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# A Media Selection Calculus

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## ABSTRACT

A convenient on-line computer system selects and schedules advertising media. The system consists of a market response model, a heuristic search routine, and a conversational input-output program. The user supplies a list of media options, a budget, and various objective and subjective data about the media options and the desired audience. The system chooses a set of options and spreads them over time, seeking to maximize total market response.

The model of market response takes into account major advertising phenomena. The population is divided into market segments. People in each segment are characterized by their sales potential and media habits. Ads placed in the media options cause people to be exposed to the advertising. The pattern of exposures in each market segment is determined by media coverage and duplication data. The exposures create a level of exposure value in each individual. However, people forget, and the retained exposure level decays in the absence of new exposures. The response of an individual, in terms of the fraction of sales potential realized by the advertiser, increases with exposure level but with diminishing returns. Total market response is a sum over people, market segments, and time periods.

A maximum-seeking, heuristic calculation starts with any schedule, adds options with a high increment of response per dollar and deletes options with a low increment per dollar until no more improvement can be found for the given budget.

An on-line system, called MEDIAC, permits the use of model and heuristic at a remote console of a time-shared computer. Communication with the computer is conversational and self-explanatory. The system is operational. Computing costs have been a fraction of a percent of the cost of the media scheduled. Improvements over previous schedules, as calculated by the model from the user's input data, have run from 5% to over 20%.

## 1. Introduction

An advertiser buys space and time in advertising media to tell prospective customers about his product. He normally hopes that the information in his advertisements will lead people to buy his product who would not otherwise do so, and that they will become satisfied customers. He presumably intends the extra sales generated to yield a net profit.

Media, therefore, play a particular role in advertising: they convey messages to prospects. Media are chosen in the course of constructing an advertising plan, the steps of which include: (1) setting the budget, (2) identifying the audience, (3) picking the advertising message, (4) preparing the copy treatment, and (5) selecting the media. The steps are not independent; message, copy, and media all depend on the audience to be reached. Budget sets the scale of the whole operation. However, once budget and audience characteristics are set, the questions of message and copy can be fairly well separated from the question of how to expose the audience to the messages efficiently. Only the media question will be taken up here, although the other planning steps affect our formulation, since provision must be made for give and take between media selection and the rest of the plan.

The media selection problem may be stated as follows: Given a set of media options, a budget, and various data about the media and the audience to be reached, which options should be used and when should they be used in order to maximize profit or some related measure of performance? By a media option we ordinarily mean a detailed specification of the place, position, size, and other outward characteristics of an advertisement, but not the message and copy treatment. Why is the media problem challenging?

It is because of the multiplicity of seemingly reasonable choices usually available, because of the complexity of advertising phenomena, and because of the quantity of media decisions that are made.

Our goal is to build a media model that will increase advertising productivity. This requires that the model lead people to make better media decisions; it requires the model to be economical to use; and it requires that the model be, in fact, used.

To establish that the model will increase productivity is, to say the least, difficult. Certain of the required inputs will be subjective. Many aspects of the advertising effectiveness process are poorly understood. The most satisfactory test of validity would be to predict outcomes (e.g., sales) and compare them with actual results, but the inherent variability in sales and the problem of relating sales to advertising when other marketing variables and competition also affect response make this difficult.

Considering these obstacles, perhaps we should give up, at least until the underlying processes are better understood. Media planners obviously do not have this option. They must do something sensible with the information they have. Furthermore, they have to do this in the midst of day to day pressures. The important questions then are: Can we isolate the most relevant phenomena for media planning, can we put them together into a consistent structure, and can we link the media planner to the structure in a practical way that increases his power to think about the problem? We shall argue that the answers to these questions are yes.

What then are some of the facts and phenomena relevant to media selection? We have said that the main purpose of media is to deliver messages to potential customers efficiently. Relevant to this are at least the following ideas:

(1) Market segments for classifying customers,

(2) Sales potentials for each segment,

(3) Exposure probabilities for each media option in each segment,

(4) Media costs.

Advertisers spread their campaigns over time. Why? One reason is that the effect of advertising tends to wear off. This is demonstrable. Vidale and Wolfe [1], for example, display data showing the effect. Another reason is that advertising is often considered most valuable near the time of purchase, and people enter and leave the market continuously. Implicit in both these reasons is the idea that people tend to forget past exposures. In addition, both sales potential and media exposure probability may vary with time of year. Therefore, we add the following phenomena:

(5) Forgetting by people exposed to advertising,

(6) Seasonality in product potential and media audience.

A recurring concern in making advertising decisions is the effect of diminishing returns. A person has only so much ability to buy a product. After some point, further advertising to him will be wasted. The phenomenon has been amply observed in practice; see, for example, Benjamin and Maitland [2]. The diminishing returns effect is one part of the more general phenomenon of customer response. We conclude that any media selection model should consider;

(7) Individual response to exposure, including the effect of diminishing returns.

Media planners and media data services frequently pay considerable attention to audience duplication. See, for example, Metheringham [3]. Discussion often centers around reach and frequency. The reach of a media schedule is usually defined as the fraction of people who are in the audience

of at least one vehicle of the schedule. Frequency is defined as the average number of times a person is in the audience of a schedule, given that he is in the audience at least once. In terms of advertising objectives, however, more important than a person being in the audience is his actual exposure to the advertising message. We wish to consider the more basic information of how many people receive zero exposures, one exposure, two exposures, etc., and further how these are spread over time. This information is needed to assess the expected response of the various individuals in the audience and so deduce the response of the market as a whole. Therefore, we take into account:

(8) The distribution of exposures over people and over time.

Finally, provision must be made for putting the exposures from different media options onto a common basis; i.e., it must be possible to assign a value to an exposure delivered by a given option. This is always done implicitly in designing a media schedule; in a formal model it is done explicitly. We therefore add consideration of:

(9) Exposure value for the exposures in each media option.

These then are minimum specifications of data and phenomena to include in a useful media model. More could be added. However, these are already more than are ordinarily used now. Most media planning is rather macroscopic with principal attention going (perhaps quite rightly) to audience potential and simple efficiency measures like cost per thousand, sometimes with a side investigation of reach. We intend to show that more phenomena can be handled with greater ease than these usually are today.

To be productive, a model must be used. To be used it should be readily available and inexpensive to operate. Modern time-shared computers

with remote on-line consoles make this possible. They permit immediate access to the computer, English language communication, user-instructing programs, and low cost per use. The user can think about his problem at the console, asking questions of the model and making changes in the schedule in a way that extends his capacity to understand and solve the problem.

To summarize, our goal is productivity; our approach is to set up a structure embodying the principal phenomena relevant to media selection and, through time-shared computing, make it easy and inexpensive to use. We cite the following reasons for believing that this approach will be productive. The computer is an enthusiastic clerk. Given a model, it can evaluate many more alternatives within reasonable time and cost limits than can people. A computer can handle complexity with ease, e.g., local media mixed with national media across several market segments. Changes are easy to make; therefore, there can be give and take between media selection and the rest of the advertising planning process. Sensitivity analyses can easily be made; i.e., data and assumptions can be changed to see whether they appreciably affect the outcome. Perhaps most important, however, a model provides a unified structure for organizing the central issues of the problem. Requirements for data and judgments are defined. Criteria are chosen and consistently applied. This seems certain to bring forth better data, more careful judgments, and more relevant criteria.

## 2. Literature Review

The literature on mathematical models for media selection starts about 1960. A simple, hypothetical media problem was formulated as a

linear program by Miller and Starr [4] at that time. Soon after came the major pioneering work on linear programming models done jointly by BBDO and CEIR. Descriptions of this may be found in Wilson [5] and Buzzell [6]. The linear programming approach has been further discussed by Day [7], Engel and Warshaw [8], Stasch [9], and Bass and Lonsdale [10].

In all published examples of these formulations, the objective function is linear in the number of exposures. This implies that the value of ten exposures to one person is the same as that of one exposure to each of ten people. Such an assumption does not seem reasonable, particularly at high levels of exposure, where additional exposures are ordinarily believed to have less value than previous ones. The effect of linearity on the solution is that the most efficient medium for generating exposures will usually be bought until some upper limit is reached, then the next most efficient will be bought until its limit is reached, and so on. Upper limits must be provided to prevent unreasonable schedules. However, this starts to look similar to picking a schedule without a model, except for the important point that the methods are systematic and explicit.

To get away from strict linearity, diminishing returns and other forms of market response were introduced. Kotler [11], for example, presents a nonlinear model. In an unpublished paper reporting on the BBDO-CEIR work, Godfrey [12] outlines a method of dealing with certain types of nonlinearities. Wilson [5] refers briefly to a method and presumably it is the same one. More recently, Brown and Warshaw [13] have published essentially the same thing. The basis of this method is a standard device for converting a nonlinear program into a linear one in the case that the

objective function is separable (i.e., is the sum of functions, each of a single variable) and, for a maximization problem, concave (i.e., the functions are linear or show diminishing returns, but never increasing returns). All three of these nonlinear models have a serious drawback in that the nonlinearities of each medium are separate. Thus, a person's increase in response from seeing an ad in LIFE is the same whether he has seen zero, one, or ten ads in some other magazine. It would seem more reasonable to expect diminishing returns with total exposure.

Several further difficulties beset most of the above formulations. First, the timing of the insertions over the planning period is usually ignored, or at least set outside the model. Exceptions are Godfrey [12] and Stasch [9] who propose to allocate over time by introducing additional variables and additional constraints. Once again, however, the borderline between setting the constraints and setting the schedule tends to blur. The next difficulty is that the treatment of audience duplication is usually weak or non-existent. Finally, a linear program permits variables to take on fractional values, whereas the number of insertions must always be an integer.

Zangwill [14] suggests handling the integrality problem by the use of integer programming, but the current state of the art in this field is not encouraging for problems of the size encountered in media selection. Furthermore, Zangwill's evaluation of the effectiveness of a media choice is done almost entirely outside the model. While this may be said to offer great flexibility, much of the appeal of a model lies in having it synthesize the effectiveness of a schedule out of events that are happening at the consumer level.

Recently, Charnes, Cooper, DeVoe, Learner, and Reinecke [15] have introduced LPII, a successor to Mediametrics. Time is considered, although not forgetting. Audience duplication is brought in under the assumption of independence between media. The objective function is a weighted combination of the magnitudes of differences between target and actual values of a set of goals. The goals might include target frequencies in each market segment and more complex quantities such as "reaching 85% of the $k^{th}$ audience segment at time $t_1$." In the examples shown, the schedule is penalized for exceeding a goal as well as for not reaching it. The choice of goals and their weights is done by the user.

Another major line of attack on media selection is microsimulation, i.e., the following of individuals through time in their media actions. An early model of this type was built by the Simulmatics Corporation [16]. The output was patterns of exposure without evaluation or optimization. Moran [17] reports a simulation model with capacity for schedule improvement but gives little detail. Brown [18] and Gensch [19] do not include time effects but do treat people individually.

The virtue of microsimulation is its potential comprehensiveness. Many phenomena can be put into the model with comparative ease. This is a mixed blessing, since the problems of model construction and testing, data gathering, and computer running time go up rapidly as detail increases. There is a danger that much of the computer time will be spent pursuing issues not really central to the decision at hand. A difficulty inherent in the simulation of individuals is that of attaining sample sizes large enough for adequate evaluation of a schedule, particularly when the schedule contains media vehicles with small audiences. Furthermore, the

search for improved schedules tends to become expensive because each separate schedule evaluation may take considerable computing time. Partly for this reason, the search for improvement is frequently left outside the computer.

The discussion so far has centered on work done in the United States. Work done in England goes back in time as far or further, has generally taken different directions, and has been of excellent quality. Lee and Burkart [20], Taylor [21], Lee [22,23], and Ellis [24] have developed a series of models motivated especially by print media. Several of the models were stimulated by problems arising at British European Airways and have been applied there. These models are much more explicit in their treatment of exposure probabilities and individual response to exposure than those previously mentioned. Under certain sets of assumptions, easily applied rules for optimal media selection are worked out mathematically. In the more complicated models, which take into account market response over time, the optimization is left as an integer programming problem.

Beale, Hughes, and Broadbent [25] describe the London Press Exchange model for media schedule assessment. This is a major model brought to the point of practical application. The authors call their model a simulation, but it is perhaps fair to say that much of their computational efficiency can be traced to clever circumvention of straight simulation. The model is flexible, computable, and has been built around a considerable base of data. One notable lack is any treatment of the effect of time; there is, for example, no forgetting. The search for schedule improvement is outside the computer, although provision is made for multiple simultaneous schedule evaluations.

There are at least two reported French media models. Steinberg, Comes, and Barache [26] present a simulation. Bertier [27] shows how some of the descrete optimization problems of media selection might be solved.

We relate the present work to our earlier paper [28]. The model there incorporates nonlinear response, market segmentation, and forgetting, and is optimized by dynamic programming. However, the latter becomes computationally prohibitive with more than one or two market segments. Although Franzi [29] has investigated separable programming methods for optimizing the model, the problem of fractional solutions remains, and in the present paper we have moved away from exact optimization to heuristic methods. The biggest change, however, is a general reworking of the model including a new, detailed treatment of the distribution of exposures over the population, and therefore of the problem of media audience duplication.

There exists a variety of commercially secret or otherwise incompletely published work. We are aware of some of it, but obviously cannot adequately review it. We would be glad to have the opportunity.

## 3. Model

The model may be described briefly as follows: The population is divided into market segments. People in each segment have their own sales potential and media habits. A media schedule consists of insertions in media options. An insertion brings about exposures to people in one or more market segments. The exposures serve to increase the exposure level of individuals in the segment. However, people are subject to forgetting and so the retained exposure level decays with time in the absence of new exposures. The response of individuals in a market segment increases with exposure level but with diminishing returns at high levels.

3.1 Media, exposure levels, and forgetting. To lay out the dimensions of the problem, let

M = number of media options under consideration.

T = number of time periods in the planning horizon.

S = number of market segments.

$$x_{jt} = \begin{cases} 1 & \text{if an insertion is made in option j in time period t.} \\ 0 & \text{if not.} \end{cases}$$

Thus, our ultimate goal will be to set the values of the $x_{jt}$ for j=1..., M and t=1,...T.

We define several terms: A media class will be a general means of communication, such as television, magazines, or newspapers. A media vehicle will be a cohesive grouping of advertising opportunities within a class, such as a particular TV show, magazine, or newspaper. A media option will be a detailed, purchasable unit within a vehicle. Examples would be: a commercial minute in BONANZA, a 4-color full page in LOOK, and a half-page in the Sunday NEW YORK TIMES. A media insertion will be a specific purchase of an option and includes specification of the time period of use. A collection of insertions over a planning period will be a media schedule.

It is assumed that a media option: (1) is available exactly once in every time period, (2) has substantial continuity of audience, and (3) has continuity in outward format. These assumptions are for conceptual convenience and are not really very restrictive. For example,if an option cannot be available in some time period, the corresponding $x_{jt}$ can be permanently set to zero. If the media planner wishes to permit multiple insertions of the same type in one time period, multiple media options, all

alike, can be created. As much detail can be included in the specification of an option as desired; for example, a geographic area can be stipulated. Several options can be grouped together and listed as one, provided that their audiences do not appreciably overlap. Ultimately the suitability of an option depends on whether the cost, exposure, and value data described below can be provided for it.

Exposure of an individual to an insertion is taken to mean that the person has perceived the presence of the ad. A number of operational measures of exposure have been developed, different measures often being appropriate for different classes of media. The particular measures to be used in a given application are selected by the media planner.

Exposure or non-exposure of an individual to an insertion is a random variable. Consider a particular person in market segment i. Let

$$z_{ijt} = \begin{cases} 1 & \text{if the person in segment } i \text{ is exposed to an insertion in media option } j \text{ in period } t. \\ 0 & \text{if not.} \end{cases}$$

The probability distribution of $z_{ijt}$ is determined by media exposure probabilities and by whether or not an insertion has been made. The arithmetic of this will be taken up below. We have been tacitly assuming that the population of interest is composed of individuals. However, for certain applications, some other basic response unit may be more appropriate and, if adopted consistently, can be used without difficulty.

We next recognize that the value of an exposure may not be the same for every option. One reason is format differences. A larger ad may convey more information. (A larger ad may be more likely to be noticed too, but that effect is covered under exposure probability.) Other reasons are differences in editorial climate, mood, and reader involvement. For example, some media vehicles are thought to be supportive

for certain products. An important reason is differences between media classes: An exposure to a 30-second radio spot is to be rated on the same scale as an exposure to a half-page newspaper ad. At present, there is a large subjective element in such appraisals, but any final media schedule, however arrived at, implicitly includes such an evaluation.

The exposure value rating can be a bridge to other parts of the advertising plan. For example, the exposure value may be affected by the proposed communications task and copy opportunities. Thus, if a capacity for demonstrating the product is important, television would rate high. If accurate color reproduction is desirable, certain magazines would rate well.

Exposure value may differ somewhat from market segment to market segment. This seems particularly likely if market segments are defined by sex, education, or life-style. Certain types of ads are routinely designed to appeal to special groups and may have much less effect on others. If such information is known in advance, it can be reflected in exposure value. Let

$e_{ij}$ = exposure value conveyed by one exposure in media option j to a person in market segment i. (exposure value/exposure)

We must emphasize that exposure value has nothing to do with cost, audience size, or exposure probability within the audience. For different options, exposure value answers the question: Given that the choice of a person seeing an ad in LIFE or the same person seeing it in LOOK, does the advertiser have any preference and, if so, what is a numerical statement of that preference? For different market segments, the question is: Given that a man sees an ad in SPORTS ILLUSTRATED and that a woman sees it there, should a different worth be assigned to the exposure?

The units for exposure value are arbitrary except that they must later be tied to a response function. It is frequently convenient to conceive of an "average" media-option-market-segment combination and assign it an exposure value of 1.0. Then values for other options and market segments can be related to this.

Exposure is assumed to increase a desirable quantity that will be called the level of retained exposure value or more simply, the <u>exposure level</u> in an individual. The amount of the increase in time period t is the sum of the exposure value contributions from each insertion seen during the period.

$\Sigma_{j=1}^{M} e_{ij}z_{ijt}$ = increase in exposure level of a particular individual in market segment i in time period t. (exposure value/capita)

We suppose that the effect of advertising wears off because of forgetting. Specifically, it is assumed that, in the absence of new input, exposure level decreases by a constant fraction each time period. Let

$y_{it}$ = exposure level of a particular individual in market segment i in time period t. (exposure value/capita)

$\alpha$ = memory constant: the fraction of $y_{it}$ retained from one time period to the next. $0 \leq \alpha < 1$

Then

$$y_{it} = \alpha y_{i,t-1} + \Sigma_{j=1}^{M} e_{ij}z_{ijt} \qquad (1)$$

For empirical evidence on retention and decay, see Zielske [30] and Simmons [31]. If desired, the memory constant can be permitted to depend on i and t and perhaps other factors. A typical pattern of $y_{it}$ over time might appear as in Figure 1. (The figure shows forgetting as a continuous

process. The model actually uses $y_{it}$ only at the descrete times ..., t, t+1, t+2, ... )

For future reference, notice that (1) can be rewritten as:

$$y_{it} = \alpha^t y_{i,0} + \Sigma_{s=1}^{t} \Sigma_{j=1}^{M} \alpha^{t-s} e_{ij} z_{ijs}$$

or, going back indefinitely, as:

$$y_{it} = \Sigma_{s=-\infty}^{t} \Sigma_{j=1}^{M} \alpha^{t-s} e_{ij} z_{ijs} \qquad (2)$$

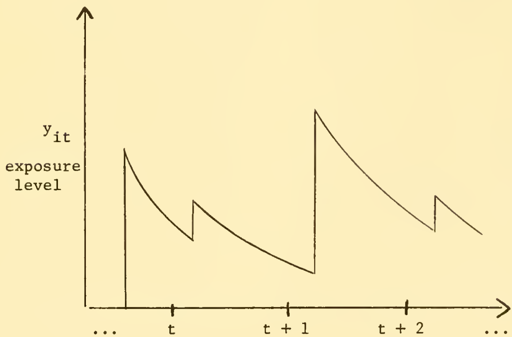


Figure 1. Exposure level, $y_{it}$, over time for some individual. Jumps represent new exposures in the time period; downward sloping portions represent forgetting.

3.2 Market response. Market response is treated as follows: Each individual has a sales potential. Sales potential varies with market segment and may also be seasonal. The fraction of sales potential realized by an advertiser in a time period depends in a nonlinear way on the person's exposure level in that time period. Exposure level varies from individual to individual within a market segment and is described by a probability distribution. Total market response is synthesized by adding up over individuals, market segments, and time.

Specifically, let

$n_i$ = number of people in market segment i

$w_{it}$ = sales potential (weight) of a person in segment i in time period t. (potential units/capita/time period)

$r(y_{it})$ = response function: the fraction of potential realized when a person has exposure level $y_{it}$

$f_{it}(.)$ = probability density of $y_{it}$

The response function r(y) might appear as in Figure 2. Let E denote the taking of expected values. Then $w_{it}E\{r(y_{it})\}$ is the average realized sales potential per person in market segment i at time t. Summing, we obtain

$$R = \Sigma_{i=1}^{S} \Sigma_{t=1}^{T} n_i w_{it} \; E\{r(y_{it})\} \tag{3}$$

= total market response. (potential units)

The specific curve to be used for r(y) will depend on the planner's judgment, and the empirical evidence available to him. Presumably the curve should show diminishing returns at high exposure levels. Some people feel that, at least in certian cases, the curve should show increasing returns at low levels. Others disagree; Simon [32], for example, argues that there is no empirical evidence to support increasing returns. A simple,

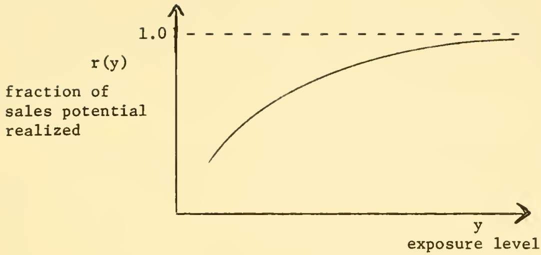


Figure 2. Possible response curve: The average fraction of an individual's sales potential realized by the advertiser as a function of the person's exposure level.

versatile function with only diminishing returns is:

$$r(y) = r_0 + a\,(1 - e^{-by}), \qquad 0 \le y < \infty \tag{4}$$

where $r_0$, a, and b are nonnegative constants specific to the product at hand. However, our work is not restricted to this curve. Conceivably, a different function $r_{it}(y_{it})$ could be used for each i and t, but, until evidence dictates otherwise, it seems best to reflect differences between market segments and time periods simply by using sales potential as a scale factor.

The units of sales potential have not been specified. We personally tend to think of response in terms of an anticipated sales rate. Then, if sales are expressed in dollars, $w_{it}$ has units of dollars/capita/time period and R is the expected total dollar sales to the market over the planning period. In allocating a fixed budget, however, only the shape of the response curve and the relative values of the sales potentials determine

the allocation. The absolute units of the $w_{it}$ are immaterial. Some media planners prefer to express sales potentials in arbitrary units. They feel they have a good idea of relative potentials but not absolute potentials.

The expected response $E\{r(y_{it})\}$ for a given market segment and time period can be expressed in terms of the moments of the distribution $f_{it}(y_{it})$. Usually only the first few moments will be needed to give a good approximation to the expected response. This will turn out to be quite convenient. For notational simplicity, we drop the subscripts i and t for the present. Let

$$\mu = E\{y\} = \text{mean of } y$$

$$\mu_n = E\{(y-\mu)^n\} = n^{th} \text{ moment of } y \text{ about the mean}, \quad n>1.$$

We can expand r(y) in a Taylor series about $\mu$:

$$r(y) = r(\mu) + \Sigma_{K=1}^{n-1} (1/k!)\ r^{(k)}\ (\mu)(y-\mu)^k + (1/n!)\ r^{(n)}(y_1)(y-\mu)^n \qquad (5)$$

where $r^{(k)}(\mu)$ is the $k^{th}$ derivative of r(y) evaluated at $y=\mu$ and $y_1$ is some value between y and $\mu$.

Taking expectations:

$$E\{r\} = r(\mu) + \Sigma_{k=2}^{n-1} (1/k!)\ r^{(k)}\ (\mu)\mu_k + (1/n!)\ E\{r^{(n)}(y_1)(y-\mu)^n\} \qquad (6)$$

In practice, we would take some number of terms as our approximation and use this last term on the right to estimate the degree of approximation. Suppose, for example, we use the exponential response of (4), and retain terms through the third moment. Then (6) becomes

$$E\{r\} = r_0 + a(1-e^{-b\mu}) + ae^{-b\mu}\{-\ (1/2)b^2\mu_2 + (1/6)b^3\mu_3\} + \varepsilon_4 \qquad (7)$$

where

$$\varepsilon_4 = -(1/24)aE\{e^{-by_1}b^4\ (y-\mu)^4\},$$

and

$$|\varepsilon_4| \leq (1/24)ab^4\ \mu_4,$$

since, at most, $e^{-by_1} = 1$, and we know $(y-\mu)^4 \geq 0$.

Before leaving the response model, we observe that its conceptual generality can be broadened considerably without adding complexity. Referring back to (3), we do not have to assume that everyone in a market segment actually has the same sales potential, $w_{it}$, nor that everybody at exposure level y responds to the same degree, r(y). The quantity $w_{it}$ can be interpreted as the average sales potential per capita in the market segment. Similarly, r(y), may be viewed as a conditional expectation, i.e., the average fraction of potential realized for a group of people having the exposure level y. Both sales potential and the fraction realized may be viewed as random variables without change in (3) if they are independent. If there is a basis for believing that sales potential and the fraction realized are not independent then this basis can be used to subdivide the market segment into more homogeneous groups.

The empirical status of our construct of retained exposure level deserves comment. We do not conceive of exposure level as a directly observable property of an individual. Perhaps something close to it is observable and, if so, this would be very helpful. Quite likely, however, the communications process and the state of the individual involve a complex of quantities. If such is the case, they are deliberately aggregated here into a single index. Even if exposure level is not observable, the model

can still, in principle, be tested empirically. Exposures are defined operationally. The sales potentials and exposure values are prespecified numbers. Therefore, if a behavioral measure of response (say, sales) is selected, it is possible to measure inputs and outputs and fit the model to data or test it against data. Essentially we would have a problem in nonlinear regression. The difficulties in doing this are substantial and we base our claim of utility on different grounds, but the idea remains a worthwhile possibility.

To summarize up to this point, our model deals with exposures, which have value, create an exposure level, but are gradually forgotten. The exposure level determines the fraction of a person's sales potential that is realized. Averaging over people and summing over market segments and time periods gives total response. Response can conveniently be expressed in terms of the moments of exposure level distribution.

3.3 Exposure arithmetic. Our next goal is to express the moments of the distribution of exposure level in terms of the media decisions, $x_{jt}$. The general plan is as follows: It will be shown that the mean and variance of exposure level depend only on the exposure probabilities of media singly and in pairs. Higher moments will be related to the first two. Therefore, the moments of the distribution can be calculated from exposure probability data that is not too difficult to gather and store. The exposure probabilities themselves will be developed in terms of the probability that a person is in the audience of the medium, the probability he will be exposed given that he is in the audience, and an audience seasonality factor.

Consider, for the moment, a single market segment and a single time period. We can then temporarily drop the corresponding subscripts i and t

and simplify notation. Suppose further that the options in which insertions have been made are j=1,...,J. Let

$y$ = exposure level of a particular individual,

$$z_j = \begin{cases} 1 & \text{if the individual is exposed to option } j, \\ 0 & \text{if not,} \end{cases}$$

$$y = \Sigma_{j=1}^{J} e_j z_j \qquad (8)$$

This expression appears to omit from y the carry over of exposure level from the previous time period, but carry over is a weighted sum of previous exposures and just adds more terms to the sum. Let

$p_j = P(z_j=1)$ = P(a person is exposed to option j)

$p_{jk} = P(z_j=1\ ,\ z_k=1)$ = P(a person is exposed to both option j and option k)

Thus $p_j$ is essentially a rating points type of measure based on exposures not just audience. The $p_{jk}$ express the pairwise duplications in the audience. The mean of y is simply

$$E\{y\} = \Sigma_{j=1}^{J} e_j p_j . \qquad (9)$$

The second moment of y is

$$E\{y^2\} = E\{(\Sigma_{j=1}^{J} e_j z_j)^2\} = \Sigma_{j=1}^{J} \Sigma_{k=1}^{J} e_j e_k E\{z_j z_k\}$$

$$= \Sigma_{j=1}^{J} e_j^2 p_j + 2 \Sigma_{j=1}^{J-1} \Sigma_{k=j+1}^{J} e_j e_k p_{jk} \qquad (10)$$

or, letting V(.) denote variance,

$$V(y) = \Sigma_{j=1}^{J} e_j^2 p_j (1-p_j) + 2 \Sigma_{j=1}^{J-1} \Sigma_{k=j+1}^{J} e_j e_k (p_{jk} - p_j p_k). \qquad (11)$$

We note that the first term on the right involves summing binomial variances as if they were independent and the second expresses the difference from strict independence.

Equations (9) and (11) give us $\mu = E(y)$ and $\mu_2 = V(y)$, the first two moments of y. An expression for $\mu_3$ can also be developed and will involve three-way overlaps among media. More generally $\mu_n$ will involve n-way overlaps. High order overlaps are expensive to collect and expensive to store in a computer. An alternative is to estimate higher moments from lower ones. For example, the first two moments can be used to determine the parameters of an analytical probability distribution such as the gamma or log normal. Then the higher moments are implied and readily deduced. So far, however, we have not found a distribution that is computationally convenient and also fits sufficiently well to live data. Instead, we have developed empirical expressions relating higher to lower moments.

Figure 3 shows plots of $\mu_n^{1/n}/\mu$ vs. $\mu_2^{1/2}/\mu$ for n=3 and 4 based on multiple-way audience overlap data in magazines. To form a distribution of y from such data we must specify a set of magazines, and, for each magazine, its $e_j$ and exposure probability for readers. In Figure 3, all $e_j$'s and exposure probabilities have been set to one. Each plotted point comes from a distribution of y defined by a set of magazines. As may be seen, straight lines give a good fit. More generally, it is assumed that we can determine a fuction:

$$\mu_n = \mu_n(\mu_2, \mu) \tag{12}$$

Next we restore time period and market segment subscripts by the following correspondences.

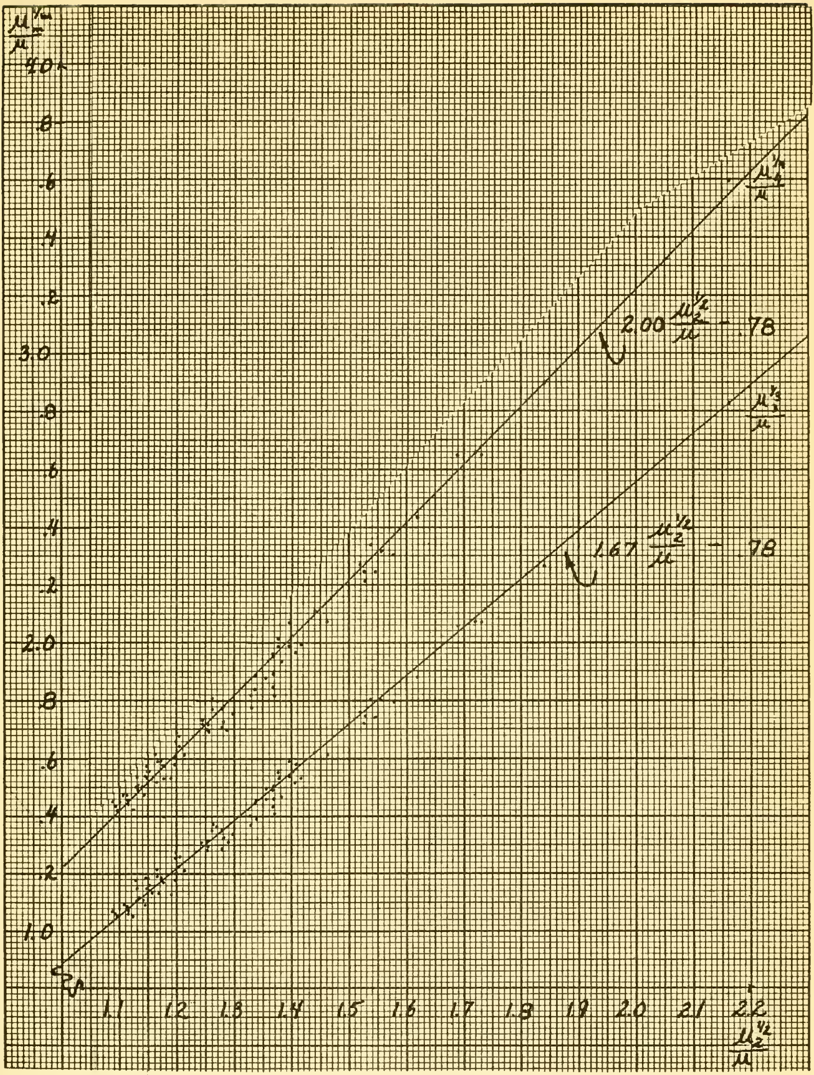


FIGURE 3. Relating the higher moments of the exposure level distribution to the lower.

$$z_j \longleftrightarrow z_{ijs}$$

$$e_j \longleftrightarrow \alpha^{t-s} e_{ij}$$

$$y = \Sigma_{j=1}^{J} e_j z_j \longleftrightarrow y_{it} = \Sigma_{s=-\infty}^{t} \Sigma_{j=1}^{M} \alpha^{t-s} e_{ij} z_{ijs}$$

$$p_j = P(z_j=1) \longleftrightarrow p_{ijt} = P(z_{ijt}=1)$$

$$p_{jk} = P(z_j=1,\ z_k=1) \longleftrightarrow p_{ijt;ks} = P(z_{ijt}=1,\ z_{iks}=1)$$

Then, using (9), (10), and (12), we obtain moments

$$\mu_{it} = E(y_{it}) = \Sigma_{s=-\infty}^{t} \Sigma_{j=1}^{M} \alpha^{t-s} e_{ij}\, p_{ijt} \tag{13a}$$

$$\mu_{2it} = V(y_{it}) = \Sigma_{s=-\infty}^{t} \Sigma_{j=1}^{M} (\alpha^{t-s} e_{ij})^2\, p_{ijt}$$

$$+ 2 \Sigma_{j=1}^{M-1} \Sigma_{k=j+1}^{M} \Sigma_{s=-\infty}^{t-1} \Sigma_{r=s+1}^{t} \alpha^{t-s} e_{ij}\, \alpha^{t-r} e_{ik}\, p_{ijs;kr} - \mu_{it}^2 \tag{13b}$$

$$\mu_{nit} = \mu_n(\mu_{2it}, \mu_{it}). \tag{13c}$$

The media exposure probabilities will be modeled further. Let

$g_{ij}$ = market coverage of the media vehicle of option j in segment i, defined as the fraction of people in segment i who are in the audiences of the vehicle of option j, averaged over a year.

$s_{jt}$ = audience seasonality, the seasonal index for the vehicle of option j in time period t. Average value over a year is 1.0.

$h_j$ = exposure probability for audience member. The probability a person is exposed to an insertion in option j given that he is in the audience of the vehicle of j.

Recalling that $x_{jt}$ is a zero-one variable indicating presence or absence of an insertion, we take

$$p_{ijt} = x_{jt}\, g_{ij}\, s_{jt}\, h_j. \tag{14a}$$

This expression implicitly assumes that media vehicle seasonality can reasonably be regarded as the same in all market segments and that $h_j$ does not change seasonally.

Next we want the duplication probabilities, $p_{ijt;ks}$. These will be modeled in two steps. First, let

$$p_{ijt;ks} = g_{ijt;ks} h_j h_k x_{jt} x_{ks} \tag{14b}$$

where

$g_{ijt;ks}$ = segment duplication: fraction of people in segment i who are in the audience of both the vehicle of option j at t and the vehicle of option k at s.

Equation (14b) assumes again that the $h_j$ are not appreciably seasonal and further that the events of being exposed to option j and being exposed to option k are independent, given that a person is in the audience of both vehicles involved. (The events of being in the audience of one vehicle and being in the audience of another are, contrary to many media models, not considered independent.)

The task of developing empirical tables of $g_{ijt;ks}$ and storing them in a computer is formidable because of the dimensionality involved and, even if seasonality were to be split off separately, the job of tabulating audience duplication by market segment would still be large. Therefore, we have developed estimating methods based on more global data. Let

$f_j$ = fraction of the total population who are in the audience of the vehicle of option j, averaged over a year.

$f_{jk}$ = fraction of the total population who are in the audience of both the vehicles of j and k, averaged over a year.

$\beta$ = an empirically determined constant.

Then we take

$$g_{ijt;ks} = [f_{jk}/(f_j+f_k)]\ [(1+\beta)(g_{ij}s_{jt} + g_{ik}s_{ks}) - \beta(f_j + f_k)]. \quad (14c)$$

A value of .53 has been empirically developed for $\beta$ by linear regression on 1965 vehicle-segment data in magazines. As a test, (14c) has then been used to predict duplications for different segments and different magazines in a different year (1967). The results are shown in Figure 4. The mean percentage error is 8.8%. Although time appears in (14c), it is suppressed in our fitting and testing, since magazines are not particularly seasonal.

Implicitly, (14c) assumes that, aside from seasonality, the fraction of people who are in the audience both of vehicle A in January and of vehicle B in November is the same as the fraction who are in the audience of both vehicles in January (or November). This is probably a rather good assumption but, in any case, we do not presently have any data one way or the other on the question.

3.4 <u>Budget constraint</u>. Let

$c_{jt}$ = cost of an insertion in media option j in time t. (dollars)

B = total budget for the planning period. (dollars)

The budget constraint is:

$$\Sigma_{j=1}^{M} \Sigma_{t=1}^{T} c_{jt}\, x_{jt} \leq B \quad (15)$$

The constraint is shown as linear in the number of insertions. Published rates are considerably more complex than this, and, worse yet for planning, the prices of many purchases are negotiated. From a mathematical point of view, a difficulty with many published rates is that they

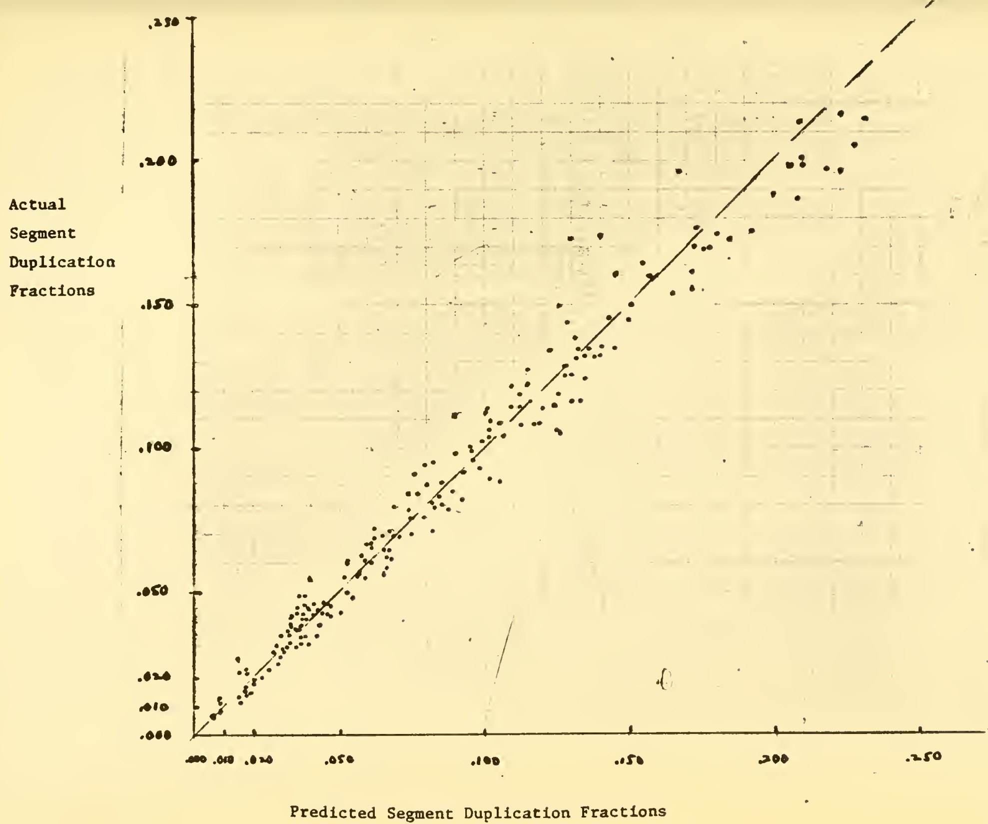


FIGURE 4.

Actual vs. Predicted Segment Duplication Fractions

are neither convex nor concave. This happens when the discount for quantity applies not only to all insertions more than a fixed amount but to all insertions. Then the effective cost per insertion may be zero or nearly zero in some places. For example, if an advertiser has bought eight insertions and the discount break is at nine, he might be able to get the ninth free because the use of nine insertions makes him eligible for a discount on all nine. However, heuristic methods for dealing with discounts will be developed below based on successive uses of the constraint (15) with changing $c_{jt}$.

3.5 End effects. The beginning and end of the planning period require special consideration. At the beginning, starting exposure levels must be specified. At the end we must find a way to evaluate advertising insertions whose effects extend beyond the planning period.

A simple and effective way to set starting exposure levels is to run the last few periods of the previous year's media schedule through the model. Ending levels for last year become starting ones for this year. Since this year's options almost always include last year's choices, the media data is readily available. If the whole previous year is run, we gain the added advantage of obtaining a comparison of the new schedule with the old under the criteria of the model.

At the other end of the planning period, we have a different problem. If we calculate response only over the planning period ($t=1,\ldots,T$), we shall under-rate the insertions during T because these also contribute to response in periods T+1, T+2, .... Only if forgetting is very rapid or sales potential very small in periods right after T will there be no problem. A variety of approaches can be taken to correct the situation. A few extra periods can be added after T without scheduling more insertions.

This solves the under-rating problem but may introduce over-rating as follows: The incremental response for adding an insertion in T is composed of an increment in T, another in T+1, another in T+2, etc. However, the amounts of these increments depend on the exposure levels in the time periods involved. For example, high exposure levels would mean small increments because of diminishing returns. With no new insertions after T, future exposure levels will be low and the response increments may be unnaturally large. This tends to produce an over-rating of the insertions in T, although the effect can be kept under control by limiting the extra periods considered.

A better but more complicated method of handling this end effect is to add extra periods but also put a schedule of new insertions into those periods. The schedule might come from various sources, but, if we are dealing with an annual plan, the most appropriate futher schedule is probably a repeat of the one the model is developing. This is a little tricky, but can be done. When an insertion is put in at t, it is also put in at t+T. In evaluating the incremental effect of that insertion only its placement at t is considered. However, the incremental evaluation of an insertion at T will assume the presence of the earlier insertion at both t and t+T.

Notationally, the end of effects will be treated as follows: Let

E = the number of extra time periods added onto the end of the planning period for evaluating response.

K+1 = the number of extra time periods added onto the beginning of the planning period to set starting exposure levels.

3.6 Mathematical program. The pieces of the model can now be pulled together and the media selection problem presented as a mathematical program. We set up the case where the objective function involves the

first n terms of the Taylor expansion (6) and end effects are treated by extending forward without new insertions. Provision is made for a set, $I_1$, of insertions that are required to be in the schedule and another set, $I_2$, required to be out.

MP. Find $x_{jt}$ $(j=1,\ldots,M\ ;\ t=1,\ldots,T)$ and maximal R subject to:

$$R = \Sigma_{i=1}^{S} \Sigma_{t=1}^{T+E} n_i w_{it} \{r(\mu_{it}) + \Sigma_{m=2}^{n} (1/m!)\, r^{(m)} (\mu_{it})\mu_{mit}\}$$

$$\Sigma_{j=1}^{M} \Sigma_{t=1}^{T} c_{jt}\, x_{jt} \leq B$$

$$\mu_{it} = \Sigma_{s=-K}^{T} \Sigma_{j=1}^{M} \alpha^{t-s}\, e_{ij}\, h_j\, g_{ij}\, s_{jt}\, x_{jt}$$

$$\mu_{2it} = \Sigma_{s=-K}^{t} \Sigma_{j=1}^{M} (\alpha^{t-s} e_{ij})^2\, h_j\, g_{ij}\, s_{jt}\, x_{jt} + 2\Sigma_{j=1}^{M-1} \Sigma_{k=j+1}^{M}$$

$$\Sigma_{s=-K}^{t-1} \Sigma_{r=s+1}^{t} \alpha^{t-s}\, e_{ij}\, \alpha^{t-r}\, e_{ik}\, h_j\, h_k\, g_{ijt;ks}\, x_{js}\, x_{kr} - \mu_{it}^{2}$$

$$\mu_{mit} = \mu_m(\mu_{it}, \mu_{2it}) \qquad m = 3,\ldots,n \qquad i = 1,\ldots,S$$
$$t = 1,\ldots,T+E$$

$$x_{jt} \in \{0,1\} \quad \text{for all } (j,t)$$
$$x_{jt} = 1 \quad (j,t) \in I_1$$
$$x_{jt} = 0 \quad (j,t) \in I_2$$

4. Heuristic Search. As a formal mathematical program, MP appears to be rather intractable. It is an integer and nonlinear. Practical problems are large; for example, we have already worked on problems involving twenty media options in ten time periods or 200 zero-one variables. We have solved a deterministic version of the model in a one market segment problem by dynamic programming (see [28]), but use of similar methods to solve MP does not appear reasonable .

Consequently we have developed heuristic search methods to find schedules that are good, possibly optimal, but not necessarily guaranteed to be optimal. The basic maximum-seeking heuristic is simply that of adding to a schedule those insertions that produce a high increment of response per dollar and deleting those that produce a low decrement of response per dollar.

<u>HS1</u>: 1. Start with any schedule (e.g. an empty one).

2. For each insertion not now in the schedule, calculate the incremental response/dollar for adding that insertion. Find the insertion with the largest value and add it to the schedule.

3. Is the budget exceeded?

   No. Return to 2

   Yes. Continue

4. For each insertion now in the schedule, calculate the decremental response/dollar for removal. Find an insertion with the smallest value. Call it I. Is the decrement/dollar for I greater than or equal to the increment/dollar of the most recently added insertion?

   Yes. Go to 5.

   No. Delete I. Return to 3.

5. Finish.

The above search can be expected to work well when the available insertions are not too widely different in cost and their costs are relatively small compared to the total budget. Pathological cases can be constructed and further heuristics developed to counter them but so far the simple procedure seems to be satisfactory.

Our confidence in the basic heuristic is based on several pieces of evidence. First a deterministic problem solved exactly by dynamic programming was solved to the same solution by the heuristic. Second the method has always given a better solution than anyone's preconceived idea of what

a schedule should be. Finally there is a theoretical reason for expecting good solutions. The objective function will usually be a concave function in the decision variables. (This might not be the case if response is strongly S-shaped but usually response will be well represented by a diminishing returns curve.) Under these circumstances a local maximum would be a global maximum if the decision variables were continuous. As it is they are integral, but if individual insertion costs are small compared to the budget, the solution will be likely to behave as it would in the continuous case. The importance of knowing that a local maximum is likely to be a global maximum lies in the fact that our search only explores solutions (schedules) in the immediate neighborhood of the solution currently at hand, i.e., the search tests for a local maximum. Our argument suggests that, once a local maximum is found, it is unlikely that some other, quite different solution will be better.

Since media costs are discrete numbers, the cost of the final schedule will not ordinarily equal the exact budget. The search HS1 will give a schedule that slightly exceeds the budget. By dropping out the last insertion added, the schedule can be made to fall slightly below the budget.

Consider next the problem of media discounts. A useful heuristic approach is to introduce media at their least cost (highest discount). If they do not appear in the schedule under these conditions they can rather safely be ignored. If they do appear, their costs can gradually be raised to whatever value actually applies. A formal procedure for this is as follows:

<u>HS2</u>: 1. Set all costs per insertion at their lowest incremental values, i.e., at the highest discount rate.

2. Solve the problem using HS1.

3. Exclude from further consideration all options not appearing in the schedule. Is the cost of each option entered at its actual average cost per insertion (including discounts) for the current schedule?

   Yes. Go to step 5

   No. Continue

4. Find the option with the largest discrepancy between the actual average cost per insertion for the current schedule and the cost being used. For this option(s) put in the actual average cost. Return to step 2.

5. Finish.

The search is operated off-line except for step 2. Visual inspection is often used to skip steps and save computing time. HS2 may not be as good as HS1. The possibility of multiple local maximum seems intuitively more severe.

5. Setting parameters. The job of supplying inputs is left to the user. Some people have claimed that, if they had all the needed data, the best schedule would be obvious. Experience contradicts this, but an interesting sidelight on data collection is that users of the model often gain valuable insights in the process of assembling the data. It is also true that gathering the input usually takes considerable effort. The purpose of this section is to indicate that the obstacles involved are surmountable. We make no pretence of covering all situations but we can suggest a few useful ideas.

A complete list of input requirements is given in the appendix. Certain items are straightforward. They have been developed many times before and the conceptual and measurement problems are minimal. In this category we put most market segment data, e.g., definition, population, sales potential, and seasonality. In constructing sales potentials, it is well to remember the use to which the numbers are to be put. For example,

if a branded product shows wide regional variations in level of distribution and this situation is fairly stable, a realistic sales potential for the model would be high where distribution is high, and low where distribution is low.

Media data that are reasonably straightforward include the list of media options, market segment coverage for each vehicle, the cost per insertion, the probability exposure to the option for a member of the audience of the vehicle, and audience duplication between pairs of vehicles. It is expected that audience duplication data may only be available for the total population, but, as previously indicated, it can usually be broken down to individual market segments by empirically developed estimating equations. Upper bounds on the number of insertions are generally the result of physical limitations on the number of issues, shows, etc. available in one time period. Policy restrictions may also enter. Ordinarily it seems desirable to let the model optimize freely without arbitrary constraints, but realistically these exist and, in addition, by permitting them in the model, they can be tested for their effect on the solution.

Media and market segment data are often pieced together from a variety of sources. One important source for consumer products is a national survey in which people are simultaneously interviewed as to demographic characteristics, product use, and media habits. National surveys, however, may yield rather small samples for individual market areas and for local or relatively rare but possibly efficient media. Sometimes a fruitful approach is to survey high potential groups directly to uncover the media they use.

The more difficult inputs are: the exposure values for the various media, the memory constant, and the individual response function.

The setting of the exposure values can be broken into three parts: (1) the setting of relative values among broad media classes, as TV, magazines, and newspapers; (2) an adjustment for individual vehicles or options within a media class, e.g., LIFE, LOOK, and NEWSWEEK; and (3) an adjustment for market segment, e.g., men, women, and children. The latter two parts are usually handled judgmentally and often represent rather small adjustments. (Recall that exposure value has nothing to do with cost, exposure probability, or sales potential but rather with whether it is preferred to have a person see an ad in one vehicle or another and whether it is thought that an exposure will have a greater effect on a person in one market segment or another in terms of increasing his percent of potential realized.)

For setting relative values among media classes, an "economic equilibrium" approach can be useful. First a portfolio of media options is formed for each media class. The portfolio is a sample from the principal vehicles that advertisers use. Then, on the basis of some standard space unit for the class, e.g. black and white full pages in magazines, a cost per thousand exposures is calculated for the class. Next it is assumed that economic forces tend approximately to equalize the value obtained from different media classes when taken as a whole. Under this hypothesis, exposure value is proportional to the reciprocal of cost per thousand. Some class can be assigned the value 1.0 and then the values for other classes are calculated.

Notice that use of this method does not imply that all media classes will be equally attractive to a given advertiser. He will have his own circumstances, particularly with respect to market segmentation, sales

potential, and media coverage of the segments. In addition, he may have special communications opportunities in a certain media class because of the particular needs of his product and these may lead him to adjust the exposure value for the class.

With respect to the memory constant and individual response, a certain number of empirical studies have been published. Zielske [30] displays data on recall vs. time, as does a more recent Simmons report [31]. A BBDO booklet [33] summarizes several studies and gives a bibliography. Examples of published work displaying diminishing returns phenomena are Benjamin and Maitland [2], who measure the effect of advertising on sales, and Rohloff [34], who measures pre-post brand choice scores. These studies tend to support the basic concepts of the model and offer insight into the range of effects to be expected. When it comes to setting values for a specific application, we have generally found that media planners are able to make judgmental estimates of the needed quantities. To aid the process we have evolved a short series of questions about response. (See computer trace in Section 6.) The answers are then used to develop a response function. As with any other part of the input, if the user feels the values of some constraints are known only within a range, he can make several runs with different values to test the sensitivity of the results.

Some companies are fortunate enough to have performed field experiments that measure the effect of advertising exposures or expenditures on sales. Such measurements can be used to calibrate the model. The particular way of doing this will depend on how the experimental results are presented, but, to illustrate, suppose that the measurements indicate

that a 10% increase in advertising spending would result in a specified sales increase and that a 10% decrease in spending would produce a specified sales decrease under the conditions of last year's media schedule. Then, using last year's schedule and all the model parameters except the response function, one can use the model to calculate exposure levels in each market segment and also levels 10% higher and lower. Then the parameters of the response function can be determined so that the model-calculated results match the experimental values at the given points. When calibrated in this way, the model makes an allocation that is consistent with the company's best information about sales response.

6. MEDIAC: An On-Line Media Selection System. The model and heuristics have been implemented on a time-shared computer. This permits a close interaction between user and model. In particular the user obtains immediate on-line access from a remote terminal, English language communication, and self-explanatory operation. Working with the system on-line gives the media planner an intuitive feel for the behavior of the model and the selection process that is difficult to obtain otherwise. The model becomes not a mysterious black box, but a routine tool that acts in rather ordinary and expectable ways.

A major advantage of time-sharing is that an organization with relatively low total computer usage can gain access to a powerful machine without incurring the elaborate overhead in personnel, space, and cost that usually accompany big machines. The computer used for the example below is an SDS940 at a commercial time-sharing firm.

We have called the on-line system MEDIAC. Its principle capabilities currently include: input, data storage, data alteration, schedule evaluation,

schedule selection, and output. With respect to size, we have solved a problem with 24 media options, 10 time periods, and 15 market segments and substantially larger problems are feasible.

The system presently uses an exponential response function of the form (4) with $r_0=0$. The a and b are fit to the answers from the response questions. Two moments are used in the Taylor series. End effects are treated by extending the planning horizon two periods without new insertions. Starting exposure levels are requested of the user. They can be generated in advance by a separate run.

The operation of MEDIAC is best demonstrated by example. A sample problem is worked out in detail below. The problem involves four media, eight time periods, and two market segments. A summary of all input data is given in Table 1. Detailed definitions of the data categories may be found in Appendix 1. The data are completely hypothetical. (We had orginally planned to show a problem whose output has been implemented but it would take up too much space.) Although the example is set up as a consumer product, it could as well have been an industrial product.

The transcript of the on-line computer session is shown in Table 2. All lines with → in front were typed in by the user at a teletype terminal. The rest was typed back by the computer, except for the explanatory notes added later at the right.

7. Discussion. We have presented a calculus for selecting advertising media. By a "calculus" we mean a system of numerical procedures for transforming data and judgments into a media schedule. The goal has been to develop a tool for today, an improvement in the state of the art relative to present practice. Our calculus uses data that is available or procurable

PRODUCT: "MINI-WIDGETS"

Budget: $350,000

Time period : 8 weeks

| Media Options (4): | 60 sec. TV Program A | 60 sec. TV Program B | 4 color page Magazine A | 4 color page Magazine B |
|---|---|---|---|---|
| Cost/insertion | $25,000 | $45,000 | $26,000 | $10,000 |
| Exposure probability for audience member | .9 | .9 | .7 | .4 |
| Exposure value | 2.0 | 2.5 | 1.5 | .75 |

Upper bounds: 1 insertion/period for each media option
Audience seasonality: none

| Market Segments (2): | Men over 20 | Women over 20 |
|---|---|---|
| Population | 45,000,000 | 50,000,000 |
| Sales potential ($/person/week) | .05 | .14 |
| Seasonality | none | none |
| Initial exposure value | 0 | 0 |

Memory constant: .6

% Potential Realized:

| | |
|---|---|
| at saturation | 15 |
| 1 average exposure | 6 |
| 2 average exposures | 9 |
| 3 average exposures | 11 |

Market Coverage:

| | Men | Women |
|---|---|---|
| TV A | .01 | .20 |
| TV B | .25 | .18 |
| Mag A | .35 | .20 |
| Mag B | .01 | .17 |

Media Vehicle Duplication:

| | TV A | TV B | Mag A | Mag B |
|---|---|---|---|---|
| TV A | .067 | .020 | .030 | .015 |
| TV B | | .110 | .070 | .025 |
| Mag A | | | .150 | .035 |
| Mag B | | | | .050 |

Table 1. Data for Sample Problem

```
-→ @COP TEL TO /ADV/
    NEW FILE
```

Begin data bank generation program.

```
 → @XFOS

 → LOAD FROM: /2INP/

   READY
 → +GO
   TYPE NO. OF DOLLARS IN BUDGET,F9.
 → 350000.,
   TYPE THE NO. OF TIME PERS,I3
 → 8,
```

The computer asks for all data needed. The F, I, and A letters refer to input format.

```
   TYPE NO. OF MKT SEGMENTS,I3
 → 2,
   TYPE NO. OF MEDIA,I4
 → 4,
    TYPE THE PERCENT OF POTENTIAL REALIZED
   AFTER COMPLETE SATURATION WITH EXPOSURES,F4
 → 15.,
   TYPE PERCENT OF POTENTIAL REALIZED
    AFTER  1AVERAGE EXPOSURES/CAPITA,F4.
 → 6.,
   TYPE PERCENT OF POTENTIAL REALIZED
    AFTER  2AVERAGE EXPOSURES/CAPITA,F4.
 → 9.,
   TYPE PERCENT OF POTENTIAL REALIZED
    AFTER  3AVERAGE EXPOSURES/CAPITA,F4.
 → 11.,
   TYPE NAME OF MKT SEG    1 A6
   MENO20
   TYPE NO. OF PEOPLE,POTENTIAL FOR SEGMENT MENO202F9.
 → 45000.,.05,
   TYPE NAME OF MKT SEG    2 A6
 → WOMO20
   TYPE NO. OF PEOPLE,POTENTIAL FOR SEGMENT WOMO202F9.
 → 50000.,.14,
    TYPE MEMORY CONSTANT,F4.
 → .6,
   TYPE NAME OF MEDIA    1 A6
 → ATELEV
   TYPE EXPOSURE VALUE,PROB. OF EXPOSURE,2F3.,OF ATELEV
 → 2.,.9,
   TYPE NAME OF MEDIA    2 A6
 → BTELEV
   TYPE EXPOSURE VALUE,PROB. OF EXPOSURE,2F3.,OF BTELEV
 → 2.5,.9,
   TYPE NAME OF MEDIA    3 A6
 → AMAGAZ
   TYPE EXPOSURE VALUE,PROB. OF EXPOSURE,2F3.,OF AMAGAZ
 → 1.5,.7,
   TYPE NAME OF MEDIA    4 A6
 → BMAGAZ
   TYPE EXPOSURE VALUE,PROB. OF EXPOSURE,2F3.,OF BMAGAZ
 → .75,.4,
   IF THERE IS NO MEDIA SEASONALITY,TYPE 1,OTHERWISE 2
 → 1,
```

TABLE 2

```
  IF SEG. COVER. OF MOST MEDIA IS 0.,TYPE 1,ELSE2
→ 2,
  TYPE MKT. SEG. COVERAGE OF ATELEV SEGMENTS
  MENOWOMO
  .XXX.XXX
→ .310.200
  TYPE MKT. SEG. COVERAGE OF BTELEV SEGMENTS
  MENOWOMO
  .XXX.XXX
→ .250.180
  TYPE MKT. SEG. COVERAGE OF AMAGAZ SEGMENTS
  MENOWOMO
  .XXX.XXX
→ .350.200
  TYPE MKT. SEG. COVERAGE OF BMAGAZ SEGMENTS
  MENOWOMO
  .XXX.XXX
→ .010.170
  TYPE COST PER INSERT F6. FOR  ATELEV
→ 25000.,
  TYPE COST PER INSERT F6. FOR  BTELEV
→ 45000.,
  TYPE COST PER INSERT F6. FOR  AMAGAZ
→ 26000.,
  TYPE COST PER INSERT F6. FOR  BMAGAZ
→ 10000.,
  TYPE NO OF SEGS WITH SEASONAL POTENTIAL
→ 0,
  TYPE NO. OF CASES(PERIODS*MEDIA) WITH
  UPPER BOUNDS NOT EQUAL TO ONE
→ 0,
  TYPE1 IF DUPLS ARE AVAI, 2MEANS INDEPENDENC
→ 1,
  TYPE DUPLICATIONS OF ATELEV WITH
  ATELBTELAMAGBMAG
  .XXX.XXX.XXX.XXX
→ .067.020.030.015
  TYPE DUPLICATIONS OF BTELEV WITH
  BTELAMAGBMAG
  .XXX.XXX.XXX
→ .110.070.025
  TYPE DUPLICATIONS OF AMAGAZ WITH
  AMAGBMAG
  .XXX.XXX
→ .150.035
  TYPE DUPLICATIONS OF BMAGAZ WITH
  BMAG
  .XXX
→ .050

  STOP

  +
  +
→ @COPY /ADV/ TO /DATA BANK/
   NEW FILE
```

The ".310" typed in was an error which will be corrected later.

The data bank for this problem is now created.

The user copies the input data onto a permanent disk file.

```
@COPY /DATA BANK/ TO /ADV/
 OLD FILE


@XFOS

LOAD FROM: /MEDIAC2/

LOAD SUBPROGRAMS FROM: "XFL1"
READY
SPACE AVAILABLE --> 23 WORDS
```

The user asks for the MEDIAC 2 media selection program which uses data from the data bank.

```
+GO
 TYPE 1 IF DATA CHANGE WANTED,OTHERWISE 2
1,
 TYPE 1 FOR COST,2 EXPOSURE VAL.,3 MKT SEG COVERAGE
 4 FOR MEMORY,5 POTENTIAL,6 UP BDS,7 MEDIA SEASONALS
8 RESPONSE,9 BUDGET,10 POTENTIAL SEASONALS,11 DUPLIC.
3,
 TYPE THE NUMBER OF CHANGES
1,
  MEDIA NO, MKT SEG,COVERAGE , 2I2,F4.
1,1,.01,
ATELEV SEG MENO20COVERAGE  .010
 IF MORE CHANGES TYPE1,ELSE2
2,
```

The user corrects his input error in coverage of "A TELEV" in segment "MENO20"

```
TYPE 1 TO DO SELECTION,3 TO END PROGRAM
1,
TYPE INITIAL EXPOSURES/CAP.IN SEGMENTMENO20 F4.
.0,
TYPE INITIAL EXPOSURES/CAP.IN SEGMENTWOMO20 F4.
.0,
TYPE 1 IF RANKING WANTED,2 FOR FULL ALLOCATION
2,
 TYPE 1 IF SOME MEDIA HAVE ALREADY BEEN SELECTED
2,
```

If some media must be in schedule, they are added here.

```
1 INSERT IN ATELEV TIME PER  1 COST    25000.REALIZED POT.     331.
1 INSERT IN ATELEV TIME PER  4 COST    50000.REALIZED POT.     654.
1 INSERT IN ATELEV TIME PER  7 COST    75000.REALIZED POT.     934.
1 INSERT IN AMAGAZ TIME PER  8 COST   101000.REALIZED POT.    1167.
1 INSERT IN AMAGAZ TIME PER  5 COST   127000.REALIZED POT.    1449.
1 INSERT IN AMAGAZ TIME PER  2 COST   153000.REALIZED POT.    1742.
1 INSERT IN AMAGAZ TIME PER  1 COST   179000.REALIZED POT.    2016.
1 INSERT IN AMAGAZ TIME PER  4 COST   205000.REALIZED POT.    2277.
1 INSERT IN AMAGAZ TIME PER  7 COST   231000.REALIZED POT.    2508.
1 INSERT IN ATELEV TIME PER  8 COST   256000.REALIZED POT.    2682.
1 INSERT IN BTELEV TIME PER  8 COST   301000.REALIZED POT.    2988.
1 INSERT IN BTELEV TIME PER  1 COST   346000.REALIZED POT.    3402.
1 INSERT IN BTELEV TIME PER  4 COST   391000.REALIZED POT.    3790.
 TYPE1 FOR DETAILED OUTPUT,ELSE2
1,
 SEGMENT TIME P EX VAL/CP  REALIZED POTENTIAL
  MENO20    1     .94800       79.
  MENO20    2     .93630       90.
  MENO20    3     .56178       65.
  MENO20    4    1.28507      117.
  MENO20    5    1.13854      114.
  MENO20    6     .68312       81.
  MENO20    7     .79537       86.
  MENO20    8    1.42522      123.
```

The MEDIAC 2 system has selected the above schedule. The user also wants to see a detailed output of the expected effects of the selected schedule in each segment.

```
   MENO20    9    .85513      92.
   MENO20   10    .51308      66.
   WOMO20    1    .97500     266.
   WOMO20    2    .79500     253.
   WOMO20    3    .47700     180.
   WOMO20    4   1.26120     375.
   WOMO20    5    .96672     323.
   WOMO20    6    .58003     224.
   WOMO20    7    .91802     310.
   WOMO20    8   1.52581     413.
   WOMO20    9    .91549     307.
   WOMO20   10    .54929     226.
  TYPE 1 IF DATA CHANGE WANTED,OTHERWISE 2
→ 1,
  TYPE 1 FOR COST,2 EXPOSURE VAL.,3 MKT SEG COVERAGE
  4 FOR MEMORY,5 POTENTIAL,6 UP BDS,7 MEDIA SEASONALS
 8 RESPONSE,9 BUDGET,10 POTENTIAL SEASONALS,11 DUPLIC.
→ 4,
 TYPE MEMORY CONSTANT,F4.
→ .15,
  IF MORE CHANGES TYPE1,ELSE2
→ 2,
 TYPE 1 TO DO SELECTION,3 TO END PROGRAM
→ 1,
 TYPE INITIAL EXPOSURES/CAP.IN SEGMENTMENO20 F4.
→ .0,
 TYPE INITIAL EXPOSURES/CAP.IN SEGMENTWOMO20 F4.
→ .0,
 TYPE 1 IF RANKING WANTED,2 FOR FULL ALLOCATION
→ 2,
  TYPE 1 IF SOME MEDIA HAVE ALREADY BEEN SELECTED
→ 2,
 1 INSERT IN ATELEV TIME PER  1 COST    25000.REALIZED POT.      137.
 1 INSERT IN ATELEV TIME PER  4 COST    50000.REALIZED POT.      274.
 1 INSERT IN ATELEV TIME PER  7 COST    75000.REALIZED POT.      411.
 1 INSERT IN AMAGAZ TIME PER  8 COST   101000.REALIZED POT.      547.
 1 INSERT IN AMAGAZ TIME PER  3 COST   127000.REALIZED POT.      683.
 1 INSERT IN AMAGAZ TIME PER  6 COST   153000.REALIZED POT.      818.
 1 INSERT IN AMAGAZ TIME PER  2 COST   179000.REALIZED POT.      950.
 1 INSERT IN AMAGAZ TIME PER  5 COST   205000.REALIZED POT.     1080.
 1 INSERT IN AMAGAZ TIME PER  1 COST   231000.REALIZED POT.     1207.
 1 INSERT IN ATELEV TIME PER  8 COST   256000.REALIZED POT.     1325.
 1 INSERT IN AMAGAZ TIME PER  4 COST   282000.REALIZED POT.     1447.
 1 INSERT IN AMAGAZ TIME PER  7 COST   308000.REALIZED POT.     1568.
 1 INSERT IN ATELEV TIME PER  2 COST   333000.REALIZED POT.     1682.
 1 INSERT IN ATELEV TIME PER  5 COST   358000.REALIZED POT.     1796.
  TYPE1 FOR DETAILED OUTPUT,ELSE2
→ 2,
  TYPE 1 IF DATA CHANGE WANTED,OTHERWISE 2
→ 2,
 TYPE 1 TO DO SELECTION,3 TO END PROGRAM
→ 3,

 STOP
```

The user wishes to change the memory constant to .15 from .6, reflecting an assumption that people will remember less from week to week. The user then asks for a redo of the media selection.

The selection has changed to using fewer media vehicles more times reflecting the slower build up to saturation due to more consumer forgetting.

along with those judgments that seem essential to define a solution. The on-line computer system is fast, easy to use, and inexpensive relative to the importance of the problem and other models of comparable scope.

There are some things the model is and some it is not. It is an allocation model; i.e., it takes a fixed budget and spreads it over time and market segments. It is not, however, a budgeting model. If market response is expressed as sales, the model appears to be capable of determining an optimal advertising budget. Such a use is unwarranted unless the model has been calibrated on sales response data. The reason is that although the allocation of a fixed budget depends on the shape of the response curve, it will be fairly insensitive to modest changes and will be completely insensitive to changes in scale factor. On the other hand, the optimal budget will be quite sensitive to such changes. For example, if the response function were multiplied by large constant, the allocation would not change but the optimal budget would change substantially.

We have in mind a number of extensions of the model and of the on-line system. These include the effect of competitive advertising, the rub-off effect of other advertising by the same firm, and the possibility of certain synergistic effects. Undoubtedly, still others will be developed.

Experience with using the model has been very encouraging. About a million and a half dollars of advertising have been scheduled and implemented in the few months that the model has been operational. Although we have found some people who definitely do not want to quantify their media decisions, we have found a growing number who find the system a distinct aid. Improvements in the objective functions, as defined by the users, have ranged from about 5% to 25% relative to previous schedules. Some model-computed schedules have

looked much like previous ones; others have been quite different. In cases that have looked different, it has been possible to find out what data or phenomena have caused the change. So far the media planner has invariably preferred the new schedule.

In trying to assess the effect of the system on the users, we find that the most important contribution at the present stage is the introduction of a relatively comprehensive logical structure. People often show a tendency to pick out one or two important issues of a problem and let information on these make the decision. The model leads people to look at many issues and ferret out the information they have on all of them. Then the model permits the information to be interrelated in a unified way. Usually, a relatively few numbers are in fact the key determinants of the decision, but not always are they the numbers thought to be important in advance.

# APPENDIX

## MEDIAC II INPUT

I. <u>Media Characteristics</u>: Data Needed for Each Media Option.

(Examples of media options are a one page black and white bleed ad in SPORTS ILLUSTRATED, a one minute spot on BONANZA, or the show BONANZA.)

1. The option's name.
2. Cost per insertion of the option.
3. Exposure probability for audience member. The probability a person is exposed to the particular ad in the vehicle given that he is in the audience of the vehicle, (e.g., the probability that a reader of SPORTS ILLUSTRATED will see the one page black and white bleed ad.)
4. Upper bounds on insertions. The maximum number of times the ad could be run in the media vehicle in each time period.
5. Audience seasonality. The audience size for each time period for the media vehicle, expressed as an index with an average value of 1.0. If audience size is not seasonal, no data need by supplied.
6. Exposure value. The value of an exposure may differ from one media option to another. Exposure value answers the question: Given the choice of a person seeing an ad in LIFE or the same person seeing it in LOOK, does the advertiser have any preference, and, if so, what is the statement of that preference? In the same manner, intermedia exposure values are also rated, e.g., an exposure to a 30-second radio spot is to be rated on the same scale as an exposure to a half page newspaper ad. The units for exposure value are

arbitrary except that they must be tied to a response function. It is best to conceive of an average media option and assign it a value of 1.0 and then assign values for other media options relative to it.

II. Market Characteristics: Data needed for each market segment.

1. Segment name.
2. Population of the segment.
3. Sales potential per person in the segment. The units in which sales potential is measured are chosen by the user.
4. Seasonality of sales potential. This is an index with a value for each time period in the advertising plan plus two time periods for ending effects. The average value over a full year is 1.00. If potential is not seasonal, no data need by supplied.
5. Initial average exposure value per person in the segment. As a substitute for this data, a list of the media insertions planned for two months before the computer generated schedule is to start will suffice; e.g., if the MEDIAC system is to plan insertions for January through December of 1968, then the planned insertions for November and December of 1967 can be used to establish initial conditions in each segment.

III. Media - Segment Data

1. Market coverage. For each media vehicle in each segment, the fraction of the segment population who will be in the audience of the media vehicle. E.g., the fraction of people in each segment who will read SPORTS ILLUSTRATED, or watch BONANZA. Essentially, this amounts to rating points in the market segment.

IV. Media Vehicle Duplications

1. Audience duplication. For each possible pair of media vehicles the fraction of people out of the total in all segments who will be in the audience of both vehicles. E.g., the fraction of people who read both SPORTS ILLUSTRATED and LIFE. Also needed is the fraction of people who will be in the audience of two appearances of the vehicle. If duplication data is not available, the system will approximate them using the assumption of independence between media.

V. Other Data Needed

1. Memory constant. The fraction of a person's exposure value that is remembered from one time period to the next.
2. The percent of potential realized after saturation with exposures.
3. The percent of potential realized when one, two and three average exposures are retained by a person. (An everage exposure is defined as an exposure to a media option with exposure value of 1.0.) These inputs may be viewed as expressing the expected effect of having one, two and three exposures presented to a person in a short period of time. When combined with the saturation level, these inputs determine the diminishing returns aspect of exposures.
4. Number of media options, market segments, and time periods.
5. Budget.

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Date Due

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| MAR 27 '78 | | |
| AG 29 '91 | | |