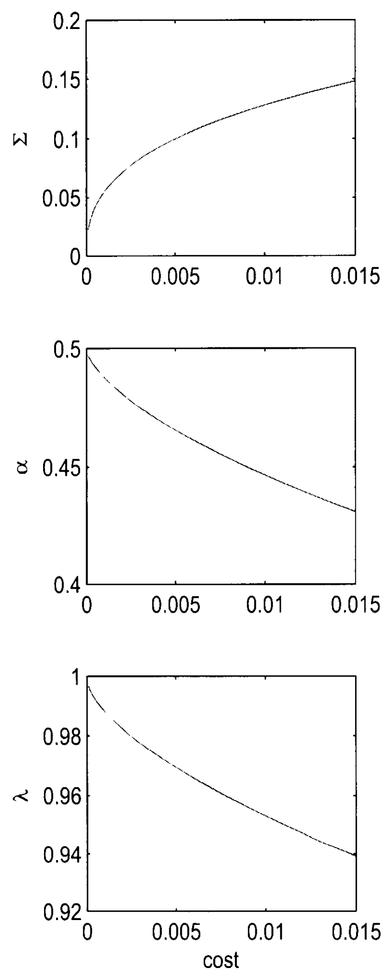

Figure 4-2: Equilibrium with various transaction cost parameters. Σ is the market maker's uncertainty. α and δ are parameters within the insider's value function. β is the insider's rate of trading size. λ is the inverse of market depth.

4.3 Conclusion

In this chapter we examine a discrete time model that nests both the CV model and the continuous trading model in chapter 3. When trading frequency increases to infinity, the model converges to the continuous trading model described in chapter 3. On the other hand, when trading cost approaches zero, the model converges to the CV model. The nesting relation between these models is illustrated in diagram 4-1. The solution to our discrete time model does not attain the strong form efficiency property, as opposed to the CV model. Some details of the comparison have been provided in this chapter.

Chapter 5

Discrete Signals at Random Times

5.1 Discrete Signals at Random Times

As noted in the previous section, close to the end of the trading interval, the insider anticipates the next signal to occur, thus he will submit orders of enormous size to use up his information from the last signal, no matter how tiny his information advantage is. It would be interesting to study a different framework, where the time of the occurrence of the next signal to the insider is not guaranteed. Put it in another way, as each private signal occurs, the insider cannot forecast the occurence of the next signal without uncertainty. Mathematically, the previous model assumes that the length of the time interval between two consecutive signals is deterministic (and fixed for simplicity). Now we want to alter this assumption such that the signals come stochastically. To maintain the stationary market assumption, we require that the random variables representing the arrival time of each new signal g are independent and identically distributed. It is standard to assume that this random variable follows an exponential distribution. However, we adopt a truncated exponential distribution with the following probability density function

$$f(t) = \frac{\phi e^{-\phi t}}{1 - e^{-\phi \bar{T}}}, \quad 0 \leq t \leq \bar{T}.$$

The benefit of this distribution is that it has finite support. The interpretation of this distribution is that if the next signal does not arrive at or before time $\bar{T}$, the insider will produce his own signal at time $\bar{T}$. This can be regarded as a model in which the insider is a firm with a R&D department. For example suppose $\bar{T}$ is two weeks, and the mean of the untruncated exponential distribution is 2 days. What happens then is that the insider expects to receive the next signal at a stochastic time, with the average at 2 days. However, if two weeks pass and no signal occurs, then the firm's R&D department acquires and reveals the signal perfectly. Potentially the acquisition of the signal imposes some cost to the R&D department; however, we ignore that cost in this paper, for simplicity.

Only the assumption on the timing of the private signals is changed. Therefore, in the derivation of the equilibrium, most steps are intact. The insider's optimization problem is the one that needs more careful calculation. As before, the insider's optimization problem is an infinite horizon problem that can be broken down into two parts. First, a discrete time dynamic programming with infinite horizon, and second, countable identical continuous time stochastic control problems within each interval between the two consecutive signals. Suppose the current time is t and this is the time at which the insider receives the l^{th} signal. The Bellman equation for the discrete time dynamic programming is

$$
\begin{aligned}
& V\left(l, g_t - g_t^m\right) \\
&= \sup_{\{\theta_s^t, t \le s \le t+\tau\}} E_t^i \left\{ \int_t^{t+\tau} e^{-r(s-t)} \left[\left(g_s^i - g_s^m\right) \theta_s - c\theta^2 \right] dt + e^{-r\tau} V\left(l+1, g_{t+\tau}, g_{t+\tau}^m\right) \right\}.
\end{aligned}
$$

We can solve this discrete time Bellman equation by considering the continuous time stochastic control problem with finite horizon $[t, t+\tau]$, as in the last section.

$$
\sup_{\{\theta_s^t, 0 \le s \le \tau\}} E_0^i \left\{ \int_0^{\tau} e^{-rs} \left[\left(g_s^i - g_s^m\right) \theta_s - c\theta^2 \right] ds + e^{-r\tau} V\left(l+1, g_\tau, g_\tau^m\right) \right\}
$$

subject to

$$\begin{aligned}
&d\left(g^{\imath}-g^{m}\right)\\
&=-\left[\kappa+\gamma\left(s\right)\nu\right]\left(g^{\imath}-g^{m}\right)dt-\lambda\left(s\right)\theta_{s}dt-\lambda\left(s\right)\sigma_{u}dB^{u}+\left[\Sigma-\Sigma_{11}^{m}\left(s\right)\right]\nu\sigma_{D}^{-1}d\tilde{B}_{s}^{\imath D}.
\end{aligned}$$

However, the horizon is random which brings a new complication. Below, we follow the approach in Richard (1975) to solve this stochastic control problem with uncertain horizon.

Suppose τ has the probability density expressed above. We define the following related functions

$$\begin{aligned}
F\left(t\right)&=\frac{1-e^{-\phi t}}{1-e^{-\phi\bar{T}}},\quad G\left(t\right)=1-F\left(t\right)=\frac{e^{-\phi t}-e^{-\phi\bar{T}}}{1-e^{-\phi\bar{T}}}\\
f\left(T,t\right)&=\frac{f\left(T\right)}{G\left(t\right)}=\frac{\phi e^{-\phi T}}{e^{-\phi t}-e^{-\phi\bar{T}}},T>t\\
h\left(t\right)&=f\left(t,t\right)=\frac{\phi e^{-\phi t}}{e^{-\phi t}-e^{-\phi\bar{T}}}.
\end{aligned}$$

where F is the cumulative distribution function, G is the tail cumulative probability distribution, and f is the conditional probability density for random time $\tau=T$ conditional on $\tau>t$. Finally h is the hazard function in survival analysis. With these notations, it can be shown that in the above per period stochastic control problem, the objective function is equivalent to the following

$$\begin{aligned}
\sup_{\{\theta_{s}^{\imath},t\leq s\leq T\}}E_{t}^{\imath}\frac{1}{G\left(t\right)}\int_{t}^{\infty}&\{G\left(T\right)e^{-rT}\left[\left(g_{T}^{\imath}-g_{T}^{m}\right)\theta_{T}-c\theta_{T}^{2}\right]\\
&+f\left(T\right)e^{-rT}V\left(l+1,g_{T},g_{T}^{m}\right)\}dT.
\end{aligned}$$

We denote the value function by $J\left(s,g_{s}^{\imath}-g_{s}^{m}\right)$. Then, the discrete time Bellman equation is simply

$$V\left(l,g_{0},g_{0}^{m}\right)=J\left(0,g_{0}^{\imath}-g_{0}^{m}\right). \tag{5.1}$$

We will address this equation later. For the finite horizon per period problem, the transversality condition is

$$J\left(\bar{T}, g^{\imath} - g^{m}\right) = e^{-r\bar{T}} E_{\bar{T}}^{\imath} V\left(l+1, g_{\bar{T}}, g_{\bar{T}}^{m}\right). \tag{5.2}$$

Proposition 12 *The value function of the infinite horizon dynamic programming is*

$$V\left(l, g^{\imath} - g^{m}\right) = \alpha\left(0\right)\left(g^{\imath} - g^{m}\right)^{2} + \delta\left(0\right)$$

where α *and* δ *are deterministic functions on interval* $\left[0, \bar{T}\right]$ *satisfying the ordinary differential equations*

$$\begin{aligned}
&- r\alpha\left(s\right) + \alpha'\left(s\right) - 2\alpha\left(s\right)\left\{\left[\kappa + \Sigma_{11}^{m}\left(s\right)\nu^{2}\sigma_{D}^{-2}\right]\right\} \\
&+ c\left\{\frac{\left[1 - 2\alpha\left(s\right)\lambda\left(s\right)\right]}{2c}\right\}^{2} + \frac{\phi e^{-\phi s}}{e^{-\phi s} - e^{-\phi\bar{T}}}\left[\alpha\left(0\right) - \alpha\left(s\right)\right] = 0,
\end{aligned}$$

$$\begin{aligned}
&- r\delta\left(s\right) + \delta'\left(s\right) + \alpha\left(s\right)\left[\left(\lambda\left(s\right)\sigma_{u}\right)^{2} + \left[\Sigma - \Sigma_{11}^{m}\left(s\right)\right]^{2}\nu^{2}\sigma_{D}^{-2}\right] \\
&+ \frac{\phi e^{-\phi s}}{e^{-\phi s} - e^{-\phi\bar{T}}}\left[\delta\left(0\right) - \delta\left(s\right) + \alpha\left(0\right)\Sigma\left(s\right)\right] = 0,
\end{aligned}$$

with the terminal value conditions

$$\begin{aligned}
\alpha\left(\bar{T}\right) &= \alpha\left(0\right), \\
\delta\left(\bar{T}\right) &= \delta\left(0\right) + \alpha\left(0\right)\Sigma\left(\bar{T}\right).
\end{aligned}$$

The insider's optimal control is given by

$$\theta_{s} = \frac{1 - 2\alpha\left(s\right)\lambda\left(s\right)}{2c}\left(g^{\imath} - g^{m}\right).$$

Figure 5-1 shows the plot with parameters $c = 0.004$, $\phi = 1$ and $\bar{T} = 2$. We mainly compare this plot with figure 2-2. In the previous model, the next signal will arrive at time $t = 1$. Now our assumptions are such that once a signal occurs, the arrival time of the next signal is distributed as a truncated exponential. with cumulative distribution function plotted in the lower-left panel. In other words, the next signal is only guaranteed to arrive before $t = 2$, however, it can arrive before any time t for $0 < t < 2$ with positive probability. It is clear that this randomness of signal arrival gives the insider the incentive to trade more aggressively at the beginning of the interval between signals, since he expects the new signal to arrive at any time. On the other hand, if the new signal has not arrived at time $t \geq 1$, then the insider will not raise his trading intensity rapidly by a large amount since he is still not sure when the next signal will occur, as opposed to the previous model with fixed arrival time. in which the insider will rapidly escalate his trading intensity just before the new signal arrives at $t = 1$. In summary, the new pattern for the insider's trading is that trading intensity is smoother with stochastic arrival of signals. Moreover, this further explains that the information asymmetry between the insider and the market maker declines more quickly in the new model.

5.2 Conclusion

We examine the case in which the arrival time of the next signal is not deterministic. With the arrival time following the truncated exponential distribution. we show that the insider trades more smoothly. He trades more aggressively right after receiving a new private signal, and less aggressively as time passes by, relative to the case in which signal arrival time is a fixed constant.

There remain several unanswered questions. One involves separation of the effect of risk aversion of the market maker from risk aversion of the insider. Either one may induce the insider to trade slowly. However. in Wang (1993) both market maker and insider are risk averse. It is interesting to study the properties of the equilibrium when only one of them is risk averse. Intuitively, risk aversion of the market maker means he will charge more for large orders, for bearing inventory risk. Risk aversion of the insider, on the other

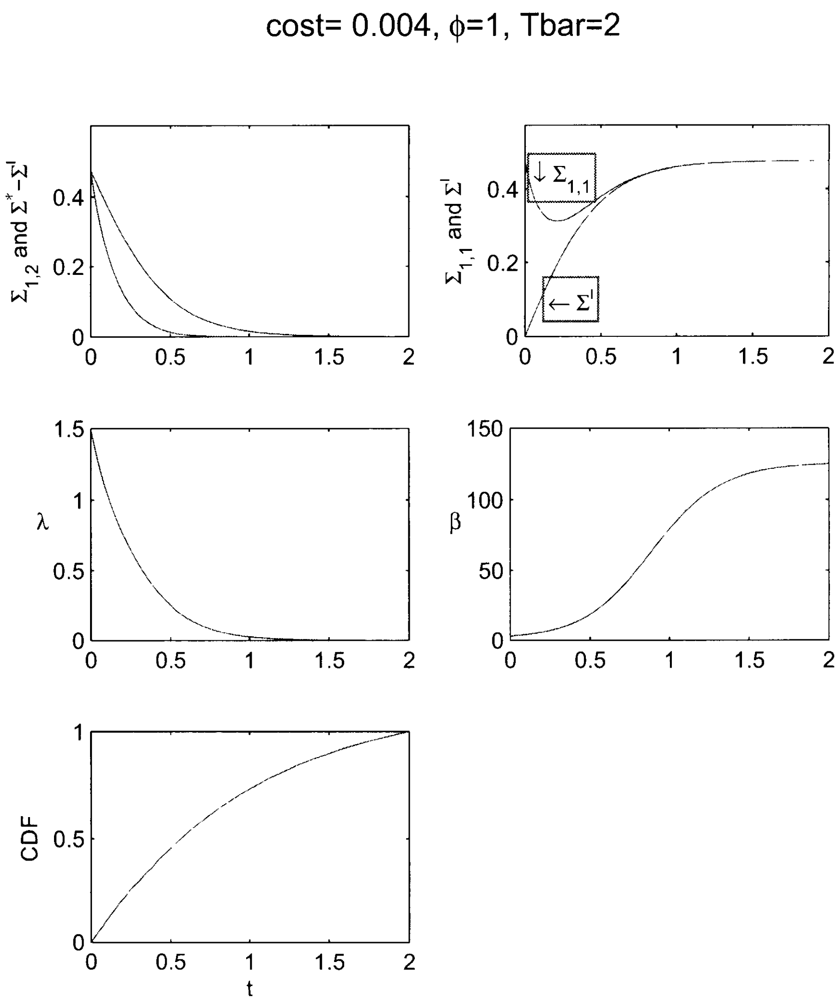

Figure 5-1: The insider receives signals at random times. The arrival time of the signal has a truncated exponential distribution with parameters $\phi = 1$, and $\bar{T} = 2$. The cost coefficient is $c = 0.004$.

hand, means that the insider will refrain from submitting large order flows since he not only seeks to utilize his private information, but also needs to take into account the precision of that private information. This concern limits his trading aggressiveness.

Appendix A

Market Maker's Pricing Rule

Proof of the market maker's pricing rule.

Proof. Under our assumption, the dividend growth is a mean-reverting process. The solution to the Orstein-Ulenbeck process is

$$g_s = e^{-K(s-t)} g_t + \left(1 - e^{-K(s-t)}\right) \bar{g} + \int_t^s e^{-K(s-u)} \sigma_g dB_u^g$$

and therefore the conditional expectation given time t filtration is

$$E_t g_s = e^{-K(s-t)} E_t [g_t] + \left(1 - e^{-K(s-t)}\right) \bar{g}.$$

To solve the SDE for the dividend process D_t, we use a integrating multiplier

$$\begin{aligned} d\left[e^{\nu t} D_t\right] &= \nu e^{\upsilon t} D_t dt + e^{\nu t} \left(\nu \left(g_t - D_t\right) dt + \sigma_D dB^D\right) \\ &= e^{\nu t} \left(\nu g_t dt + \sigma_D dB^D\right) \\ e^{\nu T} D_T &= e^{\nu t} D_t + \int_t^T e^{\nu s} \left(\nu g_s ds + \sigma_D dB_s^D\right). \end{aligned}$$

This implies that the dividend process can be represented by

$$D_T = e^{-\nu(T-t)}D_t + \int_t^T e^{-\nu(T-s)} \left[\nu g_s ds + \sigma_D dB_s^D\right].$$

Take conditional expectation we have

$$\begin{aligned}
E_t D_T &= e^{-\nu(T-t)}D_t + \int_t^T e^{-\nu(T-s)} \nu E_t\left[g_s\right] ds \\
&= e^{-\nu(T-t)}D_t + \nu \int_t^T e^{-\nu(T-s)} \left[e^{-K(s-t)}\hat{g}_t + \left(1 - e^{-K(s-t)}\right)\bar{g}\right] ds \\
&= e^{-\nu(T-t)}D_t + \nu\left(\hat{g}_t - \bar{g}\right) \int_t^T e^{-\nu(T-s)} e^{-K(s-t)} ds + \nu\bar{g} \int_t^T e^{-\nu(T-s)} ds \\
&= e^{-\nu(T-t)}D_t + \frac{\nu}{\nu - K}\left(e^{-K(T-t)} - e^{-\nu(T-t)}\right)\left(\hat{g}_t - \bar{g}\right) \\
&+ \bar{g}\left(1 - e^{-\nu(T-t)}\right)
\end{aligned}$$

Market maker sets the price to be the discounted future dividend payout

$$\begin{aligned}
P_t &= E_t \int_t^\infty e^{-r(T-t)} D_T dT \\
&= \frac{1}{\nu + r}D_t + \frac{\nu}{\nu - K}\left(\frac{1}{r + K} - \frac{1}{r + \nu}\right)\left(\hat{g}_t - \bar{g}\right) + \left(\frac{1}{r} - \frac{1}{r + \nu}\right)\bar{g} \\
&= \frac{1}{\nu + r}D_t + \frac{\nu}{(r + K)(\nu + r)}\left(\hat{g}_t - \bar{g}\right) + \left(\frac{1}{r} - \frac{1}{r + \nu}\right)\bar{g} \\
&= \frac{1}{\nu + r}D_t + \frac{\nu}{(r + K)(\nu + r)}\hat{g}_t + \left(\frac{1}{r} - \frac{1}{r + \nu} - \frac{\nu}{(r + K)(\nu + r)}\right)\bar{g} \\
&= \frac{1}{\nu + r}D_t + \frac{\nu}{(r + K)(\nu + r)}\hat{g}_t + \frac{\nu K}{r(r + \nu)(K + r)}\bar{g}
\end{aligned}$$

■

Appendix B

Proof to the Propositions

Proof of proposition 7

Proof. By standard Kalman-Bucy filtering technique (for example see Wang (1993) or Liptser and Shiryaev (2001)), we have

$$\begin{aligned} d\hat{g} &= \kappa\left(\bar{g} - \hat{g}\right)dt + \Sigma\left(t\right)\left(\beta, \nu\right)\begin{bmatrix} \sigma_u^{-2} & 0 \\ 0 & \sigma_D^{-2} \end{bmatrix}\left(d\begin{bmatrix} \tilde{Y} \\ \tilde{D} \end{bmatrix} - \begin{bmatrix} \beta\left(t\right) \\ \nu \end{bmatrix}\hat{g}dt\right) \\ &= \kappa\left(\bar{g} - \hat{g}\right)dt \\ &+ \Sigma\left(t\right)\left\{\frac{\beta\left(t\right)}{\sigma_u^2}\left[\beta\left(t\right)\left(g - \hat{g}\right)dt + \sigma_u dZ^u\right] + \frac{\nu}{\sigma_D^2}\left[\nu\left(g - \hat{g}\right)dt + \sigma_D dB^D\right]\right\} \end{aligned} \tag{B.1}$$

and

$$\Sigma'\left(t\right) = -2\kappa\Sigma\left(t\right) + \sigma_g^2 - \Sigma^2\left(t\right)\left(\frac{\beta\left(t\right)^2}{\sigma_u^2} + \frac{\nu^2}{\sigma_D^2}\right) \tag{B.2}$$

Proposition follows once λ_t and γ_t are defined. ■

Proof of proposition 8 (insider's optimization)

Proof. Let $J\left(t, g, \hat{g}\right)$ be the value function, then we can write the HJB equation of the insider as

$$
\begin{aligned}
0 = \sup J_t + J_g K(\bar{g} - g_t) + \frac{1}{2} J_{gg} (\sigma_g)^2 + J_{\hat{g}} \left[K(\bar{g} - \hat{g}) + \lambda\theta + \gamma\nu (g - \hat{g}) \right] \\
+ \frac{1}{2} J_{\hat{g}\hat{g}} \left(\lambda^2 (t) \sigma_u^2 + \gamma^2 \sigma_D^2 \right) + \left[(g - \hat{g}) \theta_t - c |\theta|^2 - \bar{c} \right] e^{-rt}
\end{aligned}
$$

First order condition is given by

$$
J_{\hat{g}} \lambda + (g - \hat{g} - 2c\theta) e^{-rt} = 0
$$

$$
\theta = \frac{1}{2c} \left(J_{\hat{g}} \lambda e^{rt} + g - \hat{g} \right) \tag{B.3}
$$

Now conjecture the value function is

$$
J(t, g, \hat{g}) = e^{-rt} \alpha(t) (g - \hat{g})^2 + e^{-rt} \delta(t).
$$

We can derive the following properties of the value function.

$$
\begin{aligned}
J_t &= -re^{-rt} \alpha(t) (g - \hat{g})^2 - re^{-rt} \delta \\
J_{\hat{g}} &= -2e^{-rt} \alpha (g - \hat{g}) \\
J_{\hat{g}\hat{g}} &= 2e^{-rt} \alpha
\end{aligned} \tag{B.4}
$$

Substitute back into equation (B.3) we can get

$$
\theta = \frac{1}{2c} (g - \hat{g}) (-2\alpha\lambda + 1) \tag{B.5}
$$

Plug the value function and the solution to the control variable θ_t back into HJB equation and we get

$$\begin{aligned}0 &= -re^{-rt}\alpha(t)(g-\hat{g})^2 - re^{-rt}\delta + 2e^{-rt}\alpha(g-\hat{g})K(\bar{g}-g_t) + \alpha e^{-rt}\sigma_g^2 \\ &- 2e^{-rt}\alpha(g-\hat{g})\left[K(\bar{g}-\hat{g}) + \frac{\lambda}{2c}(g-\hat{g})(-2\alpha\lambda+1) + \gamma\nu(g-\hat{g})\right] \\ &+ \alpha e^{-rt}\left(\lambda^2(t)\sigma_u^2 + \gamma^2\sigma_D^2\right) \\ &+ e^{-rt}\left\{\left[\frac{1}{2c}(g-\hat{g})^2(-2\alpha\lambda+1) - c\left|\frac{1}{2c}(g-\hat{g})(-2\alpha\lambda+1)\right|^2\right] - \bar{c}\right\}\end{aligned}$$

Simplify the above identity and we have

$$\begin{aligned}0 &= -r\alpha(g-\hat{g})^2 - 2\alpha(g-\hat{g})^2\left[\frac{\lambda}{2c}(-2\alpha\lambda+1) + \gamma\nu\right] - 2\alpha\kappa(g-\hat{g})^2 \\ &+ (g-\hat{g})^2\left[\frac{1}{2c}(-2\alpha\lambda+1) - c\left(\frac{1}{2c}(-2\alpha\lambda+1)\right)^2\right] \\ &- r\delta + \alpha\sigma_g^2 + \alpha\left(\lambda^2(t)\sigma_u^2 + \gamma^2\sigma_D^2\right) - \bar{c}\end{aligned}$$

There are two terms. The first term is a quadratic term of the estimation error of the market maker $(g-\hat{g})^2$, and the second term is a constant. Letting both equal to zero we obtain the following two equations

$$0 = -r\delta + \alpha\sigma_g^2 + \alpha\left(\lambda^2(t)\sigma_u^2 + \gamma^2\sigma_D^2\right) - \bar{c}$$

$$\begin{aligned}0 &= -r\alpha - 2\alpha\left[\frac{\lambda}{2c}(-2\alpha\lambda+1) + \gamma\nu + \kappa\right] \\ &+ \left[\frac{1}{2c}(-2\alpha\lambda+1) - c\left(\frac{1}{2c}(-2\alpha\lambda+1)\right)^2\right]\end{aligned}$$

where the second equation can be shown to be equivalent to the one given in the proposition.

■

proof of proposition 9.

Proof. Recall the definition of the notations

$$\lambda = \frac{\Sigma\beta}{\sigma_u^2}, \gamma = \frac{\Sigma\nu}{\sigma_D^2}$$

We can see that $\beta = \lambda\sigma_u^2/\Sigma$. In proposition (8), insider's strategy is given by $\beta = \frac{1}{2c}(-2\alpha\lambda + 1)$. By equating the two expressions, we find

$$\frac{\lambda\sigma_u^2}{\Sigma} = \frac{1 - 2\alpha\lambda}{2c}$$

Therefore we can solve α as a function of some other parameters

$$\alpha = \frac{1}{2\lambda}\left(1 - \frac{2c\lambda\sigma_u^2}{\Sigma}\right) \tag{B.6}$$

In the steady state, $\Sigma(t)$ is a constant, recall that equation (B.2) implies equation (3.7)

$$0 = -2K\Sigma + \sigma_g^2 - \Sigma^2\left(\frac{\lambda^2\sigma_u^2}{\Sigma^2} + \frac{\nu^2}{\sigma_D^2}\right)$$

which is equivalent to

$$0 = -2K\Sigma + \sigma_g^2 - \left(\lambda^2\sigma_u^2 + \frac{\nu^2}{\sigma_D^2}\Sigma^2\right)$$

$$\lambda^2 = \frac{1}{\sigma_u^2}\left(-2K\Sigma + \sigma_g^2 - \frac{\nu^2}{\sigma_D^2}\Sigma^2\right) \tag{B.7}$$

Equation (3.9) can be expressed as

$$0 = -r\delta + 2\alpha\sigma_g^2 - 2\alpha K\Sigma - \bar{c}$$

$$\delta = \frac{1}{r}\left(2\alpha\sigma_g^2 - 2\alpha K\Sigma - \bar{c}\right) \tag{B.8}$$

Equation (3.10) is equivalent to

$$
\begin{aligned}
0 &= -r\alpha - 2\alpha\left(\frac{\Sigma\nu^2}{\sigma_D^2}+\kappa\right)+\frac{1}{4c}\left(-2\alpha\lambda+1\right)^2 \\
0 &= -r\alpha - 2\alpha\left(\frac{\Sigma\nu^2}{\sigma_D^2}+\kappa\right)+\frac{1}{4}\left(\frac{\Sigma\left(1-2\alpha\lambda\right)}{2\lambda\sigma_u^2}\right)^{-1}\left(-2\alpha\lambda+1\right)^2 \\
0 &= -r\alpha - 2\alpha\left(\frac{\Sigma\nu^2}{\sigma_D^2}+\kappa\right)+\frac{\lambda\sigma_u^2}{2\Sigma}\left(-2\alpha\lambda+1\right) \\
0 &= \Sigma\left(\Sigma-2c\lambda\sigma_u^2\right)\left(r+2\kappa+2\Sigma\nu^2\sigma_D^{-2}\right)-2c\lambda^3\sigma_u^4 \qquad \text{(B.9)}
\end{aligned}
$$

The above four equations (B.6)(B.7)(B.8)(B.9) can jointly determine the steady state equilibrium values of $(\Sigma, \lambda, \alpha, \delta)$. The remain two parameters (β, γ) follows. ■ Proof of proposition 10. **Proof.** The market maker's inference is the same as the Chau and Vayanos (2007) setup, therefore we have

$$
\hat{g}_l = \left(1-\kappa h\right)\hat{g}_{l-1}+\kappa h\bar{g}+\lambda_D\left(D_l-\left(1-\nu h\right)D_{l-1}-\nu h\hat{g}_{l-1}\right)+\lambda_x\left(x_l+u_l\right)
$$

where

$$
\begin{aligned}
\lambda_D &= \frac{\left(1-\kappa h\right)\Sigma_g\nu\sigma_u^2 h}{\Sigma_g\left(\beta^2\sigma_D^2+\nu^2\sigma_u^2h^2\right)+\sigma_D^2\sigma_u^2h} \\
\lambda_x &= \frac{\left(1-\kappa h\right)\beta\Sigma_g\sigma_D^2}{\Sigma_g\left(\beta^2\sigma_D^2+\nu^2\sigma_u^2h^2\right)+\sigma_D^2\sigma_u^2h} \\
\Sigma_g &= \frac{\left(1-\kappa h\right)^2\Sigma_g\sigma_D^2\sigma_u^2h}{\Sigma_g\left(\beta^2\sigma_D^2+\nu^2\sigma_u^2h^2\right)+\sigma_D^2\sigma_u^2h}+\sigma_g^2h
\end{aligned}
$$

The Bellman equation is

$$
V\left(g_{l-1}-\hat{g}_{l-1}\right)=\max_{x_l}E\left[x_l\left(g_l-\hat{g}_l\right)-ch^{-1}x_l^2+e^{-rh}V\left(g_l,\hat{g}_l\right)\right]
$$

Market Maker's estimation error in period l is

$$g_l - \hat{g}_l = [1 - (\kappa + \nu\lambda_D)\, h]\,(g_{l-1} - \hat{g}_{l-1}) - \lambda_D \varepsilon_{D,l} - \lambda_x (x_l + u_l) + \varepsilon_{g,l}$$

Substituting into the Bellman equation, we find

$$\begin{aligned}
& B\,(g_{l-1} - \hat{g}_{l-1})^2 + C \\
&= \max_{x_l} x_l \left\{ [1 - (\kappa + \nu\lambda_D)\, h]\,[g_{l-1} - \hat{g}_{l-1}] - \lambda_x x_l \right\} - c h^{-1} x_l^2 \\
&+ e^{-rh} \left\{ B \left[([1 - (\kappa + \nu\lambda_D)\, h]\,[g_{l-1} - \hat{g}_{l-1}] - \lambda_x x_l)^2 + \lambda_D^2 \sigma_D^2 h + \lambda_x^2 \sigma_u^2 h + \sigma_g^2 h \right] + C \right\}
\end{aligned}$$

The first order equation is

$$\begin{aligned}
0 &= [1 - (\kappa + \nu\lambda_D)\, h]\,(g_{l-1} - \hat{g}_{l-1}) - 2\lambda_x x_l - 2ch^{-1} x_l \\
&\quad - 2\lambda_x e^{-rh} B\,([1 - (\kappa + \nu\lambda_D)\, h]\,(g_{l-1} - \hat{g}_{l-1}) - \lambda_x x_l)
\end{aligned}$$

which implies

$$x_l = \beta\,(g_{l-1} - \hat{g}_{l-1})$$

with

$$\beta = \frac{[1 - (\kappa + \nu\lambda_D)\, h]\,(1 - 2\lambda_x e^{-rh} B)}{2\,(\lambda_x + ch^{-1} - \lambda_x^2 e^{-rh} B)}$$

Substitute into Bellman equation, we have

$$\begin{aligned}
& B\,(g_{l-1} - \hat{g}_{l-1})^2 + C \\
&= e^{-rh} B\,(\lambda_D^2 \sigma_D^2 h + \lambda_x^2 \sigma_u^2 h + \sigma_g^2 h) + e^{-rh} C \\
&+ \frac{[1 - (\kappa + \nu\lambda_D)\, h]^2\,(1 - 2\lambda_x e^{-rh} B)^2}{4\,(\lambda_x + ch^{-1} - \lambda_x^2 e^{-rh} B)}\,(g_{l-1} - \hat{g}_{l-1})^2 \\
&+ e^{-rh} B\,[1 - (\kappa + \nu\lambda_D)\, h]^2\,[g_{l-1} - \hat{g}_{l-1}]^2
\end{aligned}$$

which yields

$$
\begin{aligned}
B &= \frac{\left[1-(\kappa+\nu\lambda_D)\,h\right]^2 \left(1-2\lambda_x e^{-rh}B\right)^2}{4\left(\lambda_x + ch^{-1} - \lambda_x^2 e^{-rh}B\right)} + e^{-rh}B\left[1-(\kappa+\nu\lambda_D)\,h\right]^2 \\
&= \left[1-(\kappa+\nu\lambda_D)\,h\right]^2 \left(\frac{\left(1-2\lambda_x e^{-rh}B\right)^2 + 4e^{-rh}B\left(\lambda_x + ch^{-1} - \lambda_x^2 e^{-rh}B\right)}{4\left(\lambda_x + ch^{-1} - \lambda_x^2 e^{-rh}B\right)}\right) \\
&= \left[1-(\kappa+\nu\lambda_D)\,h\right]^2 \frac{1+4e^{-rh}Bch^{-1}}{4\left(\lambda_x + ch^{-1} - \lambda_x^2 e^{-rh}B\right)} \\
C &= \frac{1}{1-e^{-rh}} e^{-rh}B\left(\lambda_D^2\sigma_D^2 + \lambda_x^2\sigma_u^2 + \sigma_g^2\right)h
\end{aligned}
$$

Now we write the equation for B again

$$
4\lambda_x^2 e^{-rh}B^2 + 4\left\{e^{-rh}ch^{-1}\left[1-(\kappa+\nu\lambda_D)\,h\right]^2 - \left(\lambda_x + ch^{-1}\right)\right\}B + \left[1-(\kappa+\nu\lambda_D)\,h\right]^2 = 0
$$

The quadratic equation has solution

$$
\begin{aligned}
B &= \frac{1}{2\lambda_x^2 e^{-rh}}\left[-\left\{e^{-rh}ch^{-1}\left[1-(\kappa+\nu\lambda_D)\,h\right]^2 - \left(\lambda_x + ch^{-1}\right)\right\} \pm \sqrt{\Delta}\right] \\
\Delta &= \left\{e^{-rh}ch^{-1}\left[1-(\kappa+\nu\lambda_D)\,h\right]^2 - \left(\lambda_x + ch^{-1}\right)\right\}^2 - \lambda_x^2 e^{-rh}\left[1-(\kappa+\nu\lambda_D)\,h\right]^2
\end{aligned}
$$

■

Proof of proposition 11.

Proof. Equation for β becomes

$$
\frac{\beta}{h} = \frac{1-2\lambda_x B}{2c}
$$

Equation for λ_x becomes

$$
\lambda_x = \frac{\Sigma_g}{\sigma_u^2}\frac{\beta}{h}
$$

The above two equations imply

$$
1-2\lambda_x B = \frac{2c\lambda_x\sigma_u^2}{\Sigma_g}
$$

Equation for λ_D becomes

$$\lambda_D = \frac{\Sigma_g \nu}{\sigma_D^2}$$

Equation for Σ_g becomes

$$\lambda_x^2 \sigma_u^2 \sigma_D^2 + \Sigma_g^2 \nu^2 - \sigma_g^2 \sigma_D^2 + 2\kappa \Sigma_g \sigma_D^2 = 0$$

When $h = 0$, equation for B becomes

$$(1 - 2\lambda_x B)^2 = 4cB\left[r + 2\kappa + 2\lambda_D \nu\right]$$
$$\left(\frac{2c\lambda_x \sigma_u^2}{\Sigma_g}\right)^2 = 2c\frac{\Sigma_g - 2c\lambda_x \sigma_u^2}{\lambda_x \Sigma_g}\left[r + 2\kappa + 2\frac{\Sigma_g \nu^2}{\sigma_D^2}\right]$$

Equation for C becomes

$$C = \frac{1}{r}B\left(\lambda_D^2 \sigma_D^2 + \lambda_x^2 \sigma_u^2 + \sigma_g^2\right)$$

This finishes the proof ■

Proof of proposition 2.

Proof. Formally

$$dg_s = \kappa\left(\bar{g} - g_s\right) ds + \sigma_g dB_s^g$$
$$d\tilde{D}_s = \nu g_s dt + \sigma_D dB^D$$

■

Proof of proposition 3.

Proof. Then Market maker's filtering is

$$d\begin{bmatrix} g \\ g^\imath \end{bmatrix} = \begin{bmatrix} \kappa\bar{g} \\ \kappa\bar{g} \end{bmatrix} dt + \begin{bmatrix} -\kappa & 0 \\ \Sigma\nu^2\sigma_D^{-2} & -\kappa - \Sigma\nu^2\sigma_D^{-2} \end{bmatrix} \begin{bmatrix} g \\ g^\imath \end{bmatrix} dt + \begin{bmatrix} 0 & 0 & \sigma_g \\ \Sigma\nu\sigma_D^{-1} & 0 & 0 \end{bmatrix} \begin{bmatrix} dB^D \\ dB^u \\ dB^g \end{bmatrix}$$

and the observed variables are

$$d\begin{bmatrix}\tilde{D}\\ \tilde{Y}\end{bmatrix}=\begin{bmatrix}\nu & 0\\ 0 & \beta_s\end{bmatrix}\begin{bmatrix}g\\ g^{\imath}\end{bmatrix}dt+\begin{bmatrix}\sigma_D & 0 & 0\\ 0 & \sigma_u & 0\end{bmatrix}\begin{bmatrix}dB^D\\ dB^u\\ dB^g\end{bmatrix}$$

The solution to the filtering is

$$\begin{aligned}
d\begin{bmatrix}g^m\\ g^{\imath m}\end{bmatrix} &= \begin{bmatrix}\kappa\bar{g}\\ \kappa\bar{g}\end{bmatrix}dt+\begin{bmatrix}-\kappa & 0\\ \Sigma\nu^2\sigma_D^{-2} & -\kappa-\Sigma\nu^2\sigma_D^{-2}\end{bmatrix}\begin{bmatrix}g^m\\ g^{\imath m}\end{bmatrix}dt\\
&+\left(\Sigma_s^m\begin{bmatrix}\nu & 0\\ 0 & \beta_s\end{bmatrix}+\begin{bmatrix}0 & 0\\ \Sigma\nu & 0\end{bmatrix}\right)\begin{bmatrix}\sigma_D^2 & 0\\ 0 & \sigma_u^2\end{bmatrix}^{-1}\\
&\left(\begin{bmatrix}\nu & 0\\ 0 & \beta_s\end{bmatrix}\begin{bmatrix}g-g^m\\ g^{\imath}-g^{\imath m}\end{bmatrix}dt+\begin{bmatrix}\sigma_D & 0 & 0\\ 0 & \sigma_u & 0\end{bmatrix}\begin{bmatrix}dB^D\\ dB^u\\ dB^g\end{bmatrix}\right)\\
&= \begin{bmatrix}\kappa\bar{g}\\ \kappa\bar{g}\end{bmatrix}dt+\begin{bmatrix}-\kappa & 0\\ \Sigma\nu^2\sigma_D^{-2} & -\kappa-\Sigma\nu^2\sigma_D^{-2}\end{bmatrix}\begin{bmatrix}g^m\\ g^{\imath m}\end{bmatrix}dt\\
&+\begin{bmatrix}\Sigma_{11}^m(s)\,\nu^2\sigma_D^{-2} & \Sigma_{12}^m(s)\,\beta_s^2\sigma_u^{-2}\\ \left[\Sigma_{12}^m(s)+\Sigma\right]\nu^2\sigma_D^{-2} & \Sigma_{22}^m(s)\,\beta_s^2\sigma_u^{-2}\end{bmatrix}\begin{bmatrix}g-g^m\\ g^{\imath}-g^{\imath m}\end{bmatrix}dt\\
&+\begin{bmatrix}\Sigma_{11}^m(s)\,\nu\sigma_D^{-1} & \Sigma_{12}^m(s)\,\beta_s\sigma_u^{-1}\\ \left[\Sigma_{12}^m(s)\,\nu+\Sigma\nu\right]\sigma_D^{-1} & \Sigma_{22}^m(s)\,\beta_s\sigma_u^{-1}\end{bmatrix}\begin{bmatrix}dB^D\\ dB^u\end{bmatrix}
\end{aligned}$$

and

$$d\begin{bmatrix}\tilde{B}^{m1}\\ \tilde{B}^{m2}\end{bmatrix}=\begin{bmatrix}\nu\sigma_D^{-1} & 0\\ 0 & \beta_s\sigma_u^{-1}\end{bmatrix}\begin{bmatrix}g-g^m\\ g^{\imath}-g^{\imath m}\end{bmatrix}dt+\begin{bmatrix}dB^D\\ dB^u\end{bmatrix}$$

$$\frac{d\Sigma_s^m}{ds} = \begin{bmatrix} -\kappa & 0 \\ \Sigma\nu^2\sigma_D^{-2} & -\kappa - \Sigma\nu^2\sigma_D^{-2} \end{bmatrix} \Sigma^m + \Sigma^m \begin{bmatrix} -\kappa & 0 \\ \Sigma\nu^2\sigma_D^{-2} & -\kappa - \Sigma\nu^2\sigma_D^{-2} \end{bmatrix}^T$$
$$+ \begin{bmatrix} \sigma_g^2 & 0 \\ 0 & \Sigma^2\nu^2\sigma_D^{-2} \end{bmatrix}$$
$$- \left(\Sigma_s^m \begin{bmatrix} \nu & 0 \\ 0 & \beta_s \end{bmatrix} + \begin{matrix} 0 & 0 \\ \Sigma\nu & 0 \end{matrix} \right) \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_u^2 \end{bmatrix}^{-1} \left(\begin{bmatrix} \nu & 0 \\ 0 & \beta_s \end{bmatrix} \Sigma_s^m + \begin{matrix} 0 & \Sigma\nu \\ 0 & 0 \end{matrix} \right)$$

Simplify the Riccati equation,

$$\frac{d\Sigma_s^m}{ds} = \begin{bmatrix} -\kappa & 0 \\ \Sigma\nu^2\sigma_D^{-2} & -\left(\kappa + \Sigma\nu^2\sigma_D^{-2}\right) \end{bmatrix} \begin{bmatrix} \Sigma_{11}^m & \Sigma_{12}^m \\ \Sigma_{12}^m & \Sigma_{22}^m \end{bmatrix} + \Sigma^m \begin{bmatrix} -\kappa & 0 \\ \Sigma\nu^2\sigma_D^{-2} & -\kappa - \Sigma\nu^2\sigma_D^{-2} \end{bmatrix}^T$$
$$+ \begin{bmatrix} \sigma_g^2 & 0 \\ 0 & \Sigma^2\nu^2\sigma_D^{-2} \end{bmatrix} - \begin{bmatrix} \Sigma_{11}^m\nu & \Sigma_{12}^m\beta_s \\ \Sigma_{12}^m\nu + \Sigma\nu & \Sigma_{22}^m\beta_s \end{bmatrix} \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_u^2 \end{bmatrix}^{-1} \begin{bmatrix} \Sigma_{11}^m\nu & \Sigma_{12}^m\nu + \Sigma\nu \\ \Sigma_{12}^m\beta_s & \Sigma_{22}^m\beta_s \end{bmatrix}$$
$$= \begin{bmatrix} -2\kappa\Sigma_{11}^m & -2\kappa\Sigma_{12}^m + \Sigma\nu^2\sigma_D^{-2}\Sigma_{11}^m - \Sigma\nu^2\sigma_D^{-2}\Sigma_{12}^m \\ -2\kappa\Sigma_{12}^m + \Sigma\nu^2\sigma_D^{-2}\Sigma_{11}^m - \Sigma\nu^2\sigma_D^{-2}\Sigma_{12}^m & 2\Sigma\nu^2\sigma_D^{-2}\Sigma_{12}^m - 2\left(\kappa + \Sigma\nu^2\sigma_D^{-2}\right)\Sigma_{22}^m \end{bmatrix}$$
$$+ \begin{bmatrix} \sigma_g^2 & 0 \\ 0 & \Sigma^2\nu^2\sigma_D^{-2} \end{bmatrix}$$
$$- \begin{bmatrix} \Sigma_{11}^m\nu^2\sigma_D^{-2} + \Sigma_{12}^m\beta_s^2\sigma_u^{-2} & \Sigma_{11}^m\nu\sigma_D^{-2}\Sigma_{12}^m\nu + \Sigma\nu + \Sigma_{12}^m\Sigma_{22}^m\beta_s^2\sigma_u^{-2} \\ \left(\Sigma_{12}^m\nu + \Sigma\nu\right)\sigma_D^{-2}\Sigma_{11}^m\nu + \Sigma_{22}^m\Sigma_{12}^m\beta_s^2\sigma_u^{-2} & \left(\Sigma_{12}^m\nu + \Sigma\nu\right)\sigma_D^{-2}\left(\Sigma_{12}^m\nu + \Sigma\nu\right) + \left(\Sigma_{22}^m\beta_s\right)^2\sigma_u^{-2} \end{bmatrix}$$

and therefore

$$\frac{d\Sigma_{11}^m}{ds} = -2\kappa\Sigma_{11}^m + \sigma_g^2 - \left(\Sigma_{11}^m\right)^2\nu^2\sigma_D^{-2} - \left(\Sigma_{12}^m\beta_s\right)^2\sigma_u^{-2}$$
$$\frac{d\Sigma_{12}^m}{ds} = -2\kappa\Sigma_{12}^m - \Sigma\nu^2\sigma_D^{-2}\Sigma_{12}^m - \nu^2\sigma_D^{-2}\Sigma_{11}^m\Sigma_{12}^m - \Sigma_{22}^m\Sigma_{12}^m\beta_s^2\sigma_u^{-2}$$
$$\frac{d\Sigma_{22}^m}{ds} = -2\left(\kappa + \Sigma\nu^2\sigma_D^{-2}\right)\Sigma_{22}^m - \left(\Sigma_{12}^m\right)^2\nu^2\sigma_D^{-2} - \left(\Sigma_{22}^m\beta_s\right)^2\sigma_u^{-2}$$

Think about the infinite steps. At the beginning of each interval $[t, t+\Delta]$, insider starts a brand new filtering problem, with initial observation g_t, and initial $\Sigma(t) = 0$. The Market maker. also starts a new filtering problem, however, he knows that $g_t = g_t^\imath$, therefore, at initial time $s = t$, he would force $g_t|\mathcal{F}_t^m = g_t^\imath|\mathcal{F}_t^m$ and this implies

$$E_t \begin{bmatrix} g_t \\ g_t^\imath \end{bmatrix} = \begin{bmatrix} g_t^m \\ g_t^m \end{bmatrix}$$
$$Var\left(\begin{bmatrix} g \\ g^\imath \end{bmatrix}\right) = \Sigma^m(t) = \begin{bmatrix} \Sigma_{11}^m(t) & \Sigma_{11}^m(t) \\ \Sigma_{11}^m(t) & \Sigma_{11}^m(t) \end{bmatrix}$$

Now we just need to determine g_t^m and $\Sigma_{11}^m(t)$. Suppose that he uses the last interval result on g (discards results on $g^\imath$), that is

$$g_t^m = g_{t-}^m$$
$$\Sigma_{11}^m(t) = \Sigma_{11}^m(t-)$$

If this is the case, then we can show that for all s the following hold

$$\Sigma_{11}^m(s) = \Sigma_{12}^m(s) + \Sigma(s)$$
$$\Sigma_{12}^m(s) = \Sigma_{22}^m(s)$$

and further more,

$$g^m(s) = g^{\imath m}(s)$$

holds for all $s \in [t, t+\Delta]$. To satisfy the stationary condition, one sufficient condition is that

$$\int_t^{t+\Delta} d\Sigma_{11}^m(s) = 0$$

and therefore at each end point of the interval $\Sigma_{11}^{m}(t+n\Delta)\equiv\Sigma_{11}^{m}$. It is clear that

$$dg^{m}=\kappa(\bar{g}-g^{m})\,dt+\left[\begin{array}{cc}\Sigma_{11}^{m}\nu\sigma_{D}^{-2} & \Sigma_{12}^{m}\beta_{s}\sigma_{u}^{-2}\end{array}\right]\left(\left[\begin{array}{c}\nu(g-g^{m})\\ \beta_{s}\left(g^{\imath}-g^{\imath m}\right)\end{array}\right]dt+\left[\begin{array}{c}\sigma_{D}dB^{D}\\ \sigma_{u}dB^{u}\end{array}\right]\right)$$
$$=\kappa(\bar{g}-g^{m})\,dt+\left\{\Sigma_{11}^{m}(s)\,\nu^{2}\sigma_{D}^{-2}(g-g^{m})+\Sigma_{12}^{m}(s)\,\beta_{s}^{2}\sigma_{u}^{-2}\left(g^{\imath}-g^{\imath m}\right)\right\}dt$$
$$+\Sigma_{11}^{m}(s)\,\nu\sigma_{D}^{-1}dB^{D}+\Sigma_{12}^{m}(s)\,\beta_{s}\sigma_{u}^{-1}dB^{u}$$

Therefore we can substitute the observable processes (X_t, Z_t, D_t) into the above expression, and the linear pricing rule is

$$dg^{m}=\kappa(\bar{g}-g^{m})\,dt-\Sigma_{11}^{m}(s)\,\nu^{2}\sigma_{D}^{-2}g^{m}dt$$
$$+\lambda(s)\,\theta_{s}dt+\gamma(s)\,\nu g ds+\gamma(s)\,\sigma_{D}dB^{D}+\lambda(s)\,\sigma_{u}dB^{u}$$
$$=\kappa(\bar{g}-g^{m})\,ds-\gamma(s)\,\nu g^{m}ds+\lambda(s)\left[\theta_{s}ds+\sigma_{u}dB^{u}\right]+\gamma(s)\left[dD_{s}+\nu D_{s}ds\right]$$

where $\lambda(s)\equiv\Sigma_{12}^{m}(s)\,\beta_{s}\sigma_{u}^{-2}$ and $\gamma(s)\equiv\Sigma_{11}^{m}(s)\,\nu\sigma_{D}^{-2}$. ■

Proof of proposition 4 and 5.

Proof. We have

$$d\left(g^{\imath}-g^{m}\right)=dg_{s}^{\imath}-\kappa(\bar{g}-g^{m})\,ds+\gamma(s)\,\nu g^{m}ds$$
$$-\lambda(s)\left[\theta_{s}ds+\sigma_{u}dB^{u}\right]-\gamma(s)\left\{\left(\Sigma\nu\sigma_{D}^{-2}\right)^{-1}\left[dg_{s}^{\imath}-\kappa\left(\bar{g}-g^{\imath}\right)ds\right]+\nu g_{s}^{\imath}ds\right\}$$
$$=-(\kappa+\gamma(s)\,\nu)\left(g^{\imath}-g^{m}\right)ds-\lambda(s)\left[\theta_{s}ds+\sigma_{u}dB^{u}\right]-\Sigma_{12}^{m}(s)\,\nu\sigma_{D}^{-1}d\tilde{B}_{s}^{\imath D}$$

Let $\frac{\Lambda'(s)}{\Lambda(s)}=(\kappa+\gamma(s)\,\nu)$

$$d\left(e^{\Lambda(s)}\left(g_{s}^{\imath}-g_{s}^{m}\right)\right)$$
$$=e^{\Lambda(s)}\left[\left(g^{\imath}-g^{m}\right)\Lambda'(s)\,ds+\Lambda(s)\,d\left(g^{\imath}-g^{m}\right)\right]$$
$$=e^{\Lambda(s)}\Lambda(s)\left\{-\lambda(s)\left[\theta_{s}ds+\sigma_{u}dB^{u}\right]-\Sigma_{12}^{m}(s)\,\nu\sigma_{D}^{-1}d\tilde{B}_{s}^{\imath D}\right\}$$

We have

$$\left(g^{\imath}_{(l+1)\Delta} - g^{m}_{(l+1)\Delta}\right) = e^{\Lambda(l\Delta)-\Lambda((l+1)\Delta)}\left(g^{\imath}_{l\Delta} - g^{m}_{l\Delta}\right)$$
$$+ e^{-\Lambda((l+1)\Delta)}\int_0^{\Delta} e^{\Lambda(s)}\Lambda(s)\left\{-\lambda(s)\left[\theta_s ds + \sigma_u dB^u\right] - \Sigma^m_{12}(s)\,\nu\sigma_D^{-1} d\tilde{B}^{\imath D}_s\right\}$$

By Nash equilibrium, insider knows that the pricing rule is

$$dg^m = \kappa\left(\bar{g} - g^m\right)ds - \gamma(s)\,\nu g^m ds + \lambda(s)\left[\theta_s ds + \sigma_u dB^u\right] + \gamma(s)\left[dD_s + \nu D_s ds\right]$$

Recall that

$$\begin{aligned} dg^{\imath}_s &= \kappa\left(\bar{g} - g^{\imath}_s\right)ds + \frac{\Sigma\nu}{\sigma_D} d\tilde{B}^{\imath D}_s \\ &= \kappa\left(\bar{g} - g^{\imath}_s\right)ds + \Sigma\nu\sigma_D^{-2}\left[-\nu g^{\imath}_s ds + dD_s + \nu D_s ds\right] \end{aligned}$$

This implies that

$$dD_s + \nu D_s ds = \left(\Sigma\nu\sigma_D^{-2}\right)^{-1}\left[dg^{\imath}_s - \kappa\left(\bar{g} - g^{\imath}\right)ds\right] + \nu g^{\imath}_s ds$$

then we have

$$\begin{aligned} d\left(g^{\imath} - g^m\right) &= dg^{\imath}_s - \kappa\left(\bar{g} - g^m\right)ds + \gamma(s)\,\nu g^m ds \\ &\quad - \lambda(s)\left[\theta_s ds + \sigma_u dB^u\right] - \gamma(s)\left\{\left(\Sigma\nu\sigma_D^{-2}\right)^{-1}\left[dg^{\imath}_s - \kappa\left(\bar{g} - g^{\imath}\right)ds\right] + \nu g^{\imath}_s ds\right\} \\ &= -\left(\kappa + \gamma(s)\,\nu\right)\left(g^{\imath} - g^m\right)ds - \lambda(s)\left[\theta_s ds + \sigma_u dB^u\right] - \Sigma^m_{12}(s)\,\nu\sigma_D^{-1} d\tilde{B}^{\imath D}_s \end{aligned}$$

The long term infinite horizon objective function is as follows. Suppose current time is

$t = l\Delta$, the discrete time Bellman equation is given by

$$\begin{aligned}
&V\left(l, g_t - g_t^m\right) \\
&= \sup_{\{\theta_s^t, t \leq s \leq t+\Delta\}} E_t^\imath \left\{ \int_t^{t+\Delta} e^{-r(s-t)} \left[\left(g_s - g_s^m\right)\theta_s - c\theta^2\right] dt + e^{-r\Delta} V\left(l+1, g_{t+\Delta}, g_{t+\Delta}^m\right) \right\}
\end{aligned}$$

where the state variables evolve according to

$$\begin{aligned}
g_{t+\Delta} &= e^{-\kappa\Delta} g_t + \left(1 - e^{-\kappa\Delta}\right)\bar{g} + \int_t^{t+\Delta} e^{-\kappa(s-u)} \sigma_g dB^g \\
g_{t+\Delta}^m &= \exp\left(-\tilde{\gamma}\left(t+\Delta\right)\right) \int_t^{t+\Delta} e^{\tilde{\gamma}(s)} \left[\kappa\bar{g}ds + \gamma\left(s\right) g_s^\imath ds + \lambda_s \theta_s ds\right] \\
&\quad + \exp\left(-\tilde{\gamma}\left(t+\Delta\right)\right) \int_t^{t+\Delta} e^{\tilde{\gamma}(s)} \left[\nu^{-1}\sigma_D \gamma\left(s\right) d\tilde{B}^{\imath D} + \lambda_s \sigma_u dB^u\right]
\end{aligned}$$

Notice that the state variables are both discrete time stochastic processes, observable to the insider at each discrete time $l\Delta$. The evolution of the state variables can be proved as follows. Recall that

$$\begin{aligned}
dg &= \kappa\left(\bar{g} - g\right) ds + \sigma_g dB^g \\
dg_s^m &= \kappa\left(\bar{g} - g^m\right) ds + \gamma\left(s\right)\left(g^\imath - g^m\right) ds + \lambda_s \theta_s ds + \Sigma_{11}^m\left(s\right) \nu \sigma_D^{-1} d\tilde{B}^{\imath D} + \lambda_s \sigma_u dB^u
\end{aligned}$$

where $\gamma\left(s\right) \equiv \Sigma_{11}^m\left(s\right) \nu^2 \sigma_D^{-2}$. Let $\tilde{\gamma}'\left(s\right) = \gamma\left(s\right) + \kappa$. This implies that

$$g_{t+\Delta} = e^{-\kappa\Delta} g_t + \left(1 - e^{-\kappa\Delta}\right)\bar{g} + \int_t^{t+\Delta} e^{-\kappa(s-u)} \sigma_g dB^g$$

and

$$\begin{aligned}
d\left[\exp\left(\tilde{\gamma}\left(s\right)\right) g_s^m\right] &= e^{\tilde{\gamma}(s)} dg^m + g_s^m e^{\tilde{\gamma}(s)} \tilde{\gamma}'\left(s\right) \\
&= e^{\tilde{\gamma}(s)} \left[\kappa\bar{g}ds + \gamma\left(s\right) g_s^\imath ds + \lambda_s \theta_s ds + \gamma_s \nu^{-1} \sigma_D d\tilde{B}^{\imath D} + \lambda_s \sigma_u dB^u\right]
\end{aligned}$$

this implies that

$$g^m_{t+\Delta} = \exp\left(-\tilde{\gamma}\left(t+\Delta\right)\right)\int_t^{t+\Delta} e^{\tilde{\gamma}(s)}\left[\kappa\bar{g}ds + \gamma\left(s\right)g^\imath_s ds + \lambda_s\theta_s ds\right]$$
$$+\exp\left(-\tilde{\gamma}\left(t+\Delta\right)\right)\int_t^{t+\Delta} e^{\tilde{\gamma}(s)}\left[\nu^{-1}\sigma_D\gamma\left(s\right)d\tilde{B}^{\imath D} + \lambda_s\sigma_u dB^u\right]$$

where the insider's strategy controls this state variable through

$$\int_t^{t+\Delta}\exp\left\{-\left[\tilde{\gamma}\left(t+\Delta\right)-\tilde{\gamma}\left(s\right)\right]\right\}\lambda\left(s\right)\theta_s ds.$$

Now let us consider the insider's short term (per period) objective function

$$\sup_{\{\theta^t_s, t\leq s\leq t+\Delta\}} E^\imath_t\left\{\int_t^{t+\Delta} e^{-r(s-t)}\left[\left(g_s - g^m_s\right)\theta_s - c\theta^2\right]dt + e^{-r\Delta}V\left(l+1, g_{t+\Delta}, g^m_{t+\Delta}\right)\right\}$$
$$d\left(g^\imath - g^m\right) = -\left[\kappa + \gamma\left(s\right)\nu\right]\left(g^\imath - g^m\right)dt - \lambda\left(s\right)\theta_s dt$$
$$-\lambda\left(s\right)\sigma_u dB^u + \left[\Sigma - \Sigma^m_{11}\left(s\right)\right]\nu\sigma_D^{-1}d\tilde{B}^{\imath D}_s$$

This is the following finite horizon stochastic control problem. Denote the value function by $J\left(s, g^\imath_s - g^m_s\right)$. Then the discrete time Bellman equation is just

$$V\left(l, g_t, g^m_t\right) = J\left(0, g^\imath_t - g^m_t\right).$$

We will deal with this equation later. For the finite horizon per period problem, the terminal value is

$$J\left(\Delta, g^\imath_{t+\Delta} - g^m_{t+\Delta}\right) = e^{-r\Delta}E^\imath_{t+\Delta}V\left(l+1, g_{t+\Delta}, g^m_{t+\Delta}\right)$$

Let $d \equiv g_s^\imath - g_s^m$, then the HJB equation is

$$\begin{aligned} 0 &= \sup J_s - J_d \left\{ \left[\kappa + \Sigma_{11}^m (s) \nu^2 \sigma_D^{-2}\right] \left(g^\imath - g^m\right) + \lambda (s) \theta_s \right\} \\ &+ \frac{1}{2} J_{dd} \left[(\lambda (s) \sigma_u)^2 + \left[\Sigma - \Sigma_{11}^m (s)\right]^2 \nu^2 \sigma_D^{-2} \right] \\ &+ e^{-rs} \left(g_s^\imath - g_s^m\right) \theta_s - e^{-rs} c \theta^2 \end{aligned}$$

then the FOC is

$$-J_d \lambda (s) + e^{-rs} \left(g_s^\imath - g_s^m\right) - 2 e^{-rs} c \theta = 0$$

which implies

$$\theta_s = \frac{-e^{rs} J_d \lambda (s) + \left(g_s^\imath - g_s^m\right)}{2c}$$

We can conjecture that

$$\begin{aligned} J (s, d) &= e^{-rs} \left\{ \alpha (s) \left(g^\imath - g^m\right)^2 + \delta (s) \right\} \\ J_d &= e^{-rs} \left\{ 2 \alpha (s) \left(g^\imath - g^m\right) \right\} \end{aligned}$$

Then FOC becomes

$$\begin{aligned} \theta_s &= \frac{\left[1 - 2\alpha (s) \lambda (s)\right]}{2c} \left(g^\imath - g^m\right) \\ \beta_s &= \frac{1 - 2\alpha (s) \lambda (s)}{2c} \end{aligned}$$

$$J_s = \left(-r \alpha (s) + \alpha' (s)\right) e^{-rs} \left(g^\imath - g^m\right)^2 - r e^{-rs} \delta (s) + e^{-rs} \delta' (s)$$

Plug back into HJB we have

$$0=\left(-r\alpha\left(s\right)+\alpha'\left(s\right)\right)e^{-rs}\left(g^{\imath}-g^{m}\right)^{2}-re^{-rs}\delta\left(s\right)+e^{-rs}\delta'\left(s\right)$$
$$-e^{-rs}2\alpha\left(s\right)\left(g^{\imath}-g^{m}\right)\left\{\left[\kappa+\Sigma_{11}^{m}\left(s\right)\nu^{2}\sigma_{D}^{-2}\right]\left(g^{\imath}-g^{m}\right)\right\}$$
$$+e^{-rs}\alpha\left(s\right)\left[\left(\lambda\left(s\right)\sigma_{u}\right)^{2}+\left[\Sigma-\Sigma_{11}^{m}\left(s\right)\right]^{2}\nu^{2}\sigma_{D}^{-2}\right]+e^{-rs}c\left\{\frac{\left[1-2\alpha\left(s\right)\lambda\left(s\right)\right]}{2c}\left(g_{s}^{\imath}-g_{s}^{m}\right)\right\}^{2}$$

Simplify

$$0=\left(-r\alpha\left(s\right)+\alpha'\left(s\right)\right)\left(g^{\imath}-g^{m}\right)^{2}-r\delta\left(s\right)+\delta'\left(s\right)$$
$$-2\alpha\left(s\right)\left[\kappa+\Sigma_{11}^{m}\left(s\right)\nu^{2}\sigma_{D}^{-2}\right]\left(g^{\imath}-g^{m}\right)^{2}$$
$$+\alpha\left(s\right)\left[\left(\lambda\left(s\right)\sigma_{u}\right)^{2}+\left[\Sigma-\Sigma_{11}^{m}\left(s\right)\right]^{2}\nu^{2}\sigma_{D}^{-2}\right]+c\left\{\frac{\left[1-2\alpha\left(s\right)\lambda\left(s\right)\right]}{2c}\right\}^{2}\left(g_{s}^{\imath}-g_{s}^{m}\right)^{2}$$

The following two equations are sufficient

$$0=-r\alpha\left(s\right)+\alpha'\left(s\right)-2\alpha\left(s\right)\left[\kappa+\Sigma_{11}^{m}\left(s\right)\nu^{2}\sigma_{D}^{-2}\right]+\frac{\left[1-2\alpha\left(s\right)\lambda\left(s\right)\right]^{2}}{4c}$$

$$0=-r\delta\left(s\right)+\delta'\left(s\right)+\alpha\left(s\right)\left[\left(\lambda\left(s\right)\sigma_{u}\right)^{2}+\left[\Sigma-\Sigma_{11}^{m}\left(s\right)\right]^{2}\nu^{2}\sigma_{D}^{-2}\right]$$

Now by the discrete time Bellman equation (2.10) we have

$$V\left(l,g_{t},g_{t}^{m}\right)=\alpha\left(0\right)\left(g-g^{m}\right)^{2}+\delta\left(0\right)$$

To satisfy the terminal condition (2.11), we must have

$$e^{-r\Delta}\left\{\alpha\left(\Delta\right)\left(g_{t+\Delta}^{\imath}-g_{t+\Delta}^{m}\right)^{2}+\delta\left(\Delta\right)\right\}=e^{-r\Delta}E_{t+\Delta}^{\imath}V\left(l+1,g_{t+\Delta},g_{t+\Delta}^{m}\right)$$

Substitute in the discrete time Bellman equation

$$\begin{aligned}\alpha(\Delta)\left(g^{i}_{t+\Delta}-g^{m}_{t+\Delta}\right)^{2}+\delta(\Delta) &= E^{i}_{t+\Delta}J\left(0,g_{t+\Delta}-g^{m}_{t+\Delta}\right)\\ &= E^{i}_{t+\Delta}\left\{\alpha(0)\left(g_{t+\Delta}-g^{m}_{t+\Delta}\right)^{2}+\delta(0)\right\}\end{aligned}$$

Note that

$$\begin{aligned}E^{i}_{t+\Delta}g_{t+\Delta} &= g^{i}_{t+\Delta}\\ E^{i}_{t+\Delta}g^{2}_{t+\Delta} &= Var^{i}_{t+\Delta}\left(g_{t+\Delta}\right)+\left(g^{i}_{t+\Delta}\right)^{2}=\Sigma(\Delta)+\left(g^{i}_{t+\Delta}\right)^{2}\end{aligned}$$

This implies that

$$\begin{aligned}&E^{i}_{t+\Delta}\left\{\alpha(0)\left(g_{t+\Delta}-g^{m}_{t+\Delta}\right)^{2}+\delta(0)\right\}\\ &= E^{i}_{t+\Delta}\left\{\alpha(0)\left(g^{2}_{t+\Delta}-2g_{t+\Delta}g^{m}_{t+\Delta}+\left(g^{m}_{t+\Delta}\right)^{2}\right)+\delta(0)\right\}\\ &= \alpha(0)\left(\Sigma(\Delta)+\left(g^{i}_{t+\Delta}\right)^{2}-2g^{i}_{t+\Delta}g^{m}_{t+\Delta}+\left(g^{m}_{t+\Delta}\right)^{2}\right)+\delta(0)\\ &= \alpha(0)\left(g^{i}_{t+\Delta}-g^{m}_{t+\Delta}\right)^{2}+\alpha(0)\Sigma(\Delta)+\delta(0)\end{aligned}$$

Therefore equation (2.11) becomes

$$\alpha(\Delta)\left(g^{i}_{t+\Delta}-g^{m}_{t+\Delta}\right)^{2}+\delta(\Delta)=\alpha(0)\left(g^{i}_{t+\Delta}-g^{m}_{t+\Delta}\right)^{2}+\alpha(0)\Sigma(\Delta)+\delta(0)$$

This further implies

$$\begin{aligned}\alpha(\Delta) &= \alpha(0)\\ \delta(\Delta) &= \alpha(0)\Sigma(\Delta)+\delta(0)\end{aligned}$$

■

Proof of proposition 12.

Proof. The long term infinite horizon objective function is as follows. Suppose current time is $t = l\Delta$, the discrete time Bellman equation is given by

$$\begin{aligned} & V\left(l, g_t, g_t^m\right) \\ = & \sup_{\{\theta_s^\iota, t \leq s \leq t+\tau\}} E_t^\iota \left\{ \int_t^{t+\tau} e^{-r(s-t)} \left[\left(g_s^\iota - g_s^m\right) \theta_s - c\theta^2 \right] dt + e^{-r\tau} V\left(l+1, g_{t+\tau}, g_{t+\tau}^m\right) \right\} \end{aligned}$$

To simplify the notation, we can let $t = 0$. It is clear that the state variables evolve according to

$$\begin{aligned} g_{t+\Delta} &= e^{-\kappa\Delta} g_t + \left(1 - e^{-\kappa\Delta}\right) \bar{g} + \int_t^{t+\Delta} e^{-\kappa(s-u)} \sigma_g dB^g \\ g_{t+\Delta}^m &= \exp\left(-\tilde{\gamma}\left(t+\Delta\right)\right) \int_t^{t+\Delta} e^{\tilde{\gamma}(s)} \left[\kappa \bar{g} ds + \gamma\left(s\right) g_s^\iota ds + \lambda_s \theta_s ds \right] \\ &\quad + \exp\left(-\tilde{\gamma}\left(t+\Delta\right)\right) \int_t^{t+\Delta} e^{\tilde{\gamma}(s)} \left[\nu^{-1} \sigma_D \gamma\left(s\right) d\tilde{B}^{\iota D} + \lambda_s \sigma_u dB^u \right] \end{aligned}$$

Notice that the state variables are both discrete time stochastic processes, observable to the insider at each discrete time $l\Delta$. The evolution of the state variables can be proved as follows. Recall that

$$\begin{aligned} dg &= \kappa\left(\bar{g} - g\right) ds + \sigma_g dB^g \\ dg_s^m &= \kappa\left(\bar{g} - g^m\right) ds + \gamma\left(s\right)\left(g^\iota - g^m\right) ds + \lambda_s \theta_s ds + \Sigma_{11}^m\left(s\right) \nu \sigma_D^{-1} d\tilde{B}^{\iota D} + \lambda_s \sigma_u dB^u \end{aligned}$$

where $\gamma\left(s\right) \equiv \Sigma_{11}^m\left(s\right) \nu^2 \sigma_D^{-2}$. Let $\tilde{\gamma}'\left(s\right) = \gamma\left(s\right) + \kappa$. This implies that

$$g_{t+\Delta} = e^{-\kappa\Delta} g_t + \left(1 - e^{-\kappa\Delta}\right) \bar{g} + \int_t^{t+\Delta} e^{-\kappa(s-u)} \sigma_g dB^g$$

and

$$d\left[\exp\left(\tilde{\gamma}\left(s\right)\right)g_s^m\right] = e^{\tilde{\gamma}(s)}dg^m + g_s^m e^{\tilde{\gamma}(s)}\tilde{\gamma}'\left(s\right)$$
$$= e^{\tilde{\gamma}(s)}\left[\kappa\bar{g}ds + \gamma\left(s\right)g_s^\imath ds + \lambda_s\theta_s ds + \gamma_s\nu^{-1}\sigma_D d\tilde{B}^{\imath D} + \lambda_s\sigma_u dB^u\right]$$

this implies that

$$g_{t+\Delta}^m = \exp\left(-\tilde{\gamma}\left(t+\Delta\right)\right)\int_t^{t+\Delta} e^{\tilde{\gamma}(s)}\left[\kappa\bar{g}ds + \gamma\left(s\right)g_s^\imath ds + \lambda_s\theta_s ds\right]$$
$$+ \exp\left(-\tilde{\gamma}\left(t+\Delta\right)\right)\int_t^{t+\Delta} e^{\tilde{\gamma}(s)}\left[\nu^{-1}\sigma_D\gamma\left(s\right)d\tilde{B}^{\imath D} + \lambda_s\sigma_u dB^u\right]$$

where the insider's strategy controls this state variable through

$$\int_t^{t+\Delta}\exp\left\{-\left[\tilde{\gamma}\left(t+\Delta\right)-\tilde{\gamma}\left(s\right)\right]\right\}\lambda\left(s\right)\theta_s ds.$$

Suppose $\tau \leq \bar{T}$. Let the CDF be

$$F\left(t\right) = \frac{1-e^{-\phi t}}{1-e^{-\phi\bar{T}}}$$
$$G\left(t\right) = 1-F\left(t\right) = \frac{e^{-\phi t}-e^{-\phi\bar{T}}}{1-e^{-\phi\bar{T}}}$$
$$f\left(t\right) = \frac{\phi e^{-\phi t}}{1-e^{-\phi\bar{T}}}$$
$$f\left(T,t\right) = \frac{f\left(T\right)}{G\left(t\right)} = \frac{\phi e^{-\phi T}}{e^{-\phi t}-e^{-\phi\bar{T}}}, T > t$$
$$h\left(t\right) = f\left(t,t\right) = \frac{\phi e^{-\phi t}}{e^{-\phi t}-e^{-\phi\bar{T}}}$$

Now let us consider the insider's short term (per period) objective function

$$J\left(0, g_0^\imath - g_0^m\right) = \sup_{\{\theta_s^t, 0\leq s\leq\tau\}} E_0^\imath\left\{\int_0^\tau e^{-rs}\left[\left(g_s^\imath - g_s^m\right)\theta_s - c\theta^2\right]ds + e^{-r\tau}V\left(l+1, g_\tau, g_\tau^m\right)\right\}$$

$$
\begin{aligned}
&J\left(t, g_t^\imath - g_t^m\right) \\
&= \sup_{\{\theta_s^\imath, t\le s\le T\}} E_t^\imath \int_t^\infty f(T,t)\left\{\int_t^T e^{-rs}\left[\left(g_s^\imath - g_s^m\right)\theta_s - c\theta^2\right] ds + e^{-rT} V\left(l+1, g_T, g_T^m\right)\right\} dT \\
&= \sup_{\{\theta_s^t, t\le s\le T\}} E_t^\imath \frac{1}{G(t)} \int_t^\infty \{G(T) e^{-rT}\left[\left(g_T^\imath - g_T^m\right)\theta_T - c\theta_T^2\right] \\
&+ f(T) e^{-rT} V\left(l+1, g_T, g_T^m\right)\} dT
\end{aligned}
$$

subject to

$$
\begin{aligned}
&d\left(g^\imath - g^m\right) \\
&= -\left[\kappa + \gamma(s)\nu\right]\left(g^\imath - g^m\right) dt - \lambda(s)\theta_s dt - \lambda(s)\sigma_u dB^u + \left[\Sigma - \Sigma_{11}^m(s)\right]\nu\sigma_D^{-1} d\tilde{B}_s^{\imath D} \\
&\tilde{V}\left(l+1, g_s^\imath, g_s^m\right) = E_s^\imath V\left(l+1, g_s, g_s^m\right)
\end{aligned}
$$

This is the following finite horizon stochastic control problem. Denote the value function by $J\left(s, g_s^\imath - g_s^m\right)$. Then the discrete time Bellman equation is just

$$
V\left(l, g_0, g_0^m\right) = J\left(0, g_0^\imath - g_0^m\right).
$$

We will deal with this equation later. For the finite horizon per period problem, the transversality condition is

$$
J\left(\bar{T}, g^\imath - g^m\right) = e^{-r\bar{T}} E_{\bar{T}}^\imath V\left(l+1, g_{\bar{T}}, g_{\bar{T}}^m\right)
$$

Let $d \equiv g_s^\imath - g_s^m$, then the HJB equation is

$$
\begin{aligned}
0 = &\sup J_t - J_d\left\{\left[\kappa + \Sigma_{11}^m(t)\nu^2\sigma_D^{-2}\right]\left(g^\imath - g^m\right) + \lambda(t)\theta_t\right\} \\
&+ \frac{1}{2} J_{dd}\left[\left(\lambda(t)\sigma_u\right)^2 + \left[\Sigma - \Sigma_{11}^m(t)\right]^2\nu^2\sigma_D^{-2}\right] \\
&+ \left\{e^{-rt}\left[\left(g_t^\imath - g_t^m\right)\theta_t - c\theta_t^2\right] + h(t) e^{-rt} V\left(l+1, g_t, g_t^m\right)\right\} - h(t) J
\end{aligned}
$$

then the FOC is

$$-J_d\lambda(s) + e^{-rs}\left(g_s^\imath - g_s^m\right) - 2e^{-rs}c\theta = 0$$

which implies

$$\theta_s = \frac{-e^{rs}J_d\lambda(s) + \left(g_s^\imath - g_s^m\right)}{2c}$$

We can conjecture that

$$J(s,d) = e^{-rs}\left\{\alpha(s)\left(g^\imath - g^m\right)^2 + \delta(s)\right\}$$
$$J_d = e^{-rs}\left\{2\alpha(s)\left(g^\imath - g^m\right)\right\}$$

Then FOC becomes

$$\theta_s = \frac{[1-2\alpha(s)\lambda(s)]}{2c}\left(g^\imath - g^m\right)$$
$$\beta_s = \frac{1-2\alpha(s)\lambda(s)}{2c}$$

$$J_s = \left(-r\alpha(s) + \alpha'(s)\right)e^{-rs}\left(g^\imath - g^m\right)^2 - re^{-rs}\delta(s) + e^{-rs}\delta'(s)$$

Plug back into HJB we have

$$\begin{aligned}
0 = {} & \left(-r\alpha(s) + \alpha'(s)\right)e^{-rs}\left(g^\imath - g^m\right)^2 - re^{-rs}\delta(s) + e^{-rs}\delta'(s) \\
& - e^{-rs}2\alpha(s)\left(g^\imath - g^m\right)\left\{\left[\kappa + \Sigma_{11}^m(s)\nu^2\sigma_D^{-2}\right]\left(g^\imath - g^m\right)\right\} \\
& + e^{-rs}\alpha(s)\left[(\lambda(s)\sigma_u)^2 + [\Sigma - \Sigma_{11}^m(s)]^2\nu^2\sigma_D^{-2}\right] + e^{-rs}c\left\{\frac{[1-2\alpha(s)\lambda(s)]}{2c}\left(g_s^\imath - g_s^m\right)\right\}^2 \\
& + h(s)e^{-rs}\tilde{V}\left(l+1, g_s^\imath, g_s^m\right) - h(s)e^{-rs}\left\{\alpha(s)\left(g^\imath - g^m\right)^2 + \delta(s)\right\}
\end{aligned}$$

Recall that

$$\begin{aligned}\tilde{V}\left(g_s^\imath, g_s^m\right) &= E_s J\left(0, g_s - g_s^m\right) = E_s\left\{\alpha(0)\left(g_s - g_s^m\right)^2 + \delta(0)\right\} \\ &= E_s\left\{\alpha(0)\left(g_s^2 - 2g_s g_s^m + \left(g_s^m\right)^2\right) + \delta(0)\right\} \\ &= \alpha(0)\left(\left(g_s^\imath\right)^2 + \Sigma(s) - 2g_s^\imath g_s^m + \left(g_s^m\right)^2\right) + \delta(0) \\ &= \left\{\alpha(0)\left(g_s^\imath - g_s^m\right)^2 + \delta(0)\right\} + \alpha(0)\Sigma(s)\end{aligned}$$

Simplify

$$\begin{aligned}0 &= \left(-r\alpha(s) + \alpha'(s)\right)\left(g^\imath - g^m\right)^2 - r\delta(s) + \delta'(s) \\ &- 2\alpha(s)\left(g^\imath - g^m\right)\left\{\left[\kappa + \Sigma_{11}^m(s)\nu^2\sigma_D^{-2}\right]\left(g^\imath - g^m\right)\right\} \\ &+ \alpha(s)\left[\left(\lambda(s)\sigma_u\right)^2 + \left[\Sigma - \Sigma_{11}^m(s)\right]^2\nu^2\sigma_D^{-2}\right] + c\left\{\frac{\left[1 - 2\alpha(s)\lambda(s)\right]}{2c}\left(g_s^\imath - g_s^m\right)\right\}^2 \\ &+ h(s)\left\{\alpha(0)\left(g_s^\imath - g_s^m\right)^2 + \delta(0)\right\} + h(s)\alpha(0)\Sigma(s) - h(s)\left\{\alpha(s)\left(g^\imath - g^m\right)^2 + \delta(s)\right\}\end{aligned}$$

The following two equations are sufficient

$$\begin{aligned}&- r\alpha(s) + \alpha'(s) - 2\alpha(s)\left\{\left[\kappa + \Sigma_{11}^m(s)\nu^2\sigma_D^{-2}\right]\right\} \\ &+ c\left\{\frac{\left[1 - 2\alpha(s)\lambda(s)\right]}{2c}\right\}^2 + \frac{\phi e^{-\phi s}}{e^{-\phi s} - e^{-\phi\bar{T}}}\left[\alpha(0) - \alpha(s)\right] = 0 \\ &- r\delta(s) + \delta'(s) + \alpha(s)\left[\left(\lambda(s)\sigma_u\right)^2 + \left[\Sigma - \Sigma_{11}^m(s)\right]^2\nu^2\sigma_D^{-2}\right] \\ &+ \frac{\phi e^{-\phi s}}{e^{-\phi s} - e^{-\phi\bar{T}}}\left[\delta(0) - \delta(s) + \alpha(0)\Sigma(s)\right] = 0\end{aligned}$$

The terminal condition is

$$J\left(\bar{T}, g^\imath - g^m\right) = e^{-r\bar{T}}E_{\bar{T}}^\imath V\left(l+1, g_{\bar{T}}, g_{\bar{T}}^m\right)$$

$$e^{-r\bar{T}}\left\{\alpha\left(\bar{T}\right)\left(g^\imath - g^m\right)^2 + \delta\left(\bar{T}\right)\right\} = e^{-r\bar{T}}\left\{\alpha(0)\left(g_{\bar{T}}^\imath - g_{\bar{T}}^m\right)^2 + \delta(0) + \alpha(0)\Sigma\left(\bar{T}\right)\right\}$$

This implies

$$\alpha\left(\bar{T}\right) = \alpha\left(0\right)$$
$$\delta\left(\bar{T}\right) = \delta\left(0\right) + \alpha\left(0\right)\Sigma\left(\bar{T}\right)$$

■

Appendix C

Solution to the Discrete Signal Equilibrium

We want to represent all other functions using only Σ_{22}^m and α. Notice that

$$\begin{aligned}
\beta(s) &= \frac{1-2\alpha(s)\lambda(s)}{2c} = \frac{\sigma_u^2\lambda(s)}{\Sigma_{12}^m(s)} \\
\lambda(s) &= \frac{\Sigma_{12}^m(s)}{2\left[c\sigma_u^2+\alpha(s)\Sigma_{12}^m(s)\right]} \\
\Sigma_{12}\beta &= \sigma_u^2\lambda(s) = \frac{\sigma_u^2\Sigma_{12}^m(s)}{2\left[c\sigma_u^2+\alpha(s)\Sigma_{12}^m(s)\right]}
\end{aligned}$$

we can transform equation (2.8) to

$$\begin{aligned}
\frac{d\Sigma_{22}^m}{ds} &= -2\left(\kappa+\Sigma\nu^2\sigma_D^{-2}\right)\Sigma_{22}^m - \left(\Sigma_{12}^m\right)^2\nu^2\sigma_D^{-2} - \left(\frac{\Sigma_{12}^m(s)}{2\left[c\sigma_u^2+\alpha(s)\Sigma_{12}^m(s)\right]}\right)^2\sigma_u^2 \\
\Sigma_{12}^m(0) &= \Sigma_{12}^m(\Delta)+\Sigma(\Delta)
\end{aligned}$$

and we can transform equation (2.12) to

$$0 = -r\alpha(s) + \alpha'(s) - 2\alpha(s)\left[\kappa + \left(\Sigma_{12}^{m}(s) + \Sigma(s)\right)\nu^{2}\sigma_{D}^{-2}\right] + \frac{1}{4}\frac{c\sigma_{u}^{4}}{\left[c\sigma_{u}^{2} + \alpha(s)\Sigma_{12}^{m}(s)\right]^{2}}$$

$$\alpha(0) = \alpha(\Delta)$$

Finally δ is determined by

$$\delta'(s) = r\delta(s) - \alpha(s)\left[\left(\frac{\Sigma_{12}^{m}(s)}{2\left[c\sigma_{u}^{2} + \alpha(s)\Sigma_{12}^{m}(s)\right]}\sigma_{u}\right)^{2} + \Sigma_{12}^{m}(s)^{2}\nu^{2}\sigma_{D}^{-2}\right]$$

and

$$\delta(\Delta) = \alpha(0)\Sigma(\Delta) + \delta(0).$$

Bibliography

[1] Anderson, B.D.O., and J.B. Moore, 1979 Optimal Filtering. (New Jersey: Prentice Hall).

[2] Back, K., 1992, Insider Trading in Continuous Time, Review of Financial Studies 5, 387-409.

[3] Back, K., Cao, H., and G. Willard, 2000, Imperfect Competition among Informed Traders, *Journal of Finance* 55, 2117-2155.

[4] Chau, M., 1999, Ph.D. dissertation, MIT.

[5] Chau, M. and D. Vayanos, 2008. Strong-Form Efficiency with Monopolistic Insiders, *Review of Financial Studies*, 21(5), 2275-2306.

[6] Fama, E.F., Efficient Capital Markets: A Review of Theory and Empirical Work, *Journal of Finance*, Vol. 25, No. 2, 383-417. May, 1970.

[7] Fama, Eugene (1991). Efficient Capital Markets II, *Journal of Finance*, 46, 1575-617.

[8] Kyle, A., 1985, Continuous Auctions and Insider Trading, *Econometrica*, 53, 1315-1335.

[9] Lipster and Shiryaev., 2001, Statistics of Random Processes. (New York: Springer-Verlag).

[10] Richard S. 1975. Optimal consumption, portfolio and life insurance rules for an uncertain lived individual in a continuous time model, *The Journal of Financial Economics*. 2, 187-203.

[11] Rozeff M. S. and Mir A. Zaman, 1988, Market Efficiency and Insider Trading: New Evidence, *Journal of Business*, 61, No. 1, 25-44.

[12] Wang, J., 1993, A Model of Intertemporal Asset Prices under Asymmetric Information, *Review of Economic Studies*, 60, 249-282.