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Investor Horizon and Noise in Asset Prices

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Abstract

This paper investigates how investor horizon and noise trading affect the relation between asset prices and fundamentals. We show that for low to moderate levels of noise trading, investor horizon has no appreciable effect on the degree of volatility of prices and in such cases, the opportunity cost for a rational investor to follow a myopic trading strategy is not large. Calibration of the model suggests that noise trading is able to explain the levels of excess volatility reported in the empirical literature only when the average horizon is long and prices revert slowly to fundamentals. The analysis, however, suggests that rational investors can expect to make substantial profits at the expense of noise traders if the economy is characterized by a high level of excess volatility.

Few issues in economics have generated more debate than the question of whether asset prices efficiently reflect fundamentals, e.g., discount factors and future cash flows. The traditional view has been that erratic trading by some investors, which is unrelated to fundamentals, has an insignificant effect on prices. It is argued that incentives exist for skillful, rational speculators to trade against erratic traders, and that these speculators are the marginal, price-setting investors [Friedman (1953), Fama (1965), and Samuelson (1965)]. Recent and purportedly anomalous evidence has challenged the view that asset prices vary insignificantly with noise.¹

The accumulated evidence has prompted attempts to explain the noise in prices through the incorporation of erratic trading into asset valuation models. Assuming rational investors to be short-lived, several of these papers argue that noise trading can have a considerable impact on prices.² Furthermore, the degree of myopia or short-termism in the marketplace, i.e., the average horizon of investors is viewed as a primary determinant of the level of the volatility in prices. Presumably, the presence of noise traders imposes substantial liquidation risk on the myopic investors and reduces their aggressiveness in trading against noise.

¹One apparent anomaly, first documented by Shiller (1981) and Leroy and Porter (1981), is the high volatility of changes in stock prices relative to changes in future real dividends. Another oddity, documented by Ball and Brown (1968), Foster, Olsen, and Shevlin (1984), Bernard and Thomas (1989, 1990), and Freeman and Tse (1989), among others, is the existence of predictable, abnormal stock returns following earnings announcements. Authors of some of these studies and several others, such as De Bondt and Thaler (1985, 1987), and Poterba and Summers (1988), find it difficult to accept simple models of asset valuation with homogeneous investors where price equals the sum of discounted future expected cash flows.

² See, e.g., De Long et al. (1989, 1990a, 1990b) and Froot et al. (1990).

This paper is a comparison of the trading strategies of skillful, rational investors with finite and infinite horizons. It also analyzes the effect of noise trading on asset prices as the average investor's horizon is varied. An ideal economy with a riskless asset in infinite supply, and an infinite-lived asset in finite supply is studied. Trading by some investors is erratic and can be attributed to liquidity needs or to life-cycle reasons.³ The total demand of these noise traders introduces random increments in the supply of the risky asset, and the risky asset price varies with the supply in the standard way in equilibrium. Skillful investors trade with the noise investors opportunistically; the former benefit from the supply-induced variations in investment opportunities, while the latter lose. The willingness of the skillful investors to trade is dependent on their horizons.

One extant notion of horizon is a characterization of assets: the horizon is the first date in the future that an asset will trade at its fundamental value with certainty. Ignoring default and the calling of debt, a bond, for example, has a horizon equal to its maturity; at this date, the price of the bond is equal to the principal value. Shleifer and Vishny (1990) argue that the noisy variation of asset prices increases with this horizon.⁴

³An alternative interpretation is that some traders react in a mechanistic fashion to the arrival of new information, without regard to its effect on the fundamental value of the security. Consider, for example, an announcement of a change in earnings. This change may truly be permanent but some investors, displaying a fixation on past earnings, trade as if the change is transitory.

⁴It is important in their analysis that investors may trade the asset only at the present date and at the horizon date. If skillful investors trade at the intervening dates, this result depends on the time series of the volume of erratic trading.

A second notion of horizon deals with the information set on which investors base their investment decisions: some investors may have information on short-term cash flows of a company while others may be informed of its long-term prospects. Their investment and trading policies would then reflect the horizon of their information.⁵ This interpretation suggests that when the costs of acquiring signals about future cash flows vary with the horizon of the cash flow(s) to be forecast and may also vary across individuals, different investors may end up with different information horizons.

A third interpretation of horizon characterizes the investors: horizon is the date of the most extreme future consumption which enters an investor's present utility and, therefore, his present planning. For example, investors in De Long et al. (1990) are myopic because they all have single-period horizons. A characteristic feature of their myopia is the equality of risk aversions to variations in wealth and consumption (one-period ahead). Also, the investors choose investment levels as if they will liquidate all proceeds to fund consumption in the near future. In contrast, the investors in Campbell and Kyle (1988) and Wang (1990) have infinite horizons, and these investors recognize that variations in wealth will be spread over consumptions in many future periods.

In this paper all skillful investors are assumed to be infinitely-lived so that they have infinite horizons given the third notion above. However,

⁵See for example, "Cullinet: Drawn and Quartered by Wall Street." According to the article, former chief executive officer of a now bankrupt firm and fellow at Harvard University, John J. Cullinane, advocates abandonment of quarterly reporting by firms and reporting annually in a way that would "reflect trends, not short-term blips." His firm failed because "short-term profit pressures took management's focus away from the customer and discouraged long-term product investments."

an alternate notion of horizon that characterizes the investors is introduced here. An investor has an n -period horizon when variations in future investment opportunities (caused by noisy variation in the supply of the risky asset) are foreseen only up to n periods in advance. Although a **myopic investor** stays in the market for a long period of time, he has a zero-period horizon because he does not recognize variations in opportunities next period. A **long-horizon investor**, on the other hand, has rational expectations regarding all future investment opportunities and, therefore, an infinite horizon.

Myopic behavior in this model is less extreme and, we believe, more reasonable than the behavior of a single-period lived investor. The latter investor faces considerable risk when noisy variations in price are large. Because that investor liquidates the position in the risky asset in one period in spite of this risk, he behaves as much like a noise trader as an idealized skillful arbitrageur. The alternative view offered here is that short-termism describes investors who ignore, or who do not recognize the association between future variations in wealth and in investment opportunities. For example, a large increment in the supply of the risky asset decreases its price and therefore an investor's wealth (if he holds a positive amount of the asset). However, because of the decline in the price, the return to holding the asset simultaneously increases. A myopic investor recognizes only the wealth effect, while a long-horizon investor recognizes the opportunity-set effect as well. Unlike a one-period lived investor, the myopic trader idealized here has risk aversion in wealth equal to that of the long-horizon investor.

We analyze the conditions under which short-termism contributes to the volatility of prices and also examine the ability of models that incorporate

erratic trading to explain estimates of excess volatility of common stock prices. Section I of the paper presents the basic setting of the model, which is an economy populated by erratic traders and skillful investors. The notions of noise traders' losses and excess volatility are also formalized there. The equilibria for the economies with long-horizon and myopic investors are derived in Sections II and III respectively. The general case of a representative finite-horizon investor is examined in Section IV; numerical analysis is used to characterize this setting. The sensitivity of prices to noise trading, the level of excess volatility of prices, and the losses of erratic traders are analyzed as functions of the representative horizon. Possible extensions of the model with information acquisition are discussed in Section V and Section VI concludes the paper.

I. The Economy

A. The Basic Setting

In the economy, there is a riskless asset in infinite supply with a return $R > 1$ per period and a risky asset. These assets are traded by erratic traders and skillful investors. The motives of erratic traders are exogenous to the model and their aggregate demand introduces an unexplained perturbation in the supply of the risky asset available to the skillful investors. Let Z_t represent the per capita supply available to the skillful investors at time t . Z_t is assumed to follow the autoregressive (with parameter μ) process

$$Z_t = \mu Z_{t-1} + \zeta_t, \quad (1)$$

where ζ_t is normally distributed with mean zero and variance σ_ζ^2 . This is equivalent to assuming that the risky asset is in zero net supply. However, a generalization allowing for the risky asset to be in positive supply could

be accommodated by adding a positive constant to the right hand side of (1), without altering the conclusions reached in the paper.

Skillful investors maximize expected utility of current and future consumption, given their information sets Φ_t , which are homogeneous and include only current and historical prices and dividends. The exact nature of the information and the stochastic behavior of dividends remains unspecified, although parametric restrictions are noted below. The statement of these conditions relies on the construction of the **fundamental value of the risky asset** at time t ,

$$F_t = \sum_{s=t}^{\infty} E [\tilde{U}_s | \Phi_t] / R^{s-t}. \quad (2)$$

where the dividend per unit of the risky asset in period s is U_s . Thus, F_t is the sum of the current and all expected future dividends discounted at the riskless rate. In an equilibrium without noise trading, because the risky asset is in zero net supply, the cum-dividend price of the asset at time t is equal to this fundamental value.

We assume that for any date t , conditional on Φ_t , F_t defined by (2) is finite, F_{t+1} is normally distributed with mean $R(F_t - U_t)$, and the variance-covariance matrix for (ζ_{t+1}, F_{t+1}) is

$$\begin{bmatrix} \sigma^2 & 0 \\ \sigma \zeta & \sigma_F^2 \\ 0 & \sigma_F^2 \end{bmatrix}.$$

Simple examples which satisfy this assumption are: Suppose dividend U_t follows a random walk, $U_{t+1} = U_t + u_t$, where u_t is distributed $N(0, \sigma_u^2)$ conditional on Φ_{t-1} and temporally independent. Then $F_t = RU_t / (R-1)$, with $E[F_{t+1} | \Phi_t] = RU_t / (R-1) = R(F_t - U_t)$ and $\sigma_F^2 = R^2 \sigma_u^2 / (R-1)^2$. Alternatively, let U_t be the sum of a random walk N_t , with increments u_t , and a temporally independent, identically distributed shock e_t , distributed $N(0, \sigma_e^2)$. Then

$$F_t = RN_t/(R-1) + e_t, \quad E[F_{t+1} | \Phi_t] = RN_t/(R-1) = R(F_t - U_t) \quad \text{and}$$

$$\sigma_F^2 = R^2 \sigma_u^2 / (R-1)^2 + \sigma_e^2.$$

B. The Erratic Traders' Losses and Excess Volatility

One of the aims of this work is to determine whether models that incorporate erratic trading can explain the estimates of excess volatility of common stock prices. This is achieved by examining for a given horizon whether or not levels of excess volatility reported in the literature can be sustained and whether or not the associated noise traders' losses can be considered plausible. Since the analysis focuses on the notions of both noise traders' losses and excess volatility, we now motivate these ideas in the context of the model.

Immediately after the close of time t trade, skillful investors hold Z_t units of the risky asset per capita, so that $Z_t - Z_{t-1}$ is the number of units bought (sold) by skillful investors in period t , given that this value is positive (negative). This value is net of the shares traded between noise traders, and between skillful traders. Erratic traders compensate the skillful traders for bearing this random inventory of the risky asset. In the equilibria which we study, the price of a unit of the risky asset satisfies

$$P_t + U_t = F_t - QZ_t, \quad (3)$$

where Q is a constant to be determined.⁶ Evidence of the compensation is the mispricing, i.e., the deviation QZ_t in the cum-dividend price from the

⁶In the special case that U_t follows a random walk, $F_t = RU_t/(R-1)$. Then an interpretation of equation (3) is that the (cum-dividend) price levels follow a process identical to that of the logarithms of prices in Fama and French (1988) and Poterba and Summers (1988). Price levels are the sum of a random walk and an autoregressive temporary component.

fundamental. On average, when skillful investors buy, they do so at a price that is low relative to the fundamental.

Two alternative estimates of the erratic traders' trading losses per trading period are suggested here. One is $E[QZ_t(Z_t - Z_{t-1})] = Q\sigma_\zeta^2 / (1 + \mu)$, which is the expected difference in the values of the net volume of trade by the erratic traders in, respectively, the economy where prices deviate from fundamentals, and the economy in which they do not ($Q=0$), measured per capita skillful trader. The second is the same expected difference, but measured per unit expected volume of trade: $E[QZ_t(Z_t - Z_{t-1})] / E[|Z_t - Z_{t-1}|] = Q\sigma_\zeta (\pi / (4(1 + \mu)))^{1/2}$.⁷ Because the noise traders are not formalized explicitly, these measures can not be adjusted for the gains which presumably accrue to erratic traders outside the asset market.

A variety of estimators of **excess volatility** are reported in the literature. These estimators are equivalent to the extent that they are used to provide evidence that stock prices in the United States, relative to the fundamental value of the shares, are abnormally volatile. West (1988), for example, reports values of the ratio of noise to the conditional variance of price plus dividend, which in the present notation is $Q^2\sigma_\zeta^2 / (\sigma_F^2 + Q^2\sigma_\zeta^2)$. Because the empirical literature has generally defined excess volatility in terms of the ratio of standard deviations, our choice is $EX = ((\sigma_F^2 + Q^2\sigma_\zeta^2) / \sigma_F^2)^{1/2} - 1$, i.e., EX measures the fraction by which

⁷When price is normalized to \$1, this measure can be roughly interpreted as the expected profits of a rational investor for absorbing a dollar of supply from the noise traders. This interpretation, however, requires the rational investor to have a long enough horizon so that he stays in the market till (at least a significant part of) the mispricing disappears.

the total conditional volatility of cum-dividend prices exceeds the conditional fundamental volatility.⁸

II. Equilibrium With Long-Horizon Investors⁹

At any time t , a long-horizon investor chooses a sequence of future, random consumption levels c_s to maximize the expected utility of lifetime consumption,

$$E\left[\sum_{s=t}^{\infty} -\rho^{s-t} \exp(-ac_s) \mid \Phi_t\right]. \quad (4)$$

The choice is subject to the constraint that wealth follows the process

$$W_{s+1} = (W_s - c_s - D_s P_s)R + D_s (P_{s+1} + U_{s+1}), \quad (5)$$

where W_s is given and D_s is the demand for the risky asset with unit ex-dividend price P_s . In (4), ρ is the intertemporal rate of patience and a is the absolute risk aversion with respect to gambles in consumption.¹⁰ The time homogeneity of this problem implies that one can define a value function $J(W_t, \Phi_t)$ such that the optimal consumption and investment policies are the solutions to

$$J(W_t, \Phi_t) = \underset{c_t, D_t}{\text{Maximize}} E[-\exp(-ac_t) + \rho J(W_{t+1}, \Phi_{t+1}) \mid \Phi_t] \quad (6)$$

⁸Given a range of West's measure of .7858 to .9734 [cf. Table II], EX ranges from 1.16 to 5.13.

⁹After this analysis was completed, it was brought to the authors' attention that Campbell and Kyle (1988) consider a similar model in a continuous time setting.

¹⁰A generalization allows absolute risk aversion to be heterogeneous as in Hellwig (1980) or Admati (1985). The results of this paper are not changed in a qualitative fashion by this generalization.

subject to (5). The strategy (c_t, D_t) and utility of wealth J_t are dependent on the functional form of P_t because of their dependence on the investment opportunity set, which varies over time with the supply. In turn, an equilibrium price must clear the market and is a function of the policies and utility J_t . For this reason, the parameters of J_t and P_t must be determined jointly in equilibrium.

Given the price process (3), the utility of wealth of any long-horizon investor satisfying (6) is¹¹

$$J(W_t, \Phi_t) = -\exp(-kW_t + k_0 - k_1 Z_t^2), \quad (7a)$$

where

$$k = a(R-1)/R, \quad (7b)$$

$$k_0 = -\ln(R-1) + \frac{\ln \rho + R \ln R}{R-1} - \frac{\ln(1+2k_1 \sigma_\zeta^2)}{2(R-1)}, \quad (7c)$$

and k_1 satisfies

$$Q^2 = \frac{2k_1 \sigma_F^2 (2Rk_1 \sigma_\zeta^2 + R - \mu^2)}{2k_1 (R-1) R \sigma_\zeta^2 + (R-\mu)^2}. \quad (7d)$$

For any non-zero value Q , equality (7d) has two roots, one negative and the other positive. For the negative root k_1 , the problem (6) is convex in c_t and D_t and the coefficient k_0 given by (7c) does not exist. For the positive root of k_1 , problem (6) is concave in c_t and D_t and the first order conditions identify the optimal demands. In this case, the utility level

¹¹Consider (7a) for time $t+1$. Substitute for W_{t+1} and P_{t+1} , and then Z_{t+1} using (5), (3), and (1). The expectation of the resulting expression, conditional on Φ_t , can be evaluated using knowledge of normally distributed variables. The expected value is a function of W_t , c_t and D_t , and is substituted into the right hand side of (6). The first order conditions of the maximand in (6) are linear in c_t and D_t and substituting the optimal demands into the maximand of (6), one finds J_t on the left of (6) satisfies (7).

(7a) is increasing with the mispricing of the risky asset, i.e., it is increasing with the absolute value of Z_t .

Coefficient k_1 is set independently of absolute risk aversion, but is increasing with the sensitivity of price to noise Q . The utility level (7a) is increasing with both k_0 and the benefits from current mispricing $k_1 Z_t^2$. Because the present and expected future profits of long-horizon investors from trading the mispriced asset increase with the sensitivity of price to noise, a rise in Q is beneficial to these investors. Note from (7a) that

$$\partial^2 J(W_t, \Phi_t) / \partial W_t \partial Z_t^2 = -k k_1 \exp(-k W_t - k_0 - k_1 Z_t^2), \quad (8)$$

is negative, which implies that the marginal utility of wealth is decreasing with the mispricing of the asset due to erratic trading. This occurs because the investment opportunity set improves as the contemporaneous mispricing increases. An intuitive implication of (8) is that future variations in wealth caused by mispricing appear to be less risky than they would if k_1 were zero; these variations are partially offset by the improvement of the investment opportunity set.

The demand for the risky asset is $D_t = B Z_t$ where

$$B = \frac{Q(2k_1 R \sigma_\zeta^2 + R - \mu)}{k \sigma_F^2 (2k_1 \sigma_\zeta^2 + 1) + k Q^2 \sigma_\zeta^2} \quad (9)$$

Using (7d) in (9), B can be rewritten as

$$B = \frac{2k_1 (2k_1 R \sigma_\zeta^2 + R - \mu^2)}{k Q (2k_1 R \sigma_\zeta^2 + R - \mu)}. \quad (10)$$

Equilibrium market clearing requires $D_t = Z_t$, or $B=1$. Using (7b), (7d) and (10), Q must satisfy

$$k(R-1)R\sigma_\zeta^2 Q^3 - R(k^2\sigma_F^2\sigma_\zeta^2 + (R-\mu)(\mu-1))Q^2 + k(R(\mu-1)+\mu(R-\mu))\sigma_F^2 Q - k^2\mu\sigma_F^4 = 0. \quad (11)$$

For $\mu=0$, this cubic equation degenerates to a quadratic with a single positive root representing the unique equilibrium.¹² For μ positive and less than one, the intercept is negative and the cubic has at least one positive root representing an equilibrium.¹³ Finally, if $\mu=1$, one solves the cubic for an unique equilibrium, $Q=k\sigma_F^2/(R-1)$.

In any of the cases that an equilibrium exists, one finds that as $\sigma_F^2 \rightarrow 0$, Q approaches 0; when dividends are riskless, noise has no effect on prices. However, when dividends are risky, Q is positive and the equilibrium price varies with supply in the standard way. It is not possible for infinitely-lived risk averse agents to arbitrage away the noisy variation in price, and any position taken in the asset increases consumption risk. Wang (1990) reaches a similar conclusion.¹⁴

III. Equilibrium with Myopic Investors

Consider now an economy in which skillful investors choose consumption and investment to maximize expected utility (4) subject to budget constraint (5), but also overlook or ignore future variations in investment

¹²The negative root must be rejected because it implies that k_1 is negative.

¹³For μ sufficiently close to zero, the coefficient on the first order term in the cubic is negative, so a unique equilibrium exists.

¹⁴Wang parameterizes the dividend process and considers asymmetrically informed investors. The present analysis leaves the dividend process unspecified, but require homogeneously informed investors.

opportunities due to noise. The formal feature characterizing these investors is that, at any time that they solve their lifetime problem, they ignore the noisy variation in the marginal utility of wealth, one-period ahead. That is, for these investors, the value function is given by (7a) with k_1 set to zero. Although the plans of these myopic investors are time-inconsistent, their behavior is less extreme than that of single-period-lived investors.¹⁵

Consider a myopic investor solving the lifetime problem as of time t . This investor chooses current consumption c_t and investment in shares of the risky asset d_t to

$$\begin{aligned} & \text{maximize } [-\exp(-ac_t)] + \rho E[-\exp(k_0 - kW_{t+1}) | \Phi_t], \\ & c_t, d_t \end{aligned} \quad (12)$$

subject to (5), where k_0 and k are given by (7b) and (7c). Problem (12) is solved following the steps outlined in the previous section (cf. footnote 11). The investment demand is $d_t = bZ_t$ where

$$b = \frac{Q(R - \mu)}{k(Q^2\sigma_\zeta^2 + \sigma_F^2)}. \quad (13)$$

In equilibrium, the per capita demands d_t must clear the market, i.e., $b = 1$. This equality provides an expression in Q ,

$$k\sigma_\zeta^2 Q^2 - (R - \mu)Q + k\sigma_F^2 = 0, \quad (14)$$

so that the sensitivity of price to noise at any date t is

$$Q = \frac{(R - \mu) \pm [(R - \mu)^2 - 4k^2\sigma_\zeta^2\sigma_F^2]^{1/2}}{2k\sigma_\zeta^2}. \quad (15)$$

¹⁵As noted in the Introduction, an alternative formalization of short-horizon investors is to allow repeated generations of one-period lived investors. In that case, (13) is modified so that the risk aversion with respect to gambles in wealth is a , and not k , and k_0 is set to zero.

When a real solution to (15) exists, there are two possible equilibria, characterized by the larger and smaller roots of (15), Q_+ and Q_- , each of which is positive.

The equilibrium corresponding to Q_+ appears implausible. One finds $\partial Q_- / \partial \sigma_\zeta > 0$ and $\partial Q_+ / \partial \sigma_\zeta < 0$, so that the sensitivity of price to noise trading, Q_+ , increases as the expected volume from erratic trading decreases. Moreover, from (15), for the equilibrium Q_+ , the conditional variance of price, $\text{Var}[P_t | \Phi_{t-1}] = \sigma_F^2 + Q_+^2 \sigma_\zeta^2$, rises to infinity as the expected volume due to noise trading declines to zero.

In the present setting, price varies with the supply as a mechanism by which the risk averse skillful traders are compensated for absorbing the order flow from noise traders. An increase in the volatility of erratic trading increases the volatility of the inventory of the risky asset held by the skillful traders, and increases the volatility of the cash flow stream from the risky asset.¹⁶ One, therefore, expects the compensation to increase with the volatility of supply. Recall that the measures of the compensation (the noise traders' losses) suggested above are increasing in $Q\sigma_\zeta$. Another odd quality of the Q_+ equilibrium, thus, is that as the volatility supply, and consequently the risk faced by skillful investors increases, their expected compensation declines.¹⁷

¹⁶The cash flow stream per unit risky asset is unchanged, but the volatility of the number of units held by any skillful trader increases.

¹⁷When traders are asymmetrically informed, as in Kyle (1985), the asset price varies with the volume of trade, which partially reveals the information regarding the asset's payoffs. As the volume of erratic trading increases, the sensitivity of the price to the erratic trading decreases so that the Kyle model is consistent with the Q_+ equilibrium. However, those results do not apply here because the skillful investors in the present setting are risk averse so that they require compensation for bearing more risk, and they have no preferential information which means that the net volume of trade in any period, $|Z_t - Z_{t-1}|$, is independent of the asset

Whereas long-horizon investors see opportunities in the variation of the next-period price caused by noise, myopic investors see only risk. For this reason, there is no equilibrium with linear price function (3) when the variation in erratic demand in the myopic-investor economy is large relative to the variation in cash flows, i.e., when $\sigma_{\zeta} > (R - \mu)/(2k\sigma_F)$. Therefore, the excess volatility EX in the myopic economy is bounded by $\sqrt{2} - 1$, which is independent of k and μ .¹⁸

IV. A Numerical Analysis of Equilibria With n-Period Horizon Investors

In the previous sections, investors had either a zero-period horizon or an infinite horizon. A more general setting allows horizons to be any finite number. This section is a report on the volatility of prices, and the sensitivity of prices to erratic supply when the horizon of a representative investor is n-period in length. This investor chooses consumption and investment to maximize expected utility (4) subject to budget constraint (5), but ignores the changes in the investment opportunity set caused by shifts in the supply of the risky asset beyond period n. It is shown in the Appendix that at any time t , the optimal consumption and investment, D_t^n , of the n-period-horizon investor satisfies

$$\underset{c_t, D_t^n}{\text{maximize}} \quad E[-\exp(-ac_t) + \rho J^n(W_{t+1}, \Phi_{t+1}) | \Phi_t], \quad (16)$$

subject to (5), where J^n satisfies (7), with the exception of (7d). The expression (7d) is replaced by:

payoffs.

¹⁸Substitute $\sigma_{\zeta} = (R - \mu)/(2k\sigma_F)$ into (15) and solve for Q . Use this Q to calculate EX.

$$k_{1,n} = \frac{2k_{1,n-1} \sigma_{\zeta}^2 (\nu R^2 + \mu) + \nu(R-\mu)}{2\sigma_{\zeta}^2 (2k_{1,n-1} \sigma_{\zeta}^2 R + R(1+\nu))} \quad (17)$$

where $\nu = (1+EX)^2 - 1$ and $k_{1,n}$, which is obtained iteratively, is the value of k_1 for the n-period-horizon investor, and given that $k_{1,0} = 0$.

The demand for the risky asset satisfies $D_t^n = B^n Z_t$, with B^n given by (9) with $k_{1,n}$ replacing k_1 . Equilibrium in the n-period-horizon economy is a Q which satisfies $B^n = 1$. Using (9) and the definition of ν ,

$$Q = \frac{k(2k_{1,n} \sigma_{\zeta}^2 + \nu + 1) \sigma_F^2}{2k \sigma_{\zeta}^2 R + R - \mu} \quad (18)$$

Because (17) is used iteratively to obtain $k_{1,n}$, and given that EX , and therefore, ν , in (18) are endogenous, an analytic characterization of an equilibrium with a representative finite horizon is complex. Hence, the results are numerically evaluated. For given values of μ and n , the numerical analysis is most easily accomplished by fixing the excess volatility EX a priori while leaving σ_{ζ}^2 to be determined. One iterates on (17) to determine $k_{1,n}$, then uses (18) to determine Q and finally uses ν , Q , and σ_F^2 to determine σ_{ζ}^2 . Equilibria are reported for a variety of values of μ , n , and EX .¹⁹

The model is calibrated under the assumption that the discrete periods are one year in length. The real interest rate is set at 2% ($R=1.02$).

¹⁹Standard errors of the equilibrium parameters of the model are not reported. In principle, these are derived from the standard errors of the moments of the data used to determine the exogenous coefficients.

Relative risk aversion in wealth, is set at 2.²⁰ Given a representative wealth level of \$100,000 and a riskless rate of return 2%, the value of a which sets $k (= a(R-1)/R)$ equal to $2/100000$ is $a=0.00102$. The price is normalized to be \$1 and the fundamental volatility, σ_F , is set at 0.15.²¹

For a representative value of $\mu (=0.9)$, Figure 1 portrays the relation between excess volatility and σ_ζ , and Figure 2, the relation between Q and σ_ζ for these equilibria.²² Horizons of $n=0, 3, 5, 10, 100$ years, and infinity are used. For any finite horizon n and any σ_ζ , two equilibria exist. The equilibria stand in relation to one another in the same manner as the Q_- and Q_+ stand in the myopic economy. That is, for any horizon, the equilibrium with the larger level of excess volatility corresponds to the Q_+ equilibrium, since Q and EX increase as σ_ζ declines. Another interesting feature of these equilibria is the relation between excess volatility and horizon. For a particular value of σ_ζ , one might expect Q and EX to increase as the myopia of investors increases (i.e., n declines). That is indeed the case for the equilibria with lower levels of excess volatility (Q_- equilibria). For the Q_+ equilibria, the relation is inverse: Q and EX decline as myopia increases.

Given the odd properties of the Q_+ type equilibria, we concentrate our analysis on the Q_- equilibria, i.e., for a given level of σ_ζ , those with

²⁰This is approximately the point estimate obtained by Brown and Gibbons (1985) using United States stock market data from 1926-1981.

²¹This value of fundamental volatility, with the price normalized to one, implies a volatility of returns of 15%. This number is roughly equal to the standard deviation obtained by annualizing the monthly standard deviations of real returns on common stocks as reported by Fama and Schwert (1977) in Table 2.

²² $E[QZ_{t+\tau} | \Phi_t] = \mu^\tau QZ_t$, so the half-life of the mispricing given $\mu=.9$ is about 6.5 years.

lower levels of excess volatility. Figures 1 and 2 demonstrate that the degree of myopia has only a minor influence on the level of excess volatility. Given low to moderate levels of σ_{ζ} , i.e., those levels for which equilibria exist in the, say, $n=10$ year economy, Q and EX are relatively insensitive to changes in horizon.

Figure 3 displays the relations in the myopic investor economy between excess volatility and the volatility of supply for various coefficients of reversion in the supply; $\mu = 0, .5, .7, .9, \text{ and } 1$. Figures 4 and 5 display the same relations, but respectively for the three-year horizon and long-horizon investor economies. For any horizon, for a given level of σ_{ζ} , the excess volatility increases with μ . As μ increases, the mean reversion in noise traders' holdings decreases, so that the risk faced by rational investors increases. This reduces their aggressiveness in trading against noise, and consequently Q increases with μ (i.e, prices revert more slowly to fundamentals).

West (1988a) reports estimates of excess volatility EX as large as 5 for portfolios of common stocks. These figures suggest that the excess volatility sustainable in an economy depends on the horizon of investors. For example, in a myopic-investor economy that level of excess volatility cannot exceed 0.5. When investors have a three-year horizon, equilibria exist for much larger levels of excess volatility. In particular, if μ is close to one (so that the mispricing of the risky asset reverts slowly in expectation toward the fundamental), the maximum possible excess volatility appears to be much greater than 5. In the limiting case of infinite horizon, the level of EX is unbounded. In summary, the evidence of Figures 1-5 suggests that excess volatility of the degree reported in the literature

is likely to be a characteristic of an economy of investors with moderate to long horizons.

Figure 6 reports the relation between the expected losses of erratic traders and excess volatility, when the skillful investors have infinite horizons. This figure reports the first measure of loss in Section 1.B for the same equilibria described in Figure 5 ($n=\infty$); relations are reported for μ equal to 0, .5, .7, .9 and 1.

Recall that per capita wealth of skillful traders is set at \$100,000 in the determination of the risk aversion coefficient a . Given this wealth level as a basis for comparison, the expected losses are at plausible levels only when either the excess volatility is low, or when the level of the erratic supply reverts slowly to zero. For example, given $EX=2$ and $\mu=0.5$, the expected losses are approximately \$120,000 per capita skillful trader, per year, while if $\mu=1$ the losses are close to \$15,000.

Another way to examine the plausibility of noise traders' losses is to consider the second measure of losses in Section 1.B, i.e., expected losses per unit expected volume of trade, $E[QZ_t(Z_t - Z_{t-1})]/E[|Z_t - Z_{t-1}|] = Q\sigma_f(\pi/(4(1+\mu)))^{1/2}$. As discussed in that Section, this measure can be roughly interpreted as the expected profits of a rational investor (with long horizon) for absorbing a dollar of supply from noise traders. Denoting this measure as L , it is easily shown that the relation between L and the excess volatility, EX , is given by the following expression

$$L = (\sigma_f/2)[\pi((1+EX)^2 - 1)/(1+\mu)]^{1/2}. \quad (19)$$

Thus, L is an increasing function of the fundamental volatility, σ_f and the

excess volatility EX , and a decreasing function of μ . Furthermore, the relation between L and EX does not depend on investor horizon, n .

For different values of μ , Figure 7 presents the relation between a rational investor's expected profits and the excess volatility. The expected profits appear quite big for high levels of excess volatility. For example, with $EX=5$ and $\mu=0$, $L=0.75$, i.e., the investor's expected profits are \$0.75 for every dollar of supply that he absorbs from noise traders. Even with lower levels of mean reversion, the expected profits are quite substantial (For $EX=5$, the expected profits are greater than \$0.50 per dollar traded for any value of μ). This Figure, thus, suggests that if the economy is characterized by a high level of excess volatility, rational investors can expect to make substantial gains at the expense of noise traders.

Thus, it seems that trading unrelated to fundamentals can explain the high levels of volatility of asset prices reported in the empirical literature only when mispriced assets slowly revert (in expectation) to their fundamental values and when the average horizon of investors is long. Even in such cases, noise traders' expected losses can be quite substantial.

Figure 8 reports the levels of risky asset demands per unit supply B^n for investors with horizons $n = 0, 3, 5, 10$ and 100 , and given $\mu=.9$. The calculations are made for values of Q such that $B=1$, so the reported demands are relative to that of the long-horizon investor. One sees that the finite-horizon demands are all less than the long-horizon demand, and that the difference increases with the myopia. For example, given excess volatility of 2, the demand of the ten-year investor is about 60 percent of the long-horizon demand, while that of the myopic investor is less than 30 percent of the same. These differences seem substantial. Furthermore, for

any finite horizon, the difference increases with the excess volatility, so that the relative demand of the myopic investor is less than 5 percent when EX is 5. Our conclusion is that the incentive to behave rationally, in an environment where an investor might choose to behave myopically, is large when excess volatility is at the levels reported in the literature.

Sensitivity analysis on the exogenous parameters - risk aversion, real interest rate, and fundamental volatility, indicates that the endogenous values change in intuitive ways with changes in these exogenous parameters. For example, keeping horizon and σ_{ζ} fixed, an increase in risk aversion results in a higher level of excess volatility. In the extreme, if skillful investors become very risk averse, then EX can be large, even for small σ_{ζ} . Similarly, keeping horizon and σ_{ζ} fixed, an increase in the fundamental volatility increases the excess volatility in the economy. The sensitivity analysis suggests that our conclusions above that noise trading can explain high levels of excess volatility only when average investor horizon is long and mean reversion in noise traders' holdings is low, are quite robust.

V. The Analysis of Information Acquisition

The value of services of financial analysts is determined, in part, by investor horizon, cash flow predictability, contractual arrangements, and the informativeness of prices. Consider, for example, the fact that Wall Street analysts generally forecast earnings, dividends, and other financial information only one or two years into the future. This might be interpreted as evidence that, relative to the length of life of most of corporations, investors' demand for information reflects their short horizons. An alternative hypothesis is that, for the analyst, the cost and the precision, respectively, of rudimentary information increase and

decrease with the horizon of the forecast. A third possibility is that the costs of contractual incentives which motivate the analyst to report valuable information increase with the horizon of the forecast.²³

An extension in which there exist signals of future cash flows provides a basis to formalize the first two of these three hypotheses.²⁴ In the alternative model, investors have an opportunity to acquire signals with precisions and with exogenously specified costs that vary with the horizon of the cash flow(s) to be forecast. The incentive for any single investor to purchase a given signal will then depend, in part, upon his horizon.²⁵ For this reason, the aggregate demand for short- versus long-term signals, and the degree to which prices are informative of short- versus long-term cash flows are functions of the average investor's horizon and the opportunity set of signals.

It is also desirable to consider an alternative in which the information that skillful investors have regarding noise trading is reduced. For example, consider the case of two types of investors. Each investor observes current cash flows and prices, while only the first of the two

²³Associated with this alternative is the idea that analysts are rewarded as a function of their reputation for having made precise, but idiosyncratic forecasts. See, for example, Darlin (1983). Presumably, a reputation is obtained more quickly when forecast horizons are short than when they are long, *ceteris paribus*.

²⁴The structure of this extension parallels that of the single-period model of Grossman and Stiglitz (1980). Given the cash flows to be forecast, signals are perfectly correlated across investors. The existence of idiosyncratic signals, as in the work of Hellwig (1980), Diamond and Verrecchia (1981) and Admati (1985), complicates the analysis considerably. Idiosyncratic signal errors enter as state variables in the utility function.

²⁵A common presumption is that the value investors place on signals of cash flows realized beyond their horizon is lower than on those realized within their horizon. The degree to which this is true will be identified by analysis of the extended model.

types, "the informed", observes a signal, say Y_t , informative of F_t . We conjecture that the equilibrium price is a linear function of Y_t and Z_t in this setting, so that the informed, but not the uninformed investors, can infer the contemporaneous noise Z_t . A complication in establishing equilibrium is that the utility function of the long-horizon uninformed investors will not satisfy (7); price and cash flows, and not noise directly, will enter as state variables. However, an interesting feature is likely to obtain: because the uninformed do not observe Z_t directly but infer the level of erratic trading from observation of the price, the excess volatility of price will be larger than in the present model.²⁶

VI. Conclusions

Using a model of skillful investors with finite horizons and who trade at the expense of erratic traders, this paper investigates the degree to which asset prices reflect the fundamentals. Evidence is presented that trading unrelated to fundamentals can explain the high levels of volatility of asset prices reported in the empirical literature only when average horizon of investors is long and mispriced assets slowly revert (in expectation) to their fundamental values and even in such cases noise traders' expected losses can be quite substantial. For low to moderate levels of the volatility of erratic trading, short-termism does not result in any excess volatility. In this case, the investment policies of myopic traders are virtually the same as those of long-horizon traders and

²⁶Wang (1990) analyzes such a model with infinite-horizon investors and finds that information asymmetry among investors does increase price volatility.

consequently, the sensitivity of prices to noise is invariant to the composition of the economy.

Investor horizon matters when the excess volatility in the economy is quite high. Then, the benefits of future mispricing, which are ignored by myopic traders, are substantial and the policies of the myopic and long-horizon traders deviate significantly from one another. In our view, the previous research has overemphasized the adverse consequences of short-termism by assuming myopic traders to be short-lived and thus imposing excessive liquidation risk on them.

The model developed in this paper can be used to study several other issues. For example, one can allow endogenous information acquisition in the model and then analyze the relation between horizon and incentives to acquire short- versus long-term information. A second possibility is to determine the optimal compensation contracts for money managers, an issue that we are currently investigating.

Appendix

Derivation of Equations (16) and (17)

From the analysis in section III for the myopic case, it is easily seen [cf. equation (12)] that (16) and (17) hold for horizon 0. An inductive argument is used for the remaining, positive horizons.

Suppose that (16) and (17) obtain for an $(n-1)$ -period-horizon investor. Consider an n -period horizon investor solving the lifetime problem as of time t . Because this investor ignores benefits from mispricing for periods $t+n+1$ onwards, his value-function for time $t+n+1$ is given by (7a) with k_1 set to zero. Hence his time- t problem is to choose a sequence of consumption and investment demands for times t through $t+n$ to

$$\text{maximize } E \left[\sum_{s=t}^n -\rho^{s-t} \exp(-ac_s) - \rho^{n+1-t} \exp(-kW_t + k_0) \mid \Phi_t \right], \quad (A1)$$

where k and k_0 satisfy respectively (7b) and (7c). The optimization exercise can be solved as a series of two-period problems, using dynamic programming, and beginning with that for date $t+n$. One finds that the two-period problem for time $t+1$ is the problem (16) of the $(n-1)$ -period-horizon investor. Therefore, the maximand of the latter problem is the utility of wealth J^n for the n -period-horizon investor's two-period problem (16) for time t . The expressions (16) and (17) follow (for the n -period-horizon investor) from examination of J^n so defined, and from examination of the investment demand which satisfies the first order conditions of problem (16).

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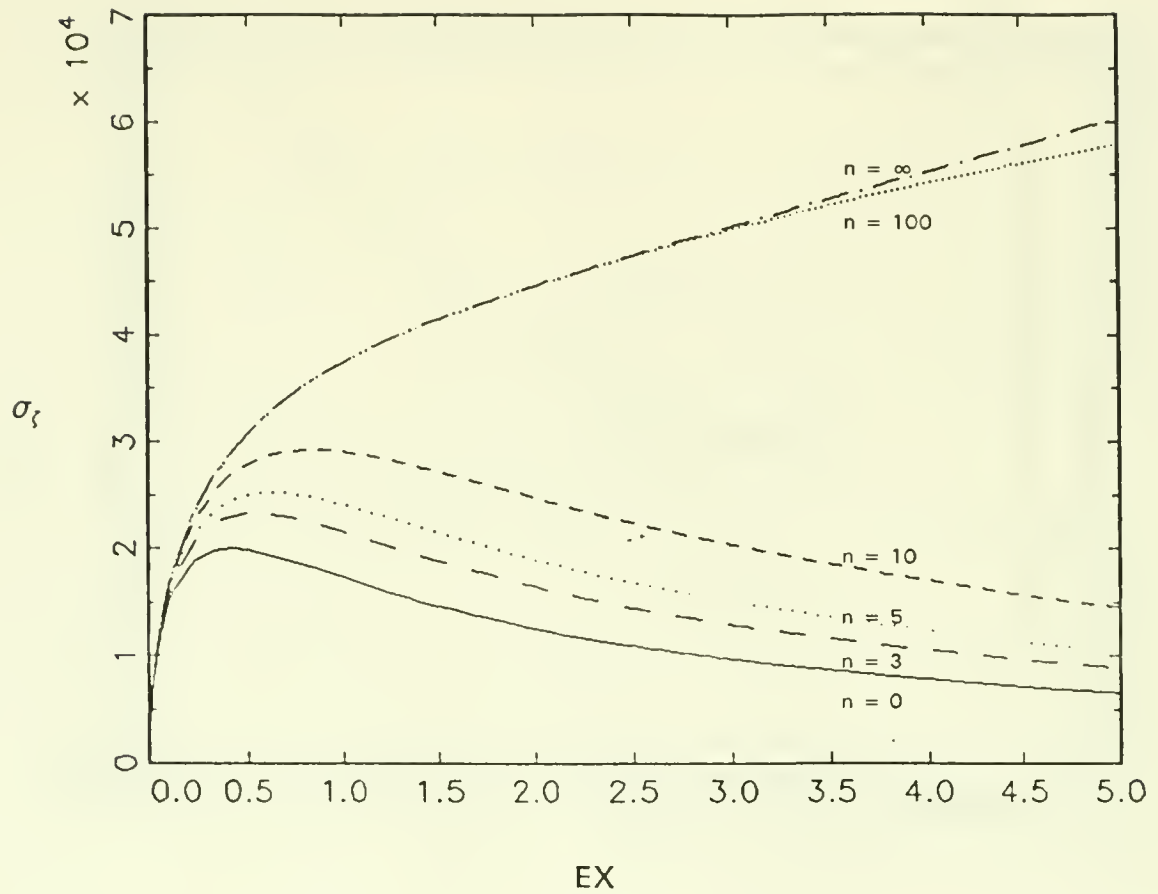


Figure 1. The relation between σ_ζ and excess volatility, EX , for different horizons, n . Fundamental volatility is set at 0.15, absolute risk aversion at 0.00102, real interest rate at 2%, and μ at 0.9.

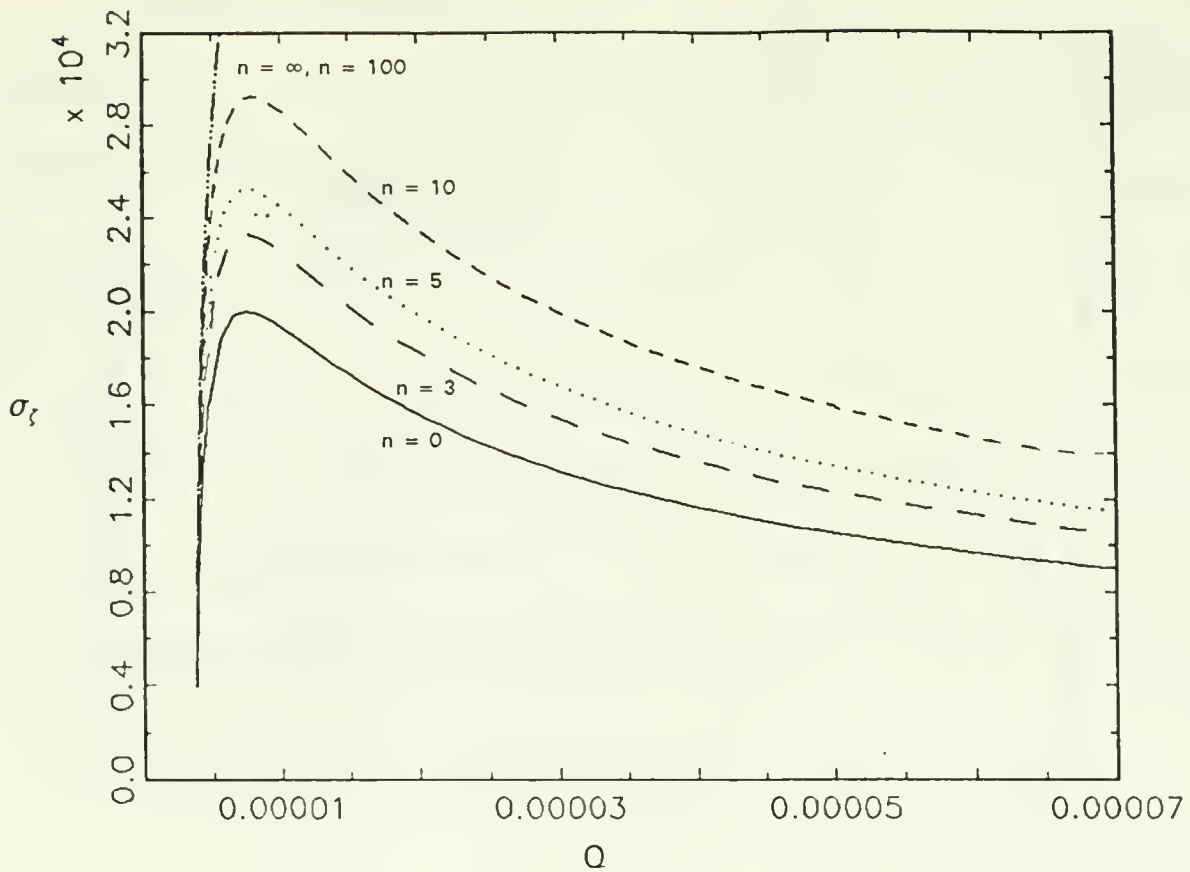


Figure 2. The relation between σ_ζ and Q , the sensitivity of prices to noise for different horizons, n . Fundamental volatility is set at 0.15, absolute risk aversion at 0.00102, real interest rate at 2%, and μ at 0.9.

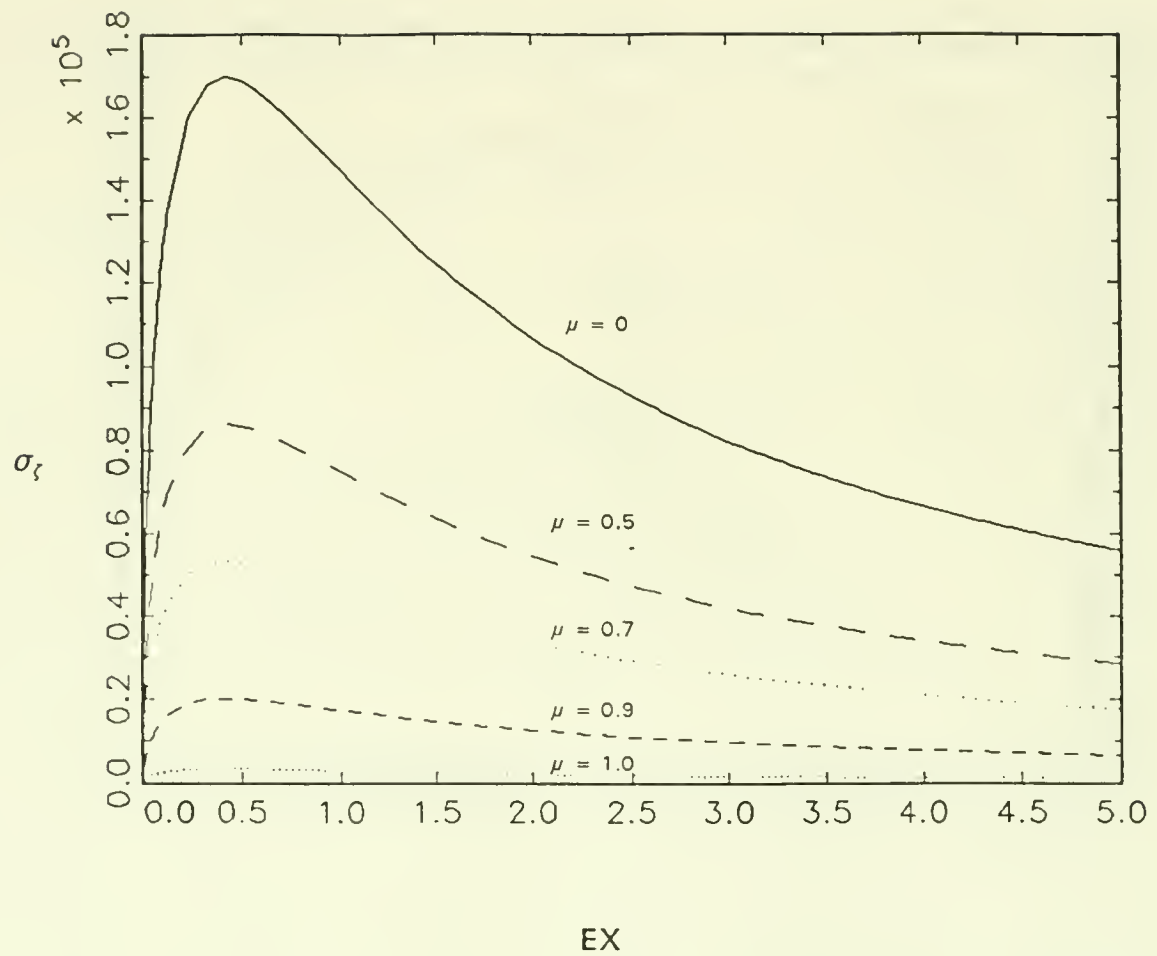


Figure 3. The relation between σ_{ζ} and excess volatility, EX, for the myopic economy ($n = 0$), for different levels of μ . Fundamental volatility is set at 0.15, absolute risk aversion at 0.00102, and real interest rate at 2%.

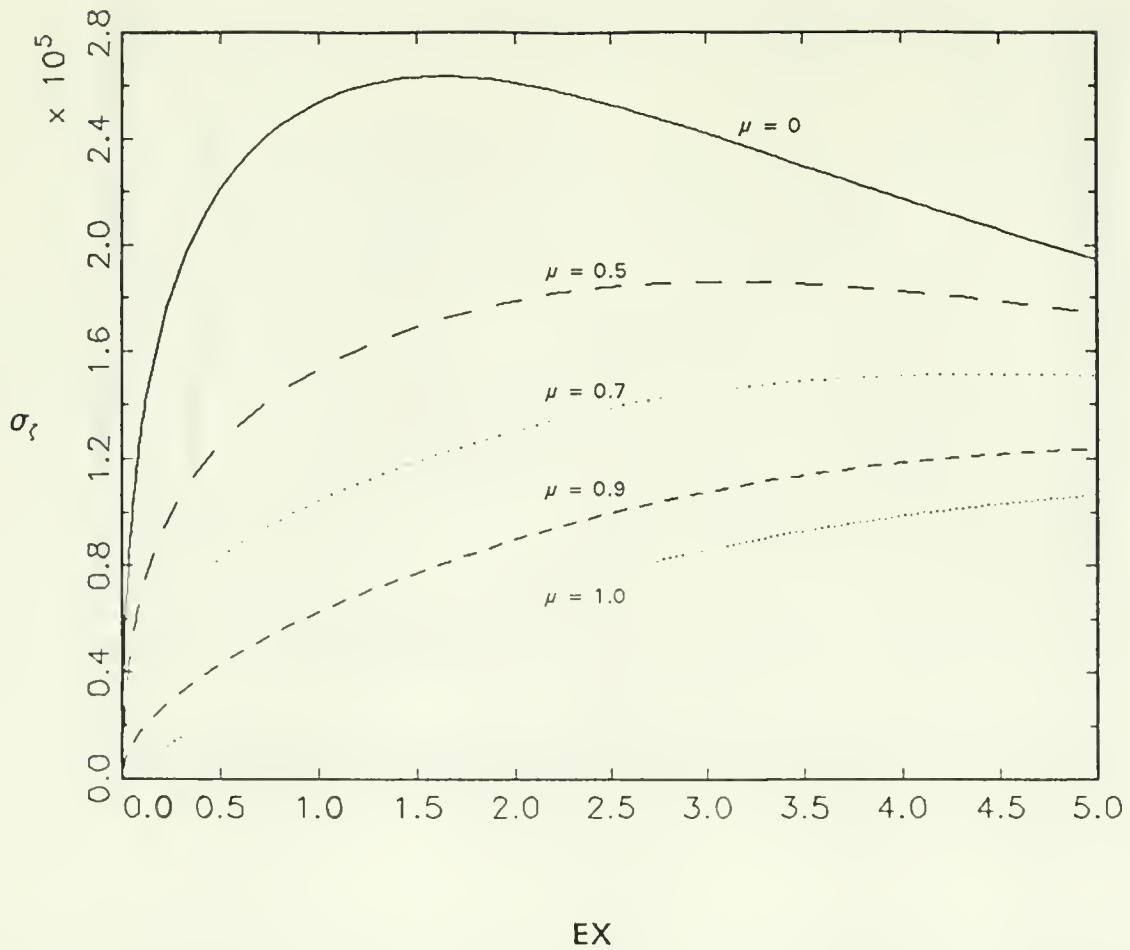


Figure 4. The relation between σ_{ζ} and excess volatility, EX , for $n = 3$, for different levels of μ . Fundamental volatility is set at 0.15, absolute risk aversion at 0.00102, and real interest rate at 2%.

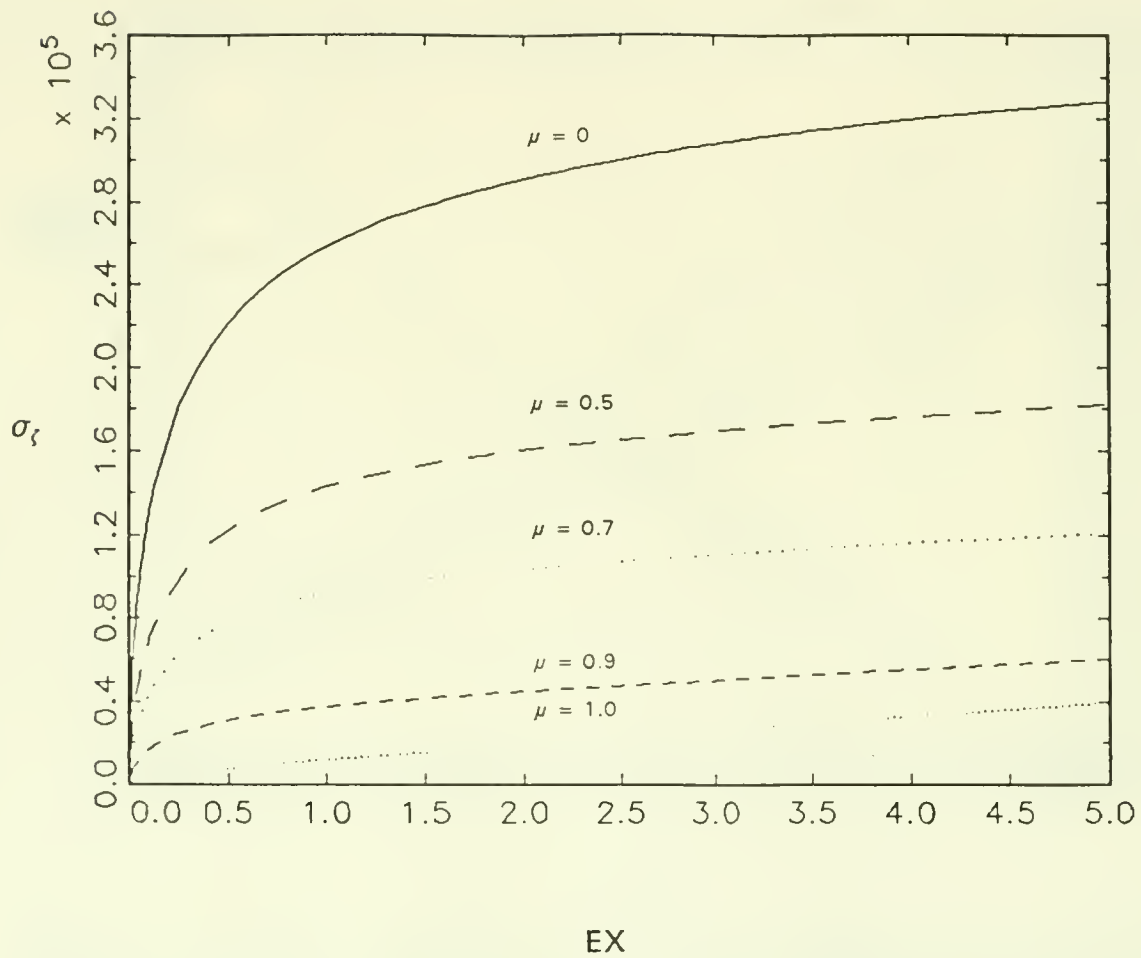


Figure 5. The relation between σ_c and excess volatility, EX, for the long-horizon economy ($n = \infty$), for different levels of μ . Fundamental volatility is set at 0.15, absolute risk aversion at 0.00102, and real interest rate at 2%.

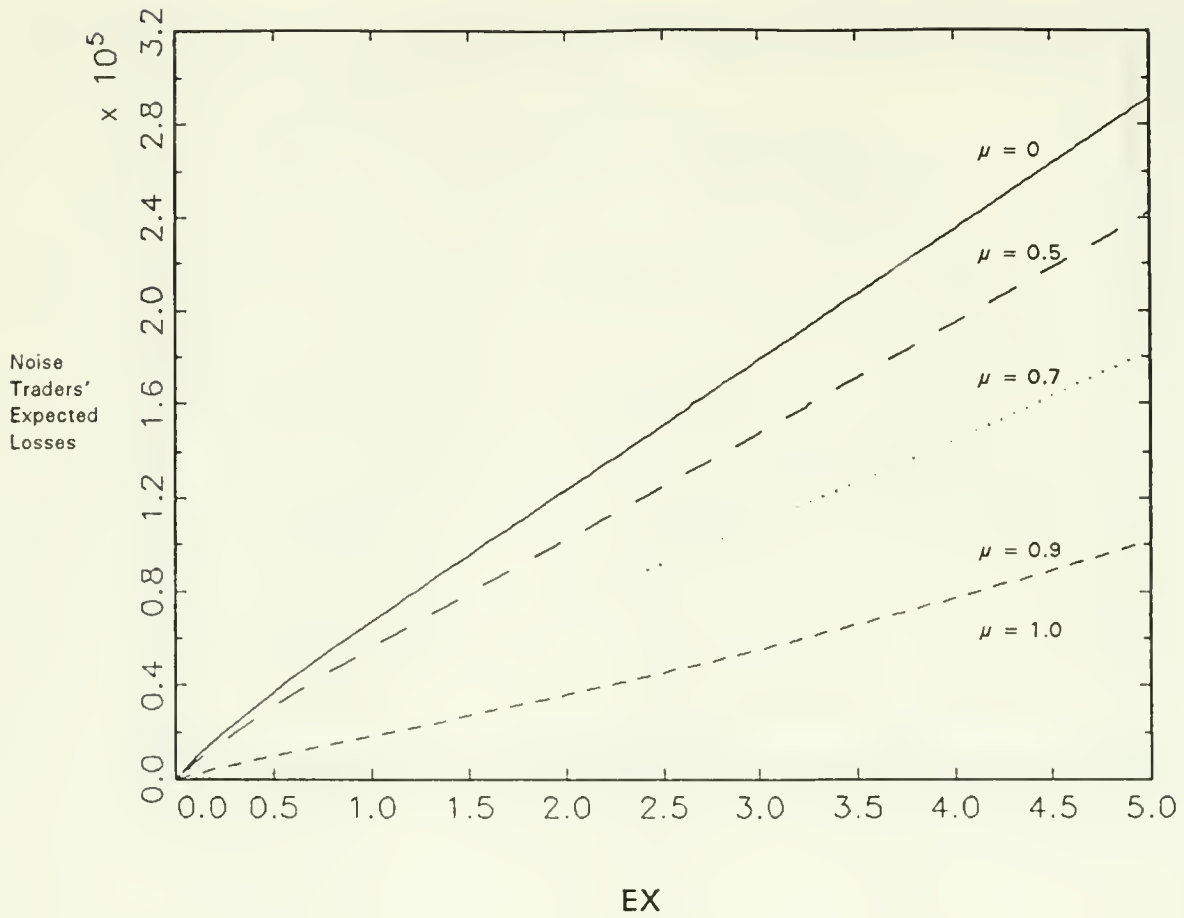


Figure 6. The relation between Noise traders' expected losses and excess volatility, EX , for the long-horizon economy ($n = \infty$), for different levels of μ . Fundamental volatility is set at 0.15, absolute risk aversion at 0.00102, and real interest rate at 2%.

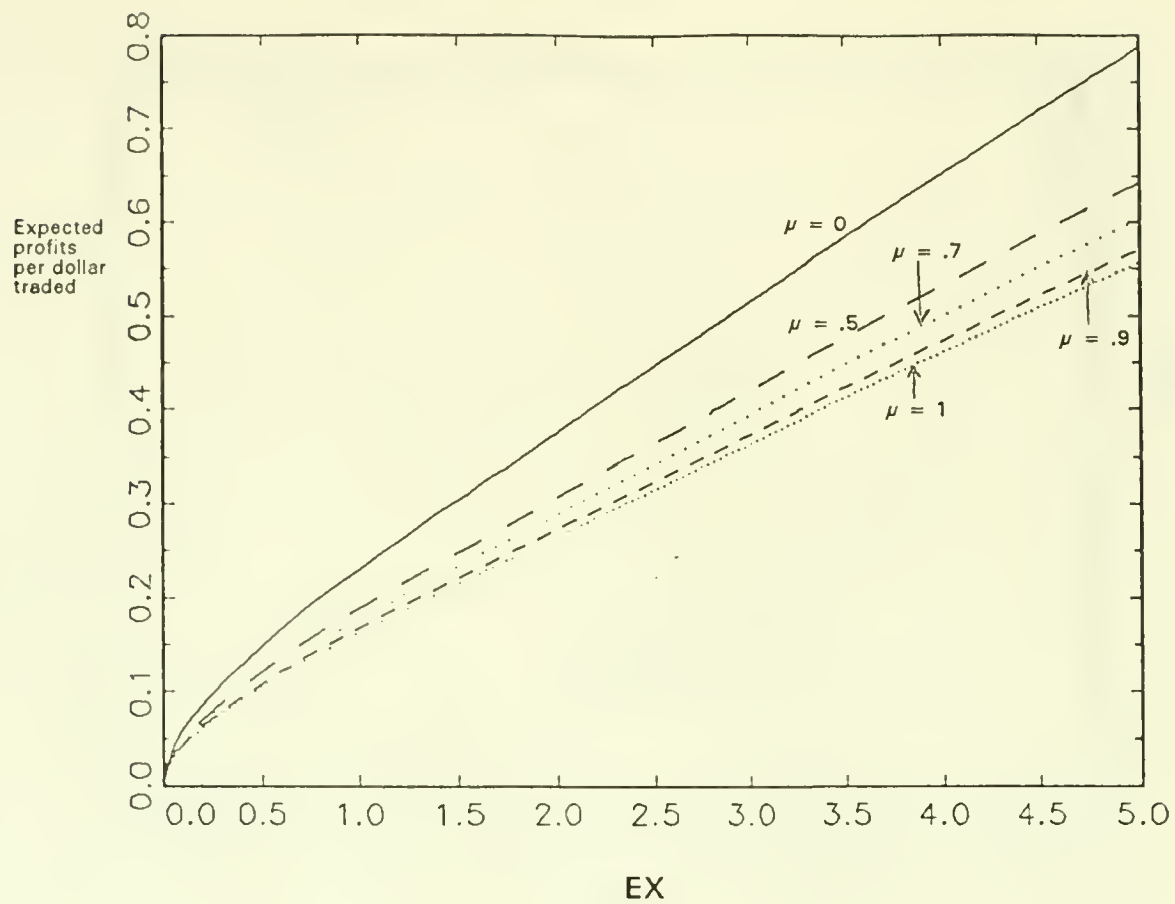


Figure 7. The relation between rational investors' expected profits per dollar traded and the excess volatility, EX. Fundamental volatility is set at 0.15.

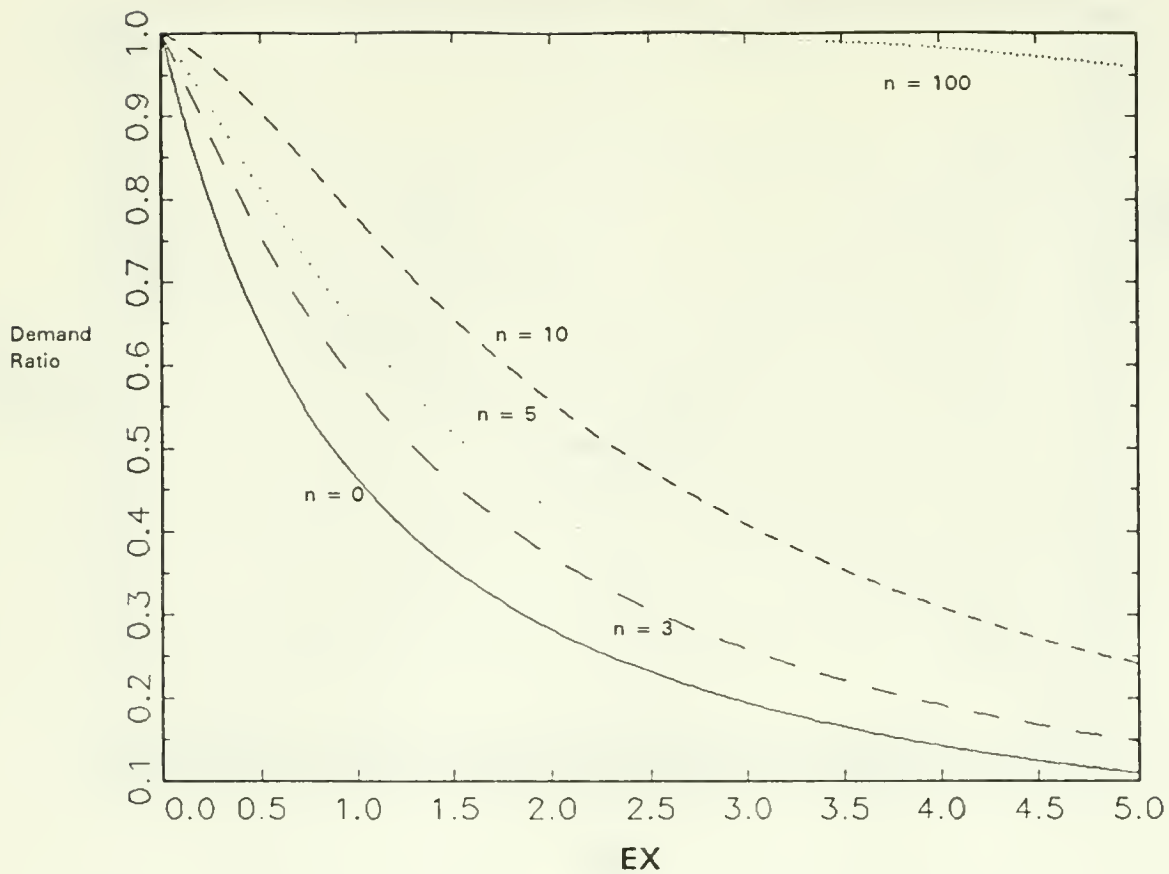


Figure 8. The ratio of the demands of the n -horizon investor and the long-horizon investor versus the excess volatility, EX . Fundamental volatility is set at 0.15, absolute risk aversion at 0.00102, and real interest rate at 2%, μ at 0.9.

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