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MEASUREMENT OF RELATIVE EFFICIENCY OF HEALTH SERVICE ORGANIZATIONS WITH DATA ENVELOPMENT ANALYSIS - A SIMULATION

by

H. David Sherman

WP 1200-81 March 1981

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Abstract

Data Envelopment Analysis (DEA), a new methodology based on linear programming concepts, provides an approach to evaluate the relative technical efficiency of nonprofit organizations which have multiple inputs and outputs. This approach potentially will identify inefficient units and the magnitude of the inefficiency to provide a basis to select inefficient units for management review or efficiency audits and to help locate areas where operations might be improved. This is believed to be an improvement over existing approaches to evaluate efficiency of such organizations and is directed toward health service organizations in this study because of the potential value of such an approach in this sector.

This paper investigates an application of DEA to an artificial data set reflecting the operations of a hospital department. The underlying technology is specified from which a set of efficient and inefficient hospital units are developed. Without knowledge of this technology, DEA accurately identifies the inefficient units when the inputs and outputs are properly specified. In contrast, the widely used single-output measures applied to this data set are found to be less reliable in this multiple output environment. The strengths and limitations of DEA are further elaborated to anticipate issues that may arise in subsequent field applications of DEA to hospitals.

I. Introduction

A manager attempting to compare and evaluate the operations of nonprofit organizations (municipalities, government agencies, universities, hospitals, etc.) generally has no single measure such as profit or return on investment available for an overall assessment. A rate of outputs to inputs can be developed for nonprofit organizations when relative prices of their outputs and relative cost of their inputs are known. These relative prices and costs can be applied to the organization's inputs and outputs to derive an output to input ratio which would yield a scalar measure that ranks individual units based on their ability to generate outputs per unit of input. Such a measure might allow managers to direct their efforts to understand and possibly improve those units that produce lower outputs per unit of input. This type of analysis is not possible with many nonprofit organizations because most of their outputs and some of their inputs

do not have objectively determined weights, like competitive market prices for outputs and costs for donated inputs.

How might such a measure be developed or evaluated for practical use? One approach is to use a weighting scheme where subjective weights are applied to the various outputs (and inputs if objective cost data is not available). Such an approach is limited by the reliability of these subjective weights. Another approach might prescribe one overriding objective to narrow the focus in a manner analogous to the way profit maximization supposedly encompasses the primary private sector objective. A single-output measure such as profit does not incorporate measurement of the other outputs that a firm may generate to meet other objectives such as employee satisfaction, reduction of pollution levels, etc., and therefore may not be sensitive to the joint character of these multiple outputs and the need for simultaneous evaluation of all these outputs. The need to explicitly consider multiple outputs is particularly important for nonprofit organizations where profit or cost minimization may represent only one of a multiple set of objectives and frequently is not the primary objective.

Another approach, perhaps more suited to the public sector activities, would segregate the problem into various component elements such as the distinction between "effectiveness" and "efficiency" that is used by the U.S. General Accounting Office in its expanded scope (or comprehensive) audits of government and other non-profit agencies. We shall follow this approach and distinguish <u>effectiveness</u> as the ability to a) state and b) attain objectives. <u>Efficiency</u> will then be defined in terms of a) the benefits and

b) the costs associated with the inputs used in pursuit of these objectives.[1] Other extensions include distinctions between "propriety" and effectiveness, etc., but these will not be pursued here.[2]

The point of the distinction we are making is that we want to underscore the fact that the focus in this paper will be on efficiency. That is, we shall assume that activities in the organizations to be studied are governed by suitably stated objectives so that we may then focus on methods of measuring efficiency, and how such measures might be evaluated en route to their use in actual applications.

A final distinction is that this paper will deal with measurement of technical efficiency in contrast to price or allocative efficiency. An organization is technically inefficient if it is possible to increase physical outputs without increasing its inputs, or if it can decrease the inputs without decreasing its outputs. Price efficiency--the purchasing of inputs at the lowest price and sale of outputs for the highest price--and allocative efficiency--the use of the correct mix of inputs based on the relative prices--are of importance but need not and will not be considered concurrently with technical inefficiency. By focusing on the physical inputs and outputs that determine technical efficiency, we can determine if the firm could become more efficient regardless of whether it is efficient with respect to price and allocation considerations.

Our applications will be oriented toward health service organizations (e.g., hospitals) where there is a great and growing need for such measures. Two key characteristics of hospitals, and

other nonprofit organizations, are 1) the presence of multiple inputs and outputs, and 2) the lack of any market value for many of these outputs (and inputs). Hospital outputs, for example, may include patient care for many types of diagnoses (generally referred to as case mix), training for nurses, medical students, interns, and residents, medical research, community education, etc. While prices for some of these outputs may be available, these are not competitively set prices as are relied upon for efficient resource allocations in the for-profit sector.

In addition to problems involved in weighting such outputs, we might note that their production is often simultaneous or joint in character. The joint character of many of these outputs (and the same is true for the inputs) has tended to place severe limits on the customary approaches to efficiency estimation and evaluation. Standard regression approaches, for example, tend to treat the outputs a) by examining them in one regression equation at a time and b) by further assuming that the inputs have requisite properties of independence in order to avoid the problems of bias or mis-specification that would otherwise result. Attempts have been made to circumvent these difficulties by various types of weighting, aggregation and reduction, but such one-at-a-time regression equations and weighting approaches are attended by difficulties and shortcomings which raise questions about reliability of the results. In any case, alternative approaches which can circumvent these difficulties would be welcome.

One such alternative, known as the method of Data Envelopment Analysis (DEA) has been developed by A. Charnes, W.W. Cooper and

E. Rhodes (CRR).[3] Since the original publication in [6], moreover, this method had been tested in a variety of applications to education and public school programs.[4] A case in point is the use of these methods to distinguish between "program efficiency" and "management efficiency" by applying DEA to date obtained from the Program Follow Through, a large scale social experiment in U.S. Public School education.

We propose to study the use of DEA in the health field as an additional possibility. Our tests will, however, differ from those noted above by more than reference only to a new area of application (health vs. education). They will also differ in that we shall generate our health related data from efficient and inefficient uses of a relevant technology which we shall introduce in an explicitly known form. Our objective will be to assess the performance of DEA in correctly identifying technically inefficient units by reference to the thus generated data without actually employing our knowledge of the underlying technology. This assessment of DEA will also be matched against another (widely used) approach which proceeds by means of the single output measure summarized by unit cost performance.

The development will proceed as follows: The next section, Section 2, provides a brief overview of the new technique referred to as Data Envelopment Analysis (DEA). Section 3 describes the construction of the artificial data set. Section 4 presents the results of applying DEA to the artificial data set. Section 5 illustrates a widely used alternative efficiency ranking system--the single output (cost) measurement technique. Section 6 is a summary of the conclusions of this analysis.

2. New Methodology for Measurement of Relative Efficiency

CCR in [6] generalized the usual input to output ratio measure of technical efficiency in terms of a fractional linear program with fractional constraints which can be summarized as follows:

Objective:

$$\max \quad h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

Constraints:

$$\text{Less than Unity Constraints} : \quad 1 \geq \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \quad ; \quad j = 1,\ldots,n \qquad (1)$$

$$\text{Positivity Constraints} : \quad 0 < u_r \; ; \; r = 1,\ldots,s$$
$$0 < v_i \; ; \; i = 1,\ldots,m$$

Data:

Outputs: y_{rj} = observed amount of $r^{\underline{th}}$ output for $j^{\underline{th}}$ hospital

Inputs: x_{ij} = observed amount of $i^{\underline{th}}$ input for $j^{\underline{th}}$ hospital

In our case the y_{ij} and x_{ij} are all positive so that the less-than-unity constraints will also be all positive by virtue of the positivity (open set) conditions imposed on the u_r and v_i choices. The latter are determined objectively from the data in terms of the above model and its related extremization. Since the hospital $j = o$ being evaluated as max. $h_o = h_o^*$ is also a member of the constraint set, it follows that a solution always exists with $0 < h_o^* \leq 1$. Finally, as is shown in CCR [6], we will have

$h_o^* = 1$ if and only if hospital $j = o$ is efficient relative to the other hospitals that are represented in the constraints. (2)

The above formulation provides a conceptually clear model which generalizes the usual single output-single input measure of efficiency(as in engineering or physics) so that it also embraces multiple output - multiple input situations. Applied to empirical data it provides a new way of estimating extremal relations as well as measuring the relative technical efficiency of decision making units (DMU's) of a non-profit variety (e.g. hospitals) where the usual market based criteria of cost, profit and price provide only one (not necessarily decisive) component for evaluating DMU efficiency.

This application of mathematical programming differs from most other applications in that the data used are the actual inputs (x_i) and outputs (y_j) of each DMU, while the decision variables which are to be calculated (u_r, v_i) are the weights to be assigned to each input or output. For the unit being evaluated, the solution sought is the set of weights which will give that unit the highest efficiency ratio, h_o^*, such that this same set of weights will result in an input-output ratio not exceeding 1 when applied to each unit in the constraint set.

The u and v values may appear to be similar to relative prices. They are not actually relative prices, but rather are relative values assigned objectively to each input and output to maximize the objective unit's efficiency rating. (In some sense, this can be interpreted as giving the objective unit the "benefit of the doubt" in

that any relative value system of u's and v's is allowed regardless of actual prices.) When a unit is found to be relatively inefficient, $h_o^* < 1$, it can be concluded that that unit cannot find another set of weights which would give it a higher rating as long as all units are subject to that same set of weights. Furthermore, such a unit is strictly inefficient compared to other efficient units in the set.

The above formulation involves a nonlinear-nonconvex programming problem. As is shown in CCR [6] however, it may be replaced by dual linear programming problems as follows to allow the use of standard liner program systems.

$$\text{Max} \quad h_o' = \sum_{r=1}^{s} \mu_r y_{ro}$$

$$\text{Subject to} \quad 0 \geq \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \omega_i x_{ij} \quad ; \quad j = 1,\ldots,n \qquad (3)$$

$$1 = \sum_{i=1}^{m} \omega_i x_{io}$$

$$0 < \mu_r, \omega_i \quad ; \quad r = 1,\ldots,s \qquad i = 1,\ldots,m$$

Particular attention might be called to the positivity conditions on the variables, which CCR ensure by introducing the conditions

$$\varepsilon \leq \mu_r, \ \varepsilon \leq \omega_i \quad \text{all r and i}$$

where $\varepsilon > 0$ is a small constant which is so small that it cannot otherwise disturb any solution involving only real numbers.[5] The adjunction of these additional conditions then produces the following problem, which is dual to (3):

$$\min \quad z_o \; - \; \varepsilon \sum_{i=1}^{m} \delta_i^- \; - \; \varepsilon \sum_{r=1}^{s} \delta_r^+$$

subject to

$$\begin{aligned} 0 &= x_{io} z_o \quad - \delta_i^- \qquad\qquad - \sum_{j=1}^{m} x_{ij} \lambda_j \; ; \; i=1,\ldots,m \\ Y_{ro} &= \qquad\qquad - \delta_r^+ \quad + \sum_{j=1}^{m} y_{rj} \lambda_j \; ; \; r=1,\ldots,s \end{aligned} \tag{4}$$

and also

$$\lambda_j, \; \delta_i^-, \; \delta_r^+ \geq 0 \quad \text{for all } j, \; i, \text{ and } r.$$

while z_o is unconstrained in sign.

We shall refer to (3) as being in "production function form" since the optimal values, ω_i^*, μ_r^* respectively represent estimates of the marginal rates of transformation (MRT) and productivity for the DMU being evaluated.[6] It is to be emphasized, however, that these estimates are derived from the data for a pertinent subset[7] of the efficient producers. The optimal values need not--and indeed generally will not--coincide with the transformation rates and productivity that might be estimated by other means (e.g., least squares regressions) for the DMU being evaluated. They are, instead, estimates of what the DMU being evaluated could have achieved if it had utilized its inputs in the manner which the behavior of the efficient firms indicates as having been possible.

The ω_i^* and μ_r^* in (3) refer to a multiple output situation so that classical (single output) production function concepts

are not applicable in a really strict sense. We shall find it convenient to adhere to these concepts, however, with the accompanying terminology found in elementary economics. Moreover, this is also not misleading. CCR have shown how to interpret these values so that the ω_i^* refer to transformations into a "virtual input" and the μ_r^* refer to transformations into a "virtual output".(8) The ω_i and μ_r values are then referred to as "virtual rates of transformation" which become "efficient transform rates" at an optimum.

We shall adhere to the above usages in referring to (3). Turning to (4) we shall refer to this member of the dual pair as the problem in "efficiency evaluation form". The reason for this designation is that min. $z_o = z_o^*$ provides a measure of efficiency by comparing like inputs and like outputs across all of the $j = 1,\ldots,n$ DMU's that are deemed to be pertinent.

For computational purposes either (3) or (4) may be employed since, at least when extreme point methods (e.g., the simplex or dual methods) are employed,--as will be true for most of the available computer codes--the solution to one problem will also generate the optimal values for the corresponding dual. Moreover, since both problems have finite optima we have access to the dual theorem of linear programming in its strong form--viz.,

$$h_o^{*\prime} = z_o^* \tag{5}$$

and further, as in CCR [6],

$$h_o^{*\prime} = h_o^* \tag{6}$$

where h_o^* is optimal for (1) and $h_o^{*\prime}$ is the optimal value for (3).

The values of δ^{+*} and δ^{-*} in (4) are determined by the linear program. As is explained in [11], the slack values, δ^{+*} and δ^{-*}, will be zero for efficient units. For inefficient units, certain of these slack values will be non-zero corresponding to the x_i's and/or y_r's. These correspond to the inputs and/or outputs which the unit being evaluated is utilizing inefficiently as well as the magnitude of this inefficiency measured in relation to the efficient units.

For purposes of the following analysis, the introduction of the conditions

$$.001 = \varepsilon \leq \mu_r \text{ and } .001 = \varepsilon \leq \omega_i \tag{7}$$

assures that wherever a condition would exist with non zero slack in (3) or (4), the solution which will be reached from this application will result in $h_o^* < 1$. Thus, once we have determined that the value $\varepsilon = .001$ (as we have) is sufficiently small so as not to distort the results, we can use the condition (2) in an analysis, i.e., that if $h_o^* = 1$, DMU_o is relatively efficient and if $h_o^* < 1$, DMU_o is relatively inefficient.

In the analysis that follows we shall focus on technical efficiency rather than, e.g., production-function estimation and related considerations. At this point we should therefore note that we seek something more than a classification into categories such as "efficient" and "inefficient". We also seek a way of using the possibilities which CCR [6] showed for DEA which provide numerical estimates of the amount of potential resource conservation or output augmentation that exists for inefficient DMU's.

Before proceeding with the DEA application, Figure 1 provides an illustration of the geometric interpretation of the DEA output for the following 4 DMU's producing output y_1 with inputs x_1 and x_2. (This example is a modified version of an illustration in [16].)

DMU	Output y_1	Input x_1	Input x_2	DEA Efficiency Rating
A	1	4	1	1.0
B	1	2	2	1.0
C	1	3	2	.857
D	1	5	1	.999

DEA defines a piecewise linear efficient production frontier defined by the most efficient DMU's in the set. Thus segment AB is the efficient segment and C and D are inefficient in that they require more inputs to produce one unit of output y_1. The information that the linear program formulation provides includes the following.

	DMU C	DMU D
h_o^* (efficiency rating)	.857	.999
Virtual rates of transformation		
u_1	.857	.999
v_1	.143	.001
v_2	.285	.995

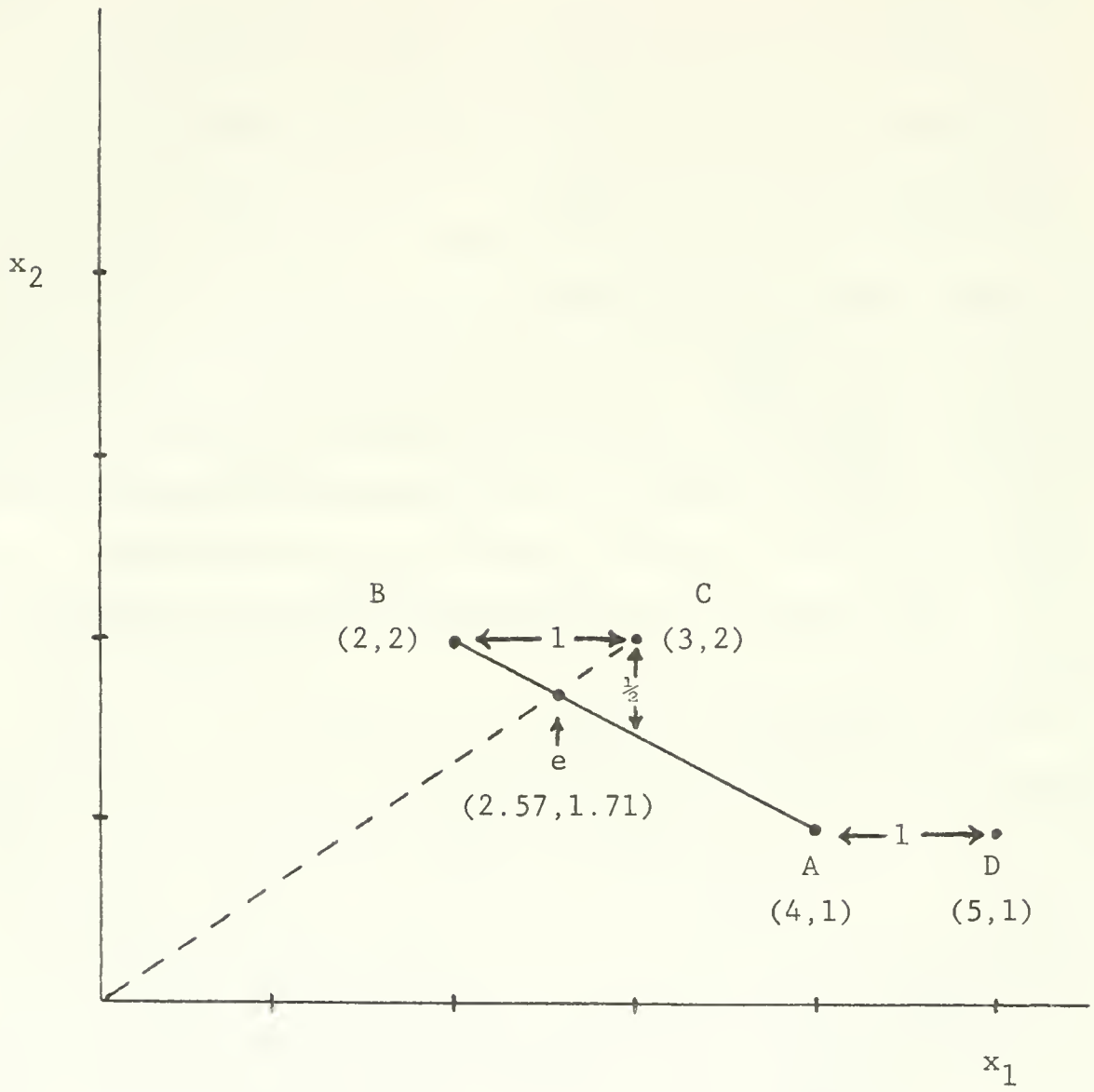

Figure 1.

Shadow price for related constraints	DMU C	DMU D
$\sum_{i=1}^{m} v_i x_{io} = 1$	.857	1.0
$u_1 \geq .001$	0	0
$v_1 \geq .001$	0	-1
$v_2 \geq .001$	0	0

For DMU C, DEA has indicated that the rate of technical substitution of inputs = $-\frac{v_1}{v_2} = -2$ which is the slope of the frontier AB. DMU C can approach the frontier by reducing its inputs by $h_o^* = .857$ to reach efficient point e. u_1, v_1, and v_2 also represent the effect on h_o^* of a one unit change in the corresponding y_1, x_1, and x_2. For example, C can reach the frontier by reducing x_1 by one unit or x_2 by 1/2 unit.

DMU D represents a special case of a unit that comprises an extreme corner of the production frontier. The efficiency rating $h_o^* = .999$ is distinct from an efficient rating of 1.0. The constraint that $u_1, v_1, v_2 > 0$ requires that each input and output be given a non-zero virtual rate of transformation. Here the results are most directly interpreted using the slack values which indicate that there is one unit of slack in v_1 corresponding to x_1 which if eliminated will move D to the efficient point A. Cases such as D will be recognizable primarily when there is an efficiency rating which is very near to,but less than 1.0. (This is assured by imposing condition (7).) Note that if the amount of x_2 used by D were less that 1, D would be rated as an efficient unit and would constitute another segment of the efficiency frontier. The interpretation of DEA results will be

expanded in the following sections where this geometric interpretation will be extended to application in a multiple input and multiple output case.

To conclude this section, we might add that we are not extending the model to the further possibilities such as the piecewise nonlinear representations in [10] which involve nonlinear segments that can accomodate both increasing and decreasing returns in different outputs at the same time. We believe that understanding will be better achieved at this point by confining attention to uses of the preceding models with their accompanying assumptions of (piecewise) constant returns to scale.

3. A Controlled Artificial Data Set

Data Generation

The set of artificial hospital data we generated for our simulation and analysis consisted of three outputs used and three inputs generated during a year as follows:[10]

Outputs	Inputs
i. Severe patient care	i. Number of hospital beds
ii. Regular patient care	ii. Staff utilized in terms of full-time equivalents (FTE's)
iii. Teaching of nurses, residents and interns	iii. Supplies in terms of dollar costs

Selection of output measures is a critical aspect of such an analysis. "Ideal" output measures might include the improvement to patients' health or the increase in interns' abilities from training. The output measures we selected are stated in terms of the activity of the hospital. These measures might be used

instead of the "ideal" measures for at least two reasons. First, the inability to obtain "ideal" measures requires use of a surrogate such as number of patients of each type of diagnosis treated. Second, the relevant parties (e.g., hospital administrators and regulators) may agree on the inputs and outputs to be measured to evaluate the hospital's efficiency. The assumptions and limitations of such alternatives are not the focus of this study, but have important implications in real data applications.

A linear model was used for these purposes and assumed to be applicable to all hospitals. That is, deviations from this structure represent inefficiencies which the DEA analysis--or any other analysis that might be used--should be able to detect.

An idealized version of the situation we have in mind is portrayed in Figure 1. Here we have assumed that all hospitals (our DMU's) are graphically represented in a one-output one-input cross section. We use x_i to represent the ith input and y_r to represent the rth output. Other outputs and inputs are assumed to be fixed at the same amounts for all hospitals with differences, if any, showing up only in this particular x_i and y_r.

The solid line represents the graph for the known production function assumed to be known and the same for all hospitals. The X's represent observations. Here the theoretically attainable production amounts are known for every input level and a measure of "absolute efficiency" is available. Thus, the efficiency of a hospital such as A could be readily computed by forming the ratio of what is theoretically obtainable in this dimension relative to what was actually attained in y_r from the x_i value used by any DMU.

For instance, the point G on the solid line relative to A reflects the absolute efficient output level (y_r) for that level of input (x_i) used by A.

Generally, we do not have access to such knowledge of the production possibilities that a technology makes available. For instance, we shall expect our observations to reflect managerial errors or deficiencies. Hence we have to be content with measures of "relative efficiency" such as DEA provides from observational data. Examples of such observations are provided by the points exhibited as X's in Figure 2.

A DEA approach such as we are using would generate a piecewise linear function which is graphically portrayed in this dimension by the broken line segments shown in Figure 1. Waving aside considerations of observational error [11] we do know that all output values must lie on or below the solid line which represents the graph of the production function in this dimension.

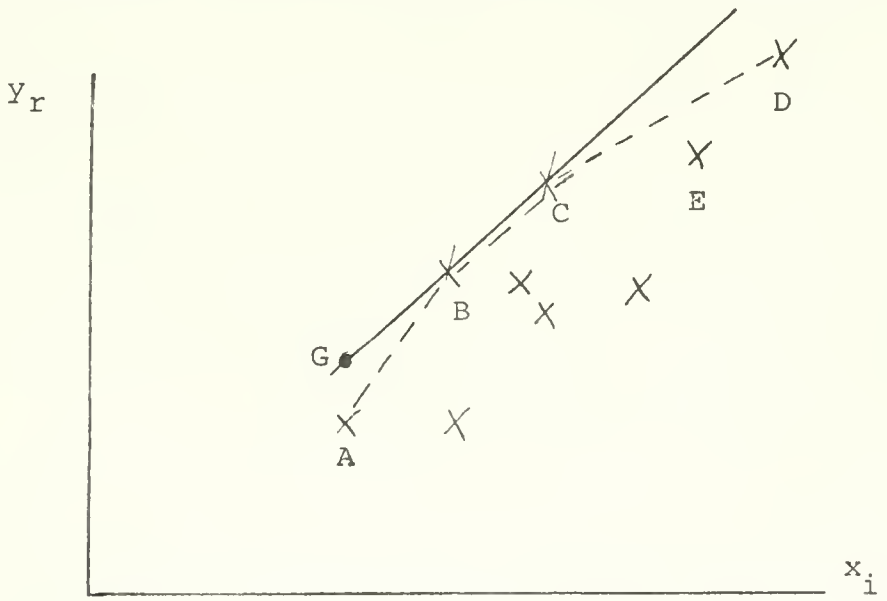

Figure 2

A One-Output/One-Input Cross Section

Thus in the situation portrayed in Figure 2, only the observations for B and C attain such a solid line position. The observations for A and D which lie on the production surface generated from a DEA analysis are thus incorrectly characterized as being efficient via that approach.(12) Furthermore, a relative efficiency measure for E will also be at an incorrect value when referred to the broken line segment between C and D rather than the portion of the solid line that lies above it.(13)

One may argue with some force that the "relative efficiency" measure yielded by DEA is about all that can be expected when knowledge of the true production function is not available. For example, the comparison of E with the piecewise linear segment interpolated between C and D is at least a comparison between E's performance and what C and D have shown to be better. It is, nevertheless, of interest to know how well DEA may perform relative to the theoretically attainable possibilities--both in correct classification and in correct measure--and this is what we propose to study in the sections that follow. In particular we shall assume that some DMU's attain levels of efficiency that are theoretically possible but others do not. Then we shall study
a) whether DEA can identify inefficient units correctly and
b) how well it can measure their inefficiency.

Model Details

For convenience of reference all details of the model and resulting data utilized are collected together in the Appendix to this article. The model and the input output relationship (and data used) to develop the model parameters are given in Exhibit 1.

These production relationships are assumed to hold for all volume levels of operations for all hospitals which, however, may use them efficiently or inefficiently. Input costs per unit are also fixed at the same amounts for all hospitals so that the resulting production activity can be converted into common dollar units. This is only done for simplicity of exposition since, of course, the DEA approach does not require such reductions into equivalent dollars.

The following two assumptions are also used (which would not exist in real data sets): One assumption is that all hospitals purchase similar inputs at the same price. The other assumption, as already noted, is that all hospitals are subject to the same "production function" which has constant returns to scale in all outputs. This provides the underlying structure which we shall henceforth refer to as the "structural model".

Via this "structural model" as represented in exhibit 1, data were developed for an assumed set of 15 hospitals based on arbitrary mixes of outputs. The related inputs required were derived from the model. The resulting data base which we shall henceforth use is shown in Exhibit 2. The first seven hospitals, H1 through H7, are efficient; i.e. the inputs and outputs are those required in the structural model. The data generated for the next eight hospitals, H8 through H15, were developed by altering the numerical values to portray various inefficiencies. The idea of course is to test the ability of DEA to identify such inefficiencies. The DEA efficiency measure would be accurate, at least as far as classification is concerned, if it isolated H8 through H15 as inefficient in this application.

The specific inefficiencies in H8 through H15 are designated by their bracketed {} values in Exhibit 2. That is, these bracketed values refer to the specifically inefficient elements in the production function. Exhibit 3 then presents an example of how the data for the efficient hospital, H1, and the inefficient hospital, H15, were calculated. H15 is designed to be inefficient in its use of inputs to treat regular patients and efficient in its use of inputs to treat severe patients and to provide training (teaching) outputs.

Certain relationships posited in the structural model are never known, such as the actual amount of staff time and supplies that are required to support each intern or nursing student at a hospital. We nevertheless explicity introduce these relationships in the simulation to determine if DEA can uncover them with the resulting input and output data. The model is simplified relative to actual hospitals insofar as it excludes inputs such as laboratory tests, surgical and intensive care units and, of course, the assumed average cost per patient is much lower than the real cost per patient. The data set is actually modeled as a prototype directed to studying the largest (and perhaps most important) single service center of a typical hospital, the medical-surgical area. It includes all the direct expenses in providing daily bedside care to medical-surgical patients but excludes ancillary services and special care such as psychiatry and obstetrics, since their inclusion might cause problems in terms of our assumption that all hospitals have the same inputs and outputs to be evaluated.

4. Application of the DEA Measure to the Hypothetical Hospital Data Set

The CCR models for securing relative efficiency measures were applied to the data set of 15 hospitals shown in Exhibit 3 by converting the fractional program to a linear form as illustrated in section 2, equations (1) and (3). In the linear program formulation, the constraint that the virtual rates of transformation be

positive was replaced by a constraint that v_i, $u_r \geq .001$ to facilitate use of the packaged programs for ordinary linear programs where positivity of these v_i and u_r values is not otherwise guaranteed.

Calculation of the relative efficiency rating was accomplished by rerunning the linear program (LP) once for each hospital with that unit's outputs in the objective function and that unit's inputs as one constraint set equal to 1 (see the previous section)[14]. Each LP run produced a set of "virtual rates of transformation", v_i and u_r described on pages 6 and 7, which at an optimum produced the requisite h_o^* value for this hospital in the objective function.[15]

Two sets of DEA efficiency evaluations were calculated, each with three inputs and three outputs. Version A, shown in Exhibit 2, used the data in a form that is most directly accessible in certain real hospital data sets. This version resulted in certain difficulties which are analyzed and reviewed en route to version B which uses the same data, but transforms two of the outputs into two new output measures, as shown in Exhibit 2. Version B is an improvement over version A and results in an accurate identification of the eight inefficient units.

The inputs are full time equivalents (FTE's), bed days days available, and supply dollars (Supply $'s). The outputs in version A are total patients treated, percentage of severe patients, and teaching units. Version B outputs are the same as Version A, but the percent of severe patients and total number of patients treated are used to calculate two different measures:

number of severe patients treated and number of regular patients treate For example, in Ex. 2 we see that H1 used 23.5 FTE's, 41,050 bed days, and $130,000 of supplies. H1's outputs in version A are 5,000 patients treated, 40% of the patients treated were severe, and 50 students trained in that year, e.g. interns. Percent severe is used as an output measure in that a higher percent represents greater input requirements and therefore reflects a patient severity index. In version B, the percent severe is used to calculate the actual number of regular and severe patients. Thus, H1 has outputs of 2,000 severe patients (40% x 5,000 patients), 3,000 regular patients, and 50 training units.

Model variables were selected to reflect the expected set of input resources that are used to produce the hospital outputs. Version A was justified as follows. The hospital produces patient care and teaching services. Patient care is provided with more complex and more resource consuming treatment for serious patients than regular (less serious) patients. The typical case mix variables reflecting severity of the illness are calculated and reported in terms of an index or percentage (%). A higher index or % reflects higher complexity of service and thus also reflects greater amounts of resources required, since these are supposed to be correlated.

Version A

The DEA efficiency rating and ranking for version A are reported in Table 1.

Recalling that the first seven hospitals are efficient (by design) we see that three efficient units (H5, H6 and H7) are misclassified as inefficient ($h_o^* < 1$). This misclassification is important for a potential DEA user who proceeds with available

Table 1

DEA Efficiency Ratings of the 15 Hospitals in the Constructed Data Base

{ } = Hospitals misclassified by DEA

Hospital	Version A* DEA Efficiency Rating	Version A** Efficiency Reference Set	Version B* DEA Efficiency Rating	Version B** Efficiency Reference Set
Efficient Units				
H1	1.0		1.0	
H2	1.0		1.0	
H3	1.0		1.0	
H4	1.0		1.0	
H5	{0.98}	H1, H2	1.0	
H6	{0.95}	H3, H4	1.0	
H7	{0.96}	H4	1.0	
Inefficient Units				
H8	0.99	H4	0.93	H1, H4, H7
H9	0.98	H2	0.98	H1, H2, H6
H10	0.80	H4	0.99	H4, H7
H11	0.83	H4	0.85	H4, H7
H12	0.97	H3, H4	0.99	H1, H4, H6
H13	0.88	H4	0.94	H3, H4
H14	0.97	H3, H4	0.99	H1, H4, H6
H15	0.87	H1	0.87	H4, H6, H7

** The efficiency reference set is determined from the DEA results and represents the set of efficient hospitals against which the related inefficient hospital is most directly compared and its degree of inefficiency is measured. Referring back to figure 1 on page 13, the efficiency reference set for point C includes points A and B.

* Versions A and B differ in the output measures used as follows.

Version A	Version B
Total Patients Treated	Severe Patients Treated
% Severe Patients	Regular Patients Treated
Teaching Units	Teaching Units

Table 2

Inputs/Outputs - Hospital 4 versus Hospital 7

Inputs	H7*	H4*	H7 ÷ H4
FTE's	51.5	25	2.06
Bed days	92,630	41,050	2.26
Supply $'s	$270,000	$140,000	1.93
Outputs in Version A			
Percent Severe	17%	40%	0.43
Total Patients Treated	12,000	5,000	2.4
Teaching Output Units	50	100	0.50
Decomposition of Percent Severe			
Severe Patients	2,000	2,000	1
Regular Patients	10,000	3,000	3.33

* from Exhibit 2.

data via the linear programming model--as will almost always be the case in practical applications--and so we amplify this result as follows:

Analysis of the DEA results for H7 shows that the inefficiency was based on comparison of H7 with H4 (See Table 1). Efficient units such as H4 against which inefficient units are most directly measured will hereafter be referred to as the efficiency reference set (ERS). Hence we examine the input and output data for these two hospitals by reference to Exhibit 2. Using only version A we obtain the input and output values shown in Table 2 under the columns headed H4 and H7, respectively. The last column of Table 2 indicated the ratios between these input and output values and provides us with a clue to the possible source of the misclassification of H7. The inputs to H7 are roughly twice those of H4 while the outputs for % Severe and Teaching Units are only half those of H4. The total patients treated figure, however, yields an H7 ÷ H4 ratio which is more closely in line with the input ratios. Following up by further decomposing % Severe into the additional categories of Severe and Regular patients noted at the bottom of Table 2 we observe wide discrepancies (all ≥1) in the resulting output ratios that DEA was prevented from considering in an explicit manner by virtue of the prior reduction to %'s.

Because h_o^* values are known to be invariant to changes in scale in a DEA analysis,[16] it is important to be clear on what is happening. This invariance applies to changes in the scale obtained by multiplying any input or any output by any positive constant. The same constant is, of course, applied to this input or output in all of the DMU's being considered. In our use of %

Severe, however, differences in scale invariance property does not apply in this case just as it would not necessarily apply to any other scale alteration that applies different constants to the same input or output in different DMU's.

Evidently a use of percentages should, in general, be avoided in a DEA analysis. Fortunately this can be done without causing troubles even when this results in increasing the number of variables, since the linear programming models can readily accomodate large numbers of inputs, outputs and DMU's.[17] The resulting values can then be used to derive any percentages or indices that may be wanted for ancillary purposes such as report summaries, etc., although the reverse proposition is, of course, not generally true.

Version B

Replacing % Severe and number of Patient Days of service with the absolute number of both Severe Patient and Regular Patients but otherwise continuing with the same inputs and outputs as in Version A[18] we arrive at a new Version B which is also shown in Exhibit 2.

Applying these new inputs and outputs we arrive at the results listed under Version B in Table 1. Evidently H1 through H7 are now all correctly classified as efficient.

The Version B DEA analysis has correctly identified the inefficient and efficient units. Once such results are obtained and are known to be reliable, they may be used in selecting the inefficient units for further study to determine if this inefficiency is justifiable and therefore does not warrant management action, or if it is due to controllable elements which may be

managed to improve efficiency in these units.[19] The inefficient units that are selected for further study might be chosen based in part on further interpretation of the degree of inefficiency which is the focus of the next section. At this point, however, a few observations and caveats are in order.

Sensitivity to Input-Output Model Specifications

The inputs and outputs included in Version B closely reflect the true production relationships, i.e., Supplies, FTE's and Bed days used are dependent on the number of regular and severe patients and the number of training units. The true production relationships are rarely known. As a result, there are alternate input-output specifications which might have been proposed as a basis for efficiency evaluation and which could lead to somewhat less reliable results than were achieved in Version B. For example, the production inputs such as supplies and FTE's may be more dependent on the number of patient days than on the number of patients. In specifying the output measure, the number of severe and regular patient days might have been used instead of number of severe and regular patients, since the true underlying production relationship is not known. If patient days are used as the output measure, efficiency will be measured based on resources consumed per day rather than per patient. It is not possible to generalize as to the effect of such alternate measures; however, the DEA evaluation can be run using alternate sets of measures to understand how alternate input and output specifications affect the results.

When alternate input and output specifications are used for DEA sensitivity analysis, the interpretation of the efficiency

measure will always be dependent on the specific set of inputs and outputs used. For example, if the patient days measure is used instead of number of patients, an inefficient hospital such as H8 will be rated as efficient. This is because H8 was designed to be inefficient with respect to the number of days per patient required, i.e., it requires two extra days per regular patient and one extra day per severe patient (see Exhibit 3). If the preferred or selected output measure is patient days of treatment, H8 may be correctly rated as efficient. This occurs because the number of patients treated within the number of days of treatment provided to a patient is not being measured with this input-output specification. Such an approach implicitly suggests that the number of days per patient is not of concern and that what is to be evaluated is the use of inputs to produce severe patient care days, regular patient care days, and teaching outputs.

Additional Interpretation.

Assuming we have agreed on the input-output specifications in Version B, let us now consider how the efficiency ratings can be further interpreted. We should perhaps underscore the fact that the h_o^* values recorded under the DEA columns in Table 1 are obtained by reference to different sets of optimal basis vectors (efficiency reference sets). Having been obtained from different efficiency reference sets the resulting h_o^* values cannot safely be used to obtain an ordering.[20] For example, we can assert that H10 with efficiency rating of .99 is more efficient than H11 with efficiency rating .85 (see table 1) because both hospitals have the same efficiency reference set comprised of H4 and H7. We cannot, however, assert that H10 is more efficient than H8 whichhas a .93 rating because H8 is being measured against a different reference set, H1, H4 and H7 than was used for H10. The DEA results have

other possible uses as resource conservation or output augmentation measures which we shall now try to elucidate.

The data in Table 3 will help provide us with easy access to this topic by reference to artificially contrived data for two additional hospitals, H16 and H17 which are to be compared with the initial 15 units as well as with each other using DEA. These were arbitrarily developed using H3 as a model of an efficient hospital. It can be immediately observed that, other things being equal,

a. H16 is less efficient than H3 because it uses $50,000 more supply inputs ($200,000 vs. $150,000) for the same quantity of other inputs and outputs; and

b. H17 is less efficient than H3 because it produces less of each output while using the same quantity of each input as H3.

Based on the expanded set of the original 15 hospitals plus these additional two hospitals, the DEA efficiency rating was calculated for H16 ($h_o^* = 0.95$) and H17 ($h_o^* = 0.84$). Using these examples where the degree of inefficiencies are clear, we will consider how the DEA output can suggest the magnitude of these inefficiencies.

Assuming we have agreed on the input-output specifications in Version B, let us now consider how the efficiency ratings can be further interpreted.

Table 3

Development of 2 Additional Hospitals, H16 and H17, Modeled After Efficient Hospital H3 to Illustrate the Interpretation of the DEA Results.

	H3 (from Ex. 3)	H16	H17
Inputs			
FTE's	26	26	26
Bed days	43,160	43,160	43,160
Supply $'s	150,000	200,000	150,000
Outputs			
Teaching outputs	100	100	80
Severe patients treated	3000	3000	2600
Regular patients treated	2000	2000	1600
DEA Efficiency Rating (based on the expanded set of 17 hospitals)	1.0	0.95	0.84
Virtual Rates of Transformation			
v_1^* - related to FTE's		.001	.001
v_2^* - related to Bed days		.0179	.01909
v_3^* - related to Supply $'s		.001	.001
u_1^* - related to Teaching output		.0075	.001
u_2^* - related to Severe Patients Treated		.0059	.0216
u_3^* - related to Regular Patients Treated		.001	.0126
Shadow price for linear program constraints with zero slack			
Denominator = 1 constraint		1.0	0.844
H3		1.0	.70
H6		-	.10
v_1		-	-.141
v_3		-50	-.584

We have from the DEA methodology that

$$h^*_o = \frac{\sum_{r=1}^{s} u^*_r y_{ro}}{\sum_{i=1}^{m} v^*_i x_{io}} \tag{8}$$

by reference to equation (3) for rating the efficiency of any DMU. Because existence is guaranteed with $0 < h^*_o \leq 1$ we can also represent this as

$$1 = \frac{\sum_{r=1}^{s} u^*_r y_{ro}}{\sum_{i=1}^{m} v^*_i (x_{io} h^*_o)} = \frac{\sum_{r=1}^{s} u^*_r (y_{ro}/h^*_o)}{\sum_{i=1}^{m} v^*_i x_{io}} \tag{9}$$

so that one may use an input adjustment approach (the middle term) or an output adjustment (the right-most term) to obtain the requisite adjustments for efficiency by the DMU being rated. Evidently $h^*_o = 1$ would result in no adjustment of the observed values while $h^*_o < 1$ would yield $\hat{x}_i = x_{io} h^*_o < x_{io}$ for the reductions from the observed values when the input adjustment is used while $\hat{y}_{ro} = y_{ro}/h^*_o > y_{ro}$ would yield the output augmentation estimates when the output adjustment estimates are used.

This approach and the slack adjustment approaches to determining the adjustments required to make an inefficient unit efficient will be illustrated using the above examples of H16 and H17 after which the analysis will be extended to the original set of hospitals. From (8) above, the meaning of $h^*_o = .95$ for H16 can be illustrated as follows.

$$h_o^* = \frac{\underset{u_1}{(.0075)}\ \underset{\text{Teaching}}{(.100)} + \underset{u_2}{(.0059)}\ \underset{\substack{\text{Severe}\\\text{Patients}}}{(30)} + \underset{u_3}{(.001)}\ \underset{\substack{\text{Regular}\\\text{Patients}}}{(20)}}{\underset{v_1}{(.001)}\ \underset{\text{FTE's}}{(26)} + \underset{v_2}{(.0179)}\ \underset{\substack{\text{Bed}\\\text{days}}}{(43.16)} + \underset{v_3}{(.001)}\ \underset{\text{Supplies}}{(200)}} = .947 \quad (10a)$$

From (9) above, we know that all the inputs can be reduced by a factor of $h_o^* \approx .95$ or all outputs can be increased by a factor of $1/h_o^* \approx 1/.95 \approx 1.05$ to make H16 efficient as follows:

$$1 \approx \frac{(.0075)(100\text{x}1.05) + (.0059)(30\text{x}1.05) + (.001)(20\text{x}1.05)}{\text{same denominator as (10a)}} \quad (10b)$$

$$\approx \frac{\text{same numerator as (10a)}}{(.001)(26\underline{\text{x}.95}) + (.0179)(43.\underline{16\text{x}.}95) + (.001)(200\text{x}.95)}$$

$$\qquad\qquad 24.70 \qquad\qquad\qquad 41 \qquad\qquad\qquad 190$$

Thus one set of possibilities to make H16 efficient is to increase the outputs or decrease the inputs to the adjusted levels in (10b). The linear program boundaries beyond which the values of u_r and v_i change as the x's and y's change would be indicated by a change in the optimal basis and these limits would have to be considered if this route were chosen .

Another more direct approach is suggested by the shadow price of the linear program constraints. Each hospital with a non-zero shadow price is included in the efficiency reference set of hospitals against which the objective hospital efficiency rating was measured. For H16 the efficiency reference set is H3. Thus, H16 was compared to H3 to calculate h_o^* and H16 should be compared to H3 above to determine the amount and location of

the inefficiency. This direct comparison earlier indicated that H16 needs to decrease supplies by $50,000 to become as efficient as H3. This intuitive result which is easily derived in this simple example can be more directly achieved in this and more complex cases from interpretation of the shadow prices of the virtual rates of transformation (u_r, v_i). This will be illustrated at a later point. It is sufficient to note at this point that the shadow price of v_3 related to supplies is -50 which corresponds to the requisite reduction of supplies by $50,000 to make H16 efficient.

H17 has an efficiency rating of $h_o^* = .84$. As in the above example, H17 can become efficient by increasing each output by a factor of $1/h_o^* = 1.19$ or decreasing its inputs by a factor of $h_o^* = 0.84$. The dual variables in table 3 indicate that the efficiency reference set includes both H3 and H6 (rather than H3 alone). Thus, although H17 was based on H3, DEA searches for reference points which will give H17 the highest efficiency rating, and in this case it is based on a linear combination of H3 and H6 with the respective weights of .70 and .10 corresponding to their dual variables. A composite of H3 and H6 can be constructed which is more efficient than H17 as follows.

(H3 Dual Variable)	(H3 Input-Output Vector*)		(H6 Dual Variable)	(H6 Input-Output Vector*)		Composit of H3 and H16
	26			36		21.8
	43.16			62.11		36.42
(.7)	150	+	(.1)	210	=	126
	100			100		80
	30			50		26
	20			20		16

* See Exhibit 3.

	(A) Composite of H3 and H6 Input-Output Vector	vs	(B) H17 Input-Output Vector	(C) Difference (B)-(A)	
Inputs					
FTE's	21.8	<	26	4.2	excess inputs
Bed Days	36,420	<	43,160	6,740	excess inputs
Supply $'s	$126,000	<	$150,000	$24,000	excess inputs
Outputs					
Teaching	80	=	80	0	
Severe Patients	2600	=	2600	0	
Regular Patients	1600	=	1600	0	

H17 produces the same outputs as a composite of two efficient units, (H3 and H6), while H17 uses more inputs than the composite to achieve that level of outputs. Thus, H17 can become efficient by decreasing its inputs by the amounts in column C above. (This adjustment can be directly calculated by use of the other dual variables in Table 3. Each input corresponding to the denominator constraint in (3) which is $\sum_{i=1}^{m} \omega_i x_{io} = 1$ is adjusted by the denominator shadow price of .844 and further adjusted by that input's related VRT shadow price. For example, H17's actual FTE's are adjusted to arrive at the composite efficient FTE level as follows: (.844)(26) - .141 = 21.80. Each of these terms is included in Table 3.

One final observation which should be noted is that the composite solution is not the same as the input-output vector of H3. While H17 was based on H3 to create an obviously inefficient unit, the degree of inefficiency and adjustment required to become efficient is based on the efficiency reference set which may include only one efficient unit, as is the case of H16, or which may include a number of efficient units. The efficiency reference set might not have included H3 in the evaluation of H17 even though we developed H17 based on H3. DEA selects the efficient reference set to give H17 the highest possible efficiency rating and will select from any of the efficient hospitals to accomplish this.

We will now consider H11 to complete the interpretation of the DEA efficiency rating and provide a more complete indication of management implications. H11 was correctly located as inefficient by DEA. From Exhibit 1 we also know that the source of the inefficiency in H11 is excess supplies and an excess number of FTE's for the realized outputs. The excess supplies amounted to $65,000 and the excess FTE's amounted to 8 units. From earlier discussion, there are three clear ways for H11 to improve its efficiency rating of $h_o^* = .85$ to the level of 1.0: 1) It can reduce its inputs to 85% of current levels, holding outputs constant; 2) It can increase its outputs to 118% (1 ÷ .85) of the current level, holding inputs constant; or 3) It can adjust its inputs and outputs. This is illustrated in Table 4, along with the DEA linear program output for H11 to be used in the following analysis.

Table 4 - Hospital 11 DEA Results

h_o^* = efficiency rating = .853
Hospital 11

Input Output Vector		(A) Objective Hospital H11	(B) Efficiency Reference Set H4	(C) H7	(D) Composite of Efficient Reference Set (1.43)(B) + (.071)(C)	(E) H11 Compared with Composite (D) - (A)	
Inputs							
x_1	FTE's	44.5	25	51.5	39.4	5.1	excess inputs
x_2	Bed days	65,260	41,050	92,630	65,260	0	excess inputs
x_3	Supply $'s	$265,000	$140,000	$270,000	$219,290	$45,710	excess inputs
Outputs							
y_1	Teaching	50	100	50	146.4	96.4	fewer outputs
y_2	Severe Patients	3000	2000	2000	30.00	-	fewer outputs
y_3	Regular Patients	5000	3000	10,000	5000	-	fewer outputs

Virtual Rates of Transformation for H11

v_1 = .001 (related to x_1)
v_2 = .0106 (related to x_2)
v_3 = .001 (related to x_3)
u_1 = .001 (related to y_1)
u_2 = .0088 (related to y_2)
u_3 = .0107 (related to y_3)

Dual Variable Related To The LP Constraints

H4 = 1.43
H7 = 0.071
v_1 = -5.11
v_3 = -45.71
u_1 = -96.4
Denom. = 1.0

This third possibility from Table 4, column E, suggests that to become efficient, i.e., to increase h_o^* to 1.0, H11 must decrease FTE's by 5.1 units, decrease supplies by $45,000, and increase teaching outputs by 96.4 units. This adjustment is calculated by comparing the composite inputs and outputs based on a weighted sum of the efficient reference units. These weights correspond to the dual variables assigned by the linear program to each of these units H7 and H4. The solution suggested in Table 4 to make H11 efficient has the following effect:

			VRT		
decrease FTE's	5.1	x	.001	=	.005
decrease supplies	45.71	x	.001	=	.046
increase teaching	95.4	x	.001	=	.095
	total increase in h_o^*				.147
efficiency of H11 before input-output adjustment =				h_o^* =	.852
					.999 ≈ 1.0

This result can be derived from non-zero dual variables of the virtual rates of transformation. Note in Table 4 that the dual variables for v_1, v_3, u_1 correspond directly to the adjustments in column (E) because the denominator constraint dual variable = 1.0.

Other alternatives exist for H11 to improve its efficiency rating. In this instance, a decrease of one FTE will increase the efficiency rating h_o^* by .001 ($=v_1$), a decrease of bed days by 1,000 will increase h_o^* by .0106, and a decrease of $1,000 of supplies will increase h_o^* by .001. Similarly, increases in an output unit will have an effect on the efficiency rating corresponding to the u_i values in Table 4. For H11, an increase of 1,000 bed days corresponds to a decrease of 10.58 FTE's or $10,580 of supplies.

Management of H11 may determine that the changes to increase H11's efficiency noted in Col. E of Table 4 are not all possible; e.g., it may be impossible to increase teaching by 95.4 units of output. Management's objective would be to select some set of implementable changes in inputs and outputs to increase efficiency by .148 which would result in an efficiency rating of 1.0.

Recall that the actual adjustment to make H11 efficient based on the underlying production function was to reduce supplies by $65,000 and FTE's by 8 units. The virtual rates of transformations defined by H4 and H7 reflect an increase of efficiency of .073 (= 65 x .001 + 8 x .001). Thus, this adjustment would appear to increase h_o^* to (.853) + (.073) = .926 which is less than the efficient evaluation expected. As the inputs and outputs are adjusted, H11 moves toward another section of the frontier which has a different efficiency reference set and different virtual rates of transformation. Such an adjustment, which we know is sufficient to make H11 efficient, is not apparent except to the extent that the LP indicates these ranges over which the VRT's are fixed. Once an adjustment goes beyond this range, DEA must be rerun to determine the new efficiency reference set.

DEA applied to this simulated data base is found to 1) accurately locate relatively inefficient units when inputs and outputs are correctly specified, 2) indicate the magnitude of inefficiency by reference to a specific efficiency reference set of units against which the objective unit's inefficiency is being measured, and 3) indicate alternative sets of adjustments to inputs and outputs to increase the efficiency of an inefficient unit to 1.0.

5. Alternative Methods to DEA for Identifying Inefficient Units

DEA has been shown to be one useful approach to locating relatively inefficient members of a group of units. Other techniques have been, and are, used to accomplish similar objectives to DEA. These techniques are generally single output oriented in contrast to DEA which is designed to incorporate multiple outputs.

Typical single output efficiency indicators for hospitals include average cost per patient day, average cost per patient, FTE's or nurses per patient day, average length of stay, supplies per patient, etc. A few of these indicators are presented in Table 5 for the same group of 15 hospitals for which DEA accurately identified the inefficient units. The first measure, average cost per patient, is portrayed under column A along with a parenthesized number which represents rank in an average cost array. Evidently, H6 would be misclassified as inefficient and so might H3 which is tied in rank with H9. Conversely, H13, known to be inefficient, would have rank 6 and hence would be mislabeled as efficient.

In real applications, the patient output is acknowledged to require some adjustment for case mix. In this data set, the relative cost of severe versus regular patients is known. Hence, using the data of Exhibit 1 we can weight the severe patient units by the ratio of severe to regular patient cost--\$170 ÷ \$130=1.3. For example, H1 would have adjusted patients of 3,000 regular + 2,000 x 1.3 severe patients for an adjusted total of 5,600 patients. This in turn would result in an adjusted average cost per patient of \$138.48 versus the unadjusted cost of \$155.10 per patient.

Table 5

Single Output Measures

Hospital	Average Cost per Patient (A)	Case Mix Adjusted Average Cost per Patient (B)	Case Mix Adjusted Average Cost per Patient Segregated into High and Low Levels of Teaching Outputs	
			Low* (C)	High* (D)
Efficient Units				
H1	$155.10 (2)	$138.48 (4)	$138.48 (4)	
H2	163.32 (5)	138.40 (3)	138.40 (3)	
H3	168.32 (7)	142.65 (8)		$142.65 (3)
H4	160.10 (4)	142.94 (9)	142.94 (2)	
H5	158.38 (3)	137.73 (2)		137.73 (2)
H6	170.15 (9)	140.12 (5)	140.12 (1)	
H7	142.60 (1)	135.81 (1)		135.81 (1)
Inefficient Units				
H8	176.95 (11)	157.99 (12)		157.99 (6)
H9	168.32 (7)	142.64 (7)	142.64 (6)	
H10	169.69 (8)	161.61 (14)		161.61 (7)
H11	170.33 (10)	153.10 (10)	153.10 (7)	
H12	178.33 (12)	155.07 (11)		155.07 (5)
H13	165.68 (6)	142.00 (6)	142.00 (5)	
H14	178.33 (12)	155.07 (11)		155.07 (4)
H15	179.74 (13)	160.48 (13)	160.48 (8)	
Mean	167.02	146.94	144.77	149.42
Standard Deviation		8.82	7.36	9.66

*Low teaching outputs were 50 units and high teaching outputs were 100 units as per Exhibit 3, Col. 6.

Continuing in this manner, the other adjusted costs and new ranking listed in column B of Table 5 are obtained. Even with these adjustments, however, trouble is still experienced, since hospitals H3 and H4, are now in the group of eight highest cost per patient hospitals, while Hl3 continues to be characterized as relatively efficient. This is more clearly illustrated in Table 6 where hospitals are listed from lowest cost (top) to highest cost (bottom) for unadjusted and adjusted cost per patient. Teaching output levels have not been adjusted for in this case mix adjusted data which explains the misranking in Table 5 column B. Note that the efficient units H3 and H4 ranked below inefficient units H9 and hl3 are the ones with higher teaching outputs (see Exhibit 3). This occurs because teaching costs have been allocated to the adjusted cost per patient day. Columns C and D segregate the case mix adjusted cost per patient between hospitals with high and low teaching outputs which result in a correct ranking for each set of hospitals. This final adjustment is often considered unnecessary in real application. Hospitals are often segregated into teaching and non-teaching groups for which intragroup cost comparisons are made as in column B, Table 5, which result in the misranking of certain units.

A further problem arising from this approach is determining the cost level that segregates efficient from inefficient hospitals. A simple rule - units above the mean cost per patient are inefficient - in the case mix adjusted data in Table 5 column B would exclude two of the eight inefficient units as candidates for study to improve efficiency. Another arbitrary rule (which has been adopted in

TABLE 6

Single Output Measure Rankings

	Average Cost per Day		Case Mix Adjusted Average Cost per Day	
	Efficient	Inefficient	Efficient	Inefficient
Lowest Cost = Most Efficient	H7		H7	
	H1		H5	
	H5		H2	
	H4		H1	
	H2		H6	
		H13		H13
	H3			H9
		H10	H3	
	H6		H4	
		H11		H11
Highest Cost = Least Efficient		H8		H12 , H14
		H12, H14		H8*
		H15		H15*
				H10*

*Hospitals more than one standard deviation over average cost

certain regulatory settings) is to examine all units more than one standard deviation above the mean cost per patient. This would exclude five of the eight inefficient units. In any case, some arbitrary rule would be required and there is no assurance that the set of units selected for remedial study will be the relatively inefficient units.

A final observation is that the levels of cost per patient between hospitals after correction for case mix and teaching level still does not provide any insights into the magnitude of the inefficiency as was available with DEA.

6. DEA Strengths and Limitations Contrasted with Single Output In put Measures

a. Robustness in Identifying Inefficient Units

DEA accurately and objectively located units which were technically inefficient with respect to their inputs and outputs compared to other units in the study This result occured when simulation inputs and outputs were properly specified as physical input and output units. The relatively efficient units identified by DEA are not, nor are they meant to be, the absolute efficient units, but rather they form efficiency reference sets against which other units' inefficiencies can be objectively measured.

The true production function in actual application is rarely known and unlike the example presented here, the function is not necessarily the same across all units. DEA seeks out relatively efficient units on the basis of repeated application of an extremal principle and mode without regard to a single production function, and

results in a relatively efficient production frontier which is piecewise linear reflecting different rates of substitution of inputs within each segment. Hence, the underlying production function or functions need not be known to use DEA and it therefore can be applied without specific knowledge of the efficient input output relationships. A further distinctive strength of DEA is the quantification of the magnitude of inefficiency and the ability to determine sets of adjustments required to improve efficiency to the level of the relatively efficient units in the set. Further study with real hospital data and hospital managers is required to understand the full extent to which this tool can be utilized by managers in seeking sources of inefficiency and the magnitude of inefficiency.

Another area requiring further research and clarification in applying DEA to real data is the sensitivity to the number of observations and the number of inputs and outputs. For example, one recently implemented hospital case mix taxonomy includes 383 diagnostic related groups (DRG's) each of which represents a different type of care requiring differing amounts of hospital resources [18][19]. If each DRG were treated as one output for DEA applications, there may be few or no units isolated as relatively inefficient when the sample of hospitals is small.

Single output measures (in contrast to DEA) require that an arbitrary or subjective cutoff be designated as the point of relative inefficiency, e.g., units with average cost over one standard deviation beyond the mean might be designated as the relatively inefficient units for review. Even when this cut-off point can be established in some objective manner, other problems associated

with this approach severely limit its reliability. As illustrated, the inability to evaluate multiple outputs simultaneously may result in the inclusion of technically efficient units in the inefficient set. Specifically, this will occur where there is no accurate set of weights which can be applied to combine all outputs into a single adjusted output measure. Currently, no such set of weights is available to combine patient outputs into a weighted patient output.[21] Similarly, there is no set of weights to combine patient care, teaching, and research outputs of hospitals. In addition, the single output measure is input oriented and therefore does not consider the possibility of output adjustments as a means to improve efficiency.

The weights that were assigned to each of the multiple outputs to obtain a single output efficiency measure result in a weighted output measure for which it is difficult to interpret the meaning of differences between various units' output levels. In contrast, DEA can utilize absolute units of each output which can be directly interpreted.

A similar but more basic issue arises with respect to the need to collapse all the inputs into a single measure such as dollars. In Table 3, where costs per patient were listed, it was assumed that all hospitals paid the same price for each input. (In DEA, no such assumption is necessary, as the physical quantities of each input can be used.) Real hospital input data measured in actual dollars will reflect different costs incurred for similar inputs. This cost variation which relates to price efficiency could further confound the cost per patient ranking considered in Table 5. Such a ranking would reflect a combination of technical, price and allocative efficiency and would therefore not be beneficial in

isolating technically inefficient units where there is potential for improved utilization of resources. For example, a unit which has a relatively low cost per patient may be highly efficient in its ability to purchase inputs at a low cost. This unit may still be technically inefficient suggesting that fewer of these inputs may be needed to produce the same outputs, suggesting that costs might be further reduced.

One problem which may affect both DEA and single output analysis is that data may be available only in the form of indices or percentages (such as an index of severity or quality). In such cases, the only alternative available will be to weight the related input or output and use this adjusted value rather than the raw physical input or output in the analysis. The results in such cases will be only as reliable as the indices or weights applied. In addition, the interpretation of the results will be more complex.

DEA appears to be most useful at evaluating the technical efficiency as distinguished from issues of price and allocative efficiency. For this purpose, DEA appears to be more appropriate than the widely used single output measure approach. The results of the DEA analysis would nevertheless be tempered by price and allocative efficiency considerations. For example, DEA will suggest alternative routes toward improving efficiency, i.e., alternative sets of adjustments of inputs and outputs. The routes which are chosen by management would naturally include consideration of the cost associated with these alternatives. Along these lines, DEA may be a useful analytic tool which management of inefficient units can use to evaluate the impact of alternative sets of input and output adjustments on that units relative technical efficiency.

In addition, where a unit is inefficient because of certain unmanageable characteristics such as the size of the plant or the demand level for its services, DEA can be reapplied to evaluate the efficiency with respect to manageable elements. For example, if the plant is known to be excessive with no ability to reduce the size, a revised DEA evaluation can be completed using the ideal reduced plant input level to determine if the unit is still relatively inefficient when this uncontrollable element is assumed to be corrected in the analysis.

b. Constant Returns to Scale

The version of DEA applied herein assumes constant returns and therefore does not correct for units which might be inefficient solely due to their size. This problem can be addressed by comparing only units of similar output levels; however, caution is needed here as a hospital may experience increasing returns with respect to certain outputs and decreasing returns with respect to other outputs. Single output measures are subject to similar limitations. This issue is often addressed with respect to single output measures by controlling for size and by ignoring the returns to scale with respect to the other outputs. An expanded version of DEA which incorporates economies of scale is under development (see [10]).

c. Potential Application of DEA

DEA appears to be a promising method of evaluating relative technical efficiency of nonprofit organizations with multiple inputs and outputs, and where there are no market prices to objectively collapse these inputs and outputs into common units. This approach provides a potentially reliable basis for evaluating the relative efficiency of a set of similar units and specifically to select less efficient units for audit or management review. It provides both a method of identifying relatively inefficient units, and information about the magnitude of inefficiency which provides a basis for possible management action to improve efficiency. Thus, DEA appears to be appropriate for the hospital manager that wishes to compare one hospital's operations with other comparable hospitals much like one might use profit or return on investment to compare firms in an industry. In addition, with further field research this tool may indicate that DEA is an alternative basis for establishing health care reimbursement rates which determine the amount paid to a hospital for each service rendered. Current reimbursement systems primarily reimburse hospitals based on their actual or projected costs. Using DEA, reimbursement rates may be established based on the input-output relationships of the relatively efficient units, thereby filtering out inefficient units from the rate setting base.

This paper has illustrated some of the potential benefits and limitations of a new efficiency measurement technique--Data Envelopment Analysis. DEA clearly presents new possibilities for efficiency evaluation, and particularly in the nonprofit sector. Broader understanding of the feasibility of potential applications will require going beyond the data simulation phase to

real health care data applications of DEA. Of greatest interest in field evaluation would be the practical limitations imposed by real data, as well as operational, political and legal constraints that may be encountered among potential health care users of DEA.

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Exhibit 1

Structural Model (Efficient Hospital Operations): Efficient Input-Output and Cost Relationships Assumed in the Hospital Production Model to Create the Artificial Data Base in Exhibit 3.

	Full Time Equivalents of Labor (FTE's)	Bed days	Supply $'s	Efficient Cost of Outputs
	Amount of Input Required to Efficiently Produce One Unit of Output			
Outputs				
Regular Patients	.004/patient	7	$20/patient	$130/regular pat.(1)
Severe Patients	.005/patient	9	$30/patient	$170/severe pat.(2)
Training Outputs	.03/training unit/yr.	-	$200/training unit/yr.	$500/training unit /yr.(3)
	Cost of Inputs			
	$10,000/FTE/yr.	$10/bed day	(measured in $'s)	

Other Assumptions:

a) Vacancy Rate - Efficient hospitals will have 5% of total beds vacant during the year (available for emergencies).

b) There are no regional cost differences for bed days, supplies, and FTE's, and that the mix of FTE's and supplies are similar between hospitals.

c) Cost of unused bed days = $10/bed day.

(1) Cost/regular patient = (.004 FTE/patient)($10,000/FTE) + (7 bed days/patient) ($10/bed day) + ($20 supplies/patient) = $130.

(2) Cost/severe patient = (.005 FTE/patient)($10,000/FTE) + (9 bed days/patient) ($10/bed day) + ($30 supplies/patient) = $170.

(3) Cost/training unit = (.03 FTE/training unit)($10,000/FTE) + ($200 supplies/training unit) = $500.

Exhibit 2

Constructed Data Base

{ } = Inefficient use of inputs compared to a structural model of efficient input-output relationship described in Ex. 1.

	Inputs (Version A and B)			Outputs (Version A)			Outputs (Version B)		Actual Inputs Used							
									FTE		Bed days		Supply $		Training	
	FTE	Bed Days	Supply $'s	Total* Pat.s	% Sev.** Pat.s	Teach. Units	Reg. Pat.s	Sev. Pat.s	Reg. Pat.s	Sev. Pat.s	Reg. Pat.s	Sev. Pat.s	Reg. Pat.s	Sev. Pat.s	Supply $'s	FTE	Vacancy Rate
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
H1	23.5	41050	$130,000	5000	40	50	3000	2000	.004	.005	7	9	20	30	200	.03	.05
H2	24.5	43160	140,000	5000	60	50	2000	3000	.004	.005	7	9	20	30	200	.03	.05
H3	26.0	43160	150,000	5000	60	100	2000	3000	.004	.005	7	9	20	30	200	.03	.05
H4	25.0	41050	140,000	5000	40	100	3000	2000	.004	.005	7	9	20	30	200	.03	.05
H5	28.5	50530	160,000	6000	50	50	3000	3000	.004	.005	7	9	20	30	200	.03	.05
H6	36.0	62105	210,000	7000	71	100	2000	5000	.004	.005	7	9	20	30	200	.03	.05
H7	51.5	92630	270,000	12000	17	50	10000	2000	.004	.005	7	9	20	30	200	.03	.05
H8	25.0	49475	140,000	5000	14	100	3000	2000	.004	.005	{9}	{10}	20	30	200	.03	.05
H9	24.5	43160	165,000	5000	60	50	2000	3000	.004	.005	7	9	{25}	{35}	200	.03	.05
H10	77.0	92630	340,000	12000	17	100	10000	2000	{.006}	{.007}	7	9	{25}	{35}	200	.03	.05
H11	44.5	65260	265,000	8000	38	50	5000	3000	{.005}	{.006}	7	9	{30}	{35}	200	.03	.05
H12	30.0	60000	170,000	6000	50	100	3000	3000	.004	.005	7	9	20	30	200	.03	{.20}
H13	43.5	81110	245,000	9000	56	50	4000	5000	.004	.005	7	9	20	30	{300}	{.05}	{.10}
H14	30.0	60000	170,000	6000	50	100	3000	3000	.004	.005	{9}	9	20	30	200	.03	{.10}
H15	26.5	47370	160,000	5000	40	50	2000	3000	{.005}	.005	{9}	9	{30}	30	200	.03	.05

* Total Patients = Col. 7 + Col. 8

** % Severe = Col. 8 ÷ Col. 4

Note: Outputs (Col.s 4 - 8) and input usage (Col.s 9 - 17) are arbitrarily assigned so that H1 - H7 are efficient based on the production model in Ex. 1 and H8 - H15 are inefficient based on the same model. Inputs (Col. 1, 2, 3) are derived from Col.s 4 - 17 as indicated in Ex. 2.

Exhibit 3

Example of Construction of Data Base for Hospitals H1 (efficient) and H15 (inefficient)

	H1 Efficient	H15 Inefficient	Difference Between H1 and H15	Inputs Required for Each Unit of the Related Output: FTE H1	FTE H15	Bed days H1	Bed days H15	Supply $'s H1	Supply $'s H15	Vacancy Rate H1	Vacancy Rate H15
Outputs											
Regular patients	3000	3000	-	.004	≠ {.005}	7	≠ {9}	$20	≠ {$30}		
Severe patients	2000	2000	-	.005	= .005	9	= 9	30	= 30	5%	5%
Teaching units	50	50	-	.03	= .03	-	-	200	= 200		
(outputs are identical for H1 and H15)											
Total Inputs Required											
FTE	23.5(1)	26.5(2)	3								
Bed days	41,050(3)	47,370(4)	6,320								
Supplies	$130,000(5)	$160,000(6)	$30,000								
Total Cost	$775,500(7)	$898,700(8)	$123,200								

(1) H1-FTE's = (3000 Reg. Pat.)(.004) + (2000 Sev. Pat.)(.005) + (50 Teach. Units)(.03) = 23.5

(2) H15-FTE's = (3000 Reg. Pat.)(.005) + (2000 Sev. Pat.)(.005) + (50 Teach. Units)(.03) = 26.5

(3) H1-Bed days = [(3000 Reg. Pat.)(7) + (2000 Sev. Pat.)(9)] ÷ (.95 Vacancy factor) = 39,000 ÷ .95 = 41,050

(4) H15-Bed days = [(3000 Reg. Pat.)(9) + (2000 Sev. Pat.)(9)] ÷ (.95 Vacancy factor) = 45,000 ÷ .95 = 47,368

(5) H1-Supply $'s = (3000 Reg. Pat.)(20) + (2000 Sev. Pat.)(30) + (50 Teach. Units)(200) = $130,000

(6) H15-Supply $'s = (3000 Reg. Pat.)(30) + (2000 Sev. Pat.)(30) + (50 Teach. Units)(200) = $160,000

(7) H1-Total Cost = (23.5 FTE)($10,000/FTE) + (41,050 Bed days x $10/bed day) + $130,000 Supplies = $775,500

(8) H15-Total Cost = (26.5 FTE)($10,000/FTE) + (47,368 Bed days x $10/bed day) + $160,000 Supplies = $898,700

Exhibit 4

Example of Linear Program Format for DEA Analysis*

Objective Hospital is Hospital 8 (H8)

Version B

LINEAR: HOS 8

OBJECTIVES	FTE	BED DAYS	SUPP $	TEACHING#	SEVEREPAT	REGULARPAT		
OBJ-001	0	0	0	100	20	30		
CONSTRAINTS	**FTE**	**BED DAYS**	**SUPP $**	**TEACHING#**	**SEVEREPAT**	**REGULARPAT**	**RELATION****	**RHS-001**
DEM01	25	49.47	140	0	0	0	EQ	1
H1	-23.5	-41.05	-130	50	20	30	LE	0
H2	-24.5	-43.16	-140	50	30	20	LE	0
H3	-26	-43.16	-150	100	30	20	LE	0
H4	-25	-41.05	-140	100	20	30	LE	0
H5	-28.5	-50.53	-160	50	30	30	LE	0
H6	-36	-62.11	-210	100	50	20	LE	0
H7	-51.5	-92.63	-270	50	20	100	LE	0
H8	-25	-49.47	-140	100	20	30	LE	0
H9	-24.5	-43.16	-165	50	30	20	LE	0
H10	-77	-92.63	-340	100	20	100	LE	0
H11	-44.5	-65.26	-265	50	30	50	LE	0
H12	-30	-60	-170	100	30	30	LE	0
H13	-43.5	-81.11	-245	50	50	40	LE	0
H14	-30	-60	-170	100	30	30	LE	0
H15	-26.5	-47.37	-160	50	20	30	LE	0
V1	1	0	0	0	0	0	GE	1E-3
V2	0	1	0	0	0	0	GE	1E-3
V3	0	0	1	0	0	0	GE	1E-3
U1	0	0	0	1	0	0	GE	1E-3
U2	0	0	0	0	1	0	GE	1E-3
U3	0	0	0	0	0	1	GE	1E-3

* This corresponds to the equation (3) format.

** EQ (=)
LE (≤)
GE (≥)

Footnotes

(1) Note, for example, that an organization may be highly effective and also inefficient.

(2) See N.C. Churchill and W.W. Cooper [12] for further discussion.

(3) See [5], [6] , and [7]. See also the earlier work by S. Carlson [4].

(4) See [1], [5] and [7].

(5) We shall use the value $\varepsilon = 0.001$ in the discussion for numerical convenience. Although still smaller values may be used, a series of checks needs to be made in any case (as we have done) to ensure that the numerical value assigned to ε does not alter the resulting optimal solutions.

(6) The marginal rate of transformation is defined as the slope of the production possibility frontier - $\frac{dy_2}{dy_1}$ and represents the relative change in one output which is required for a change in another output holding inputs constant. The marginal rate of productivity or the rate of technical substitution is the slope of the isoquant $\frac{dx_2}{dx_1}$ representing the rate of change in one input required for a change in another unit with output held constant. See [13] Ch. 3 for further discussion.

(7) As determined by the model and the computing procedures utilized.

(8) The analogy is with respect to concepts like "virtual work" or "virtual displacements" in physics and engineering. See A Charnes and W.W. Cooper [9] for other uses of these concepts in mathematical programming.

(10) These outputs represent simplified breakdowns of true hospital outputs where more severe types of patient diagnoses require more intense utilization of inputs than less severe (in this case "regular") diagnoses.

(11) Methods for dealing with these considerations are not very far advanced.

(12) Note that statistical approaches such as least square regressions, etc. would fail even more badly. Econometric approaches to address this problem for the single-output case are reviewed in [13].

(13) To put the matter differently, DEA would provide a conservative measure with E being at least as inefficient as the DEA analysis suggests.

(14) A new computer code for effecting these computations without requiring complete LP reruns has been developed by J. Kennington in association with A. Charnes, W.W. Cooper, and A. and W. Bessent. See [15]. See also [3] for an earlier formulation.

(15) The format of the linear program is illustrated in Exhibit 4 for hospital 8.

(16) This is proved in [16].

(17) An efficient computer code for effecting the wanted computations and data printouts has been developed by Jeff Kennington in association with A. and W. Bessent and A. Charnes and W.W. Cooper. See [15].

(18) These are shown under the columns headed "Outputs Version B" in Exhibit 2.

(19) The inefficiency might, for example, be justifiable if a hospital were inefficient because it is operating in a low population area where continued operations are deemed necessary for public health policy reasons.

(20) See CCR [6] for further discussion.

(21) An example of a comprehensive attempt to ascertain the cost of care is one developed at Yale University and which is now being applied to New Jersey hospitals. Here costs have been estimated for 383 Diagnostic-related groups (DRG's). See, e.g., [19].

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