WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

Findings of Forward Discount Bias Interpreted in Light of Exchange Rate Survey Data*

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Working Paper #1906-87 June 29, 1987

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Abstract

Survey data on exchange rate expectations are used to divide the forward discount into expected depreciation and a risk premium. Our starting point is the common test of whether the forward discount is an unbiased predictor of future changes in the spot rate. We use the surveys to decompose the bias into a portion attributable to the risk premium and a portion attributable to systematic prediction errors. The survey data suggest that our findings of both unconditional and conditional bias are overwhelmingly due to systematic expectational errors. Regressions of future changes in the spot rate against the forward discount do not yield insights into the sign, size or variability of the risk premium as is usually thought. We test directly the hypothesis of perfect substitutability, and find support for it in that changes in the forward discount reflect, one for one, changes in expected depreciation. The "random-walk" view that expected depreciation is zero is thus rejected; expected depreciation is even significantly more variable than the risk premium. Investors would do better if they always reduced fractionally the magnitude of expected depreciation. This is the same result that Bilson and many others have found with forward market data, but now it cannot be attributed to a risk premium.

* This is an extensively revised version of NBER Working Paper No. 1963. We would like to thank Greg Connor, Alberto Giovannini, Robert Hodrick, Joe Mattey and many other participants at various seminars for helpful comments; Barbara Bruer, John Calverley, Louise Cordova, Kathryn Dominguez, Laura Knoy, Stephen Marris, and Phil Young for help in obtaining data, the National Science Foundation (under grant no. SES-8218300), the Institute for Business and Economic Research at U. C. Berkeley, and the Alfred P. Sloan Foundation for research support.

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1. Introduction

The forward exchange rate is surely the jack-of-all-trades of international financial economics. Whenever researchers need a variable representing investor expectations of future spot rates, the forward rate is the first to come to mind. On the other hand, the forward rate is frequently used in efforts to extract the empirically elusive foreign exchange risk premium.

These two conflicting roles are most evident in the large literature testing whether the forward discount is an unbiased predictor of the future change in the spot exchange rate.[1] Most of the studies that test the unbiasedness hypothesis reject it, and they generally agree on the direction of bias. They tend to disagree, however, about whether the bias is evidence of a risk premium or of a violation of rational expectations. For example, studies by Longworth (1981) and Bilson (1981a) assume that investors are risk neutral, so that the systematic component of exchange rate changes in excess of the forward discount is interpreted as evidence of a failure of rational expectations. On the other hand, Hsieh (1984) and most others attribute the same systematic component to a time-varying risk premium that separates the forward discount from expected depreciation.

Investigations by Fama (1984) and Hodrick and Srivastava (1986) have recently gone a step further, interpreting the bias not only as evidence of a non-zero risk premium, but also as evidence that the variance of the risk premium is greater than the variance of expected depreciation. Bilson

[1] References include Tryon (1979), Levich (1979), Bilson (1981a), Longworth (1981), Hsieh (1984), Fama (1984), Huang (1984), Park (1984) and Hodrick and Srivastava (1986). For a recent survey of the literature and additional citations see Boothe and Longworth (1986).

(1985) interprets this view as a new "empirical paradigm" that comes close to assuming static expectations: changes in expected depreciation are small or zero, and changes in the forward discount instead reflect predominantly changes in the risk premium. Often cited in support of this view is the work of Meese and Rogoff (1983), who find that a random walk model consistently forecasts future spot rates better than alternative models, including the forward rate.

But one cannot address without additional information the basic issues of whether systematic expectational errors or the risk premium are alone responsible for the repeatedly biased forecasts of the forward discount (or whether it is some combination of the two), let alone whether the risk premium is more variable than expected depreciation. In this paper we use survey data on exchange rate expectations in an attempt to help resolve these issues. The data come from three surveys: one conducted by American Express Banking Corporation of London irregularly between 1976 and 1985; another conducted by the *Economist's Financial Report*, also from London, at regular six-week intervals since 1981; and a third conducted by Money Market Services (MMS) of Redwood City, California, every two weeks beginning in January 1983 and every week beginning in October 1984. Frankel and Froot (1985, 1987) discuss the data and use it to estimate models of how investors form their expectations.[2] In this paper we use the surveys to divide the forward discount into its two components - expected depreciation and the risk premium - in order to shed light on the proper interpretation of the large literature that finds bias in the predictions of the forward rate.

We want to be skeptical of the accuracy of the survey data, to allow for the possibility that they measure true investor expectations with error. Such measurement error could arise in a number of ways. We will follow the existing literature in talking as if there exists a single expectation that is homogeneously held by investors, which we measure by the median survey response. But, in fact, different survey respondents report different answers, suggesting that if there is a single true expectation, it is measured with error. Another possible source of measurement error in our expected depreciation series is that the expected future spot rate may not be recorded by the survey at precisely the same moment as the contemporaneous spot rate is recorded.[3] Our econometric tests

[2] Another paper that uses the MMS surveys is Dominguez (1986).

[3] To measure the contemporaneous spot rate, we experimented with different approximations to the precise survey and forecast dates of the Amex survey, which was conducted by mail over a period of up to a month. We used the average of the 30 days during the survey and also the mid-point of the survey period to construct reference sets. Both gave very similar results, so that only results from the former sample were reported. In the case of the *Economist* and MMS surveys, which constitute

allow for measurement error in the data, provided the error is random. There is an analogy with the rational expectations approach which uses *ex post* exchange rate changes rather than survey data, and assumes that the error in measuring true expected depreciation, usually attributed to "news," is random. One of our findings below is that the expectational errors contained in *ex post* sample exchange rate changes are not uncorrelated with the forward discount. This, of course, could be consistent with a failure of investor rationality, but it is also consistent with "peso problems," learning on the part of investors, or the presence of nonstationarities in the sample. A second advantage to our approach to measuring expectations: aside from the question of bias, the absolute magnitude of our measurement errors is much less that that of the expectational errors in the *ex post* changes.

The paper is organized as follows. In Section 2, we perform the standard regression test of forward discount bias. We then use the surveys to separate the bias into a component attributable to systematic expectational errors and a component attributable to the risk premium. Sections 3 and 4 in turn test whether the component attributable to the risk premium and the component attributable to systematic expectational errors are statistically significant. Section 5 concludes.

2. The Regression of Forward Discount Bias

The most popular test of forward market unbiasedness is a regression of the future change in the spot rate on the forward discount:

$$\Delta s_{t+k} = \alpha + \beta fd_t^k + \eta_{t+k} \tag{1}$$

where Δs_{t+k} is the percentage depreciation of the currency (the change in the log of the spot price of foreign exchange) over k periods and fd_t^k is the current k-period forward discount (the log of the forward rate minus the log of the spot rate). The null hypothesis is that $\beta = 1$. (Some authors include $\alpha = 0$ in the null hypothesis as well.) In other words, the realized spot rate is equal to the forward rate plus a purely random error term, η_{t+k}. A second but equivalent specification is a regression of the forward rate prediction error on the forward discount:

$$fd_t^k - \Delta s_{t+k} = \alpha_1 + \beta_1 fd_t^k + \eta_{t+k} \tag{2}$$

most of our data set, this issue hardly arises to begin with, as they were conducted by telephone on a known day.

where $\alpha_1 = -\alpha$ and $\beta_1 = 1 - \beta$. The null hypothesis is now that $\alpha_1 = \beta_1 = 0$: the left-hand side variable is purely random.

Most tests of equation (1) have rejected the null hypothesis, finding β to be significantly less than one. Often the estimate of β is close to zero or negative.[4] Authors disagree, however, on the reason for this finding of bias. Longworth (1981) and Bilson (1981a), for example, assume that there is no risk premium, so that the forward discount accurately measures investors' expectations; they therefore interpret the bias as a rejection of the rational expectations hypothesis. Bilson describes the finding of β closer to zero than to one as a finding of "excessive speculation," meaning that investors would do better to reduce the absolute magnitude of their expected exchange rate changes. In the special case of $\beta = 0$, the exchange rate follows a random walk, and investors would do better to choose $\Delta s^e_{t+k} = 0$. On the other hand, Hsieh (1984) and most others assume that investors did not make systematic prediction errors in the sample; they interpret the bias as evidence of a time-varying risk premium. Fama (1984) and Hodrick and Srivastava (1986) describe the finding that $\beta < 1/2$ as a finding that the risk premium accounts for more of the variation in the forward discount than does expected depreciation. In the special case of $\beta = 0$, the assumption of rational expectations implies that $\Delta s^e_{t+k} = 0$, so that, as Bilson (1985) points out, the risk premium would account for *all* of the variation in the forward discount.

2.1. Standard Results Reproduced

We begin by reproducing the standard OLS regression results for equation (1) on sample periods that correspond precisely to those that we will be using for the survey data.[5] We report these results, in part, to show that the results obtained when we use the survey data below cannot be attributed to small sample size unless one is also prepared to attribute the usual finding of forward discount bias to small sample size.

In these and subsequent regressions, we pool across currencies in order to maximize sample size. (The four currencies in the MMS survey are the pound, mark, Swiss franc and yen, each against the dollar. The other two surveys include these four exchange rates and the French franc as

[4] The finding that forward rates are poor predictors of future spot rates is not limited to the foreign exchange market. In their study of the expectations hypothesis of the term structure, for example, Shiller, Campbell and Schoenholtz (1983) conclude that changes in the premium paid on longer-term bills over short-term bills are useless for predicting future changes in short-term interest rates.

[5] DRI provided us with daily forward and spot exchange rates, computed as the average of the noon-time bid and ask rates.

well.) As usual, we must allow for contemporaneous correlation in the error terms across currencies, in addition to allowing for the moving average error process induced by overlapping observations ($k > 1$). We report standard errors that assume conditional homoskedasticity, in this case because they were consistently larger than the estimated standard errors that allow for conditional heteroskedasticity. We also at times pool across different forecast horizons to maximize the power of the tests, requiring correction for a third kind of correlation in the errors. We are not aware of this having been done before, even in the standard forward discount regression. Each of these econometric issues is discussed at greater length in the appendices.

Table 1 presents the standard forward discount unbiasedness regressions (equation (1)) for our sample periods.[6] Note that in the *Economist* and Amex data sets, in which forecasts horizons were stacked, the standard errors fell in the aggregated regressions by 14 and 31 percent, respectively, in comparison with regressions that used the shorter-term predictions alone.

Most of the coefficients fall into the range reported by previous studies. There is ample evidence to reject unbiasedness: most of the coefficients are significantly less than one. More than half of the coefficients are even significantly less than zero, a finding of many other authors as well. In the two MMS data sets, the coefficients have unusually large absolute values, lending support to the observation by Gregory and McCurdy (1984) that the regression relation in equation (1) may be unstable. The F-tests also indicate that the unbiasedness hypothesis fails in most of the data sets.

At this point, we could interpret the results as reflecting systematic prediction errors. Under this interpretation, it follows that agents would do better by placing more weight on the contemporaneous spot rate and less weight on other factors in forming predictions of the future spot rate, the view discussed by Bilson (1981a). On the other hand, we could interpret the results as evidence of a time-varying risk premium. Then the conclusions would be that changes in expected depreciation are not correlated (or are negatively correlated) with changes in the forward discount and, from equation (3), that the variance of the risk premium is greater than the variance of expected depreciation.

[6] Regressions were estimated with dummies for each country, which we do not report to save space. For the regressions which pool over different forecast horizons (marked Economist Data and Amex Data), each country was allowed its own constant term for every forecast horizon.

2.2. Decomposition of the Forward Discount Bias Coefficient

The survey data, however, let us go a step further with the results of Table 1. We can now allocate part of the deviation from the null hypothesis of $\beta = 1$ to each of the alternatives: failure of rationality and the presence of a risk premium. The probability limit of the coefficient β in equation (1) is:

$$\beta = \frac{\text{cov}(\eta^k_{t+k}, fd^k_t) + \text{cov}(\Delta s^e_{t+k}, fd^k_t)}{\text{var}(fd^k_t)} \tag{3}$$

where η^k_{t+k} is market participants' expectational error, and Δs^e_{t+k} is the market expectation. We use the definition of the risk premium

$$rp^k_t = fd^k_t - \Delta s^e_{t+k} \tag{4}$$

and a little algebra to write β as equal to 1 (the null hypothesis) minus a term arising from any failure of rational expectations, minus another term arising from the risk premium:

$$\beta = 1 - \beta_{re} - \beta_{rp} \tag{5}$$

where

$$\beta_{re} = \frac{\text{cov}(\eta^k_{t+k}, fd^k_t)}{\text{var}(fd^k_t)}$$

$$\beta_{rp} = \frac{\text{var}(rp^k_t) + \text{cov}(\Delta s^e_{t+k}, rp^k_t)}{\text{var}(fd^k_t)}.$$

With the help of the survey data, both terms are observable. By inspection, $\beta_{re} = 0$ if there are no systematic prediction errors in the sample, and $\beta_{rp} = 0$ if there is no risk premium (or, somewhat more weakly, if the risk premium is uncorrelated with the forward discount).

The results of the decomposition are reported in Table 2a. First, β_{re} is very large in size when compared to β_{rp}, often by more than an order of magnitude. In all of the regressions, the lion's share of the deviation from the null hypothesis consists of systematic expectational errors. For example, in the *Economist* data, our largest survey sample with 525 observations, $\beta_{re} = 1.49$ and $\beta_{rp} = 0.08$. Second, while β_{re} is greater than zero in all cases, β_{rp} is sometimes negative, implying in equation (5) that the effect of the survey risk premium is to push the estimate of the standard coefficient β in the direction *above* one. In these cases, the risk premia do not explain a positive share of the forward discount's bias. The positive values for β_{re}, on the other hand, suggest the

possibility that investors tended to overreact to other information, in the sense that respondents might have improved their forecasting by placing more weight on the contemporaneous spot rate and less weight on the forward rate. Third, to the extent that the surveys are from different sources and cover different periods of time, they provide independent information, rendering their agreement on the relative importance and sign of the expectational errors all the more forceful. To check if the level of aggregation in Table 2a is hiding important diversity across currencies, Table 2b reports the decomposition for each currency in every data set. Here the results are the same: expectational errors are consistently large and positive, and the risk premium appears to explain no positive portion of the bias.

While the qualitative results above are of interest, we would like to know whether they are statistically significant, whether we can formally reject the two obvious polar hypotheses: (a) that the results in Table 4 are attributable to expectational errors, i.e. that the point estimates in column (1) are statistically significant; and (b) that they are attributable to the presence of the risk premium, i.e. that the point estimates in column (2) are statistically significant. We test these two hypotheses in turn in the following sections.

2.3. The Variance of Expected Depreciation vs. Variance of the Risk Premium

Notice that for most of the sample periods in Table 1, β is significantly less than 1/2. It is precisely on the basis of such estimates that Fama (1984) and Hodrick and Srivastava (1986) have claimed that expected depreciation is less variable than the exchange risk premium. We state the Fama-Hodrick-Srivastava (FHS) interpretation of the results as:

$$\mathbf{var}(\Delta s^e_{t+k}) < \mathbf{var}(rp^k_t) \tag{6}$$

To see how they arrive at this inequality, we use the definition of the risk premium in equation (4) to write the FHS proposition as

$$\mathbf{var}(\Delta s^e_{t+k}) < \mathbf{var}(rp^k_t) + \mathbf{var}(fd^k_t) - 2\mathbf{cov}(fd^k_t, \Delta s^e_{t+k}).$$

or

$$\mathbf{cov}(fd^k_t, \Delta s^e_{t+k}) < \frac{\mathbf{var}(fd^k_t)}{2} \tag{6'}$$

Under the assumption that the prediction error, η_{t+k}^k, is uncorrelated with fd_t^k (the usual rational expectations assumption), the regression coefficient β, as given by equation (3), becomes

$$\beta = \frac{\mathrm{cov}(\Delta s_{t+k}^e, fd_t^k)}{\mathrm{var}(fd_t^k)}. \tag{7}$$

Thus a finding of $\beta < 1/2$ implies the variance inequalities in equation (6). Added intuition is offered by recalling the special case $\beta = 0$: the variation in fd_t^k then consists entirely of variation in rp_t^k, and not at all variation in Δs_{t+k}^e.

We can use expectations as measured by the survey data to investigate the FHS claim directly, without assuming there is no systematic component to the prediction errors. Table 3 shows the variance of expected changes in the spot rate, as measured by the surveys, and the variance of the risk premia, for each data set and broken down by currency. The magnitude of *ex post* exchange rate changes (column 1) dwarfs that of the forward discount (column 2).[7] For example, the reported variance of annualized spot rate changes of 2 percent represents a standard deviation of about 14 percent. By comparison, the variance of expected depreciation is around .25 percent, a standard deviation of 5 percent.

The variance of expected depreciation is comparable in size to the variance of the risk premium (column 4), and is larger in 36 of the 40 samples calculated in Table 3. Thus "random walk" expectations ($\Delta s_{t+k}^e = 0$) do not appear to be supported by the survey data. We test formally the Fama (1984) hypothesis that the variance of expected depreciation is less than the variance of the risk premium in section 3. We can see from Table 3 that both are several times larger than the variance of the forward discount. Thus the relative stability of the forward discount masks greater variability in its two components, corroborating Fama's finding that the risk premium is negatively correlated with the expected change in the spot rate.[8,9]

3. A Direct Test of Perfect Substitutability

In the previous section we offered point estimates of the bias in the forward discount, which

[7] This empirical regularity has often been noted; e.g., Mussa (1979).

[8] This correlation is, however, biased downward by any measurement error that might be present in the surveys. If such error is purely random, then the covariance of expected depreciation and the risk premium may be written as $\mathrm{cov}(\Delta s_{t+k}^e, rp_t^k) - \mathrm{var}(\epsilon_t)$, where Δs_{t+k}^e and rp_t^k are the "true" values of expected depreciation and of the risk premium, respectively, and ϵ_t is the measurement error component of the survey.

[9] The low variance of the risk premium reported in Table 3 also has implications for the ability of tests of serial correlation in the forward rate errors, $fd_t^k - \Delta s_{t+k}$, to detect evidence of a time-varying risk premium. See the NBER working paper version of this paper (p. 10).

suggested that more of the bias was due to a failure of rational expectations than to a time-varying risk premium. In this section we formally test for the existence of the time-varying risk premium. In the next section we will formally test rational expectations.

Analogously to the standard regression equation equation, we regress our measure of expected depreciation against the forward discount:

$$\Delta \hat{s}^e_{t+1} = \alpha_2 + \beta_2 fd^k_t + \epsilon_t. \tag{8}$$

The null hypothesis that the correlation of the risk premium with the forward discount is zero implies $\beta_2 = 1$. By inspection, $\beta_2 = 1 - \beta_{rp}$, so that a finding of $\beta_2 = 1$ would imply that the results in column (2) of Table 2a are not statistically different from zero. Equation (8) also allows us to test the hypothesis of no constant risk premium either: $\alpha_2 = 0$. The hypothesis that the risk premium is identically zero is given by $\Delta \hat{s}^e_{t+1} = fd^k_t$. We should therefore interpret the regression error ϵ_t as random measurement error in the surveys. That is, $\Delta \hat{s}^e_{t+1} = \Delta s^e_{t+k} + \epsilon_t$, where Δs^e_{t+k} is the unobservable market expected change in the spot rate. Note also that in a test of equation (8) using the survey data, the properties of the error term, ϵ_t, will be invariant to any "peso problems," which affect instead the ex post distribution of actual spot rate changes.

Another way of stating the null hypothesis is the proposition that domestic and foreign assets are perfect substitutes in investor's portfolios. Assuming that covered interest parity holds, the forward discount fd^k_t is equal to the differential between domestic and foreign nominal interest rates $i^k_t - i^{*k}_t$. The null hypothesis then becomes a statement of uncovered interest parity: $\Delta s^e_{t+k} = i^k_t - i^{*k}_t$. In other words, investors are so responsive to differences in expected rates of return as to eliminate them.[10]

We can also use equation (8) to test formally the FHS hypothesis that the variance of the risk premium is greater than the variance of expected depreciation. This is the inequality (6), which we found to be violated by point estimates in Table 3. (Although random measurement error in the survey data would tend to overstate each of these variances individually, it does not affect the estimate of their difference.) The probability limit of the coefficient β_2 is:

$$\beta_2 = \frac{\mathrm{cov}(\Delta \hat{s}^e_{t+1}, fd^k_t)}{\mathrm{var}(fd^k_t)} = \frac{\mathrm{cov}(\Delta s^e_{t+k}, fd^k_t)}{\mathrm{var}(fd^k_t)} \tag{9}$$

[10] For tests of uncovered interest parity similar to the tests of conditional bias in the forward discount that we considered in section 3, see Cumby and Obstfeld (1981).

where we have used the assumption that the measurement error ϵ_t is uncorrelated with the forward discount fd_t^k (analogously to the derivation of equation (7)). If follows from equation (9) that only if $\beta_2 < 1/2$ does the FHS inequality (6') hold; if β_2 is significantly greater than 1/2, the variance of expected depreciation exceeds that of the risk premium.

Table 4 reports the OLS regressions of equation (8). In some respects the data provide evidence in favor of perfect substitutability. Contrary to the hypothesis of a risk premium that is correlated with the forward discount, all of the estimates of β_2 are statistically indistinguishable from one (with the sole exception of the MMS three-month sample). In the *Economist* and Amex data sets which aggregate across time horizons, the estimates are 0.99 and 0.96, respectively.[11] Expectations seem to move very strongly with the forward rate. With the exception of the MMS data, the coefficients are estimated with surprising precision.

In terms of our decomposition of the forward discount bias coefficient, Table 4 shows the values of β_{rp} in column 2 of Table 2a are statistically far from one and are not significantly different from zero. Thus the rejection of unbiasedness found in the previous section cannot be explained entirely by the risk premium at any reasonable level of confidence. Indeed, in spite of the fact that the survey risk premium has substantial magnitude, we cannot reject the hypothesis that the risk premium explains *no* positive portion of the bias.

There is strong evidence of a constant term in the risk premium however: α_2 is large and statistically greater than zero. Each of the F-tests reported in Table 6 rejects the parity relation at a level of significance that is less than 0.1 percent. Charts 1-4 make apparent the high *average* level of the risk premium (as well as its lack of correlation with the usual measure of the risk premium, the forward discount prediction errors).[12] Thus the qualitatively small values of β_{rp} reported in Tables 2a and 2b should not be taken to imply that the survey responses include no information about the future spot rate beyond that contained in the forward rate.[13]

[11] For the *Economist* six-month and twelve-month and the Amex twelve-month data sets, the estimates of β_2 from equation (8) do not exactly correspond to $1 - \beta_{rp}$ in Tables 2a and 2b. This is because Table 4 includes a few survey observations for which actual future spot rates have not yet been realized, whereas these observations were left out of the decomposition in Tables 2a and 2b for purposes of comparability. If we had used the smaller samples in Table 4, the regression coefficients would have been .92 and 1.03, for the *Economist* and Amex data sets, respectively.

[12] The degree to which the surveys qualitatively corroborate one another is striking. For example, the risk premium in the *Economist* data (Chart 1) is negative during the entire sample, except for a short period from late 1984 until mid-1985. The MMS three-month sample (Chart 2) reports that the risk premium did not become positive until the last quarter of 1984, while MMS one-month data (Chart 3) shows the risk premium then remained positive until mid-1985. That the surveys agree on the nature and timing of major swings in the risk premium is some evidence that the particularities of each group of respondents do not influence the results.

[13] In Table 2 of the NBER working paper version of this study, we reported mean values of the risk premium as measured

Table 4 also reports a t-test of the hypothesis that $\beta_2 = 1/2$. In six out of nine cases the data strongly reject the hypothesis that the variance of the true risk premium is greater than or equal to that of true expected depreciation; we have rather $\mathrm{var}(\Delta s^e_{t+k}) > \mathrm{var}(rp^k_t)$. Indeed, the finding that $\beta_2 = 1$ implies that:

$$\mathrm{var}(rp^k_t) + cov(\Delta s^e_{t+k}, rp^k_t) = 0 \qquad (10).$$

Thus we cannot reject the hypothesis that the covariance of true expected depreciation and the true risk premium is negative (as Fama found), nor can we reject the extreme hypothesis that the variance of the true risk premium is zero.

Under the null hypothesis that there is no time-varying risk premium and the regression error ϵ_t is equation (8) is random measurement error, we can use the R^2's from the regressions to obtain an estimate of the relative importance of the measurement error component in the survey data. The R^2 statistics in Table 4 are relatively high, suggesting that measurement error is relatively small. For example, under this interpretation of the R^2s, measurement error accounts for about 10 percent of the variability in expected depreciation from the *Economist* data. For a standard of comparison, the R^2 for the same sample period in Table 1 (which uses *ex post* exchange rate changes as a noisy measure of expectations) implies that 84 percent of the variability in the measure is noise.[14]

In Table 5 we correct for the potential serial correlation problem in the *Economist* and MMS data sets by employing a Three-Stage-Least-Squares estimator that allows for contemporaneous correlation (SUR) as well as first order auto-regressive disturbances.[15] 3SLS is consistent here because there are no overlapping observations - predictions by the forward rate and the surveys are observed contemporaneously - and it has the advantage of being asymptotically efficient. The results reported in Table 5 show that this correction does not change the nature of the results; all but one of the coefficients remain close to one, and there is clear evidence that the variance of expected depreciation is greater than that of the risk premium (while there is no evidence for the alternative that the variance of the risk premium is greater).

by the survey data. They were different from zero at the 99 percent level for almost all survey sources, currencies and sample periods.

[14] In both cases, the R^2 statistics include the explanatory power of the constant terms for each currency and forecast horizon.

[15] Unfortunately, the highly irregular spacing of the Amex data sets did not permit an auto-regressive correction in this case.

4. Tests of Rational Expectations

In the previous section we formally tested the hypothesis that there exists no time-varying risk premium that could explain the findings of bias in the forward discount. In this section we formally test the hypothesis that there exist systematic expectational errors that *can* explain those findings.

4.1. A Test of Excessive Speculation

Perhaps the most powerful test of rational expectations is one which asks whether investors would do better if they placed more or less weight on the contemporaneous spot rate as opposed to all other variables in their information set.[16] This test is performed by a regression of the expectational prediction error on expected depreciation:

$$\Delta \hat{s}^e_{t+1} - \Delta s_{t+k} = a + d \Delta \hat{s}^e_{t+1} + v^k_{t+k} \tag{11}$$

where the null hypothesis is $a = 0$ and $d = 0$.[17] This is the equation that Bilson (1981a) and others had in mind, which we already termed a test of "excessive" speculation, with the difference that we are measuring expected depreciation by the survey data instead of by the ambiguous forward discount.

Our tests are reported in Table 6. The findings consistently indicate that $d > 0$, so that investors could on average do better by giving more weight to the contemporaneous spot rate and less weight to other information they deem pertinent. In other words, the excessive speculation hypothesis is upheld. F-tests of the hypothesis that there are no systematic expectational errors, $a = d = 0$, reject at the one percent level for all of the survey data sets.

The results in Table 8 would appear to constitute a resounding rejection of rationality in the survey expectations. Up until this point, our test statistics have been robust to the presence of

[16] Frankel and Froot (1986) test whether the survey expectations place too little weight on the contemporaneous spot rate and too much weight on specific pieces of information such as the lagged spot rate, the long-run equilibrium exchange rate, and the lagged expected spot rate.

[17] To see how the alternative in equation (11) is too much or too little weight on all variables in the information set other than the contemporaneous spot rate, assume expectations are formed as a linear combination of the current spot rate, s_t, and any linear combination of variables in the information set, I_t:

$$s^e_{t+k} = \pi_1 I_t + (1 - \pi_1) s_t$$

If the actual process is:

$$s_{t+k} = \pi_2 I_t + (1 - \pi_2) s_t - \nu^k_{t+k}$$

Then equation (11) can be rewritten as

$$\Delta \hat{s}^e_{t+1} - s_{t+k} = a + (\pi_1 - \pi_2)(I_t - s_t) + \nu^k_{t+k}.$$

Rational expectations is the case in which the coefficient $\pi_1 - \pi_2$ is zero. A positive value implies $\pi_1 > \pi_2$: investors put insufficient weight on s_t and too much weight on other information.

random measurement error in the survey data because expectations have appeared only on the left-hand side of the equation. But now expectations appear also on the right-hand side; as a result, under the null hypothesis, measurement error biases toward one our estimate of d in equation (11). To demonstrate this effect, suppose again that expected depreciation as recorded by the survey is equal to the market's true expectation, Δs^e_{t+k}, plus an error term:

$$\Delta \hat{s}^e_{t+1} = \Delta s^e_{t+k} + \epsilon_t \tag{12}$$

where ϵ_t is iid and $E(\epsilon_t|\Delta s^e_{t+k}) = 0$. The actual spot rate change can then be expressed as the sum of the true market expectation plus a prediction error:

$$\Delta s_{t+k} = \Delta s^e_{t+k} + \eta^k_{t+k} \qquad (13).$$

Using these facts, the coefficient d in equation (11) converges in probability to:

$$d = \frac{\mathbf{var}(\epsilon_t) - \mathbf{cov}(\eta^k_{t+k}, \Delta s^e_{t+k})}{\mathbf{var}(\epsilon_t) + \mathbf{var}(\Delta s^e_{t+k})} \tag{14}$$

Measurement error therefore biases our OLS estimates toward one. Indeed, in the limiting case in which the measurement error accounts for all of the variability of expected depreciation in the survey – in other words, no information at all about the "true" market expectation is contained in the surveys – the parameter estimate would be statistically indistinguishable from one. In Table 6, 13 of 15 estimates of d are *greater* than one; in five cases the difference is statistically significant. This result suggests that measurement error is not the source of our rejection of rational expectations. However, we shall now see that stronger evidence can be obtained.

4.2. Another Test of Excessive Speculation

Another test of rational expectations that is free of the problem of measurement error is to replace $\Delta \hat{s}^e_{t+1}$ on the right-hand side of equation (11) with the forward discount fd^k_t:

$$\Delta \hat{s}^e_{t+1} - \Delta s_{t+k} = \alpha_1 + \beta_1 fd_t + \epsilon_t - \eta_{t+k}. \tag{15}$$

where the residual, $\epsilon_t - \eta_{t+k}$, is the measurement error in the surveys less the unexpected change in the spot rate.

There are several reasons for making the substitution in equation (15). We know from our results in section 3 that expected depreciation is highly correlated with fd^k_t. Because fd^k_t is free

of measurement error, it is a good candidate for an exogenous "instrumental variable." Indeed, if we as econometricians can look the forward discount up precisely in the newspaper, we can also do so as prospective speculators. A finding of $\beta_1 > 0$ in either equation (9) or (13) suggests that a speculator could have made excess profits by betting against the market. But the strategy to "bet against the market" is far more practical if expressed as "bet against the (observable) forward discount" than as "do the opposite of whatever you would have otherwise done."

Equation (15) has additional relevance in the context of our decomposition of the forward rate unbiasedness regression in section 3: the coefficient, β_1, is precisely equal to the deviation from unbiasedness due to systematic prediction errors, β_{re}. Thus equation (24) can tell us whether the large positive values of β_{re} found in column (1) of Tables 3a and 3b, are statistically significant.

Table 7 reports OLS regressions of equation (15). We now see that the point estimates of β_{re} in Tables 3a and 3b are measured with precision. The data continue to reject statistically the hypothesis of rational expectations, $\alpha_1 = 0$, $\beta_1 = 0$. They reject $\beta_1 = 0$, in favor of the alternative of excessive speculation. (Because the measurement error has been purged, the levels of significance are necessarily lower than those of Table 6.) Thus the result that β_{re} is significantly greater than zero seems robust across different forecast horizons and different survey samples. In terms of the decomposition of the typical forward rate unbiasedness test in Table 3a, we can now reject the hypothesis that all of the bias is attributable to the survey risk premium. Put differently, we cannot reject the hypothesis that none of the bias is due to repeated expectational errors made by survey respondents. Recall that this finding need not mean that investors are irrational. If they are learning about a new exchange rate process, or if there is a "peso problem" with the distribution of the error term, then one could not expect them to foresee errors in the sample period, even though the errors appear to be systematic *ex post*.

5. Conclusions

Our general conclusion is that, contrary to what is assumed in conventional practice, the systematic portion of forward discount prediction errors do not capture a time-varying risk premium. This result was qualitatively clear from the point estimates in section 2 or from the charts. But we can now make several statements that are more precise statistically.

(1) We reject the hypothesis that all of the bias in the forward discount is due to the risk premium. This is the same thing as rejecting the hypothesis that none of the bias is due to the presence of systematic expectational errors.

(2) We cannot reject the hypothesis that *all* of the bias is attributable to these systematic expectational errors, and none to a time-varying risk premium.

(3) The implication of (1) and (2) is that changes in the forward discount reflect, one-for-one, changes in expected depreciation, as perfect substitutability among assets denominated in different currencies would imply.

(4) We reject the claim that the variance of the risk premium is greater than the variance of expected depreciation. The reverse appears to be the case: the variance of expected depreciation is large in comparison with both the variance of the risk premium and the variance of the forward discount.

(5) Because the survey risk premium appears to be uncorrelated with the forward discount, we cannot reject the hypothesis that the market risk premium we are trying to measure is constant. We do find a substantial *average* level of the risk premium. But it does not vary positively with the forward discount as conventionally thought.

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CHART 1

FORWARD RATE ERRORS & THE RISK PREMIUM

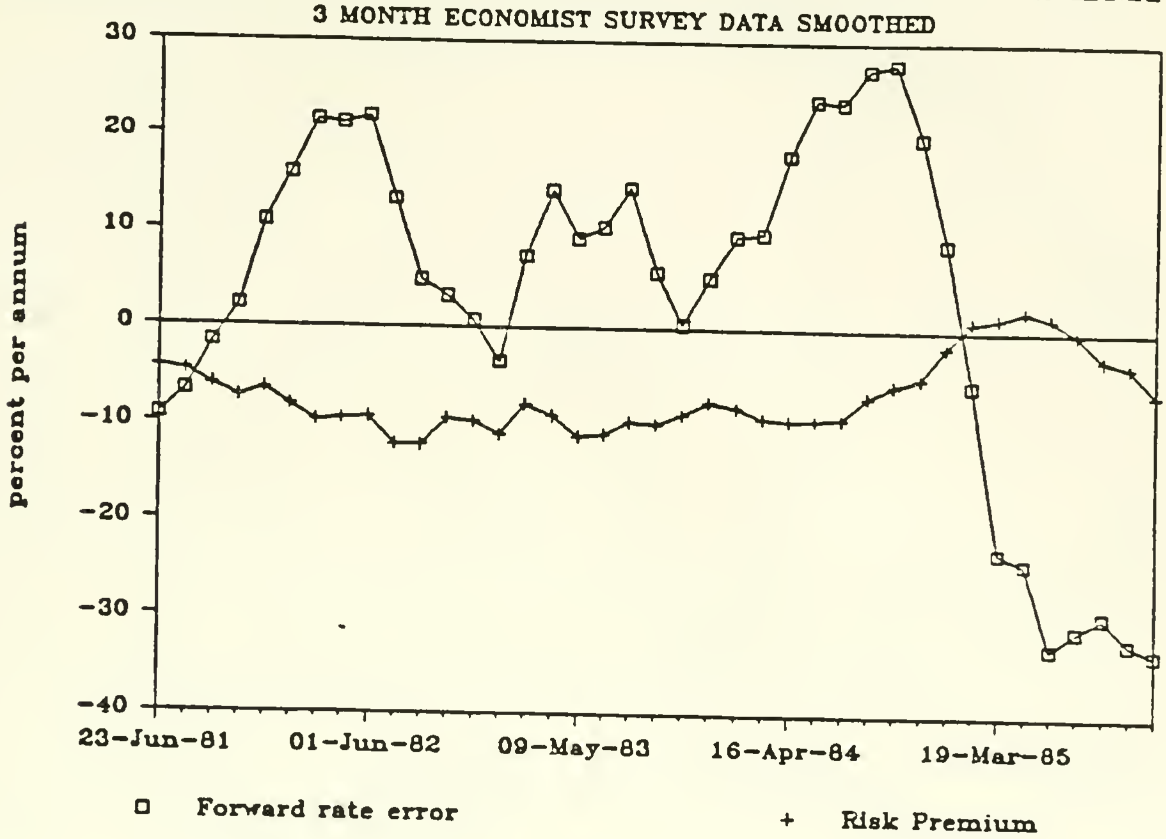

CHART 3

FORWARD RATE ERRORS & THE RISK PREMIUM

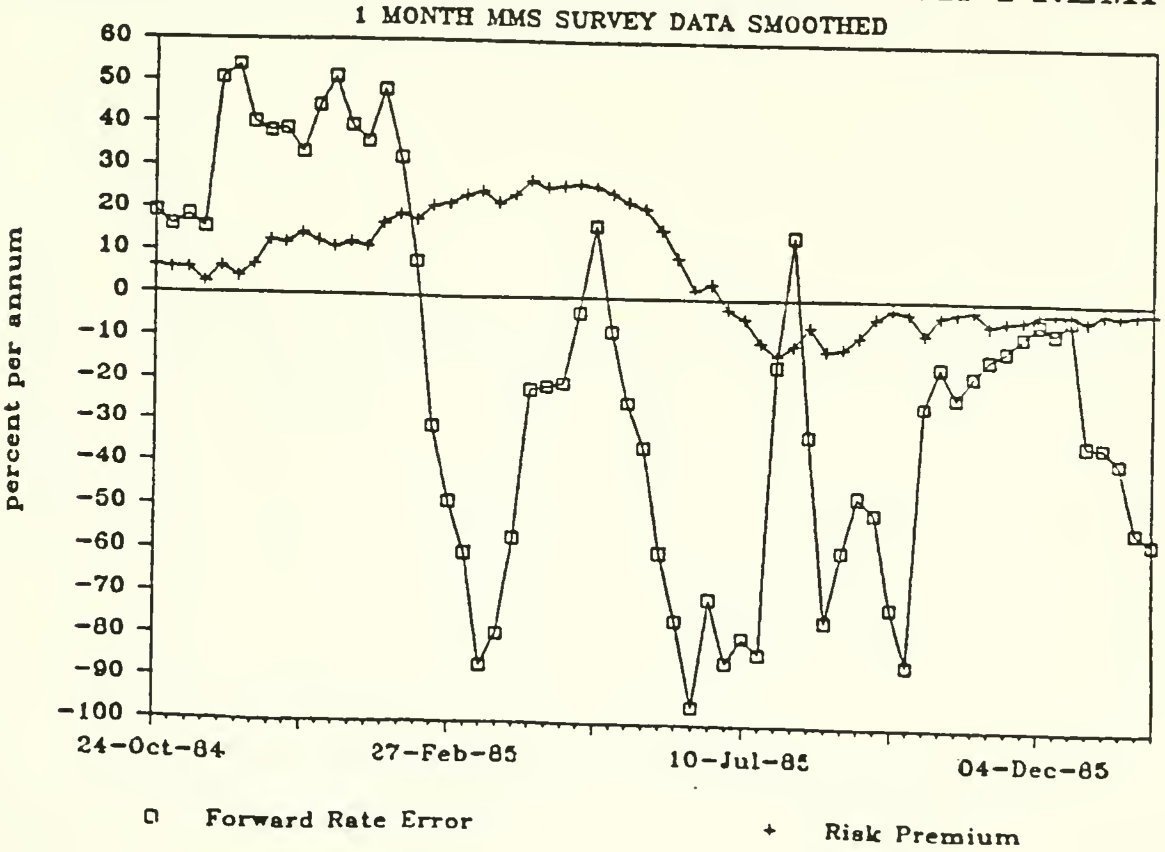

CHART 2

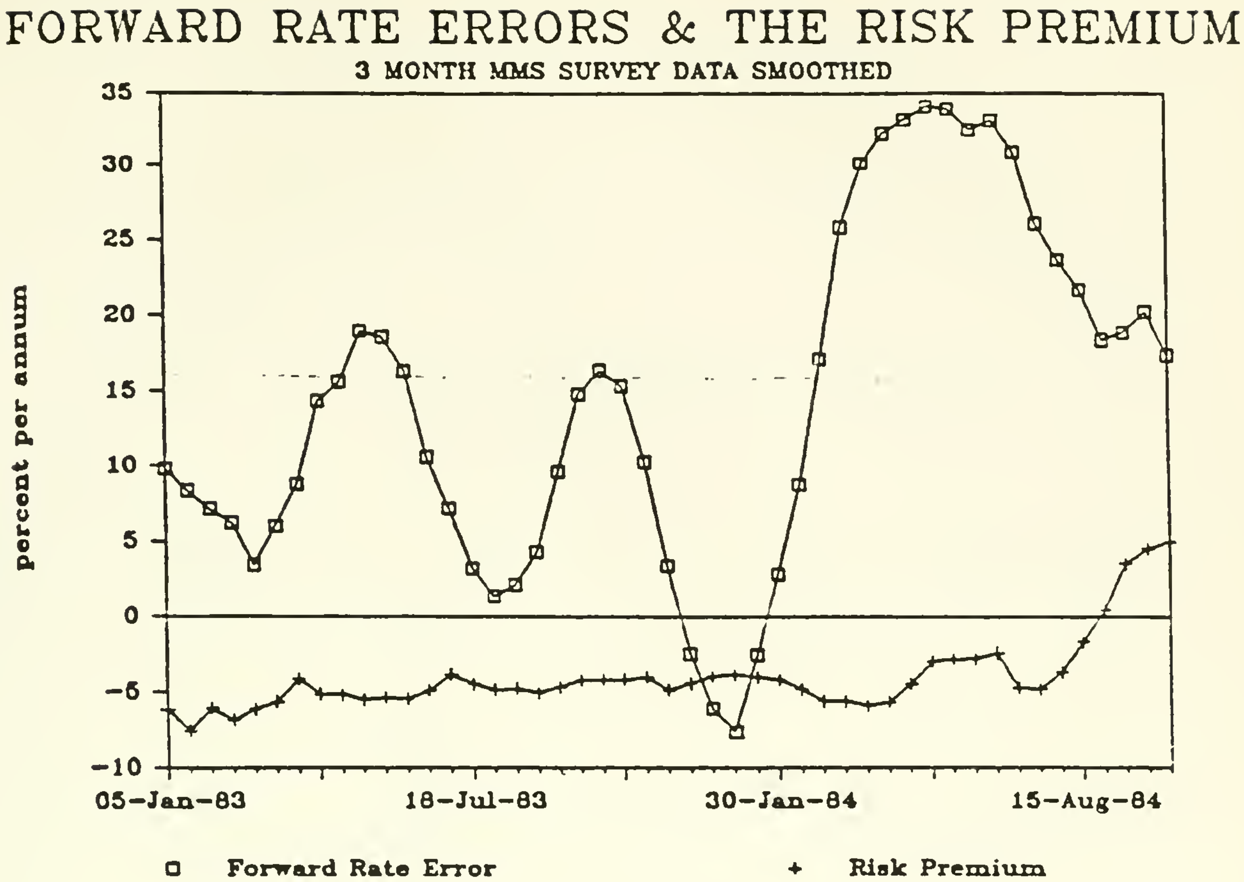

CHART 4

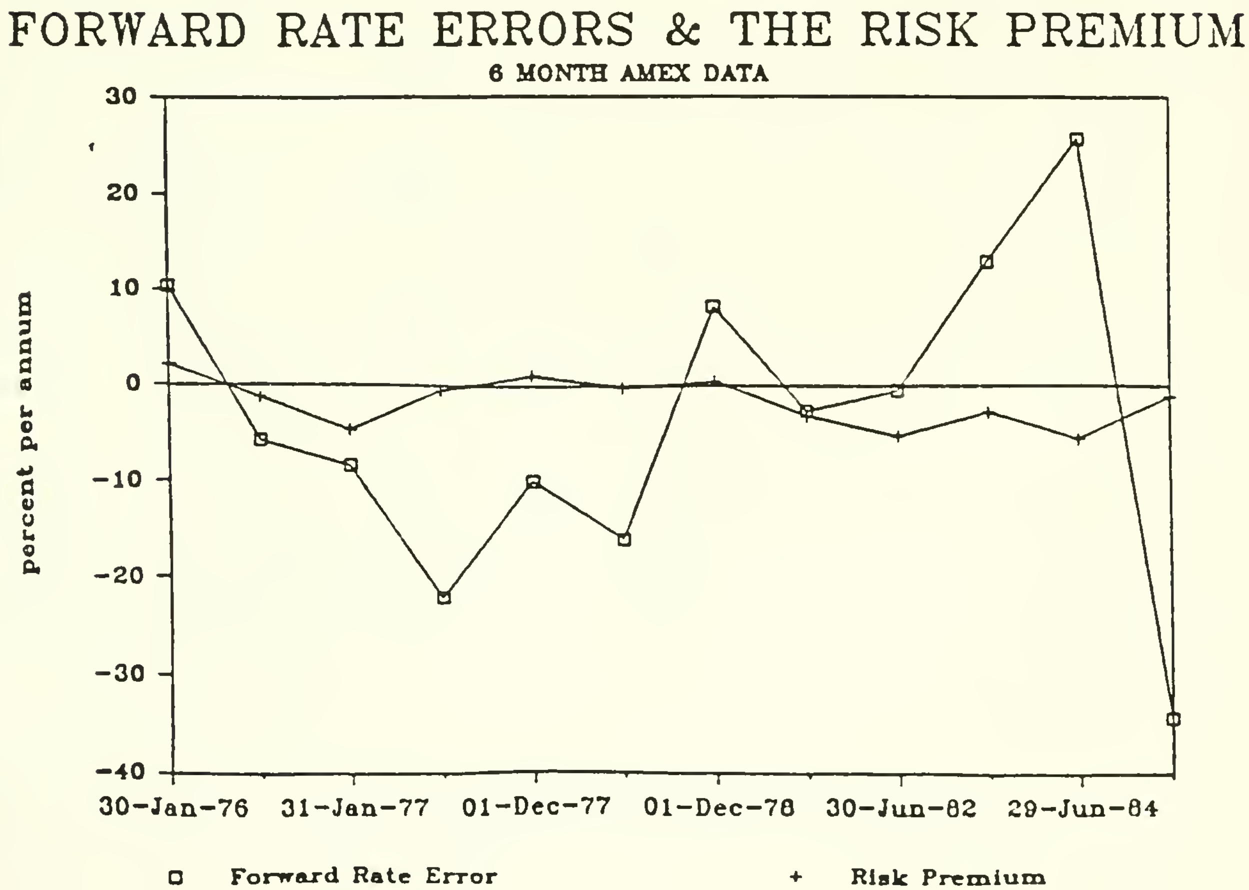

TABLE 1
TESTS OF FORWARD DISCOUNT UNBIASEDNESS

OLS Regressions of Δs_{t+k} on fd_t^k

Data Set	Dates	$\hat{B}$	t: $\hat{B}=0$	t: $\hat{B}=1$	R^2	DF	F test $\hat{a}=0$, $\hat{B}=1$	Prob > F
Economist Data	6/81-12/85	-0.5684 (1.0171)	-0.56	-1.54	0.16	509	2.12	0.007
Econ 3 Month	6/81-12/85	-1.2090 (1.1598)	-1.04	-1.91 *	0.01	184	1.29	0.282
Econ 6 Month	6/81-12/85	-1.9919 (1.4445)	-1.37	-2.06 **	0.07	174	1.47	0.191
Econ 12 Month	6/81-12/85	0.2592 (1.2733)	0.20	-0.56	0.29	149	3.23	0.005
MMS 1 Month	10/84-2/86	-14.5529 (6.0030)	-2.43 **	-2.59 **	0.21	171	2.67	0.024
MMS 3 Month	1/83-10/84	-6.2540 (2.1508)	-2.91 ***	-3.37 ***	0.50	183	12.01	0.000
AMEX Data	1/76-7/85	-2.2107 (0.9620)	-2.30 **	-3.34 ***	0.23	86	2.60	0.007
AMEX 6 Month	1/76-7/85	-2.4181 (1.2616)	-1.92 *	-2.71 ***	0.26	45	2.42	0.041
AMEX 12 Month	1/76-7/85	-2.1077 (1.0549)	-2.03 **	-2.97 ***	0.21	40	1.66	0.157

Notes: Method of Moments standard errors are in parentheses. * Represents significance at the 10% level, ** and *** represent significance at the 5% and 1% levels, respectively.

TABLE 2a

COMPONENTS OF THE FAILURE OF THE UNBIASEDNESS HYPOTHESIS

In Regressions of Δs_{t+1} on fd_t

Data Set	Approximate Dates	N	Failure of Rational Expectations (1) $\hat{\beta}_{re}$	Existence of Risk Premium (2) $\hat{\beta}_{rp}$	Regression Coefficient 1-(1)-(2) $\hat{\beta}$
Economist Data	6/81-12/85	525	1.49	0.08	-0.57
Econ 3 Month	6/81-12/85	190	2.51	-0.30	-1.21
Econ 6 Month	6/81-12/85	180	2.99	0.00	-1.99
Econ 12 Month	6/81-12/85	155	0.52	0.19	0.29
MMS 1 Month	10/84-2/86	175	15.39	0.16	-14.55
MMS 3 Month	1/83-10/84	133	5.07	1.18	-5.25
AMEX Data	1/76-7/85	97	3.23	-0.03	-2.21
AMEX 6 Month	1/76-7/85	51	3.63	-0.22	-2.42
AMEX 12 Month	1/76-7/84	46	3.11	0.03	-2.14

TABLE 2b

COMPONENTS OF THE FAILURE OF THE UNBIASEDNESS HYPOTHESIS

In Regressions of Δs_{t+1} on fd_t

Data Set	Approximate Dates	N	Failure of Rational Expectations (1) $\hat{B}_{re}$	Existence of Risk Premium (2) $\hat{B}_{rp}$	Regression Coefficient 1-(1)-(2) $\hat{B}$
ECON 3 MONTH	6/81-12/85	190	2.51	-0.30	-1.21
UK		38	7.31	-1.11	-5.20
FR		38	-1.75	0.47	2.28
WG		38	7.69	-1.64	-5.05
SW		38	5.03	-0.63	-3.40
JA		38	4.66	-0.73	-2.93
ECON 6 MONTH	6/81-12/85	180	2.99	0.00	-1.98
UK		36	7.04	-0.17	-5.87
FR		36	-1.31	0.21	2.10
WG		36	10.16	-0.38	-8.77
SW		36	5.75	-0.01	-4.74
JA		36	4.69	-0.18	-3.51
ECON 12 MONTH	6/81-12/85	155	0.52	0.19	0.29
UK		31	1.37	0.93	-1.29
FR		31	-1.45	0.16	2.29
WG		31	-0.13	0.16	0.96
SW		31	0.96	0.25	-0.21
JA		31	3.09	-0.04	-2.05
MMS 1 MONTH	10/84-2/86	176	15.39	0.16	-14.55
UK		44	21.23	0.06	-20.28
WG		44	10.34	-8.95	-0.39
SW		44	10.15	-2.39	-6.76
JA		44	4.58	7.10	-10.68
MMS 3 MONTH	1/83-10/84	188	6.07	1.18	6.25
UK		47	7.90	0.27	-7.16
WG		47	4.96	2.52	-6.48
SW		47	7.90	0.09	-6.98
JA		47	3.43	2.14	-4.57
AMEX 6 MONTH	1/76-7/85	51	3.63	-0.22	-2.42
UK		12	2.76	-0.15	-1.60
FR		9	1.09	-0.03	-0.06
WG		12	4.73	-0.63	-3.15
SW		9	5.53	-0.33	-4.20
JA		9	4.53	-0.10	-3.43
AMEX 12 MONTH	1/76-7/84	46	3.11	0.03	2.14
UK		11	2.53	-0.09	-1.45
FR		8	0.48	0.32	0.20
WG		11	2.53	-0.40	-2.13
SW		8	3.99	0.78	-3.77
JA		8	5.36	0.12	-4.47

TABLE 3

COMPARISON OF VARIANCES OF EXPECTED DEPRECIATION AND THE RISK PREMIUM

(x 10^2 per annum)

FORECAST HORIZON	SURVEY SOURCE	DATES	N	(1) Variance of $\Delta s(t)$	(2) Variance of $fd(t)$	(3) Variance of $E[\Delta s(t+1)]$	(4) Variance of $rp(t)$	(5) (4)-(5)
1 WEEK								
TOTAL	MMS	10/84-2/86	247	2.756		0.346		
UK			62	3.809		0.429		
WG			62	2.506	NA	0.251	NA	NA
SW			61	3.271		0.404		
JA			62	1.564		0.264		
2 WEEKS								
TOTAL	MMS	1/83-10/84	187	0.703		0.113		
UK			47	0.765		0.111		
WG			47	0.886	NA	0.122	NA	NA
SW			46	0.640		0.114		
JA			47	0.533		0.084		
1 MONTH								
TOTAL	MMS	10/84-2/86	176	2.285	0.008	0.258	0.240	0.018
UK			44	2.839	0.002	0.359	0.353	0.001
WG			44	2.091	0.000	0.224	0.219	0.005
SW			44	2.595	0.001	0.228	0.224	0.004
JA			44	1.676	0.001	0.129	0.137	-0.008
3 MONTHS								
TOTAL	MMS	1/83-10/84	187	0.610	0.014	0.067	0.062	0.005
UK			47	0.647	0.004	0.036	0.034	0.002
WG			47	0.702	0.002	0.045	0.054	-0.009
SW			46	0.447	0.002	0.118	0.116	0.002
JA			47	0.470	0.004	0.036	0.048	-0.012
TOTAL	ECONOMIST	6/81-12/85	190	1.651	0.051	0.178	0.121	0.056
UK			38	1.433	-0.014	0.188	0.142	0.046
FR			38	1.783	0.037	0.092	0.090	0.002
WG			38	1.557	0.004	0.129	0.111	0.017
SW			38	2.096	0.011	0.109	0.084	0.025
JA			38	1.440	0.020	0.198	0.149	0.049
6 MONTHS								
TOTAL	ECONOMIST	6/81-12/85	190	2.004	0.093	0.173	0.082	0.091
UK			38	1.554	0.021	0.110	0.077	0.033
FR			38	2.251	0.050	0.097	0.072	0.025
WG			38	1.913	0.006	0.082	0.070	0.013
SW			38	2.345	0.016	0.087	0.070	0.017
JA			38	1.798	0.033	0.101	0.053	0.048
TOTAL	AMEX	1/76-8/85	51	1.658	0.111	0.134	0.084	0.051
UK			12	1.395	0.074	0.131	0.035	0.096
FR			9	1.583	0.033	0.074	0.039	0.035
WG			12	1.347	0.025	0.150	0.094	0.056
SW			9	2.213	0.043	0.133	0.110	0.072
JA			9	1.679	0.056	0.095	0.023	0.067
12 MONTHS								
TOTAL	ECONOMIST	6/81-12/85	195	1.368	0.155	0.215	0.092	0.123
UK			38	1.319	0.027	0.132	0.113	0.019
FR			38	1.432	0.053	0.092	0.069	0.023
WG			38	1.088	0.008	0.071	0.057	0.014
SW			38	1.340	0.023	0.078	0.058	0.020
JA			38	1.096	0.046	0.115	0.052	0.063
TOTAL	AMEX	1/76-8/85	51	1.446	0.192	0.195	0.129	0.066
UK			12	1.315	0.111	0.198	0.071	0.127
FR			9	1.130	0.055	0.040	0.020	0.020
WG			12	0.768	0.039	0.228	0.164	. 0.064
SW			9	1.949	0.064	0.178	0.195	-0.018
JA			9	2.564	0.065	0.109	0.065	0.044

Note: The variance of the forward rate prediction error and its correlation with the risk premium are reported in the NBER Working Paper.

TABLE 4

TESTS OF PERFECT SUBSTITUTABILITY

OLS Regressions of Δs^{e}_{t+k} on fd^{k}_{t}

Data Set	Dates	B	t: B=.5	t: B=1	R	DF	DW	F test a=0, B=1	Prob > F
Economist Data	6/81-12/85	0.9880 (0.1465)	3.33 ***	-0.08	0.89	554	1.44	28.61	0.000
Econ 3 Month	6/81-12/85	1.3037 (0.2557)	3.14 ***	1.19	0.70	184	1.56	16.55	0.000
Econ 6 Month	6/81-12/85	1.0326 (0.1694)	3.14 ***	0.19	0.89	184	1.37	52.06	0.000
Econ 12 Month	6/81-12/85	0.9296 (0.1499)	2.86 ***	-0.48	0.91	184	1.44	65.82	0.000
MMS 1 Month	10/84-2/86	0.8416 (1.7275)	0.20	-0.09	0.21	171	1.02	6.79	0.000
MMS 3 Month	1/83-10/84	-0.1816 (0.4293)	-1.59	-2.75 ***	0.73	182	1.50	14.60	0.000
AMEX Data	1/76-7/85	0.9605 (0.2495)	1.85 *	-0.16	0.64	91	0.74	5.38	0.000
AMEX 6 Month	1/76-7/85	1.2165 (0.2085)	3.44 ***	1.04	0.71	45	1.45	6.32	0.000
AMEX 12 Month	1/76-7/85	0.8770 (0.2755)	1.37	-0.45	0.61	45	0.51	8.10	0.000

Notes: Method of Moments standard errors are in parentheses. * Represents significance at the 10% level, ** and *** represent significance at the 5% and 1% levels, respectively.

TABLE 5

TESTS OF PERFECT SUBSTITUTABILITY

3SLS Regressions of Δs^{e}_{t+k} on fd^{k}_{t}

Data Set	Dates	B	t: B=.5	t: B=1	average p(1)	DF	Prob > F a=0, B=1
Economist 3 Month	6/81-12/85	0.8723 (0.1327)	2.81 ***	-0.96	0.13	184	0.000
Economist 6 Month	6/81-12/85	0.8768 (0.0780)	4.83 ***	-1.58	0.32	184	0.000
Economist 12 Month	6/81-12/85	0.8378 (0.0793)	4.26 ***	-2.04 **	0.27	184	0.000
MMS 1 Month	10/84-2/86	-1.1535 (1.0445)	-1.58	-2.06 **	0.21	171	NA
MMS 3 Month	1/83-10/84	0.4672 (0.3354)	-0.10	-1.59	0.33	179	0.000

(1) Average p is the mean across countries of the first order auto-regressive coefficients.

Notes: Asymptotic standard errors are in parentheses. * Represents significance at the 10% level, ** and *** represent significance at the 5% and 1% levels, respectively.

TABLE 6
TESTS OF EXCESSIVE SPECULATION

Regressions of $\Delta \hat{s}^{e}_{t+k} - s_{t+k}$ on $\Delta \hat{s}^{e}_{t+k}$

Data Set	Dates	B	t: B=0	t: B=1	R	DF	DW	F test a=0, B=0	Prob > F
Economist Data	6/81-12/85	1.0162 (0.4104)	2.48 **	0.04	0.49	509		4.79	0.000
Econ 3 Month	6/81-12/85	1.6141 (0.4664)	3.46 ***	1.32	0.26	184		2.91	0.010
Econ 6 Month	6/81-12/85	2.5325 (0.6746)	3.75 ***	2.27 **	0.41	174		3.54	0.002
Econ 12 Month	6/81-12/85	-0.3005 (0.5241)	-0.57	-2.48 **	0.67	149		6.32	0.000
MMS 1 Week, 1 Month	10/84-2/86	1.2561 (0.3544)	3.54 ***	0.72	0.24	414		6.07	0.000
MMS 1 Week	10/84-2/86	1.1476 (0.2939)	3.90 ***	0.50	0.14	242	1.84	3.97	0.002
MMS 1 Week, SUR	10/84-2/86	0.7858 (0.1109)	7.09 ***	-1.93 *	0.19	239		12.42	0.000
MMS 1 Month	10/84-2/86	1.3068 (0.4741)	2.76 ***	0.65	0.23	171		3.11	0.010
MMS 2 Week, 3 Month	1/83-10/84	1.0494 (0.3159)	3.32 ***	0.16	0.59	365		7.37	0.000
MMS 2 Week	1/83-10/84	1.0594 (0.2870)	3.59 ***	0.21	0.23	192	1.74	5.40	0.000
MMS 2 Week, SUR	1/83-10/84	1.0469 (0.1813)	5.77 ***	0.26	0.16	179		9.42	0.000
MMS 3 Month	1/83-10/84	1.0465 (0.3895)	2.69 ***	0.12	0.63	182		7.59	0.000
AMEX Data	1/76-7/85	2.6082 (0.5123)	5.09 ***	3.14 ***	0.23	86		4.71	0.000
AMEX 6 Month	1/76-7/85	2.5693 (0.7358)	3.49 ***	2.13 **	0.37	45		4.22	0.002
AMEX 12 Month	1/76-7/85	2.6382 (0.5312)	4.54 ***	2.82 ***	0.50	40		4.24	0.002

Notes: All regressions except those marked SUR are estimated using OLS, with Method of Moments standard errors (in parentheses). SUR regressions report asymptotic standard errors. Durbin-Watson statistics are reported for data sets in which the forecast horizon is equal to the sampling interval. * Represents significance at the 10% level. ** and *** represent significance at the 5% and 1% levels, respectively.

TABLE 7
TESTS OF RATIONAL EXPECTATIONS

OLS Regressions of $\Delta s^e_{t+1} - \Delta s_{t+1}$ on fd

Data Set	Dates	B	t: B=0	R	DF	F test a=0, B=0	Prob > F
Economist Data	6/81-12/85	1.4903 (1.0580)	1.41	0.48	509	4.75	0.000
Econ 3 Month	6/81-12/85	2.5127 (1.2918)	1.95 *	0.14	184	1.31	0.256
Econ 6 Month	6/81-12/85	2.9866 (1.5974)	1.87 *	0.28	174	1.46	0.194
Econ 12 Month	6/81-12/85	0.5174 (1.2290)	0.42	0.67	149	6.01	0.000
MMS 1 Month	10/84-2/86	15.3945 (6.3520)	2.42 **	0.20	171	2.54	0.030
MMS 3 Month	1/83-10/84	6.0725 (2.3392)	2.60 **	0.66	182	11.93	0.000
AMEX Data	1/76-7/85	3.2452 (1.1675)	2.72 ***	0.33	86	2.69	0.005
AMEX 6 Month	1/76-7/85	3.6346 (1.3437)	2.70 ***	0.26	45	3.30	0.009
AMEX 12 Month	1/76-7/85	3.1091 (1.2954)	2.40 **	0.25	40	1.48	0.210

Notes: Method of Moments standard errors are in parentheses. * Represents significance at the 10% level, ** and *** represent significance at the 5% and 1% levels, respectively.

Econometric Issues

APPENDIX 1: GENERAL

Estimation of most of our equations is performed using OLS. We stack different countries, and in some cases different forecast horizons, into a single equation. The complicated correlation pattern of the residuals, however, renders the OLS standard errors incorrect in finite samples. Several types of correlation are present.

First, there is serial correlation induced by a sampling interval shorter than the corresponding forecast horizon (up to eight times). This is the usual case in which overlapping observations imply that, under the null hypothesis, the error term is a moving average process of an order equal to the frequency of sampling interval divided by the frequency of the horizon, minus one. Hansen and Hodrick (1980) propose using a method of moments (MoM) estimator for the standard errors in precisely the application studied here.

Second, in order to take advantage of the fact that the surveys covered four or five currencies simultaneously, we pooled the regressions across countries. This type of pooling induces contemporaneous correlation in the residuals.[1] Normally, Seemingly Unrelated Regressions should be used to exploit this correlation efficiently. We use SUR later; here, however, the serial correlation induced by overlapping observations makes SUR inconsistent.

The basic model may be written as:

$$y^k_{t,i} = x^k_{t-k,i}\beta + v^k_{t,i} \tag{14}$$

where k is the number of periods in the forecast horizon and i indexes the currency. We account for the two types of correlation in the residuals with a MoM estimate of the covariance matrix of $\hat{\beta}$:

$$\hat{\Theta}^1 = (X_{NT}'X_{NT})^{-1}X_{NT}'\hat{\Omega}X_{NT}(X_{NT}'X_{NT})^{-1} \tag{14'}$$

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[14] Each currency in our pooled regressions was given its own constant term. This modeling strategy seemed most reasonable in view of the differences across currencies in the magnitudes of both ex post spot rate changes and the forward discount (see Table 1 of NBER Working Paper version of this study).

where X_{NT} is the matrix of regressors of size N (countries) times T (time). The (i,j)th element of the unrestricted covariance matrix, $\hat{\Omega}$ is:

$$\hat{\omega}(i,j) = \frac{1}{NT-k} \sum_{l=0}^{N-1} \sum_{t=k+1}^{T} \hat{v}_{t+lT} \hat{v}_{t-k+lT} \text{ for } mT-r \leq k \leq mT+r \ ; \quad m=0, \ldots, N-1$$

$$= 0 \qquad \text{otherwise .} \tag{15}$$

where r is the order of the MA process, $\hat{v}_{t+lT}$ is the OLS residual, and $k = |i-j|$. In some cases, this unrestricted estimate of Ω uses well over 100 degrees of freedom.[15] We therefore estimated a restricted covariance matrix, $\tilde{\Omega}$ with typical element:

$$\tilde{\omega}(t+lT, t-k+pT) = \frac{1}{N-1} \sum_{l=0}^{N} \hat{\omega}(t+lT, t-k+pT) \quad \text{if } \lambda = p \text{ and } -r \leq k \leq r$$

$$= \frac{2}{N(N-1)} \sum_{p=0}^{N-1} \sum_{l=0}^{N-1} \hat{\omega}(t+lt, t-k+pT) \quad \text{if } \lambda \neq p \text{ and } -r \leq k \leq r$$

$$= 0 \qquad \text{otherwise .} \tag{16}$$

These restrictions have the effect of averaging the own-currency and cross-currency autocorrelation functions of the OLS residuals, respectively, bringing the number of independent covariance parameters down to $2r$.

Tests of forward discount unbiasedness also provide an opportunity to aggregate across different forecast horizons (though we are unaware of anyone who has done this, even with the standard forward discount data), adding a third pattern of correlation in the residuals. Such stacking seems appropriate in this case because we wish to study the predictive power of the forward discount generally, rather than at any particular time horizon. Moreover, a MoM estimator which incorporates several forecast horizons has appeal beyond the particular application

[15] The number of independent parameters in the covariance matrix does not affect the asymptotic covariance, as long as these parameters are estimated consistently (see Hansen (1982)). Nevertheless, one suspects that the small-sample properties of the MoM estimator worsen as the number of nuisance parameters to be estimated increases.

studied here because it is computationally simpler than competing techniques and at the same time can be more efficient than single k-step-ahead forecasting equations estimated with MoM.

To demonstrate the precise nature of the correlation induced by such aggregation, consider the stochastic process, Y_t, which is stationary and ergodic in first differences and has finite second moments. We denote the k period change in y from period $t-k$ to t as y_t^k, and the h period change as $y_t^h = \sum_{i=0}^{n-1} y_{t-ik}^k$, where $h = nk$ for any positive integer n.[16] We then define the innovations, v_t^k and v_t^h as:

$$v_t^k = y_t^k - E(\, y_t^k \mid \phi_{t-k}) \tag{17}$$

$$v_t^h = y_t^h - E(\, y_t^h \mid \phi_{t-h})$$

where ϕ_t includes present and lagged values of the vector of right-hand-side variables, x_t^k. These facts allow us to write the covariance matrix of the innovations as:

$$\Sigma = E\left[\begin{bmatrix} v_t^k \\ v_t^h \end{bmatrix} [v_t^{k\prime} v_t^{h\prime}] \right] = \begin{bmatrix} \Delta^k & \Delta^{hk} \\ \Delta^{hk\prime} & \Delta^h \end{bmatrix} \tag{18}$$

where the (i,j)th element of each submatrix of Σ is equal to the corresponding autocovariance function, evaluated at $q = i - j$:

$$\begin{aligned} \Delta_{i,j}^k &= E(\, v_t^k v_{t+q}^k) = \lambda_q^k \quad \text{if } \mid q \mid < k \\ &= 0 \quad \text{otherwise}, \end{aligned} \tag{19}$$

$$\begin{aligned} \Delta_{i,j}^h &= E(\, v_t^h v_{t+q}^h) = \lambda_q^h \quad \text{if } \mid q \mid < h \\ &= 0 \quad \text{otherwise}, \end{aligned}$$

[16] The following example can easily be generalized to allow h and k to be any positive integers. It is also possible to combine in a similar fashion more than two different forecast horizons. Indeed, we combine three horizons in the Economist data estimates in the regressions below. Because these extensions yield no additional insights and come at the cost of more complicated algebra, however, we retain the simple example above.

$$\Delta_{i\,j}^{hk} = E(\,v_t^k v_{t+q}^h\,) = \lambda_q^{hk} \text{ if } 0 \le q < k \tag{20}$$
$$= E(\,v_t^k v_{t+q}^h\,) = \lambda_q^{hk} \text{ if } -h < q < 0$$
$$= 0 \text{ otherwise .}$$

In this context consider the aggregated model:

$$\mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{v}_t \tag{21}$$

where $\mathbf{y}_t' \equiv [y_{t+k}^{k}{}' \;\; y_{t+h}^{h}{}']$, $\mathbf{x}_t' \equiv [x_t^{k}{}' \;\; x_t^{h}{}']$ and $\mathbf{v}_t' \equiv [v_{t+k}^{k}{}' \;\; v_{t+h}^{h}{}']$. The OLS estimate of β then has the usual MoM estimate of the sample covariance matrix:

$$\hat{\Theta}^2 = (x_{2NT}'x_{2NT})^{-1}x_{2NT}'\hat{\Sigma}x_{2NT}(x_{2NT}'x_{2NT})^{-1}$$

where $\hat{\Sigma}$ is a consistent estimate of Σ, and is formed by using the OLS residuals to estimate the autocovariance and crosscovariance functions in equations (19) and (20).

One might think that by stacking forecast horizons, as we do in equation (21), greater asymptotic efficiency always results than if only the shorter-term forecasts are used, in other words, that $\hat{\Theta}^1 - \hat{\Theta}^2$ is positive semidefinite. After all, the sample size has doubled, and the only additional estimates we require are nuisance parameters of the covariance matrix. This intuition would be correct for asymptotically efficient estimation strategies, such as maximum likelihood. But because OLS weights each observation equally, the MoM covariance estimates reflect the average precision of the data. It follows that if the longer-term forecasts are sufficiently imprecise relative to the shorter-term forecasts, the precision of the estimate of β drops: we could actually lose efficiency by adding more data. In the appendix we demonstrate this potential loss in asymptotic efficiency, and show how it is related to the disparity in forecast horizons. Efficiency is most likely to increase if the longer-term forecast horizon is a relatively small multiple of the shorter-term horizon. Indeed, in the forthcoming regressions we find a marked increase in precision from stacking across forecast horizons when $r = 2$ (in the Economist and Amex samples), but little or no increase in precision when $r = 4$ or 6 (in

the MMS samples).

Finally, the above MoM estimates of the covariance matrix need not be positive definite in small samples. Newey and West (1985) offer a corrected estimate of the covariance matrix that discounts the jth order autocovariance by $1-(j/(m+1))$, making the covariance matrix positive definite in finite sample. Nevertheless, for any given sample size, there remains the question of how small m must be to guarantee positive definiteness. In the upcoming regressions we tried $m = r$ (which Newey and West themselves suggest) and $m = 2r$; we report standard errors using the latter value of m because they were consistently larger than those using the former.[17] ,[18]

APPENDIX 2: EFFICIENCY AND POOLING OVER FORECAST HORIZONS

In this appendix we show how the asymptotic efficiency of the method-of-moments estimator is affected by aggregating over forecast horizons. Consider the model:

$$y_{t+k}^{k} = x_t^{k}\beta + \epsilon_{t+k}^{k} \tag{A1}$$

where $y_{t+k}^{k} = Y_{t+k} - Y_t$ and the error term is orthogonal to the present and past values of x and y, $E(\epsilon_{t+k}^{k} \mid x_t^{k}, x_{t-1}^{k}, \ldots, y_t^{k}, y_{t-1}^{k}, \ldots) = 0$. Our example below considers the simple case of a single regressor, x_t^{k}, but may easily be extended to a vector of righthand-side variables. Define the iid innovations $\upsilon_{t+k} = E(y_t^{k} \mid x_t^{k}, x_{t-1}^{k}, \ldots, y_t^{k}, y_{t-1}^{k}, \ldots)$ and $\eta_{t+k} = E(x_t^{k} \mid x_t^{k}, x_{t-1}^{k}, \ldots, y_t^{k}, y_{t-1}^{k}, \ldots)$. If x and y are jointly covariance stationary, then the Wold decomposition implies that:

$$y_{t+k}^{k} = \sum_{i=0}^{\infty}\delta_i \upsilon_{t+k-i} + \sum_{i=0}^{\infty}\gamma_i \eta_{t+k-i} + D_y^{k} \tag{A2}$$

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[17] For the two aggregated MMS data sets in Table 6, a value of $m = r$ was used after finding that $m = 2r$ resulted in a nonpositive semi-definite covariance matrix. This correction reduced the standard errors in these two regressions by an average of only 3 percent.

[18] The regressions reported in the text assume homoscedasticity. Concern often arises about heteroscedasticity in the prediction errors. We tried a heteroscedasticity-consistent estimator and found that the correction resulted in lower estimated standard errors. We choose the conservative route of reporting the results with the larger estimated standard errors.

$$E(y_{t+k}^k \mid \phi_t) = \sum_{i=k}^{\infty} \delta_i \upsilon_{t+k-i} + \sum_{i=k}^{\infty} \gamma_i \eta_{t+k-i} + D_y^k$$

where D_y^k is the deterministic component of y, and ϕ_t includes past and present values of x and y. We are primarily concerned with the case in which x_t^k is the best unbiased forecast of y_{t+k}^k. That is, under the null hypothesis of forward discount unbiasedness, fd_t^k is an efficient predictor of the future spot rate change, Δs_{t+k}. Thus we assume that x_t^k already contains all relevant information for forecasting y_{t+k}^k, so that $E(y_{t+k}^k \mid \phi_t) = E(y_{t+k}^k \mid x_t^k)$.

We define analogously the h period change in Y_{t+h} as $y_{t+h}^h = \sum_{j=0}^{n-1} \sum_{i=0}^{h-jk-1} y_{t+h-jk-i}^k$, where $h \equiv nk$. Using equations (A1) and (A2) we then have:

$$y_{t+h}^h = \sum_{j=0}^{n-1}\sum_{i=0}^{\infty} \delta_i \upsilon_{t+h-jk-i} + \sum_{j=0}^{n-1}\sum_{i=0}^{\infty} \gamma_i \eta_{t+h-jk-i} + D_y^h \tag{A2'}$$

$$E(y_{t+h}^h \mid \phi_t) = \sum_{j=0}^{n-1} \sum_{i=h-jk}^{\infty} \delta_i \upsilon_{t+h-jk-i} + \sum_{j=0}^{n-1} \sum_{i=h-jk}^{\infty} \gamma_i \eta_{t+h-jk-i} + D_y^h$$

These facts imply that the k and h period prediction errors, ϵ_{t+k}^k and ϵ_{t+h}^h, respectively, are stationary with finite second moments. If we assume that $q_t^k = \epsilon_{t+k}^k x_t^k$ and $q_t^h = \epsilon_{t+h}^h x_t^h$ are stationary with finite variance, then $E(q_t^k q_{t+j}^k) = 0$ for $j \geq k$, and $E(q_t^h q_{t+j}^h) = 0$ for $j \geq h$. Thus q_t^k can be expressed as a $k-1$ order moving average process:

$$q_t^k = \sum_{i=0}^{k-1} a_i \nu_{t+k-i} \tag{A3}$$

Similarly, from equation (A2') we have that q_t^h may be written as a $h-1$ order moving average process:

$$q_t^h = \sum_{j=0}^{n-1} \sum_{i=0}^{h-1} d_i \nu_{t+h-jk-i} \tag{A3'}$$

$$= \sum_{j=0}^{n-1} \sum_{i=0}^{k} (c_{h-jk-i} + a_{k-i}) v_{t+h-jk-i} \tag{A4}$$

where $c_{h-jk-i} = \sum_{m=j}^{n-2} d_{(n-m)k-i}$. The covariance generating function for q_t^k is denoted by $\lambda^a(z)$, where

$$\lambda^a(z) = \sum_{s=1-k}^{k-1} \lambda_s^a = \sigma_v^2 \sum_{s=1-k}^{k-1} \sum_{j=0}^{k-1} a_j a_{j+s} \ . \tag{A5}$$

Using equation (A4), the covariance generating function for q_t^h can be written as:

$$\lambda^d(z) = \lambda^c(z) + \frac{n(n+1)}{2} \lambda^a(z) + \frac{n(n-1)}{2} \lambda_0^a + 2\lambda^{ac1}(z) \tag{A5'}$$

where $\lambda^{ac1}(z)$ is a complicated generating function of the a_i's and c_i's which we need not specify here. Finally, the covariance generating function, $\lambda^{ad}(z) = \sigma_v^2 \sum_{s=1-k}^{h-1} \sum_{j=0}^{h-1} a_j d_{j+s}$ can be rewritten as:

$$\lambda^{ad}(z) = \frac{(n+1)}{2} \lambda^a(z) + \frac{(n-1)}{2} \lambda_0^a + \lambda^{ac2}(z) \tag{A6}$$

where $\lambda^{ac2}(z)$ is another generating function of the a_i's and c_i's.

Now consider the asymptotic MoM covariance matrix of $\sqrt{T}(\hat{\beta} - \beta)$ from equation (A1):

$$\Theta^1 = (\lambda_0^k)^{-2} \lambda^a(z) \tag{A7}$$

where $\lambda_0^k = \lim_{t\to\infty} T^{-1} \sum_{t=1}^{T} x_t^k x_t^k$. If we add in the longer-term forecast data, our model is that of equation (21) above, with asymptotic covariance matrix:

$$\Theta^2 = (\lambda_0^h + \lambda_0^k)^{-2} (\lambda^d(z) + \lambda^a(z) + 2\lambda^{ad}(z)) \tag{A8}$$

By substitution, we have that $\Theta^1 > \Theta^2$ if and only if:

$$\frac{\lambda_0^h}{\lambda_0^k} \left(1 + \frac{\lambda_0^h}{\lambda_0^k} \right) \tag{A9}$$

$$> n^2 \tfrac{1}{2}\left(1+\frac{\lambda_0^a}{\lambda^a(z)}\right) + n\ \tfrac{1}{2}\left(3+\frac{\lambda_0^a}{\lambda^a(z)}\right) + \frac{\lambda^a(z)+\lambda^c(z)+2(\lambda^{ac^1}(z)+\lambda^{ac^2}(z))-\lambda_0^a}{\lambda^a(z)}$$

Equation (A9) says that the variance of the longer-term data, λ_0^h, must increase at a rate the same as or greater than the relative forecasting interval, n, if we are to gain by adding longer-term forecasts to data sets of only shorter-term forecasts. Thus as the forecasting interval increases, we require correspondingly greater variability of the regressors in order to compensate for the greater variability of the forecast errors.

One might think that the result in equation (A9) is a consequence of weighting the more imprecise longer-term predictions equally with the predictions of shorter-term. Perhaps if we downweighted the longer-term data, we would always gain in efficiency. It turns out that this is not the case. In the remaining space, we construct a consistent, optimally weighted estimator and show that the efficiency of this estimator may still worsen asymptotically by adding in the longer-term forecasts.

In most circumstances, GLS represents the optimal weighting strategy when the data have different levels of precision. GLS is, however, inconsistent when used on a model with overlapping observations. Thus we consider instead a weighted least squares estimator which is optimal within a class of consistent estimators. Consider a transformation of the model in equation (21), which stacks the shorter- and longer-term data:

$$W\mathbf{y}_t = W\mathbf{x}_t\beta + W\mathbf{v}_t \tag{A10}$$

where W is a diagonal matrix. The MoM estimate of β in equation (A10), $\hat{\beta}_W$, will be consistent for any arbitrary diagonal matrix W. To see this, note that the MoM estimate of equation (A10), $\hat{\beta}_W$, may be written as:

$$\sqrt{T}\,(\hat{\beta}_W - \beta) = \left(\frac{\mathbf{x}_{2NT}'W^2\mathbf{x}_{2NT}}{T}\right)^{-1} \frac{\mathbf{x}_{2NT}'W^2\mathbf{v}}{\sqrt{T}} \tag{A11}$$

$$= \left(\frac{1}{T}\sum_{i=1}^{2T} x_i^2 w_{ii}^2\right)^{-1} \frac{1}{\sqrt{T}} \sum_{i=1}^{2T} x_i v_i w_{ii}^2$$

The final term in equation (A11) converges in probability to zero, provided that the error term in equation (A10) is conditionally independent of the contemporaneous value of the regressor, $E(v_t \mid x_t) = 0$ (this is just the Gauss-Markov assumption required for the consistency of OLS in estimating equation (A10)). Suppose now that we choose W optimally in order to maximize the gain in efficiency from adding longer-term forecasts to our shorter-term data. That is:

$$\max_W \; \Theta^1 - \Theta^3 . \tag{A12}$$

where Θ^3 is the MoM asymptotic covariance matrix of $\hat{\beta}_W$:

$$\Theta^3 = (x_{2NT}' W^2 x_{2NT})^{-1} x_{2NT}' W^2 \Omega W^2 x_{2NT} (x_{2NT}' W^2 x_{2NT})^{-1} \tag{A13}$$

By normalizing the weight on every shorter-term data point to one, it is straightforward to show that the optimal weight placed on each longer-term observation is:

$$w_h^* = \left(\frac{\lambda_0^h \lambda^a(z) - \lambda_0^k \lambda^{ad}(z)}{\lambda_0^k \lambda^d(z) - \lambda_0^h \lambda^{ad}(z)} \right)^{1/2} \tag{A14}$$

Note that w_h^* will always be positive if the data sets are uncorrelated, i.e. if $\lambda^{ad}(z) = 0$. In other words, appropriately weighted independent information can always improve efficiency, no matter how imprecise the new information may be. But, the nature of the correlation between contemporaneous longer-term and shorter-term predictions implies that the optimal weight given to longer-term data may be zero. In particular, w_h^* will be zero if the numerator in equation (A14) becomes negative. This occurs if n is too large in comparison with the relative variance of the longer-term forecasts. Using equations (A5'), (A6) and (A14), it can be shown that a sufficient condition for w_h^* to be zero is:

$$\frac{(n+1)}{2} > \frac{\lambda_0^h}{\lambda_0^k} . \tag{A15}$$

Thus, while the standard errors reported in the text indicate that for small values of n one may obtain improvements in efficiency, this result is not likely to apply MoM estimation of data with considerably longer forecast horizons, even when the data are downweighted to account for the greater variance of the longer-term forecast errors. It is worth stressing in closing that this potential loss in efficiency is a direct consequence of our limited information MoM estimation strategy. Full information techniques, such as maximum likelihood estimation, will consistently achieve nonzero gains in asymptotic efficiency with the addition of longer-term data.