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by

John Heaton

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ABSTRACT

In this paper I develop and empirically analyze a continuous-time, linear-quadratic, representative consumer model in which the consumer has time-nonseparable preferences of several forms. Within this framework I show how time aggregation and time nonseparabilities in preferences over consumption streams can interact. I show that the behavior of both seasonally adjusted and unadjusted consumption data is consistent with a model of time-nonseparable preferences in which the consumption goods are durable and in which individuals develop habit over the flow of services from the good. The presence of time nonseparabilities in preferences is important because the data does not support a version of the model that focuses solely upon time aggregation and ignores time nonseparabilities in preferences by making preferences time additive.

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I. Introduction

There has been an extensive amount of work examining whether aggregate consumption expenditures are consistent with the restrictions implied by the permanent income hypothesis. A strict interpretation of this hypothesis predicts that aggregate consumption should be a martingale as shown by Hall (1978). This implication has been questioned by a number of authors who have found that consumption changes are predictable over time (see, for example, Flavin (1981), Hayashi (1982) and Hall and Mishkin (1982)). However recently Christiano, Eichenbaum and Marshall (1991) have argued that there is little evidence against the martingale hypothesis using aggregate consumption data, once the fact that the data is time averaged is taken into account.

If consumers are assumed to make consumption decisions quite frequently, so that the martingale hypothesis applies to consumption at a much finer interval than observed data, time averaging of the consumption data implies that consumption changes will be predictable. Under further assumptions, time averaging the data implies that the first-order autocorrelation in consumption changes should be 0.25 and information lagged *two* periods should not be useful in predicting consumption changes. Christiano, Eichenbaum and Marshall (1991) show that these implications are remarkably consistent with quarterly observations of seasonally-adjusted consumption so that rejections of the martingale hypothesis could be due to the fact that the data is time averaged. However, I show that these results are sensitive to the data used in the analysis. In particular, monthly observations of seasonally adjusted consumption and quarterly observations of seasonally unadjusted consumption are at odds with the martingale model, even with an account for the effects of time averaging of the data.

The martingale hypothesis and its implications for time averaged data that are exploited by Christiano, Eichenbaum and Marshall (1991), are derived under the strong assumption that there is a representative consumer with time-separable preferences over consumption in continuous-time. This assumption about preferences has also been imposed in most other investigations of the effects of time averaging of the consumption data[1]. In a continuous-time environment the assumption that preferences are time separable is far from appealing since it implies that an individual's preferences over consumption at one instant are unaffected by consumption the instant before. In this setting it seems more reasonable to assume that preferences are time-nonseparable[2].

In this paper, I show that with the introduction of time-nonseparable preferences, the different dynamics of monthly and quarterly seasonally adjusted consumption data can be easily explained. This occurs because over short periods of time time nonseparabilities are very important. However as the data is averaged over longer periods of time, the model's implications tend to be consistent with a model of time-separable preferences. I also show that the same type of model is consistent with seasonally unadjusted consumption data.

In conducting this study, I develop a continuous-time linear-quadratic permanent income model in which the representative consumer has time-nonseparable preferences. Although the implications of the model are derived under very general forms of time-nonseparable preferences, I show that without further restrictions upon preferences it is not possible to identify the preferences of the consumer using discrete-time data. As a result, I examine several specific forms of time nonseparabilities in preferences.

The first form of time-nonseparable preferences captures the notion that consumption is substitutable over time or that the consumption goods are durable. Discrete-time versions of this type of model have been considered by Dunn and Singleton (1985), Eichenbaum and Hansen (1990), Hansen (1987) and Ogaki (1988), for example. The second form of time nonseparability is a model of habit persistence like those studied by Constantinides (1990), Detemple and Zapatero (1991), Novales (1990), Ryder and Heal (1973) and Sundaresan (1989). This preference specification implies that consumption is complementary over time.

Using seasonally adjusted observations of consumption expenditures on nondurables and services, I show that there is strong evidence for the model where consumption is substitutable over time (or that the consumption goods are durable) and that this model reconciles the conflict between monthly and quarterly consumption data. Further, I find no evidence for habit persistence alone, however there is some weak evidence for habit persistence if habit is assumed to develop over the flow of services created by the durable nature of the consumption goods. Using seasonally unadjusted data I show that there is also very strong evidence in favor of a model where consumption is substitutable over time. Also there is evidence for habit persistence at seasonally frequencies, but again the durable nature of the goods must be modeled.

I also apply the model to durable goods expenditures. Using this data there is strong evidence for habit persistence that forms over the flow of services from the durable goods. I examine whether the model can help to explain some of the durable goods puzzles discussed by Mankiw (1982) and more recently by Caballero (1990). I show that the model provides only a partial resolution of these puzzles.

The rest of the paper is organized as follows. In section II I examine the implications of the martingale hypothesis for time averaged data and I show that the model is inconsistent with monthly seasonally-adjusted data and quarterly seasonally-unadjusted data. I also discuss whether these results could be due solely to measurement error in the consumption data. In section III I develop a model of time-nonseparable preferences and capital accumulation and develop its implications for consumption. I show that it is necessary to focus upon several specific parametric forms of the preferences. In section IV I discuss the implications of two examples of time-nonseparable preferences that capture notions of substitution and complementarity over time. In section V I present the empirical results of applying these examples to consumption data. Section VI concludes the paper.

II. Time-Separable Preferences and Time Aggregation

Consider a situation in which there is a representative consumer with time-separable preferences over consumption facing a constant interest rate that equals the consumer's pure rate of time preference. In this case the euler equation for the consumer implies that the marginal utility of consumption is a martingale (see, for example Hall (1978)). Under the further restriction that preferences are quadratic, with a constant bliss point, the model implies that consumption, $\{c(t):t=0,1,2, \ldots\}$, is a martingale.

A typical way to investigate this sharp prediction is to construct observations of $\bar{c}(t+1) - \bar{c}(t)$ using consumption data observed at quarterly frequencies (for example). A test is then conducted of whether $E\{c(t+1) - c(t) | \mathcal{F}(t)\} = 0$ where $\mathcal{F}(t)$ denotes the consumer's information set at time t (see, for example, Hall (1978), Flavin (1981) and Hayashi (1982)). A potential problem with this approach (as emphasized recently by Christiano, Eichenbaum and Marshall (1991)) is that the available aggregate consumption data consists of observations of consumption expenditures over a period. In other words, at time t observed consumption is $\bar{c}(t)=\int_{t-1}^{t} c(\tau)d\tau$. If consumption within the observation interval is not viewed by the consumer as being perfectly substitutable, then the use of this *time averaged* data could lead to spurious rejections of the martingale implication for consumption.

The fact that the consumption data is time averaged implies that $\{\bar{c}(t)-\bar{c}(t-1): t=0,1,2, \dots\}$ follows a first-order moving average process with first-order autocorrelation[3] of 0.25. This *temporal aggregation* problem could help to explain some of the rejections of the martingale hypothesis for aggregate consumption. This is exactly what Christiano, Eichenbaum and Marshall (1991) find using quarterly data.

II.A Tests of the Model Using Seasonally Adjusted Consumption Data

Following Flavin (1982) and Christiano, Eichenbaum and Marshall (1991), I assume that the model applies to detrended consumption where μ is the trend parameter[4]. Table 2.1 gives maximum likelihood estimates of the MA(1) model: $\bar{c}(t) - \bar{c}(t-1) = \theta_0\varepsilon(t) + \theta_1\varepsilon(t-1)$, where $E\{\varepsilon(t)^2\} = 1$ and $E\{\varepsilon(t)\varepsilon(\tau)\} = 0$ for $\tau \neq t$. The first-order autocorrelation of $\bar{c}(t) - \bar{c}(t-1)$, R(1), is restricted to 0.25 so that the θ_0 implies θ_1. The estimates are reported for quarterly per capita seasonally adjusted consumption expenditures on nondurables and services. Estimates of the MA(1) model with θ_1 unrestricted are given in table 2.2. In both tables estimated parameters are reported for the period 1952,1 to 1986,4 and 1959,1 to 1986,4. The latter subsample was used since this matches the period of the monthly data. Likelihood ratio tests of the restriction that R(1) = 0.25 yield probability values of 0.73 for the data set from 1952,1 to 1986,4 and 0.78 for data from 1959,1 to 1986,4. This occurs because R(1) is very close to 0.25 in each case.

The model also implies that information lagged two periods should not be useful in predicting the consumption change today. In particular, if z(t-2) is a set of instruments that are members of $\mathcal{F}(t)$, and allowing $\bar{c}(t)-\bar{c}(t-1)$ to have a constant mean value[5] of *m*, the model implies that:

$$(2.1) \qquad E\{[\bar{c}(t)-\bar{c}(t-1)-m]z(t-2)\} = 0 .$$

Letting $z(t) = [1,\bar{c}(t-2)-\bar{c}(t-3),\bar{c}(t-3)-\bar{c}(t-4),\bar{c}(t-4)-\bar{c}(t-5),\bar{c}(t-5)-\bar{c}(t-6)]'$, a test of the restriction (2.1) can be performed using the GMM criterion function where the parameter *m* is estimated using GMM and the moment condition (2.1)[6]. If (2.1) is true, then the minimized GMM criterion function is (asymptotically) a chi-squared random variable with 4 degrees of freedom. The resulting P-value of the implied test of (2.1) is 0.065 for the data set from 1952,1 to 1986,4 and 0.083 for the data set from 1959,1 to 1986,4. As a result, there is not substantial evidence against the restriction (2.1). This indicates that the time-separable model is reasonably consistent with quarterly seasonally adjusted consumption, due to the fact that the data is time averaged. This success was noted by Christiano, Eichenbaum and Marshall (1991).

Now consider *monthly* measures of seasonally adjusted consumption expenditures on nondurables plus services[7]. If the only difficulty with the time-separable model is that the consumption data is time averaged, the results should be robust to different intervals of time averaging. Table 2.3 reports estimates of the trend and the moving average parameters of $\bar{c}(t)-\bar{c}(t-1)$ with and without the 0.25 restriction on R(1) using monthly data. A likelihood ratio test of the restriction yields a probability value of essentially zero. As a result, the model is inconsistent with the monthly data[8]. This occurs because the first order autocorrelation of the first difference of monthly consumption is -0.188 with a standard error of 0.052, which is significantly *negative*.

II.B. Tests of the Model Using Quarterly Seasonally Unadjusted Consumption Data

The process of seasonal adjustment used to construct seasonally adjusted data changes the correlation structure of the observed series in a fundamental way (for a discussion of this issue see, for example, Miron (1986) and Ferson and Harvey (1991)). This is a serious issue since a large part of the success of the time-separable model with seasonally adjusted quarterly data is due to the fact that R(1) is estimated to be close to 0.25. It is important to examine whether this result is sensitive to the process of seasonal adjustment.

As a first pass, consider the autocorrelation structure of first differences of seasonally unadjusted consumption with trend and seasonal dummies removed[9]. Table 2.4 gives estimates of the autocorrelation function using quarterly expenditures on nondurables and services[10]. There are two important things to notice. First, unlike the seasonally adjusted quarterly data, the first-order autocorrelation is not close to 0.25. Hence with a different manner of accounting for seasonality, the time-separable model is *not consistent* with the data, even at quarterly frequencies. Second, note that the autocorrelation value at the fourth lag is significantly positive and large. This indicates that the use of seasonal dummies does not remove all of the seasonality in the data and some addition must be made to the model to account for this behavior.

III.C. Measurement Error

Before turning to a model based explanation of these different results, the problem of measurement error in the consumption series must be addressed. Consider a world in which the discrete-time version of the time-separable model is correct at monthly frequencies, but the monthly data is contaminated with i.i.d. measurement error. In this case, observed consumption differences at monthly frequencies would be given by:

$$(2.2) \quad \bar{c}(t) - \bar{c}(t-1) = u_1(t) + u_2(t) - u_2(t-1)$$

where u_1 is the model error and u_2 is the i.i.d. measurement error[11]. Notice that in this case consumption differences are negatively correlated over time. Time averaging from monthly to quarterly frequencies eliminates much of the effect of measurement error and hence would explain the different results using seasonally adjusted monthly and quarterly data.

However, an i.i.d. model for measurement error in consumption data is not very reasonable due to the methods used to construct the data. This point has been stressed in several recent papers Bell and Wilcox (1991) and Wilcox (1991). I will provide a brief summary of the conclusions from these studies. A detailed discussion of these issues can be found in Bell and Wilcox (1991) and Wilcox (1991).

The measure of aggregate consumption used in this paper is personal consumption expenditures on nondurables and services taken from the U.S. National Income and Product Accounts. This measure of consumption is constructed by the Bureau of Economic Analysis in the Department of Commerce. An important ingredient in the construction of personal consumption expenditures is the monthly estimates of retail sales constructed by the Census Bureau. In estimating retail sales, the Census Bureau receives reports of retail sales from all large retail establishments every month and from a sample of small retail establishments. The small companies report in rotating panels every three months. The sampling error induced by this sampling scheme is an important source of measurement error. This sampling error is correlated over time for two reasons.

First, the small retail establishments reporting each month report both their current month's sales and their sales for the previous month. The final estimate of a month's retail sales reported by the Census Bureau are based on the sales reported by the panel of that month and the panel reporting the following month. Due to the reporting overlap, there is very strong autocorrelation in the sampling error from one month to the next. Given their information on the methods used in sampling the small retail establishments, the Census Bureau has attempted to measure the autocorrelation in the sampling error induced by the two month reporting of each panel. Not surprisingly, the autocorrelation is quite close to one (see Bell and Wilcox (1991)). A second source of autocorrelation in the sampling error is the use of a rotating panel. This induces correlation between the current month's sampling error and the sampling error 3 months back.

As discussed by Bell and Wilcox (1991), the strong positive autocorrelation induced by the sampling practices of the Census Bureau implies that the first-order autocorrelation in $\bar{c}(t) - \bar{c}(t-1)$, due to the presence measurement error in (2.2), is likely to be very small. As a result, measurement *cannot* explain the negative correlation of the first-differences of monthly consumption. Further measurement error cannot explain the correlation structure of seasonally unadjusted quarterly consumption data.

III. A Model with Time-Nonseparable Preferences

A problem with the model of preferences that underlies the MA(1) model for $\{\bar{c}(t) - \bar{c}(t-1): t=0,1,2, \ldots\}$ is the assumption that the consumer has time-separable preferences over consumption in continuous time. This assumption implies that the consumer's preferences over consumption at time t are unaffected (through preferences) by consumption in the instant before time t. A reasonable alternative to this model is to allow preferences to be *time-nonseparable.*

In this section, I present a model of time-nonseparable preferences and capital accumulation and derive its implications for observations of time-averaged consumption. Time nonseparability in preferences is introduced by specifying a mapping from current and past consumption goods into a process called *services.* The representative consumer is assumed to have time-separable preferences over services. The mapping from consumption into services is a type of Gorman-Lancaster[12] technology in which the consumption goods are viewed as bundled claims to characteristics that the consumer cares about[13].

I develop the implications of the model under a very general specification of preferences. I show that without further restriction on preferences, discrete-time data will not reveal the preference structure. This implies that preferences must be restricted in some way. In the development of the model I ignore several technical issues. The appendix provides a discussion of these issues.

III.A. Preferences Over Services and Consumption

The preferences of the consumer are assumed to be time separable over a stochastic process, $s \equiv \{s(t): 0 \leq t < \infty\}$, called *services*. The representative consumer evaluates s via the utility function:

$$(3.1) \quad U(s) \equiv -(1/2)E\{\int_0^\infty exp(-\rho t)(s(t)-b(t))^2 dt\}, \ \rho > 0$$

where $\{b(t): 0 \leq t < \infty\}$ is a deterministic process describing the bliss point movement and ρ is the pure rate of time preference[14]. I assume that each element of the space of feasible services is restricted such that: $E\{\int_0^\infty exp(-\rho t)s(t)^2 dt\} < \infty$. Under the further assumption that the deterministic bliss point satisfies $\int_0^\infty exp(-\rho t)b(t)^2 dt < \infty$, the preferences of the consumer are well defined.

Time nonseparability in the consumer's preferences over consumption is introduced by making $s(t)$ a linear function of current and past consumption given by a convolution between a nonrandom distribution g and consumption $c \equiv \{c(t): 0 \leq t < \infty\}$[15]:

$$(3.2) \quad s(t) = g * c(t)$$

where * denotes convolution of two distributions. To insure that $s(t)$ is a function of current and past consumption, g is assumed to be one sided with weight only on the nonnegative real line. For many examples, the convolution in (3.2) is given by the integral: $s(t) = \int_0^t g(\tau)c(t-\tau)d\tau$, however this integral need not be interpretable using standard notions of integration. For example, the nonseparability could involve a comparison of current consumption to past consumption as in a model of habit persistence. In this case, g would include a dirac delta function.

III. B. Capital Accumulation and Equilibrium

The model is completed by assuming that there is a capital accumulation technology that allows the transfer of consumption over time at the constant instantaneous rate ρ:

$$(3.3) \quad Dk(t) = \rho k(t) + e(t) - c(t),\ k(0) \text{ given,}$$

where $k(t)$ is the level of the capital stock at time t, $k(0)$ is an initial condition and $e(t)$ is the endowment process of the consumption good. The endowment process is also assumed to satisfy $E\{\int_0^\infty exp(-\rho t)e(t)^2 dt\} < \infty$. This assumption and the restrictions on c discussed in the appendix, implies that $E\{\int_0^\infty exp(-\rho t)k(t)^2 dt\} < \infty$. As in Christiano, Eichenbaum and Marshall (1991), Hansen (1987), Hansen, Heaton and Sargent (1991), and Hansen and Sargent (1990), this assumption is used instead of a nonnegativity restriction on the capital stock which would make the model analytically intractable.

Following Hansen (1987) and Sargent (1987) I am assuming that the rate of return on capital is equal to the pure rate of time preference. This restriction on the real rate of interest was also imposed by Flavin (1981) and Hayashi (1982). Christiano, Eichenbaum and Marshall (1991) discuss this assumption in the context of the time separable version of this model. They show that this assumption is necessary for the model to imply positive consumption and capital in a deterministic world. These results carry over directly to this model, when applied to services. The restriction that consumption is positive in a deterministic world imposes a set of restrictions on g, as I will discuss in section IV.

The problem facing the consumer is to maximize the objective function (3.1) subject to the mapping (3.2) and the capital accumulation technology (3.3), by the choice of c(t). The Lagrangian for this problem is given by:

$$(3.4) \qquad \mathcal{L} = -E\left\{\int_0^\infty exp(-\rho t)\left\{(1/2)[g*c(t) - b(t)]^2 - \lambda(t)\left[k(t) - k(0) - \rho\int_0^t k(\tau)d\tau - \int_0^t e(\tau)d\tau + \int_0^t c(\tau)d\tau\right]\right\}\right\}$$

where $\lambda(t)$ is the value of the Lagrange multiplier at time t.

The first-order conditions for the choice of $k(t)$ and c(t) for each $t\geq 0$, are given by:

$$(3.5) \qquad E\left\{-g^f*[g*c(t) - b(t)] \mid \mathcal{F}(t)\right\} + E\left\{\int_0^\infty exp(-\rho t)\lambda(t+\tau)d\tau \mid \mathcal{F}(t)\right\} = 0,$$

and

$$(3.6) \qquad \lambda(t) - \rho E\left\{\int_0^\infty exp(-\rho\tau)\lambda(t+\tau)d\tau \mid \mathcal{F}(t)\right\} = 0.$$

where g^f is given by:

$$(3.7) \qquad g^f = \begin{cases} exp(\rho t)g(-t) \text{ if } t \leq 0 \\ 0 \text{ otherwise} \end{cases}.$$

Although it is not possible to completely characterize the solution to this problem for general forms of g, a simple implication for services can be derived.

Notice that (3.6) implies that $\lambda(t)$ is a martingale, so that $E\{\lambda(t+\tau)|\mathcal{F}(t)\} = \lambda(t)$. Also (3.5) and (3.6) imply that

$$(3.8) \qquad E\left\{g^f*[g*c(t) - b(t)]|\mathcal{F}(t)\right\} = \lambda(t)/\rho.$$

The left side of (3.8) gives the marginal utility of a unit of the consumption good at time t. As a result, the model implies that the marginal utility of consumption is a martingale and:

$$(3.9) \qquad g^f*[s(t+\tau) - b(t+\tau)] = g^f*[s(t) - b(t)] + u(t+\tau), \ \tau>0 .$$

where $E\{u(t+\tau)|\mathcal{F}(t)\} = 0$ and where I have substituted in the fact that $g*c(t)=s(t)$.

Under reasonable assumptions on g (see the appendix), g^f has a one-sided forward looking inverse denoted g_i^f. Applying g_i^f to (3.9) yields:

$$(3.10) \qquad s(t+\tau) - b(t+\tau) = s(t) - b(t) + g_i^f*u(t+\tau).$$

Since g_i^f is forward looking, (3.10) implies that $E\{s(t+\tau) - b(t+\tau)|\mathcal{F}(t)\} = s(t) - b(t)$ for $\tau>0$, so that $s-b$ is a continuous-time martingale. I denote the time derivative of $(s-b)[t]$ as: $D(s-b)[t] = D\xi(t)$ and I assume that $E\{D\xi(t)^2\} = \sigma^2dt$.

III.C. Implications for Time-Averaged Consumption

The assumption that g^f has a one-sided inverse also implies that there exists a one-sided inverse of g, g^s, so that the convolution in (3.2) can be inverted to yield a mapping from services to consumption:

$$(3.11) \quad g^s * s = c.$$

Recalling that $D[s-b](t) = D\xi(t)$, (3.10) and (3.11) imply the following representation for the first difference of consumption:

$$(3.12) \quad c(t) - c(t-1) = g^s * \int_{t-1}^{t} D\xi(\tau) + g^s * [b(t) - b(t-1)].$$

Observed consumption, $\{\bar{c}(t):\ t=0,1,2,\ \ldots\}$, consists of averages of consumption expenditures over a unit of time: $\bar{c}(t) = \int_{t-1}^{t} c(\tau)$.

Averaging (3.12) over a unit of time, we obtain the following general representation for observed consumption:

$$(3.13) \quad \bar{c}(t) - \bar{c}(t-1) = g^s * w(t) + g^s * \{\int_{t-1}^{t} [b(\tau) - b(\tau-1)]d\tau\}$$

$$\text{where } w(t) = \int_{t-1}^{t} \int_{\tau-1}^{\tau} D\xi(r)d\tau$$

$$= \int_{t-1}^{t} (t-\tau)D\xi(\tau) + \int_{t-2}^{t-1} (\tau-t+2)D\xi(\tau).$$

Notice that $w(t)$ is a time-averaged martingale difference so that $w(t)$ has an MA(1) structure with first-order autocorrelation[16] of 0.25. The first difference in consumption is obtained by "filtering" $w(t) + [b(t) - b(t-1)]$ through g^s. In the special case where $c(t) = s(t)$ and b is a constant, $\bar{c}(t) - \bar{c}(t-1) = w(t)$ and (3.13) reduces to the MA(1) model examined in section II.

III.D. Identification of g with Sampled Data

There are several identification issues in making inferences about g using (3.13) and observed consumption data. The first involves the presence of the bliss point b(t) in (3.13). If an unobservable stochastic process for the bliss point were added to be model, then the dynamics of consumption could be completely explained by this bliss point movement and the mapping g^s in (3.13) could be set to the identity mapping. The strategy taken here is to minimize the role of unobservables and to try to explain consumption based upon the mapping g^s. As a result, I assume that the bliss point moves in a deterministic fashion. One way to interpret the introduction of time nonseparabilities in preferences is as an attempt to link bliss point movements to observable variables. An advantage to this approach is that it is possible to give an economic interpretation to the required bliss point movement.

Even with the strong assumption that the bliss point is deterministic, there is a further identification issue. To see this, let b(t) be constant, then (3.13) implies a continuous-time moving average representation for $\bar{c}(t) - \bar{c}(t-1)$ of the form:

$$(3.14) \quad \bar{c}(t) - \bar{c}(t-1) = g^c * D\xi(t).$$

where $g^c = h*g^s$, $h(t) = t$ for $t\in[0,1]$, $h(t) = 2 - t$ for $t \in[1,2]$ and $h(t) = 0$ otherwise. If a continuous record of $\bar{c}(t) - \bar{c}(t-1)$ were available, then the functions g^c and g^s could be exactly identified.

With the discrete-time data, $\{\bar{c}(t)-\bar{c}(t-1):t=0,1,...\}$,the best we can do is to identify the discrete-time moving average representation for time-averaged consumption differences:

(3.15) $\quad \bar{c}(t) - \bar{c}(t-1) = \sum_{j=0}^{\infty} G^c(j)\varepsilon(t-j)$

where $E\{\varepsilon(t)^2\} = 1$ and $E\{\varepsilon(t)\varepsilon(\tau)\} = 0$ for $\tau \neq t$. We would like to be able to map the sequence $\{G^c\}$ to the function g^c that describes the time-nonseparabilities. Of course without further restriction it will not be possible to identify completely the function g^c. However, note that (3.15) is exactly the relationship that arises in a discrete-time version of the model in which the decision interval of the consumer is set equal to the interval of the data. In the discrete-time model, the function G(j) reflects the *discrete-time* time-nonseparabilities in preferences.

Marcet (1991) shows how the sequence $\{G^c(j)\}$ is constructed as a function of g^c. To construct this mapping, let A be the closure (in L^2) of the set of all finite linear combinations of the functions of $t \in \mathbb{R}^1$: $g^c(t-1)$, $g^c(t-2)$, $g^c(t-3)$, ... Let $a = g^c - Proj(g^c \mid A)$ where *Proj* denotes the L^2 projection operator. Note that $a(t)$ is in general a convolution of $g^c(\tau)$ at many values of τ. Marcet (1991) shows that $G^c(j)$ is given by:

$$
\begin{aligned}
G^c(j) &= \frac{\int_0^{\infty} g^c(t+j)a(t)dt}{\int_0^{\infty} a(t)^2 dt} \\
&= -\hat{a}*g^c(j)/\int_0^{\infty} a(t)^2 dt \qquad (3.16) \\
&= -\hat{a}*h*g^s(j).
\end{aligned}
$$

where $\hat{a}(t) = a(-t)$.

If $G^c(j)$ were an average of the function $g^s(t)$ for values of t near j, then we would expect the discrete-time interpretation to be reasonably good. However (3.16) implies that the $G^c(j)$ is an average of the function $g^s(t)$

for all values of t and $G^c(j)$ will, in general, lead to poor inferences about the structure of preferences. For example, consider the time-separable model. In this case, $G^c(1)$ is positive since observed consumption differences are positively correlated over time. The discrete-time interpretation would be that preferences are time nonseparable, which they are not.

Since the function G^c will not in general allow us to obtain a correct economic interpretation of preferences, I next focus upon several simple forms of g^s that summarize intuitive types of time-nonseparability in preferences. These restricted forms of g^s allow for the estimation of the *continuous-time* form of g^s. In the next section I outline two basic forms of time-nonseparable preferences that capture notions of substitution and complementarity over time. In section V, I show that these preferences do a good job in explaining the difficulties with the time additive model that I pointed out in section II.

IV. Two Polar Cases of Time-Nonseparabilities

Two specific examples of time-nonseparabilities are of interest. The first example captures the notion that consumption is substitutable over time. In the second example consumption is complementary over time. Throughout this section, I assume that the bliss point is constant.

IV.A. Exponential Depreciation

In the continuous-time model of section III, the assumption that preferences are time separable would imply that consumption is not substitutable from one instant to the next, so that the consumer cares a great deal about the exact timing of consumption. A natural way to introduce time-nonseparable preferences is to assume that consumption can be easily substituted over short periods of time or that preferences are continuous in the dimension of the timing of consumption. This intuitive restriction on preferences in continuous-time models has recently been suggested by Huang and Kreps (1987) and Hindy and Huang (1991). Another reason to consider preference structures where consumption can be readily substituted over short periods of time is that most consumption goods are relatively durable. In particular, the nondurables and services consumption series examined in section II includes several components that should be thought of as durable (for example clothing and shoes).

To capture these ideas, let g be given by:

$$(4.1) \quad g(t) = \begin{cases} 0 \text{ if } t < 0 \\ exp(\delta t) \ t \geq 0 \end{cases}, \text{ where } \delta < 0.$$

The fact that g(t) is positive for $t \geq 0$, implies that consumption can be easily substituted over time and the model satisfies the continuity requirement of Huang and Kreps (1987) Hindy and Huang (1991). The consumption good can also be interpreted as a durable good where δ governs the depreciation of the good.

To apply the results of section III, the mapping g^s of (3.13) must be found. The Laplace transform of g^s is given by $\tilde{g}^s(\zeta) = 1/\tilde{g}(\zeta)$ where $\tilde{g}(\zeta)$ is the Laplace transform of g. In this case $\tilde{g}^s(\zeta) = \zeta - \delta$, which is the Laplace transform of the operator $(D-\delta)$. As a result, the implication of section III that $s(t)$ is a martingale implies that:

$$(4.2) \quad c(t) = Ds(t) - \delta s(t)$$

$$= D\xi(t) - \delta s(t).$$

Note that the goods process has a component that is the time derivative of a martingale. As a result, the consumption process is not a standard stochastic process but is a *generalized* stochastic process.[17] The consumer can tolerate very erratic consumption in this model, because the consumer "consumes" a weighted average of past consumption that is relatively smooth.

In (4.1), δ is restricted to be less than 0. In a deterministic world, the model implies that services are constant over time. In this situation consumption is also constant over time and when $\delta < 0$, (4.2) implies that consumption is positive. Further, the assumption that $\delta < 0$ implies that the model satisfies assumptions 1 through 4 of the appendix that are required to apply the results of section III.

Using (3.13) the exponential depreciation model implies that:

$$(4.3) \qquad \bar{c}(t) - \bar{c}(t-1) = (D-\delta)w(t)$$
$$= \int_0^1 (1-\delta\tau)D\xi(t-\tau) - \int_0^1 [1+\delta(1-\tau)]D\xi(t-1-\tau)$$

As in the time-separable case, first differences in time-averaged consumption follow an MA(1). In this case however, the first-order autocorrelation value need *not* be 0.25 (as in the time-separable case) nor even positive. The first-order autocorrelation of the first differences in consumption is[18]:

$$(4.4) \qquad R(1) \equiv \frac{\delta^2/6 - 1}{2 + (2/3)\delta^2} .$$

R(1) is plotted in Figure 1. Note that as δ goes to $-\infty$, the value of R(1) goes to 0.25 since, as δ is driven to $-\infty$, the consumption good becomes instantly perishable and preferences are time separable. Notice also that if $\delta = -\sqrt{6}$ (a half life of 0.28 periods for the consumption good) then $R(1) = 0$ and a discrete-time martingale model would fit the consumption data. This is a further example of the identification problem discussed in section III.D.

IV.B. Habit Persistence

Suppose that the consumer cares about the level of consumption today relative to an average of past consumption so that the consumer develops an acceptable level of consumption over time. *Habit persistence* of this form has been studied, for example, by Constantinides (1990), Detemple and Zapatero (1991), Novales (1990), Pollack (1970), Ryder and Heal (1973), and Sundaresan (1989). Following Constantinides (1990) I model habit persistence by assuming that $s(t)$ is of the form:

$$(4.5) \qquad s(t) = c(t) - \alpha(-\gamma)\int_{(0,\infty)} exp(\gamma\tau)c(t-\tau)d\tau, \quad \gamma<0, \ 0<\alpha<1$$

Note that the term $(-\gamma)\int_0^{\infty} exp(\gamma\tau)c(t-\tau)d\tau$ is a weighted average of past consumption, and α gives the proportion of this average that is compared to current consumption to arrive at the level of services today. Unlike the model of exponential depreciation, habit persistence implies that consumption is complementary over time.

In this example, g is given by:

$$(4.6) \qquad g = \Delta - \alpha\eta$$

where Δ is the dirac delta function and η is given by:

$$\eta(t) = \begin{cases} 0 \text{ if } t \le 0 \\ exp(\gamma t) \;\; t > 0 \end{cases} \quad . \tag{4.7}$$

The Laplace transform of g in the case is: $\tilde{g}(\zeta) = [\zeta-(1-\alpha)\gamma]/[\zeta-\gamma]$ so that $\tilde{g}^{s}(\zeta) = 1/\tilde{g}(\zeta) = [\zeta-\gamma]/[\zeta-(1-\alpha)\gamma]$, which is the Laplace transform of the operator $[D-\gamma]/[D-(1-\alpha)\gamma]$. Using (3.13) this implies that:

$$D[\bar{c}(t) - \bar{c}(t-1)] = (1-\alpha)\gamma[\bar{c}(t) - \bar{c}(t-1)] + \int_0^1(1-\gamma\tau)D\xi(t-\tau) - \int_0^1[1+\gamma(1-\tau)]D\xi(t-1-\tau) \; . \tag{4.8}$$

This continuous-time autoregressive model for $\bar{c}(t) - \bar{c}(t-1)$ implies that the discrete-time observations $\{\bar{c}(t)-\bar{c}(t-1)\colon t = 0,1,2, \ldots\}$ satisfy an ARMA(1,2) model where the autoregressive parameter is given by $exp[(1-\alpha)\gamma]$. Independent of the effects of time-averaging the data, habit persistence induces smoothness into consumption in the sense that the first difference of consumption is positively autocorrelated[19]. The fact that consumption is time-averaged reinforces this effect. Also, unlike the case of exponential depreciation, the habit persistence model implies higher order dynamics for consumption than the time-separable model.

To satisfy the assumptions of the appendix, the parameters of the habit persistence model must be restricted such that $\gamma<\rho/2$ and $\gamma^{*}\equiv(1-\alpha)\gamma<\rho/2$. In (4.5) these parameters are further restricted for two reasons. First in a world of certainty, s is constant (at level $\bar{s}$, say) and consumption at time t is given by:

$$(4.9) \quad c(t) = \left\{exp(\gamma^{*}t) - (\gamma/\gamma^{*})[exp(\gamma^{*}t) - 1]\right\}\bar{s}.$$

Notice that if $\gamma^{*} < 0$, then $c(t)$ tends to $(\gamma/\gamma^{*})\bar{s}$ which is positive only if $\gamma<0$. If $\gamma^{*} = 0$ (by setting $\alpha = 1$), then $c(t) = \bar{s}(1 - \gamma t)$ which is again positive (for large t) only if $\gamma<0$. If $\gamma^{*} > 0$ then, for large t, c(t) is approximately $exp(\gamma^{*}t)(1-\gamma/\gamma^{*})\bar{s} = exp(\gamma^{*}t)[\alpha/(\alpha-1)]\bar{s}$. For this to be positive, α must be large that one[20] and $\gamma^{*}>0$ implies that $\gamma < 0$. Assumption 3 of the appendix is satisfied as long as $\gamma^{*}<\rho/2$, which allows $\alpha \geq 1$. When $\alpha = 1$, consumption grows linearly and when $\alpha > 1$, consumption grows geometrically at the rate γ^{*}. This type of explosive consumption path under extreme habit persistence has been emphasized by Becker and Murphy (1988). To support the growth in consumption, the capital stock grows along with consumption since the capital stock at time t is given by:

$$(4.10) \quad k(t) = \int_{0}^{\infty} exp(-\rho\tau)c(t+\tau)d\tau - \int_{0}^{\infty} exp(-\rho\tau)e(t+\tau)d\tau .$$

When the the endowment is constant over time at the level $\bar{e}$, $k(t) = \int_{0}^{\infty} exp(-\rho\tau)c(t+\tau)d\tau - \bar{e}/\rho$, so that the capital stock grows along with consumption.

The restriction that capital and consumption are positive in a deterministic world is consistent with $\alpha > 1$ as long as γ is negative. However in this situation if there is an initial level of services in the model, they will not die out when $\alpha > 1$ and it would be difficult to analyze the model empirically. The assumption that $\alpha < 1$ in (4.5) avoids this problem[21].

V. Empirical Results with Time-Nonseparable Preferences

In this section I report the results of fitting the exponential depreciation and habit persistence models to seasonally adjusted and seasonally unadjusted consumption expenditures. In order to estimate these models some account must be made for the trend in consumption data. Following Christiano, Eichenbaum and Marshall (1991)[22], I assume that the model applies to detrended services $s(t) \equiv exp(-\mu t)s^{*}(t)$ where $\mu > 0$ and $s^{*}(t)$ is the trending level of services at time t. This assumption can be directly modeled by assuming that the bliss point follows the geometric trend $exp(\mu t)$. Given that the mappings from consumption to services are linear, trending consumption $c^{*}(t)$ inherits the trend $exp(\mu t)$. The trend parameter μ is estimated along with the rest of the parameters for each model.

V.A. Seasonally Adjusted Expenditures on Nondurables and Services

The exponential depreciation model implies that R(1) is negative if $|\delta| < \sqrt{6}$ (see (4.4)), so that the model is consistent with the fact that first differences of monthly consumption are negatively autocorrelated. As δ becomes more negative, R(1) becomes positive which would then explain the fact that the first differences of quarterly consumption are positively autocorrelated. To investigate the exponential depreciation model, parameterize the moving average process implied by the exponential depreciation model (see (4.3)) as $\bar{c}(t) - \bar{c}(t-1) = \theta_0\varepsilon(t) + \theta_1\varepsilon(t-1)$. This MA(1) model can then be parameterized in terms of θ_0 and δ, where δ ties down the parameter θ_1 through its affect on R(1) given in (4.4). Table 5.1 gives maximum-likelihood estimates of the MA(1) model using the seasonally adjusted monthly expenditures on nondurables and services. The estimated value of δ is -1.36, implying that the consumption goods have a half-life of 0.51 months.

If a quarter is used as the basic time interval, the value of δ implied by the *monthly* estimation is 3*(-1.358) = -4.07. A likelihood ratio test of of this restriction on the quarterly data versus an unrestricted MA(1) model gives a P-value of 0.101 for the data from 1952,1 to 1986,4 and a P-value of 0.281 for the data from 1959,1 to 1986,4. As a result, neither data set rejects this restriction at the 10% significance level and the monthly and quarterly first-order autocorrelation values can be reconciled with a simple exponential depreciation model. The exponential depreciation model fits the first-order negative correlation in monthly consumption differences and time averaging the data to quarterly frequencies implies a model consistent with the quarterly data.

Maximum likelihood estimation[23] of the habit-persistence model using the quarterly consumption series yielded parameter estimates of α that were not significantly different from zero. Habit persistence implies that first differences in consumption are positively correlated over time, but this aspect of the data is well accounted for by the fact that the consumption data is time averaged. In fitting the habit persistence model to monthly data the model did very poorly since the monthly data requires a model that induces negative correlation in the first difference of consumption.

The nonseparability induced by habit persistence captures the notion that people develop a level of acceptable consumption over time. However the fact that g(t) is negative for t > 0 implies that consumption from one instant to the next is not substitutable[24]. A potentially better way to specify the habit persistence model is to first define an *intermediate* *service* process, denoted $\tilde{s}$, which captures the fact that consumption is durable or that consumption can be substituted over short periods of time. The consumer is then assumed to develop habit over the intermediate services process.

The intermediate service process is generated according to:

$$(5.1) \quad \tilde{s}(t) = \int_0^{\infty} exp(\delta\tau)c(t-\tau)d\tau, \ \delta < 0 \ .$$

Again, habit persistence is modeled using (4.5), except that habit develops over $\tilde{s}$ instead of consumption directly:

$$(5.2) \quad s(t) = \tilde{s}(t) - \alpha(-\gamma)\int_0^{\infty} exp(\gamma\tau)\tilde{s}\ (t-\tau)d\tau, \ \gamma < 0, \ 0 < \alpha < 1 \ .$$

This model nests the exponential depreciation model and allows the habit persistence effects to develop much more slowly. I will refer to this model as habit persistence with exponential depreciation. For slow rates of depreciation (δ close to zero) the model implies that the first-order autocorrelation of $\bar{c}(t) - \bar{c}(t-1)$ is negative. Further, for low rates of depreciation, the model satisfies the requirement of Huang and Kreps (1987) and Hindy and Huang (1991) that consumption be easily substitutable from one instant to the next. As δ is driven to $-\infty$, the model approaches the habit persistence model.

Table 5.2 reports results of the estimation of the habit persistence with exponential depreciation model using monthly data[25]. In comparing table 5.2 to table 5.1 notice that there is some improvement in the likelihood function in adding the habit persistence effects to the exponential depreciation model.

A formal test of the exponential depreciation model relative to the habit persistence with exponential depreciation model, is a test of $\alpha = 0$. When $\alpha = 0$ the parameter γ is not identified and the regularity conditions for the likelihood ratio test break down. Davies (1977) has suggested a test in this situation where the test statistic is found by evaluating the likelihood ratio statistic at all possible values of γ and taking the supremum of the resulting values. A lower bound on the P-value of this test can be found by using a chi-square distribution with one degree of freedom and the likelihood ratio statistic based upon the results of tables 5.1 and 5.2. This lower bound is 0.067. As a result, the addition of habit persistence to the exponential depreciation model does not result in a dramatic improvement in the fit of the model

Table 5.3 reports estimated half-life values for the exponential depreciation and habit-persistence effects using the parameter estimates of table 5.2. The half-life of the habit persistence effect is estimated to be relatively large possibly reflecting longer-run habit formation, which seems reasonable. However, the half-life is not precisely estimated. It seems that the degree of habit persistence is difficult to measure with this data set.

In sum, there is strong evidence in favor of the exponential depreciation model[26]. There is some weak evidence for habit persistence in conjunction with the exponential depreciation model. The lack of evidence for habit persistence alone is due to the fact that the consumption data is time averaged and this explains the persistence in the first difference of consumption that the habit persistence model predicts.

V.B. Seasonally Adjusted Expenditures on Durables

In examining time-nonseparable preferences it is natural to consider expenditures on durable goods. Table 5.4 reports estimates of the exponential depreciation model using seasonally-adjusted monthly expenditures on durables. Notice that the estimated value of δ is in fact more negative than the estimate using nondurables and services (table 5.1) which implies that the durable goods are less durable than nondurables and services. The half-life of the durable good implied by this estimate of δ is 0.42 months which seems very unreasonable for durable goods. This puzzle was first discussed by Mankiw (1982). The results of table 5.4 reinforce this puzzle since they account for the fact that the consumption data is time averaged which raises the first-order autocorrelation in the first difference of consumption.

The presence of habit persistence could in principle explain this puzzle since habit persistence induces smoothness into the consumption series. Table 5.5 reports estimates of the habit persistence with exponential depreciation model using the monthly expenditures on durables. Notice first that the log likelihood significantly improves when habit persistence is added to the exponential depreciation model (compare tables 5.4 and 5.5) so that there is much more evidence in favor of habit persistence using durable goods. Also notice that the estimate of δ is much lower and the implied half-life for the durable good is 1.50 months which is somewhat more reasonable, but still too small.

The estimates reported in table 5.4 impose a set of restrictions on the corresponding model for quarterly data. In particular the implied quarterly values of δ and γ are three times the values reported in table 5.5 and the value of α is a third of the value reported in table 5.5. A likelihood ratio test of these restrictions against an unrestricted habit persistence with exponential depreciation model using quarterly expenditures on durables results in a P-value of 0.087 for the data from 1952,1 to 1986,4 and 0.358 for the data from 1959,1 to 1986,4. As with the model of exponential depreciation for nondurables and services, the model of habit persistence with exponential depreciation is consistent with time averaging the monthly data to form the quarterly data.

Caballero (1990) has pointed out that there are higher order dynamics in durables than that captured by the exponential depreciation model. The habit persistence with exponential depreciation model captures some of this as evidenced by the improvement in the likelihood function when habit persistence is added to the model, however the model does not do a complete job. To see this consider table 5.6 which reports likelihood ratio statistics and corresponding P-values of tests of the habit persistence with exponential depreciation model against higher order ARMA models (the habit persistence with exponential depreciation model implies a restricted ARMA(1,2) model for $\bar{c}(t) - \bar{c}(t-1)$). Notice that the higher order ARMA models provide significant evidence against the model. This could perhaps be due to lumpy adjustment and other costs of adjustment as argued, for example, by Caballero (1991).

V.C. Seasonally Unadjusted Expenditures on Nondurables and Services

In this section, I examine whether exponential depreciation model is also consistent with the seasonally unadjusted data examined in section II. Given the weak evidence for habit persistence found with seasonally adjusted expenditures on nondurables and services, I examine whether the seasonally unadjusted data provides more evidence for habit persistence.

As I noted in section II, the seasonal pattern of consumption requires that the models of section III and IV be modified in some way. The first part of the model for seasonals that I use is formed by assuming that the bliss point follows a deterministic seasonal pattern which induces an exact seasonal pattern in consumption. Seasonal bliss point movement has been used in a similar way by Miron (1986) and Ferson and Harvey (1991). In the continuous-time model that is being examined here there is an aliasing problem since there is an infinite dimensional class of deterministic continuous-time functions that will match any deterministic discrete-time function. I assume that the bliss point follows a seasonal pattern in continuous time with frequencies corresponding exactly to the seasonal frequencies of the observed data.

Since the seasonally unadjusted data is quarterly, the bliss point is assumed to be governed by:

$$(5.3) \qquad b(t) = \sum_{j=1}^{2}\{\phi_j cos(t\pi j/2)+\beta_j sin(t\pi j/2)\}$$

where ϕ_1, ϕ_2, β_1 and β_2 are constants. This model of b(t) and (3.13) imply that:

$$(5.4) \qquad \bar{c}(t) - \bar{c}(t-1) = g^s*w(t) + g^s*B(t)$$

$$\text{where} \qquad B(t) = \sum_{j=1}^{2}\{\hat{\alpha}_j[sin(t\pi j/2) + sin((t-2)\pi j/2)] \\ -\hat{\beta}_j[cos(t\pi j/2 + cos((t-2)\pi j/2)]\}$$

and where $\hat{\alpha}_j = 2\alpha_j/\pi$ and $\hat{\beta}_j = 2\beta_j/\pi$. Note that $g^s*B(t)$, observed at the integers, can be represented using seasonal dummies.

In order to capture the durable nature of the goods and to capture the negative first-order autocorrelation noted in table 2.4, I again use the intermediate service process $\tilde{s}$ of (5.2). As in the habit persistence with exponential depreciation model that I considered above, the consumer is assumed to compare the current level of $\tilde{s}$ to an average of past intermediate services. In this case instead of letting the habit stock be a weighted average of all past consumption, assume that the habit stock is given by the level of the intermediate service process of exactly one year ago.[27] In other words $s(t)$ is given by:

$$(5.5) \qquad s(t) = \tilde{s}(t) - \alpha\tilde{s}(t-4)$$

where the basic time period is assumed to be a quarter. The Laplace transform of the distribution g^s for this case is given by: $\tilde{g}^s(\zeta) = (\zeta-\delta)/[1-\alpha exp(-4\zeta)]$ which is the Laplace transfer of the operator $(D-\delta)/(1-\alpha L^4)$. As a result:

$$(5.6) \qquad \bar{c}(t) - \bar{c}(t-1) = \frac{\int_0^1(1-\delta\tau)D\xi(t-\tau) - \int_0^1[1+\delta(1-\tau)]D\xi(t-1-\tau)}{(1 - \alpha L^4)} + \frac{(D-\delta)B(t)}{(1 - \alpha L^4)} .$$

The representation given in (5.6) implies that first differences of time-averaged consumption have two pieces. The first is a first-order moving-average with a first-order seasonal-autoregressive structure. The second piece is a set of deterministic seasonal dummies. Notice that if the model of (5.6) is correct, then seasonal adjustment removes some of the dynamics of consumption that are due to the preferences of the consumer and applying the model to seasonally adjusted data will lead to misleading inferences about the structure of preferences[28].

Table 5.7 gives estimates of the parameters[29] of (5.6) using quarterly seasonally unadjusted expenditures on nondurables and services. A likelihood ratio test of $\alpha=0$ results in a P-value of 0.01% so that there is strong evidence for the presence of habit persistence in this case. This confirms the findings in table 2.4 that the inclusion of seasonal dummies does not completely remove all seasonal effects. Notice that (5.6) implies that first-differences in consumption follow an ARMA(4,1) where all but the fourth-order autoregressive parameters are zero. Estimation of an unrestricted ARMA(4,1) model for the random piece of the first difference in consumption yields a log likelihood value of 150.02. A likelihood ratio test of the model given in (5.6) against this alternative yields a P-value of 0.184. This indicates that the model is doing a reasonable job in representing the seasonally unadjusted consumption data.

The estimated value of δ in table 5.7 is much closer to zero than the value found with seasonally adjusted data. The implied half-life using unadjusted data is 0.38 quarters versus 0.17 quarters using seasonally adjusted data. Estimation of a model with just seasonal habit persistence ($\delta=-\infty$) results in a log-likelihood value of 140.61. Testing this restriction against the model with δ unrestricted, results in a P-value of 0.0002 for the likelihood ratio test.

The seasonally unadjusted data at quarterly frequencies indicates that durability in the goods is important and there is evidence for *seasonal* habit persistence[30]. The model with time-separable preferences does a very poor job in fitting this *quarterly* data set. In section V.A, the evidence against the time-separable model and in favor of the exponential depreciation model came in the form of negative first-order autocorrelation in monthly data and the changing structure of monthly and quarterly data. Unfortunately, the Department of Commerce does not publish monthly, seasonally-unadjusted data, so that the corresponding exercise with seasonally unadjusted data cannot be performed.

VI. Concluding Remarks

Time averaging of aggregate consumption data is an important problem since it destroys the information structure implied by any economic model and can lead to misleading analysis about the underlying economic structure. Christiano, Eichenbaum and Marshall (1991) showed that once this problem is taken into account, there is little evidence in quarterly data against the martingale hypothesis for consumption. However there is a great deal of evidence against the martingale hypothesis using monthly seasonally adjusted data and quarterly seasonally unadjusted data.

In this paper I have shown that the changing structure of monthly and quarterly consumption expenditures on nondurables and services can be reconciled by a model in which consumption is substitutable over time, or in which the consumption goods are durable. The model of preferences is also sensible given the continuous-time perspective that I used to analyze the time-averaged consumption data. The empirical findings support the continuous-time theoretical developments of Huang and Kreps (1987) and Hindy and Huang (1991).

In addition to this model of substitution over time, I found some weak evidence for habit formation when habit is modeled as developing over the flow of services created by the durable nature of the goods. Using expenditures on durable goods I found further evidence for this type of model of preferences. Quarterly seasonally unadjusted consumption expenditures on nondurables and services provided stronger evidence for a model of preferences in which consumption is substitutable over time and in which there is habit formation at seasonal frequencies.

In general the analysis of the paper has implications for investigations of the temporal aggregation issue and models of preferences in other contexts. First in investigations of the effects of time averaging of the consumption data it is not appropriate to assume that preferences are time separable. Also the empirical results with seasonally-unadjusted data indicate that in addition to accounting for time averaging of the data, it is important to examine whether results are robust to the process of seasonal adjustment.

Second I have shown that the fact that the data is time averaged introduces a serious identification problem in fitting models of time-nonseparable preferences to consumption data. In particular, the smoothness in consumption due to time averaging could spuriously be interpreted as being due to habit persistence in a discrete-time model. I found weak evidence for habit persistence using seasonally adjusted consumption expenditures on nondurables and services because I accounted for the fact that the consumption data is time averaged. As a result, in fitting models of time nonseparable preferences using aggregate consumption data, the fact that the consumption data is time averaged should be taken into account.

The analysis in this paper has two major drawbacks: linearity and a constant exogenously given interest rate. These assumptions were imposed so that the implications of the model for time averaged data could be easily developed. In Heaton (1991), I have extended some of the analysis of this paper to an endowment economy where asset returns are endogenous and the representative consumer's period utility is not quadratic (CRRA). I have found implications about the structure of preferences similar to the results of this paper. However the model in Heaton (1991) assumes that endowments are exogenously given and makes strong assumptions on other dimensions.

It would be interesting to determine whether the results of this paper are robust to the introduction of endogenous production. Also more nonlinear time-nonseparabilities have been proposed by Abel (1990), for example. An extension of the analysis of this to that class of preferences would also be interesting.

Footnotes

[1]See, for example, Grossman, Melino and Shiller (1985), Hansen and Singleton (1991), Litzenberger and Ronn (1986), Naik and Ronn (1987) and Hall (1988).

[2]See Huang and Kreps (1987) and Hindy and Huang (1991) for theoretical arguments in favor of time-nonseparable preferences in continuous-time models.

[3]This result was originally derived by Working (1960). See section III and Christiano, Eichenbaum and Marshall (1991) for a derivation of this result.

[4]In section V, I discuss this assumption further.

[5]The mean m can be introduced by allowing differences in the bliss point of the consumer to be constant over time. I allow for this possibility so as to match the results reported in Christiano, Eichenbaum and Marshall (1991).

[6]For a discussion of GMM see Hansen (1982). For a detailed discussion of the test performed here see Christiano, Eichenbaum and Marshall (1991). In detrending the consumption data to perform this GMM estimation I used the trend estimate from unrestricted MA(1) estimation reported in table 2.2.

[7]Note that the quarterly and monthly data are consistent in the sense that quarterly averages of the monthly data yield the quarterly data used in the paper.

[8]This fact was also noted by Ermini (1989).

[9]In section V, I show that this is consistent with a time separable model with a deterministic bliss point.

[10]The trend value and seasonal dummies were removed using a likelihood function in which the residuals were assumed to be white noise. In table 2.4, estimates are reported using the sample period 1959,1 to 1986,4. Seasonally unadjusted data is available from 1946. The longer data set was not used because the simple trend model used in this paper could not account for what appears to be a dampening of the variance of the seasonal component of the series over the longer time period. This did not occur over the shorter time period used in this paper. Modeling the change in the seasonal behavior of the series is beyond the scope of this paper.

[11]This model also occurs in a situation where the bliss point in the quadratic preferences of the consumer is stochastic and i.i.d. over time (see Sargent (1987)). I discuss this issue further in Section III.

[12]Unlike most analyses of Gorman-Lancaster technologies, I ignore nonnegativity constraints.

[13]Discrete-time versions of this specification of preferences have been considered by many authors. See, for example, Dunn and Singleton (1986), Eichenbaum and Hansen (1990), Eichenbaum, Hansen and Richard (1987) and Hansen (1987).

[14]The assumption that the preferences of the consumer can be represented with a quadratic utility function is made for convenience. This assumption along with the linear constraint on capital accumulation described below implies linear laws of motion for the endogenous variables that can be easily analyzed. In particular, the implications of the model for time-averaged data can be studied without much difficulty. The quadratic assumption for preferences could be viewed as an approximation to a different utility function. For a discussion of this approximation issue see, for example, Christiano (1990).

[15]Notice that $s(t)$ is a process that gives the contribution to services from consumption purchases from time 0 forward. To simplify the exposition, I have set initial conditions for services to zero. The issue of initial conditions is discussed in the appendix.

[16]For a discussion of this result and a derivation of the representation for $w(t)$ see, for example, Grossman, Melino and Shiller (1987).

[17]See Gel'fand and Vilenkin (1964) for a discussion of generalized stochastic processes. As it stands, this model does not directly fit within the framework of section III since consumption is not a standard stochastic process. However, with a simple modification the analysis of section III can be applied. The idea is to think of the consumer as choosing accumulated consumption at time t, where accumulated consumption is given by: $c_a(t) = \int_0^t c(\tau)d\tau$. The mapping from consumption to services can be easily modified to be a mapping from accumulated consumption to services. To interpret the capital accumulation problem integrate (3.3) over time to yield a law of motion for another capital stock where accumulated consumption is the control. This remapping of the problem is discussed further in Heaton (1989) and Hansen, Heaton and Sargent (1991).

[18]The variance of of $\bar{c}(t) - \bar{c}(t-1)$ is given by $\int_0^1(1-\delta\tau)^2d\tau + \int_0^1(1+\delta(1-\tau))^2d\tau$ and the autocovariance is given by $-\int_0^1(1-\delta\tau)(1+\delta(1-\tau))d\tau$ (see Rozanov (1967)).

[19]Sundaresan (1989) and Detemple and Zapatero (1991) investigate whether habit persistence results in smooth consumption in the sense that habit persistence may lower the volatility of consumption for a given volatility in the endowment process.

[20]Assuming that $\alpha \geq 0$ so that we are considering a model of habit persistence.

[21]In the estimation of the habit persistence model reported in section V I searched for values of α in an unrestricted manner. I found no evidence for values of $\alpha > 1$.

[22]Actually Christiano, Eichenbaum and Marshall (1991) follow a slightly different strategy for the trend than the one used here section. The difference is very minor and does not affect the results presented here.

[23]The habit persistence and habit persistence with exponential depreciation models were fit using the frequency domain approximation to the likelihood function suggested by Hannan (1970).

[24]This implies that the habit persistence model does not satisfy the continuity restriction of Huang and Kreps(1987) and Hindy and Huang(1991).

[25]Maximum likelihood estimates of the habit persistence with exponential depreciation durability model for the two quarterly data sets yielded very marginal improvement in the likelihood function over the pure exponential depreciation model. As a result, I do not report the results here.

[26]Gallant and Tauchen (1990) and Eichenbaum and Hansen (1990) also find evidence for durability in the nondurables and services data. The results reported here reinforce these results since they account for the fact that the consumption data is time averaged.

[27]A more general model would be to set $s(t) = s^*(t) - [\alpha(1-\gamma)/(1-\gamma L^4)]s^*(t-4)$ so that the habit stock is a weighted average of services one year ago, two years ago and so on. A likelihood ratio test of $\gamma = 0$ yields a P-value of 0.26 (with γ allowed to be negative in the unrestricted estimation). When γ is set to zero, we get the model given in (5.5). Since the more complicated model provides very insignificant improvement in the likelihood function, it is not discussed.

[28]Heaton (1989) also examines models in which the durable good has a finite life. This induces peaks in the spectral density of consumption differences. If these peaks are close to the seasonal frequencies, then seasonal adjustment will also remove dynamics that are induced by the nature of the consumption goods themselves.

[29]The estimated values of the seasonal dummies are omitted for simplicity.

[30]Osborn (1988) and Ferson and Harvey (1991) also suggest that there is an important role for habit persistence in fitting seasonally unadjusted data.

Appendix

This appendix provides technical support for section III and gives a more general development of some of the results.

Let $(\Omega,\mathcal{F},Pr)$ be the underlying probability space. Information in the economy is represented by a sequence of sub sigma-algebras of $\mathcal{F}$: $F\equiv\{\mathcal{F}(t): t\in[0,\infty)\}$ where $\mathcal{F}(t)\subseteq\mathcal{F}(s)$ for $t \leq s$. Let $\Omega^{+}\equiv\mathbb{R}^{1}_{+}\times\Omega$ and $\mathcal{F}^{+}$ be the product sigma-algebra given by $\mathcal{F}\times\mathcal{B}^{1}_{+}$ where $\mathcal{B}^{1}_{+}$ denotes the Borel sets of $\mathbb{R}^{1}_{+}$. Let λ be a measure on $\mathbb{R}^{1}_{+}$ that has density $exp(-\rho t)$ with respect to Lebesgue measure, where $\rho > 0$. I denote the product measure given by $Pr\times\lambda$, as Pr^{+}. Let $\mathcal{P}$ be the predictable sigma-algebra (see Chung and Williams (1983) or Elliot (1982)) of subsets of Ω^{+}. Services are required to be measurable with respect to $\mathcal{P}$ and square integrable, i.e., an element of $\mathcal{L}^{2}(\Omega^{+},\mathcal{P},Pr^{+})$. Since the consumer makes choices over services, requiring them to be predictable implies that, at t, the consumer can use only information generated by $\{\mathcal{F}_{s}: s < t\}$.

The consumer evaluates $s \in \mathcal{L}^{2}(\Omega^{+},\mathcal{P},Pr^{+})$ with the utility function:

$$(A.1)\qquad U(s) \equiv -(1/2)E\{\int_{0}^{\infty}exp(-\rho t)[s(t)-b(t)]^{2}dt\}$$

Let g be a nonrandom distribution on $\mathbb{R}^{1}_{+}$, then part of services are generated by:

$$(A.2)\qquad s^{1} = g*c$$

The distribution g often puts weight on all of $\mathbb{R}^{1}$, in which case, (A.2) is defined by setting c to be zero for $\tau < 0$. In order to allow for initial conditions for the service process, let $s^{2}(t)$ be a nonrandom member of

$\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$. $s^2(t)$ gives the contribution to services at time t from the initial stock of services. The service process at time t is then given by: $s(t) = s^1(t) + s^2(t)$, $t\geq 0$.

Since the preferences of the representative consumer are defined over elements of $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$, some restriction needs to be placed upon the space of consumption processes and the distribution g such that the convolution given by (A.2) results in a member of $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$.

The first restriction that I impose insures that the mapping yields a member of $\mathcal{L}^2(\Omega^+,\mathcal{F}^+,Pr^+)$. To describe this restriction, define a *discounted* version of g, denoted g′ as: $g' \equiv gh$, where $h(t) = exp(-\varepsilon t)$ for $t\geq 0$ and zero otherwise and $\varepsilon = \rho/2$. Similarly let $c' \equiv ch$, $(Dc)' \equiv Dch$ and so on.

Assumption 1: The space of admissible consumption processes is restricted to be: $C^\ell \equiv \{D^\ell c \in \mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)\}$ where $\ell \geq 0$ and ℓ is the smallest integer such that $(\varepsilon^2+\omega^2)^{-\ell}\, \hat{g}'(\omega)\hat{g}'(\omega)^*$ is essentially bounded and where $\hat{g}'$ is the Fourier transform of g′.

Assumption 1 implies that (A.2) maps the space of admissible consumption processes into $\mathcal{L}^2(\Omega^+,\mathcal{F}^+,Pr^+)$, since:

$$\text{(A.3)}\quad E\{\int_0^\infty exp(-\rho t)[\, g*c(t)\,]^2 dt\} = E\{\int_0^\infty [g'*c(t)]^2 dt\}$$
$$= E\{\int_{-\infty}^{+\infty} \hat{g}'(\omega)\hat{g}'(\omega)^*\ \widehat{c'}(\omega)\ \widehat{c'}(\omega)^* d\omega\}.$$

where $\widehat{c'}(\omega)$ is the Fourier transform of c′. The second equality in (A.3) follows from the Parseval formula. Note that $(\varepsilon + D)(D^j c)'(t) = (D^{j+1}c)'$ and we have:

(A.4) $E\{\int_0^\infty exp(-\rho t)[\ g*c(t)\ ]^2 dt\}$

$$= E\{\int_{-\infty}^{+\infty} (\varepsilon^2+\omega^2)^{-\ell}\ \hat{g}'(\omega)\hat{g}'(\omega)^*(\widehat{D^\ell c})'(\omega)(\widehat{Dc^\ell})'(\omega)^* d\omega.$$

Assumption 1 and the Hölder inequality imply that (A.5) is finite.

To insure that the mapping (A.2) satisfies the information structure of the economy, the following additional assumption is imposed upon g:

Assumption 2: The Laplace transform of g, $\tilde{g}(\zeta)$, is analytic in the half plane: $\{\zeta:\mathrm{Re}(\zeta) > \rho/2\}$. Further, for each real $\sigma' > \rho/2$ and $\zeta = \sigma + i\omega$ where $\sigma > \sigma'$, $|\tilde{g}(\zeta)| \le |\mathcal{P}_K(\zeta)|$ for some polynomial $\mathcal{P}_K$ which depends on compact sets $K\subset[\sigma',\infty)$ where $\sigma\in K$.

Using Theorem 2.5 of Beltrami and Wohlers (1966, p. 51), Assumptions 1 and 2, imply that g is one-sided, putting weight only on $\mathbb{R}^1_+$ and maps (possibly a subset of) $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$ into $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$.

The problem facing the consumer is to maximize the objective function (A.1) subject to the mapping (A.2) and the capital accumulation technology (3.3) by choice of $D^\ell c(t)$. Because the control variable of the problem is not necessarily consumption itself but is a derivative of c, it is convenient to rewrite the mapping (A.2) as: $s^1 = g_\ell * D^\ell c$. The distribution g_ℓ can always be found using a result that is analogous to integration by parts (see, for example, Beltrami and Wohlers (1966, p. 28)).

The representative consumer solves the following resource allocation problem:

(MP) Max $(-1/2)E\{\int_0^\infty exp(-\rho t)[s(t)-b(t)]^2 dt\}$

subject to: $s(t) = g_\ell * D^\ell c + s^2(t)$ and

$$Dk(t) = \rho k(t) + e(t) - c(t),\ k(0) \text{ given}$$

by choice of a $D^{\ell}c$ process in C^{ℓ}. Letting $y_j(t) \equiv D^j c$ for j=0,2, ... ℓ, the Lagrangian for this problem is given by:

$$\mathcal{L} = -E\Bigg\{\int_0^{\infty} exp(-\rho t)\Bigg\{(1/2)\ [g_{\ell} * y_{\ell}(t) + s^2(t) - b(t)]^2 - \lambda_k(t)\Big[k(t) - k(0) - \rho\int_0^t k(\tau)d\tau - \int_0^t e(\tau)d\tau + \int_0^t y_0(\tau)d\tau\Big] - \sum_{j=0}^{\ell-1} \lambda_j(t)\Big[y_j(t) - y_j(0) - \int_0^t y_{j+1}(\tau)d\tau\Big]\Bigg\}\Bigg\}$$

where λ_k and λ_j, j=0,1, ..., ℓ-1 are Lagrange multipliers.

The first-order conditions for the choice of $k(t)$ and y_j, j=0,1, ... ℓ for each t≥0, are then given by:

$$\text{(A.5)}\qquad E\Big\{-g_{\ell}^{f} * [g_{\ell} * y_0(t) + s^2(t) - b(t)] \mid \mathcal{F}(t)\Big\} - E\Big\{\int_0^{\infty} exp(-\rho t)\lambda_{\ell-1}(t+\tau)d\tau \mid \mathcal{F}(t)\Big\} = 0,$$

$$\text{(A.6)}\qquad \lambda_j - E\Big\{\int_0^{\infty} exp(-\rho\tau)\lambda_{j-1}(t+\tau)d\tau \mid \mathcal{F}(t)\Big\} = 0,\ j = 1,2,\ \dots\ \ell-1,$$

$$\text{(A.7)}\qquad \lambda_0 + E\Big\{\int_0^{\infty} exp(-\rho\tau)\lambda_k(t+\tau)d\tau \mid \mathcal{F}(t)\Big\} = 0$$

and

$$\text{(A.8)}\qquad \lambda_k(t) - \rho E\Big\{\int_0^{\infty} exp(-\rho\tau)\lambda_k(t+\tau)d\tau \mid \mathcal{F}(t)\Big\} = 0.$$

$$\text{where } g_{\ell}^{f} = \begin{cases} exp(-\rho t)g_{\ell}(-t) & \text{if } t \le 0 \\ 0 & \text{otherwise} \end{cases}.$$

The solution for $\lambda_k(t)$, to (A.8) is a martingale, hence $E\{\lambda_k(t+\tau) \mid \mathcal{F}(t)\} = \lambda_k(t)$. Relations (A.6) and (A.7) then imply that $\lambda_j(t) = -\lambda_k(t)/\rho^{j+1}$ for j=0,1,2, ... ℓ-1. As a result we have:

$$(A.9) \quad E\left\{-g_\ell^f*[g_\ell*y_0(t) + s^2(t) - b(t)] \mid \mathcal{F}(t)\right\} = \lambda_k(t)/\rho^\ell.$$

As in section III, the left side of (A.9) is the marginal utility of a unit of the consumption good at time t and the marginal utility of consumption is a martingale. To use this result, a one-sided forward looking inverse for g_ℓ^f is needed. This inverse exists under the following restriction on g (see Beltrami and Wohlers (1966)).

Assumption 3: $1/\tilde{g}(\zeta)$ is analytic for $\{\zeta\colon \mathrm{Re}(\zeta) > \rho/2\}$. Further, for each real $\sigma' > \rho/2$ and $\zeta = \sigma + i\omega$ where $\sigma > \sigma'$, $|1/\tilde{g}(\zeta)| \le |\mathcal{P}_K(\zeta)|$ for some polynomial $\mathcal{P}_K$ which depends on compact sets $K\subset[\sigma',\infty)$ where $\sigma\in K$.

This assumption also implies that there is distribution g^s such that: $g^s*g = \Delta$, where Δ is the Dirac delta function. As in section III we have:

$$(A.10) \quad c(t+1) - c(t) = g^s*\int_{t-1}^{t} D\xi(\tau) + g^s*[b(t+1) - b(t)] - g^s*[s^2(t+1) - s^2(t)]$$

In the text the initial condition, $s^2(t)$ is ignored. This is justified (asymtotically) under the following assumption:

Assumption 4: There exists an $\nu > 0$ such that $\lim_{t\to\infty} \exp(\nu t) g^s*s^2(t) = 0$.

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TABLE 2.1

ESTIMATES OF MA(1) MODEL FOR FIRST-DIFFERENCES
OF PER CAPITA, SEASONALLY-ADJUSTED QUARTERLY CONSUMPTION
EXPENDITURES ON NONDURABLES AND SERVICES.
FIRST-ORDER AUTOCORRELATION RESTRICTED TO 0.25

Parameter	Parameter Estimates[a]	
	52,1 to 86,4	59,1 to 86,4
μ ($\times 10^2$)	0.475 (0.052)	0.485 (0.060)
θ_0	0.224 (0.016)	0.253 (0.019)
Log-Likelihood	92.491	67.046

[a]Standard errors are in parentheses.

TABLE 2.2

ESTIMATES OF MA(1) MODEL FOR FIRST-DIFFERENCES
OF PER CAPITA, SEASONALLY-ADJUSTED QUARTERLY CONSUMPTION
EXPENDITURES ON NONDURABLES AND SERVICES.
UNRESTRICTED ESTIMATION

Parameter	Parameter Estimates[a]	
	52,1 to 86,4	59,1 to 86,4
μ ($\times 10^2$)	0.474 (0.053)	0.487 (0.059)
θ_0	0.224 (0.016)	0.252 (0.019)
θ_1	0.062 (0.019)	0.060 (0.023)
Log-Likelihood	92.494	67.092

[a]Standard errors are in parentheses.

TABLE 2.3

ESTIMATES OF MA(1) MODEL FOR FIRST-DIFFERENCES
OF PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION
EXPENDITURES ON NONDURABLES AND SERVICES, 59,1 TO 86,12

Parameter	Estimation with 0.25 Restriction on R(1)[a]	Unrestricted[a]
μ $(\times 10^2)$	0.156 (0.027)	0.164 (0.017)
θ_0	0.248 (0.015)	0.216 (0.010)
θ_1	——	-0.047 (0.011)
Log-Likelihood	211.33	254.54

[a]Standard Errors are in parentheses.

TABLE 2.4

AUTOCORRELATION VALUES FOR FIRST-DIFFERENCES
OF PER CAPITA, SEASONALLY-UNADJUSTED QUARTERLY CONSUMPTION
EXPENDITURES ON NONDURABLES AND SERVICES, 59,1 TO 86,4
TREND AND SEASONAL DUMMIES REMOVED

Order of Autocorelation	Autocorrelation[a]
1	-0.059 (0.087)
2	-0.027 (0.083)
3	0.167 (0.107)
4	0.327 (0.080)
5	-0.073 (0.086)

[a]Standard errors are in parentheses. These were calculated using one year of lags in the Newey-West (1987) procedure.

TABLE 5.1

ESTIMATES OF EXPONENTIAL DEPRECIATION MODEL USING PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION EXPENDITURES ON NONDURABLES AND SERVICES, 59,1 TO 86,12

Parameter	Estimated Value[a]
μ $(\times 10^2)$	0.164 (0.017)
θ_0	0.215 (0.010)
δ	-1.358 (0.161)

Log-Likelihood 254.54

[a]Standard errors are in parentheses

TABLE 5.2

MAXIMUM LIKELIHOOD ESTIMATES OF PARAMETERS OF HABIT PERSISTENCE
WITH EXPONENTIAL DEPRECIATION MODEL, USING
PER CAPITA, SEASONALLY-ADJUSTED MONTHLY
CONSUMPTION EXPENDITURES ON NONDURABLES AND SERVICES,
59,1 TO 86,12

Parameter	Parameter Estimates[a]
μ ($\times 10^2$)	0.156 (0.024)
σ	0.132 (0.010)
δ	-1.558 (0.743)
γ	-0.211 (0.414)
α	0.438 (0.308)
Log-Likelihood	256.22

[a]Standard errors are in parentheses

TABLE 5.3

ESTIMATES OF HALF-LIVES IN MONTHS
FOR DURABILITY AND HABIT PERSISTENCE EFFECTS WITH MONTHLY DATA

Depreciation Parameter	Estimates[a]
δ	0.656 (0.104)
γ	3.293 (6.473)

[a]Standard errors in parentheses

TABLE 5.4

ESTIMATES OF EXPONENTIAL DEPRECIATION MODEL
USING PER CAPITA, SEASONALLY-ADJUSTED MONTHLY
CONSUMPTION EXPENDITURES ON DURABLES, 59,1 TO 86,12

Parameter	Estimated Value[a]
μ $(\times 10^2)$	0.389 (0.036)
θ_0	0.137 (0.009)
δ	-1.647 (0.280)
Log-Likelihood 278.66	

[a]Standard errors are in parentheses

TABLE 5.5

MAXIMUM LIKELIHOOD ESTIMATES OF PARAMETERS FOR HABIT PERSISTENCE
WITH EXPONENTIAL DEPRECIATION MODEL, USING
PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION
EXPENDITURES ON DURABLES, 59,1 TO 86,12

Parameter	Parameter Estimates[a]
μ $(\times 10^2)$	0.390 (0.035)
σ	0.047 (0.005)
δ	-0.461 (0.137)
γ	-4.176 (0.059)
α	0.754 (0.069)
Log-Likelihood	281.87

[a]Standard errors are in parentheses

TABLE 5.6

LIKELIHOOD RATIO TESTS OF HABIT PERSISTENCE
WITH EXPONENTIAL DEPRECIATION MODEL
AGAINST UNRESTRICTED ARMA MODELS
USING PER CAPITA, SEASONALLY-ADJUSTED MONTHLY
CONSUMPTION EXPENDITURES ON DURABLES, 59,1 TO 86,12

ARMA Model	Likelihood Ratio Value	P-Value
ARMA(1,2)	3.52	0.061
ARMA(1,3)	8.32	0.016
ARMA(1,4)	12.4	0.006
ARMA(1,5)	14.6	0.006
ARMA(1,6)	16.3	0.006

TABLE 5.7

ESTIMATES OF EXPONENTIAL DEPRECIATION MODEL
WITH SEASONAL HABIT FORMATION
USING PER CAPITA, SEASONALLY-UNADJUSTED
QUARTERLY CONSUMPTION EXPENDITURES ON NONDURABLES AND SERVICES,
59,1 TO 86,4.
ESTIMATES OF SEASONAL DUMMIES OMITTED

Parameter	Estimated Value[a]
μ $(\times 10^2)$	0.408 (0.074)
θ_0	0.127 (0.010)
δ	-1.802 (0.414)
α	0.349 (0.087)
Log-Likelihood	147.60

[a]Standard errors are in parentheses

Figure 1: R(1) implied by Exponential Depreciation Model