Macroeconomic Models

for

Computer Simulation

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CONTENTS

PREFACE

This text is designed to serve as a supplement to intermediate-level courses in macroeconomic theory. Its raison d'etre is that simulation techniques can be profitably employed in the teaching and learning of macroeconomics. These methods are useful, first, because they make it possible to consider fairly complex systems, systems which go well beyond the usual textbook models in embodying many of the basic element of economic reality. Second, it has been the author's experience that students enjoy simulation. They find that actual numerical solutions to models provide more insights and ideas than complex general formulae. Where interest and involvement are high, learning is facilitated.

The models discussed in this text were designed and implemented on the TROLL system at M.I.T. Chapter II provides a brief introduction for students to the use of this system, and two appendices show how the models and their associated data files "look" in TROLL. The instructor should be familiar with the TROLL manual. Other simulation systems do exist that can handle these models, but TROLL should be used if possible.

We then examine some static models. Chapter III presents two models that illustrate the fact that markets are interdependent. They are models of general economic equilibrium in competitive economies involving two good and two factors of production. Chapter IV presents

two variants of the standard textbook IS-LM macro model. We have incorporated the government budget constraint in these models, though in a rather simple fashion.

The remainder of the text is concerned with a sequence of dynamic models. Chapter V is a compact discussion of difference equations and distributed lags. It is certainly not necessary that students master all of the mathematic presented, but they should have a good grasp of the key concepts before reading subsequent chapters. Chapter VI discusses a basically simple model that involves the multiplier, the accelerator, and natural barriers to the motion of GNP. This chapter is necessary for those that follow, as the models presented in Chapters VII - IX are essentially modifications of the simple model of Chapter VI.

In Chapter VII, distributed lags are added at several points in the system. In Chapter VIII, the basic model is further modified to take account of businesses' production decisions. Changes in final goods inventories, both planned and unplanned, are introduced. The model presented in Chapter IX adds a market for real cash balances which determines the rate of interest as an endogenous variable. The government budget constraint is not considered in this model.

We shall attempt to explain and motivate all the models presented in this text. But our discussion will be confined exclusively to those models. This is not designed to be the only textbook in a macroeconomics course. It is assumed that students know some macroeconomics, and it is further assumed that they are receiving more general information

than that presented here. The combination of the usual readings and lectures and some experience with simulation models should produce a deeper understanding of macroeconomics than could the conventional materials alone.

Each chapter contains a discussion of the kinds of exercises that can be performed with the models it presents. These discussions are not intended to be exhaustive. Especially in the case of the last three dynamic models (Chapters VII - IX), there are a variety of interesting things that can be explored. Short papers should be assigned on several models. Part of the assignment might involve one or more of the specific problems mentioned. Students should, however, feel free to explore the models on their own; the assignments should be understood to be somewhat open-ended. Curiosity and learning go hand in hand.

Few works as long as this one are truly the work of only one individual . This text is definitely not. I am indebted to the Edwin Land Foundation for considerable financial support. Charles Revier, Stephen Fisher, and Daniel Luria provided able research assistance. Robert Solow's comments caused me to make major changes in the models of Chapter IV. Edwin Kuh provided the inspiration and incentive for this project, worked with me on the design of the models, and incisively criticised earlier drafts of this text. Mark Eisner and the rest of the TROLL staff helped us considerably, and Mark wrote the second part of Chapter II. Finally, I would like to thank the students who endured my experiments with the materials presented herein.

Part I

Introduction

CHAPTER I

Introduction and Overview

Models

Any discussion of observable phenomena makes use of models. These may be explicit or implicit. That is, the models may be presented in complete detail, or they may be implicitly present in statements about the world. Newtonian physics is based upon an explicit model, while the assertion that deficit financing of government spending is the road to economic ruin derives from an implicit model of the economic system.

The crippling difficulty with implicit models is that their implications cannot be logically deduced and tested. Implicit models cannot be proved or disporoved. Disciplines that aspire to the status of sciences are concerned with developing explicit models of the parts of reality that concern them. Such models can be rigorously tested, applied, and checked for internal consistency.

One can divide explicit models into verbal and mathematical models. Mathematical models have the advantages of clarity and definiteness; the language of mathematics is often more precise than English or any other spoken language. In addition, mathematical models of complex phenomena are usually a good deal easier to analyze than the corresponding verbal models. The tools of mathematics and the methods of deductive reasoning are in some fundamental sense the same, but the former are often easier to apply.

Another useful distinction is between static and dynamic models. Static models do not involve time in an essential way. Such models react instantly to changes in their environments. The usual models of price theory are static in this sense. In the theory of household behavior, for instance, no mention is made of the time it takes a household to react to changes in relative prices. Dynamic models describe systems that evolve over time. Almost all of the physical sciences are based on dynamic models. A model that describes changes in the velocity of a falling ball is dynamic; a model whose only predictio is that a ball with no forces acting to hold it up will fall is static.

It is possible to use static models in situations where the phenomena being considered are essentially dynamic. We say that a dynamic system is stable if it returns to a condition of either rest or steady evoluation after it is disturbed. A pendulum is a stable system; a ball at rest on a mountain top is not. Static models can be used to compare the equilibrium positions of stable systems - indeed, this is their main use in economics. Models of equilibrium situations tell us where a dynamic system is tending, though they will be unable to say anything about the path to the new equilibrium.

Variables

There are two basic types of variables in all models. Exogenous variables are those determined outside the model being considered.* Rainfall is exogenous to most economic models, though it is endogenous in meteorological models. Endogenous variables are those determined within the model being considered. (It is perhaps clearer to say that endogenous variables are those whose values are determined by the model under consideration). Notice that variables are defined as endogenous or exogenous with regard to a particular model; this distinction does not refer to any particular characteristic of the variable. Gross National Product may sensibly be taken as exogenous to a model explaining the price of beet sugar, but it will be an endogenous variable in models concerned with the level of aggregate economic activity.

We may distinguish two kinds of exogenous variables. First, there are those quantities that come from outside the system and that cannot be readily controlled. An example would be rainfall in economic models. Such variables are simply termed exogenous. A second class of variables is comprised of those directly under the control of policy-makers. The usual example here is government spending. These quantities are usually refered to as policy variables.

In dynamic models, the distinction is made between endogenous and predetermined variables. All exogenous variables are predetermined. So are lagged values of endogenous variables. Consider a model in which consumer spending last quarter influences consumer spending

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* Students of econometrics will recall that exogenous variables are assumed to be distributed independently of the disturbance terms in behavioral equations.

this quarter. The equation for consumer spending will have as one of its variables lagged spending, and that quantity is a predetermined variable.

Constants that influences a model's behavior are called parameters. Two classes of parameters must be distinguished. The first are structural or behavioral parameters. An example would be the marginal propensity to consume, a constant that is inherently part of most macroeconomic systems. Changes in structural parameters represent changes inthe system, while changes in policy parameters represent actions upon the system. An example of a policy parameter might be personal income tax rates. Clearly there is no hard and fast distinction between policy variables and policy parameters. In what follows, we shall generally speak of all policy instruments as parameters. We do this mainly because it is compatiable with the useage of the TROLL computer system, discussed in detail in the next chapter.

Some of this discussion may be clarified by two simple examples. Consider a simple, static, Keynesian macroeconomic model:

$$(1.1) \quad Y = C + I + G$$

$$C = a + bY$$

$$I = c + dY$$

Consumption is denoted by C, investment by I, government spending by G, and gross national product by Y. In this model, C, I, and Y are endogenous. G is exogenous; it is either a policy variable or a policy parameter, depending on convention. The constants a, b, c, and d are structural parameters of the system.

Suppose now that the economy is a dynamic one. A simple extension of model (1.1) is the following:

$$(1.2) \quad Y(t) = C(t) + I(t) + G(t)$$

$$C(t) = aY(t) + bC(t - 1)$$

$$I(t) + c[Y(t) - Y(t - 1)] + dI(t - 1)$$

The quantities in parentheses refer to time periods. Thus Y(t), C(t), and I(t) relate to the current time period. These are our endogenous variables, and G(t) is exogenous as before. The constants a, b, c and d are still of course, structural parameters. The new elements are the predetermined variables that appar in model (1.2); these are C(t - 1), I(t - 1), and Y(t - 1). The last two equations relate quantities in different time periods, and they are called difference equations. In both these equations, the lagged values of the left-hand variables act to spread or distribute the influence of Y over time. These equations are said to embody distributed lags. Difference equation systems, especially systems with distributed lags, are very important in dynamic economics, and Chapter V will examine them in some detail.

Analysis

How does one work with a model? Verbal models can only be manipulated verbally, according to the rules of deductive logic. For simple models, this is quite satisfactory. But when the models become

at all complicated, this is difficult or impossible and often leads to errors. Mathematical models of complicated situations are easier to work with than equivalent verbal models.

Mathematical models can be examined two ways. The first approach is to solve the model <u>analytically</u>. The model is manipulated until the endogenous variables are written as functions of the predetermined variables and the parameters determining the structure of the system. A model expressed in this way is said to be a <u>reduced form</u>. The original expression of most models involves equations in which endogenous variables are functions of bothendogenous and predetermined variables; this is said tobe the <u>structural form</u>. Both models (1.1) and (1.2) are written in structural form. The reduced form of (1.1) is

$$Y = (a + c + G)/(1 - b - d)$$

$$(1.3) \quad C = a + b(a + c + G)/(1 - b - d)$$

$$I = c + d(a + c + G)/(1 - b - d).$$

Here the only predetermined variable is G, which is exogenous. The reduced form of (1.2) would involve C(t), I(t) and Y(t) as functions of the predetermined variables G(t), Y(t - 1) , C(t - 1), and I(t - 1). Of this list, only G(t) is exogenous.

Once the reduced form is obtained, it is usually fairly easy to examine the effects of changes in the predetermined variables or parameters. In static models, simple algebra is involved. In dynamic models, the reduced form represents the first step in the solution of

of a set of difference or differential equations, and the effects of equilibrium and dynamic behavior can be found.

When models are at all complicated, analytical solution poses a number of problems. Sometimes, no analytical solution exists. This is often the case when the structural equations are not linear in the endogenous variables. When an analytical solution can be obtained, it describes the system's behavior under all possible conditions. Generality is to be prized, but is often purchased at the cost of complexity. Analytical solutions are often quite complicated and it is very difficult for most people to obtain much insight from complex formulae.

The alternative to analytical solution is simulation Simulation does not involve examining a model's behavior under all possible conditions. Rather, simulation consists in generating the response of a system to particular changes in exogenous conditions or to particular changes in the structure of the system itself. Instead of formulae, simulation yields numbers - either as tables or graphs. Simulation tends to yield greater intuitive understanding of complex models than analytical methods, but this understanding relates only to the particular set of numerical values used in the simulations.

Whether this is a great handicap or not depends on circumstances. If the marginal propensity to consume has always been between .85 and .97, a clear understanding of how the economy behaves when this parameter takes on values in this range will be more useful than a complicated

formula that permits solution for the case where MPC = .25. On the other hand, if it is desired to pick the best tax rate for some purpose, and no constraints on the rate are present, an analytical solution may well yield more insight.

How does simulation work? A detailed description of the methods used would take us far afield, but we can easily indicate the general principles involved. Consider a static mathematical model. All such models that we will be concenred with may be written as a system of simultaneous equations. There will be N equations and the N unknown endogenous variables. Most general - purpose simulation programs use iterative techniques to solve such systems.* The user supplies an initial guess as to the values of the endogenous variables. On the first interaction, the computer uses these values to generate a set of trial values, another set of guesses. These are then used on the second iteration to generate anotherset of values, and so on. When there is a little difference between Nth such guess and the N + 1st guess, the N + 1st guess is taken as the solution. If the system has been correctly set up, the sequence of guesses will converge to the true solution.

The process of solution of dynamic models is similar. The system is solved for the first period using an iterative method. The

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* When the model can be expressed as a system of simultaneous linear equations, exact solutions based on matrix algebra can be used efficiently.

solution values of the endogenous variables will be functions of the exogenous variables, the structural and policy parameters, and the lagged values of the endogenous variables supplied by the user as initial conditions. The solution values of the endogenous variables for the first period are then used as predetermined variables in the solution for the second period, and so on. In order to examine the dynamic properties of a system, many periods may need to be simulated in this fashion.

This Text

The models discussed in this text were designed and implemented on the TROLL system at M.I.T. Chapter II provides a brief introduction to the use of this system with these models, and two appendices show how the models and their associated data files "look" in TROLL.

In part II, we present four static models. The models of Chapter III illustrate market interdependence. The economies they depict are more often discussed in microeconomics than in macroeconomics, but analyzing them provides a good foundation for macroeconomics. Chapter IV presents two variants of the standard textbook IS-LM model. These models are based on the original theory of Keynes and its exposition by J. R. Hicks.

Part III of this text is concerned with a sequence of dynamic models. We first discuss economic dynamics in general and distributed lags in particular. The concepts developed in Chapter V will be useful in the subsequent chapters. We then discuss the simplest possible model involving both the

multiplier of static analysis and the accelerator of business cycle theory. Chapter VII presents a modification of the model in Chapter VI. The changes involve adding distributed lags at several points in the system. This model, and the ones presented in succeeding chapters, would be quite difficult to solve analytically. In Chapter VIII, the basic model is further modified to take account of businesses' production decisions. Changes in inventories, both planned and unplanned, are introduced. The first three dynamic models treat the interest rate as exogenous. Chapter IX adds a market for real cash balances which determines the rate of interest as an endogenous variable. This is a rather complex model; it mirrors most of the basic features of full-blown econometric models of the U. S. economy.

CHAPTER II

Simulation on the TROLL System

The TROLL time-shared computer system is rather complex. It can perform a variety of functions useful to economists in general and econometricians in particular. TROLL makes possible the estimation and simulation of large models on a routine basis. This chapter is not an introduction to the TROLL system. Our intent is rather to provide the reader with just the information needed to handle the models discussed in this text.

As this is being written in July of 1970, TROLL is available only on the CTSS time-sharing system, implemented on an IBM 1094 at M.I.T. Sometime early in 1971, the 7094 system will be discarded as an improved version of TROLL will be running on the IBM 360/67 time sharing syatem at M.I.T. The first section of this chapter discusses the use of 7094 TROLL. The second section, written by TROLL project director Mark Eisner, translates this discussion into 360/67 TROLL. You should read only the section that describes the system you will be working with.

We shall assume that the user wants to examine a model already installed in the system. Further, we assume that the user knows how to obtain access to TROLL. This will involve turning on a console, linking it to the central computer, and typing one or more passwords or other messages. Messages typed in at the console are in lower case and

and underlined, whereas the computer responses are in capitals.

Note that user responses during an actual console session must not be underlined.

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We shall examine a hypothetical console session. Assume the user has logged into the CTSS system and has gained access to TROLL. (How this is to be done will be specified.) The computer then types

GOOD EVENING

TROLL is now entered, and you are at the controller level.

The other level of 7094 TROLL is the phase level. The phases to be employed must be specified at the controller level, as well as the model you will analyze. The controller will then send you to the phases, where the work is performed. At the controller level, you have three commands at your disposal: DEFAULT, EXECUTE, AND QUIT:

DEFAULT - By typing default archnm , you are telling the system to look for data in the archive named "archnm". The archive name corresponding to each model described herein will be indicated below.

EXECUTE - The execute command allows the user to specify the model and phases he wishes to use. The phases of interest to us are simulation and simulation output. The list of phases must be terminated by an asterisk (*).

QUIT - When you have finished with TROLL, type quit to return to the command level of the CTSS system. The correct way to leave that system will be specified in class.

Let us suppose, for purposes of concreteness, that we wish to use the ISLM1 model (described in Chapter IV) and the associated data files in archive islm. The dialogue would then proceed as follows.

TYPE COMMAND - default islm

You have told the computer which data files you will be working with. Had you wished to work with either of the GE models (described in Chapter III), you would hafve typed default geneq to call for archive geneq. Data for the DYNEC models (ChaptersVI - IX) are in archive dynec. To specify which model the system should employ and what operations you will perform, you must proceed as follows.

TYPE COMMAND - execute islm1 simulation simout *

TROLL now knows which model to use. It also knows that you will perform some simulations and then look at your output in the simulation output phase. (TROLL has a number of other phases, but these will not concern us here.) Note the asterisk at the end of the phase list. Use of asterisks to end lists is a convention throughout 7094 TROLL.

TROLL will accept commands in a string. That is, you could have typed the above commands all on one line in response to the first "TYPE COMMANDS -" request as follows:

TYPE COMMANDS - default islm execute islml simulation simout *

If TROLL discovers an error in a command, for instance, a misspelled word, that command and all that follow it are ignored, but all that precede it are accepted. TROLL will print out a message explaining the error. If you notice an error before hitting the carriage return, you may type @ to delete (cause the computer to ignore) an entire line. You then start the line over. If the error is not so serious, you may use a number of #'s to tell the computer to ignore the preceding character or characters. For instance, the following line

then was @ now is not### thee# time for all

will be read by TROLL as

now is the time for all

After you have given default and execute commands as above, the computer will type

SIMULATION PHASE

SETUP SIMULATION

TYPE COMMAND - continue

RUNNAME - doodle

You may pick any collection of six or fewer letters as your run-name. This will identify your output for later examination in the simulation output phase.

TIME BOUNDS

START DATE-2

FINISH DATE-5

You have told the system that you will obtain three solutions to the static ISLM1 model. The first will be labeled year 2, and the last labeled year 5. All static models should have a start date of 2, and the dynamic models should be started in year 3.

CURUSE = 3

SIMULATION CONTROL

TYPE COMMANDS-

At this point the user has several options. First,the methods used by the system to find a solution may be altered. This is necessary for the GE1. GE2, ISLM1, and ISML2 models, but not for DYNEC1 - DYNEC4. If you are using either GE model, you must type the following:

tune divtune 5 5 10. ignore convtune 5 5 500 .0001
damp step continue

For the ISLM models, the following command must be issued here:

tune convtune 5 5 1000 .0001 damp .3 continue

Each model has a parameter file stored with it, giving values for important quantities in the model. Parameters correspond to policy variables that might be altered by the government, to exogeneous influences such as population, and to structural characteristics of the system such as the marginal propensity to consume. If you wish to find the solution corresponding to the pre-set parameter values, type process 1 in response to the system's request for a command after you have "tuned". The solution values will be stored and labeled year 2 for your later inspection. The computer will then type

SIMULATION CONTROL

TYPE COMMAND-

Suppose now that you want to change one or more of the parameters and see how this affects the solution. You would then type the command replace, followed by a list of the parameters to be changed and their new values. To raise G to 145. and lower MPC to .85, you would type

replace g 145. mpc .85 *

Any number of parameter-value pairs may be included, but the list must be terminated with an asterisk. Each new value must contain a decimal point, and it must be a postive number. Remember that once a parameter has been changed, it remains set to the new value until you leave the simulation phase, unless you "replace" it again. To observe the current values of any of the parameters, you may type

look parametername1 parametername2 ... *

followed by a carriage return after any "TYPE COMMANDS-" request from the computer.

After you have changed parameter values, the dialogue will procede as follows:

TYPE COMMAND - process 1

SIMULATION CONTROL

TYPE COMMAND-

The computer has now found solutions for years 2 and 3. If this is enough, type close in response to this request. If solutions for years 4 and 5 are desired, repeat the sequence of changing parameter values and processing as above. When year 5 has been considered, the computer will type

FINISH DATE HAS BEEN REACHED

TYPE COMMAND- close

Typing close causes the results of your simulation to be saved in files which can be identified by your runname.

If at any time TROLL indicates a failure to find a solution and asks for a request, type kill. You should then change some of the parameter values and try again.

In the case of static models, the one-year-at-a-time approach outlined above is probably the best way to proceed. The solution for year 4 will not depend at all on the solution for year 3; it will be determined entirely by the parameter values prevailing in year 4.

To examine the impact of parameter changes in dynamic models, however, this is not a useful approach. Changing something like the marginal propensity to consume will affect the evolution of the system through time. Instead of typing process 1 after a parameter change, one should type process n, where n is some number greater than one, or process finish. The latter command will cause the simulation phase to compute the evolution of the system from the year being considered until the finish date. Typing process 20, on the other hand, will cause the machine to examine what happens to the model in the next twenty periods - unless, of course, the finish date is reached before twenty periods have been considered.

In fact, the following approach may well be the best one for the dynamic models. At the controller level, instead of typing simulation simout* after execute amd the model name, one might type

simulation simout simulation simout simulation simout*

You might then replace parameters, type process finish, and look at the output. When you leave the simulation output phase, TROLL will place you back in the simulation phase automatically, and you can repeat the process.

Let us now examine the simulation output phase. After the user has typed close in the simulation phase, the computer will type

SIMULATION OUTPUT PHASE

RUN NAME: doodle

The runname tells the computer which simulation's output you wish to examine. There are two ways you can look at the results of your simulations. You may request tables of numbers, or you may ask for a graph of one or more variables over time.

To obtain tabular output, the dialogue is as follows.

TYPE COMMAND- pttype

TYPE PLOT OPTIONS-simulated

These two user responses tell the machine to print in tabular form the simulated values of the endogenous variables of the system.

RANGE: all

Your response to this request tells TROLL which periods you wish to examine. The response all will cause data for all years for which simulation has been performed to be printed. If you want to look at only a sub-set of these years, type the first year and the last year of

the sub-set, separated by one or more spaces. Thus typing 3 5 followed by a carriage return would cause data for years three, four, and five to be printed.

VARIABLES: name1 name2 name3 ...*

The names name1 - name3 are the actual variable names from the model for which you want to see data. The names should be separated by one or more spaces, and an asterisk should follow the last name in the list. If you want to see all the endogenous variables, type all * followed by a carriage return.

If you want graphical output, your response to the first request from the computer after you have given it the runname is as follows:

TYPE COMMAND-pgtype

TYPE PLOT OPTIONS-title this is the title//*

The above response will cause the words "this is the title" to be printed at the top of your graph. If you do not want a title, type an asterisk in response to this request.

PLOT TYPE- simulated

RANGE: all

These two responses are the same as when tabular output is desired. Next you must tell TROLL the scale limits on the graph. You can specify them yourself by typing the values - for instance

SET LOWER SCALE LIMIT-100

SET UPPER SCALE LIMIT-300

Alternatively, you can let TROLL decide on the scale as follows.

SET LOWER SCALE LIMIT-fix

SET UPPER SCALE LIMIT-fix

Finally, you must tell TROLL the variables you desire to plot. The maximum number is five per graph. Remember to have only variables of roughly the same magnitude on the same graph. For instance, do not try to plot GNP and interest rates on the same graph. The variables should be separated by a space and the list terminated with an asterisk.

VARIABLE LIST- c i yd *

These variables are present in the islm1 model.

SET PAPER AND HIT RETURN

You should use the carriage roller to set the paper to the top of a new page and press the carriage return. TROLL will then type your graph.

After it has typed a graph or a table, the system will print

TYPE COMMAND-

If you want another graph or table, type pptype or pgtype and proceed as outlined above. If you desire no more output, type quit. If you have completed your phase list, the conversation proceeds as follows.

CONTROLLER

TYPE COMMAND-

You may then type quit to leave TROLL, or execute to continue work. If the phase list you initially gave the controller was

simulation simout simulation simout*

typing quit in the simulation output phase the first time you enter that phase (that is, after your first exit from the simulation phase) will return you to the simulation phase.

If you are ever hopelessly confused, type $$ (superquit) and you will be returned to the controller. You can then type a new default and execute command to start over, or you can type quit to leave TROLL. Superquit should be used only as a last resort.

360/37 TROLL

We shall examine a hypothetical console session. Assume the user has logged into the time-sharing system and has gained access to TROLL. (How this is to be done will be specified). The computer then types

GOOD EVENING

TROLL is now entered, and you are at the controller level. At this level commands are entered which identify the model to be simulated and the data library from which set-up data are to be obtained. Here you tell the computer that you will do a simulation. The commands at this level are as follows:

ARCHIVE By typing default archnm, you are telling the system to look for data in the archive named archnm. The archive name corresponding to each model described herein will be indicated.

LOADMOD By typing loadmod modelname, you specify the model on which you wish to perform simulation experiments.

SIMULATE By typing simulate startdate, you cause the system to enter the simulation phase. The "startdate" argument specifies the time period in which simulation will begin.

All commands in 360 TROLL have the following form

command arg, arg, arg, arg;

That is, a command name followed by a set of arguments. Each argument

is followed by a comma except the last argument, which is followed by a semi-colon. If some arguments contain lists, each element in the list is separated by one or more blanks.

Let us suppose, for purposes of concreteness, that we wish to use the islml model (described in Chapter IV) and the associated data files in archive islm. The dialogue would then proceed as follows.

TROLL COMMAND - classname islm ;

You have told the computer which data files you will be working with. Had you wished to employ either of the GE models (described in Chapter III), you would have typed classname geneq to call for archive geneq. Data for the DYNEC models (Chapters VI - IX) are in archive dynec.

To specify which model the system should employ and what operations you will perform, you must proceed as follows.

TROLL COMMAND - loadmod islml;

TROLL COMMAND - simulate 2 ;

TROLL now knows which model to use. It also knows that you will perform some simulations.

TROLL will accept commands in a string. That is, you could have typed the above commands all on one line in response to the first "TROLL COMMANDS-" request, as follows:

TROLL COMMAND - archive islm; loadmod islml; simulate 2;

If TROLL discovers an error in a command, for instance, a misspelled word, that command and all that follow it are ignored, but all that precede it are accepted. TROLL will print out a message explaining the error. If you notice an error before hitting the carriage return, you may type @ to delete (cause the computer to ignore) an entire line. You then start the line over. If the error is not so serious, you may use a number of #'s to tell the computer to ignore the preceding character or characters. For instance, the following line

then was @ now is not ### thee# time for all

will be read by TROLL as

now is the time for all

After you have given the SIMULATE command with a startdate of 2, you have told the system to perform simulations on the static macroeconomic model ISLM1, labeling the first solution with a date of 2, the second with a date of 3, and so on. All static models should have a startdate of 2, and the dynamic models should be started in year 3. The system will continue with the request

SIMULATION COMMANDS -

At this point the user has several options. First, the methods used by the system to find a solution may be altered. This is necessary for the GE1, GE2, ISLM1, and ISLM2 models, but not for DYNEC1, - DYNEC4. If you are using either GE model, you must type the following:

<u>tune</u> <u>divtune</u> <u>5</u> <u>5</u> <u>10.</u> <u>ignore</u> <u>convtune</u> <u>5</u> <u>5</u> <u>500</u> <u>.0001</u> <u>damp</u> <u>step</u> <u>continue</u>

For the ISLM models, the following command must be issued here:

<u>tune</u> <u>convtune</u> <u>5</u> <u>5</u> <u>1000</u> <u>.0001</u> <u>damp</u> <u>.3</u> <u>continue</u>

Each model has a <u>parameter</u> <u>file</u> stored with it, giving values for important quantitites in the model. Parameters correspond to policy variables that might be altered by the government, to exogenous influences such as population, and to structural characteristics of the system such as the marginal propensity to consume. If you wish to find the solution corresponding to the pre-set parameter values, type <u>dosim</u> <u>1;</u> in response to the system's request for a command after you have "tuned". The solution values will be stored and labeled year 2 for your later inspection. The computer will then type

SIMULATION COMMAND-

Suppose now that you want to change one or more of the parameters and see how this affects the solution. You would then type the command <u>setval</u> followed by a list of the parameters to be changed and their new values. To raise G to 145. and lower MPC to .85, you would type

<u>setvalue</u> <u>g</u> <u>145.</u> <u>mpc</u> <u>.85</u> <u>;</u>

Any number of parameter-value pairs may be included, but the list must be terminated with a semi-colon. Remember that once a parameter has

been changed, it remains set to the new value until you leave the simulation phase, unless you use "setvalue" again. To observe the current values of any of the parameters, you may type

lkvalue parametername1 parametername2

followed by a carriage return after any "SIMULATION COMMAND-" request from the computer.

After you have changed parameter values, the dialogue will proceed as follows:

SIMULATION COMMAND- dosim 1 ;

The computer has now found two solutions, labeled as year 2 and year 3. If more solutions are desired, repeat the sequence of changing parameter values and processing as above. To terminate simulation processing, you use the SAVESIM and QUIT commands. By typing savesim runname, the results produced in previous simulations will be saved for later examination. The runname, which serves to identify this output, can be any collection of up to eight letters. By typing quit, the user leaves the simulation phase and returns to the controller level.

In the current example, the dialogue to save output under the name isout and cease simulation would be

SIMULATION COMMAND - savesim isout;

SIMULATION COMMAND - quit;

If at any time during simulation TROLL indicates a failure to find a solution and asks for a request, type kill. You should then change some

of the parameter values and try again.

In the case of static models, the one-year-at-a-time approach outlined above is probably the best way to proceed. The solution for year 4 will not depend at all on the solution for year 3; it will be determined entirely by the parameter values prevailing in year 4.

To examine the impact of parameter changes in dynamic models, however, this is not a useful approach. Changing something like the marginal propensity to consume will affect the evolution of the system through time. Instead of typing dosim 1 after a parameter change, one should type dosim n, where n is ome number greater than one. Typing dosim 20,on the other hand, will cause the machine to examine what happens to the model in the next twenty periods.

In fact, the following approach may well be the best one for the dynamic models. After giving the archive and model names, proceed as follows:

TROLL COMMAND- simulate 3;setvalue "parameter-value list one";

SIMULATE COMMAND - dosim "number of periods" ;

SIMULATE COMMAND - savesim "runname one" ; quit

One can repeat this sequence of commands for a different parameter - value list and a different runname. The results of two or more such simulations with the same archive avd model can then be compared in the simulation output phase, to which we now turn.

Two controller-level commands allow you to examine the output of simulation runs:

TABSIMV - This command generates tables of results by variables across various runnames. It is useful for comparing the results of different experiments with the dynamic models, or for presenting one set of experiments on the static models. The command can generate not only the simulated values for any run, but also first differences and error values in relation to other runs.

PLSIMV - This command produces the same information as the table command, but in the form of a graph rather than a table.

To obtain tabular output, the dialogue is as follows:

TROLL COMMAND- tbsimv

ENTER TABLE TYPES- simdata ,

The argument "table types" specifies what kind of information is to be generated. Here the user has requested only simulated data. For other possibilities, see the TROLL Manual.

ENTER RANGE - all ,

Your response to this request tells TROLL which periods you wish to examine. The response all will cause data for all years for which simulation has been performed to be printed. If you want to look at only a sub-set of these years, type the first year and the last year of the sub-set, separated by one or more spaces. Thus typing 3 5 , followed by a carriage return would cause data for years three, four, and five to be printed.

ENTER VARIABLES: name1 name2 name3 ... ,

The names name1 - name3 are the actual variable names from the model for which you want to see data. The names should be separated by one or more spaces, and a comma should follow the last name in the list. If you want to see all the endogenous variables, type all , followed by a carriage return.

ENTER SIMOUT SETS: run1 run2 run3 ...;

The names run1 - runN are the runnames attached to the simulation runs from which data is to be extracted. The runnames should be separated by one or more spaces, and the list should be terminated by a semicolon.

The sequence of commands just illustrated will cause a table of the simulated data from all the indicated runs to be printed. Very often in using the DYNEC's, it is of interest to compare one or more runs with a control solution. Suppose run1 is the runname of the control solution, and runs with runnames run2 - runN are to be compared to it. The necessary sequence of commands is the following:

TROLL COMMAND - simcontrol run1 tbsimv simdata simerr simpct,
ENTER RANGE - all, "list of variable names", run2, run3
... runN ;

To obtain graphical output, enter the TROLL command plsimv. After that the dialogue can be exactly the same as the table command. After

a graph is ready to be printed, the message

SET PAPER, HIT RETURN

will be printed. Use the carriage roller to set the paper to the top of a new page and press the carriage return.

If you wish to have a title on either a graph or a table, the controller level command <u>title</u> followed by the desired title and terminated by a semi-colon will produce this title on the next graph or page printed.

To leave TROLL, simply type the command <u>quit</u> when at the controller level.

If you are ever hopelessly confused, type <u>$$</u> (superquit) and you will be returned to the controller. You can then start over again or leave TROLL. Superquit should be used only as a last resort.

PART II

Static Models

CHAPTER III

Two Models of Market Interdependence

Introduction

In price theory, we examine what happens in individual markets when external influences like factor prices and consumer incomes change. Most of microeconomics proceeds on a ceteris paribus basis; all other things are assumed constant. In analyzing the real world, this assumption is rarely met. What goes on in one market will affect other markets. There may be feedback effects; changes in one market may trigger changes in other markets, and these may in turn affect the first market. A fundamental fact of economic life is that markets are interdependent.

Consider a tax on coffee. Partial equilibrium analysis, the usual approach of microeconomics, says that such a tax will act to raise the price of coffee and lower the quantity purchased. Partial analysis stops there, for that is indeed all that happens if all other prices and money incomes in the economy are unchanged. A general equilibrium analysis, on the other hand, looks at the effects of the tax on other markets and at the implications of those effects for the coffee market. The tax on coffee will cause consumers to demand more tea at the going price. Unless tea is in perfectly elastic supply, the price of tea will rise. This will tend to move consumers back to coffee, and the eventual fall in the quantity of coffee demanded will be less than the fall predicted by partial analysis.

In this example, the qualitative implications of partial and general analysis were the same: both predicted a fall in the amount of coffee demanded. The difference lay in the quantitative results: the general analysis predicted a smaller fall. The qualitative implications of general and partial analysis may differ also. Suppose all households decide to save a larger fraction of their after-tax income. Partial analysis says that savings would rise. A general analysis says that lower consumer spending on goods and services will result in less demand on the part of producers for factors of production. This will lower consumers' incomes, and may well result in lower absolute savings. This is, of course, the famous paradox of thrift.

Partial equilibrium analysis would probably yield very good results in our first example. The importance of other markets is probably slight. In the second example, though, partial equilibrium analysis would badly mislead. Macroeconomics is based on general equilibrium analysis; its first proposition is that the interaction of the markets in an economy must be considered to properly understand the economy's behavior.

In this chapter, we shall present two simple static models that illustrate market interdependence. Both models consider simple economics with two factors of production, land and labor, in perfectly inelastic supply. The only goods produced are two consumer goods, food and clothing. All markets are perfectly competitive, and all prices are perfectly variable. These are models of competitive general equilibrium. They do not formally resemble the usual models encountered in macroeconomics, but they represent the logical antecedents of such models, since they focus on the interactions among markets.

In model GE1, prices are determined by the intersection of domestic demand and supply schedules; the economy does not trade abroad. Model GE2 describes a small country that is free to trade with other countries. The ratio of the price of clothing to the price of food is determined in the world market, and the actions of the small country being modeled cannot affect these prices.

We shall briefly consider the general structure of static models of competitive general equilibrium. In the first place, there is no growth. Such models describe equilibrium with capital stocks, population, and technology given. Second, all factor and product prices are perfectly flexible. As we shall discuss in the next chapter, this is not a very realistic assumption. Among its implications is that all factors of production are always fully-employed. Modern macroeconomics assumes that prices and wages are fairly sticky, and it places great weight on income changes and on income effects in demand equations. Finally, money is not present as a store of value.

The general static competitive equilibrium model with no trade considers an economy with M products and N factors of production. There are thus M product prices and N factor prices, for a total of 2(M+N) unknowns. The amount of each product supplied will depend on all product and factor prices under competition, so we have M product supply curves. Similarly, there will be N factor supply equations, reflecting household decisions, and N factor demand equations, embodying firms' production decisions. Both sets of schedules will depend, in general, on all (M+N) prices. There will also be M product demand functions, reflecting household tastes and incomes. But one of these equations is redundant.

Household income must equal household spending, so if all prices, the quantities of all factors of production, and the amounts of M-1 products are given, the amount of the Mth product demanded is determined by the budget constraint.

Thus such models have 2(M+N) unknowns and 2(M+N) - 1 independent equations. Two approaches to this problem can be taken. The first is to take one price as given and equal to unity. This eliminates one of the unknowns, but it does not really change the model, since only relative prices affect decisions in a world where money is not a form of wealth. The second approach, which we take here, is to assume that, via one mechanism or another, the government keeps the level of prices fixed. This implies that some index of prices is constant and adds the needed equation.

When international trade is permitted, it is usually assumed that the money value of exports must equal the money value of imports. If deficits or surpluses in the balance of payments were to be permitted, a static equilibrium model would no longer be appropriate. For a small country, the usual assumption is that the price ratios of traded goods are determined on the world market, and that they will be unaffected by the country's actions. Suppose R goods can be traded. We then have R new unknowns: the net exports of each good. If the R-1 price ratios are given exogenously to the model, these plus the equation requiring the balance of trade to be zero will determine the R net exports.

To obtain versions of these models that can be easily understood, we have had to make a large number of simplifying assumptions. Not everything that is true for GE1 and GE2 will be true for all static

general equilibrium models. In particular, the assumption that factor supplies do not depend on relative prices is quite special, though often made. But enough of the properties of these two models are of general validity that their study can be quite instructive.

We shall discuss the structure of the two models in some detail in the next two sections. We then discuss the sort of exercises that can be performed with these models. Table III.1 describes the variables used in both models. Tables III.2 and III.3 list the equations that describe the two economies. The notation is fairly standard, with one exception: x**y means x to the y power. The TROLL versions of the models are given in Appendix A, and archive geneq is described in Appendix B. This archive contains data used to find solutions to the two systems. The parameter values stored in the system are given below.

The Structure of GE1

The equations in Table III.2 are equilibrium conditions for a static economy that does not trade abroad. Several of them clearly do not represent decision rules. All that can be said is that when the economy is in equilibrium, the relations written must hold. We thus have a static model as discussed in Chapter I, a model that describes equilibria but offers no information on how they are attained.

Consider Table III.2. The first two equations are the Cobb-Douglas production functions for the food and clothing industries. The general form of a Cobb-Douglas production function with factor inputs land (T) and labor (L) is the following:

$$(3.1) \quad Q = A L^b T^c.$$

An increase in A permits more output for given real inputs, and we interpret the multiplicative constants AC and AF as indicators of technology. An innovation in the clothing industry, for instance, would increase AC.

Suppose that L and T in equation (3.1) are replaced by kL and kT respectively. Then the left-hand side of (3.1) is multiplied by k^{b+c}. If b+c is less than one, we have decreasing returns to scale, while if this sum exceeds unity we have increasing returns to scale. In both production functions in GE1, b+c=1, and we have constant returns to scale. This means, for instance, that doubling both inputs will exactly double output.

We assume that food and clothing production are carried on by perfectly competitive industries. This means that the marginal value products of labor (land) in the two industries must both equal the wage (rental) rate. Let P be the price of the final product in either industry. Then equation (3.1) yields the following expressions for the marginal value products:

$$(3.2) \quad \begin{aligned} MVP_L &= P\ b\ Q/L \\ MVP_T &= P\ c\ Q/T \end{aligned}$$

The sets of equations labeled II and III in Table III.2 are easily interpreted. In block II, the marginal value products in the two industries are equated, and the equations are solved for the amounts of labor and land employed by the clothing industry. In block III, the marginal value products of the two factors in the clothing industry are equated to the two factor prices.

Equations (3.2) yield a comparison of the technologies of the two industries. Under constant returns to scale, c = 1 - b. Set the two marginal value products in (3.2) equal to the factor prices w and r and divide. This yields

$$(3.3) \quad \frac{L}{T} = \frac{b}{1-b}\,\frac{r}{w}.$$

Since b in the clothing industry equals .75, while b in the food industry is only .30, it is clear that for given factor prices the ratio of labor to land in the clothing industry, LC/TC, will be greater than the ratio of labor to land in the food industry. We say that the clothing industry is more <u>labor-intensive</u> than the food industry. This, of course, implies that the food industry is more <u>land-intensive</u> than the clothing industry.

It is assumed that all labor available and all land available will be offered to the labor and land markets regardless of the wage and rental rates. The supply curves in both markets are therefore verticle lines. As mentioned above, this is a special assumption. Since prices by hypothesis are perfectly flexible, this implies that both factors of production will be fully employed at all times. In block IV, supply is set equal to demand in both markets by requiring full employment of land and labor.

The next block consists of two identities that determine the incomes of landlords and workers.

We assume that the demand functions for food and clothing have unit price elasticity, another special assumption. That is, the fractions of total income spent on food and clothing by workers and landlords are fixed. These fractions add to unity, as there is no saving. The first equation in block VI determines the price of clothing by the condition that the total amount of money spent on clothing by workers and landlords must equal the total amount of money received by producers of clothing. (This becomes obvious if both sides of the equation are multiplied by QC.)

The second equation in block VI determines the price of food from the requirement that the level of prices remain constant. The measure of the level of prices remain constant. The measure of the level of prices chosen is the following price index:

$$(3.4) \quad I = [\, PC\, QC_0 + PF\, QF_0 \,]/[\, PC_0\, QC_0 + PF_0\, QF_0 \,],$$

where the "0" subscripts refer to a base period. In the base year chosen, the quantitites on the right of (3.4) took on the following values:

$$PC_0 = 1.046 \qquad QC_0 = 61{,}000$$

$$PF_0 = 1.000 \qquad QF_0 = 50{,}000$$

Setting I equal to one, substituting the values into (3.4), and solving for PF, we obtain the second equation in block VI.

In block VII, the incomes of both factors of production and the tastes of both classes are used to determine how much of each good each class consumes. The distribution of consumption between workers and

landlords is of interest in its own right, but the quantities calculated in block VII do not feed back into the rest of the system; it is only the total demand curves that matter.

The parameters of GE1 have been pre-set to the folliwing values:

LT	=	100,000	WTC	=	.400	AC	=	1.0
TT	=	100,000	RTC	=	.755	AF	=	1.0

Notice that these parameters provide information that is basic to any economy: factor endowments (LT and TT), tastes of consumers (WTC and RTC), and technologies (AC and AF).

Corresponding to these parameter values is the following equilibrium solution:

TC	=	31,118	W	=	.677	QC	=	60,839	RCC	=	36,832
TF	=	68,822	R	=	.5102	QF	=	50,157	RCF	=	12,500
LC	=	76,026	PF	=	1.000	WI	=	62,769	WCC	=	24,007
LF	=	23,974	PC	=	1.046	RI	=	51,020	WCF	=	37,661

The Structure of GE2

The economy modeled in GE2 is free to trade abroad in food and clothing at fixed world prices. Thus the ratio of the clothing price to the food price, IRPCF, is a parameter in this model, rather than an endogenous variable.

Comparing Tables III.2 and III.3, it is clear that the main differences between the two models arise in block VII, which is not present in GE2. In GE2, the equations in blocks I - III relate to the domestic production

of food and clothing, as they did in GE1. But production of either commodity need not equal consumption of that commodity, since trade is now possible. We must distinguish between QCP and QFP, the production of food and clothing, and QCC and QFC, the consumption of food and clothing. The first equation in block VIII determines the quantity of clothing consumed from the same identity used to obtain PC in GE1. (Multiply both sides by PC to clarify this.)

The quantity of food consumed is determined from the requirement that trade be balanced. That is, the value of exports must equal the value of imports. This equation reduces to

$$PF*NXF + PC*NXC = 0,$$

where NXC and NXF are the net exports of food and clothing. For this equation to hold, either both NXC and NXF must be zero, or one must be positive and the other negative. The country either exports nothing, or it exports only one commodity.

Model GE2 has two more unknowns that do not appear in model GE1: these are the differences between domestic production and consumption in the two industries. Two additional conditions are supplied with which to determine these quantities: the ratio of the food price to the clothing price is given as a parameter, and the requirement that the country's balance of trade must balance is supplied as an equation.

For this model, the parameters are pre-set at the following values:

LT = 100,000	WTC = .400	AC = 1.0	IRPCF = 1.046
TT = 100,000	RTC = .755	AF = 1.0	

of food and clothing, [illegible]

[illegible]

Corresponding to these parameter values is the following equilibrium solution:

TC	=	31,203	W	=	.6278	QCP	=	60,864	WI	=	62,784
TF	=	68,797	R	=	.5100	QCC	=	60,828	RI	=	51,005
LC	=	76,048	PF	=	1.000	NXC	=	37.07	WCC	=	24,010
LF	=	23,952	PC	=	1.406	QFP	=	50,130	WCF	=	37,670
						QFC	=	50,169	RCC	=	36,817
						NXF	=	-38.78	RCF	=	12,496

Notice that there is essentially no trade and that all other variables are approximately the same as in the initial solution to GE1.

When trade is permitted, the possibility of specialization is present. That is, under some conbinations of parameters the economy will seek to devote all its resources to production of one of the commodities. Consumption of the other commodity would then be entirely imported. Intuitively, if IRPCF and AC are large and AF is small, the economy is likely to specialize in the production of clothing, importing all the food it uses.

The model as formulated in Table III.3 (and in Appendix A) will not be soluble when specialization is indicated. Some of the equations presented there must be replaced with inequalities if specialization is to be permitted, and several variables must be allowed to become zero. If the parameters are chosen such that the economy modeled should produce only one good, no simulation program will be able to find a solution to GE2 as it is presently set up. To avoid this, parameter values should be chosen so that the following inequalities hold:

$$.66 < \frac{IRPCF \quad AC}{AF} \left[\frac{LT}{TT}\right]^{.45} < 1.55.$$

If the lower inequality is violated, the economy should specialize in food, and if the upper inequality is violated equilibrium will involve specialization in clothing. If both inequalities hold, the economy will produce both goods, and a solution to the model can be found.

Computer Analysis and Exercises

As mentioned above, both GE1 and GE2 are static models. In the TROLL system, both should be solved for year 2 and succeeding years. The solution for year N will depend only on the parameters prevailing in year N, not on the solution for year N-1. The years are purely arbitrary labels attached by the computer to separate and unrelated solutions.

Normally, if the parameters are chosen sensibly, there will be no difficulty solving the models on TROLL, provided the simulation algorithm has been "tuned" as described in Chapter II. If trouble is encountered, try again with parameters closer to the pre-set values.

The basic exercise that can be performed with GE1 and GE2 involves varying the parameters one at a time by at least 10%. Each parameter change will cause several of the endogenous variables of the system to change. It should be possible to intuitively explain these changes in terms of the markets of the economy. A comparison of general and partial equilibrium solutions may be undertaken, and the reasons for the differences between them may be detailed.

It is instructive also to compare the two models. Is the economy always better off if it is allowed to trade? (This question raises a very fundamental issue: how do you measure whether the economy is, in fact, better off? You should think about this.) What parameters in GE2 do not affect production, and why? In general, how do the two economies react differently to the same parameter changes, and why?

As you work with these models, keep in mind that they are quite simple, in the sense that any real economy has a large number of factor and product markets, rather than two of each sort. Many of these markets are not perfectly competitive. We have ignored all dynamic aspects, including population growth and capital accumulation. Yet GE1 and GE2 are fairly complex in another sense. The repercussions of any parameter change are not usually immediately obvious; considerable thought is often required to interpret what happens.

Table III.1

Notation Used in GE Models*

I. Parameters

LT	Total labor force
TT	Total supply of land
AC	Coefficient of level of technology in clothes-making (pure number)
AF	Coefficient of level of technology in food-making (pure number)
WTC	Fraction of workers' budgets spent on clothing (fraction)
RTC	Fraction of landlords budgets spent on clothing (fraction)
IRPCF	Ratio of clothing price to food price on the world market [GE2 only] (pure number)

II. Production

QC	Production and consumption of clothing [GE1 only]
QF	Production and consumption of food [GE1 only]
QCP	Production of clothing [GE2 only]
QFP	Production of food [GE2 only]
LC	Labor employed by the clothing industry
TC	Land employed by the clothing industry
LF	Labor employed by the food industry
TF	Land employed by the food industry

III. Prices and Incomes

W	Wage rate (dollars/unit)
R	Rental rate (dollars/unit)
PC	Price of clothing (dollars/unit)
PF	Price of food (dollars/unit)
WI	Workers' income (dollars)
RI	Landlords' income (dollars)

IV. Consumption

QCC	Consumption of clothing [GE2 only]
QFC	Consumption of food [GE2 only]
WCC	Workers' consumption of clothing
WCF	Workers' consumption of food

(Table III.1, Continued)

RCC	Landlords consumption of clothing
RCF	Landlords consumption of food

V. Foreign Trade

NXC	Net exports of clothing [GE2 only]
NXF	Net exports of food [GE2 only]

*Unless indicated otherwise, all variables are to be thought of as being in physical units, such as tons, manhours, yards, or acres.

Table III.2

GE1: No Foreign Trade

I. Production Functions

```
QC = AC*(LC**.75)*(TC**.25)
QF = AF*(LF**.30)*(TF**.70)
```

II. Equality of Marginal Value Products

```
LC = (.75*QC*PC)/(.30*PF*QF/LF)
TC = (.25*QC*PC)/(.70*PF*QF/LF)
```

III. Factor Prices Equal Marginal Value Products

```
W = .75*PC*QC/LC
R = .25*PC*QC/TC
```

IV. Factor Market Clearing

```
LF = LT - LC
TF = TT - TC
```

V. Income by Class

```
WI = W*LT
RI = R*TT
```

(Table III.2, Continued)

VI. Determination of Prices

PC = (WTC*WI + RTC*RI)/QC

PF = 2.276 - 1.22*PC

VII. Consumption by Class

WCC = WTC*WI/PC

WCF = WI-WTC*WI

RCC = RTC*RI/PC

RCF = RI-RTC*RI

Table III.3

GE2: Free Trade at Fixed World Price Ratio

I. Production Functions

QCP = AC*(LC**.75)*(TC**.25)

QFP = AF*(LF**.30)*(TF**.70)

II. Equality of Marginal Value Products

LC = (.75*QCP*PC)/(.30*PF*QFP/LF)

TC = (.25*QCP*PC/(.70*PF*QFP/LF)

III. Factor Prices Equal Marginal Value Products

W = .75*PC*QCP/LC

R = .25*PC*QCP/TC

IV. Factor Market Clearing

LF = LT - LC

TF = TT - TC

V. Income by Class

WI = W*LT

RI = R*TT

(Table III.3, Continued)

VI. Determination of Prices

PC = IRPCF*PF

PF = 2.276 - 1.22*PC

VII. Consumption by Class

WCC = WTC*WI/PC

WCF = WI - WTC*WI

RCC = RTC*RI/WI

RCF = RI - RTC*RI

VIII. Domestic Demand and Foreign Trade

QCC = (WTC*WI + RTC*RI)/PC

QFC = (QFP + PC*QCP) - PC*QCC

NXC = QCP - QCC

NXF = QFP - QFC

CHAPTER IV

Two Hicksian Macroeconomic Models

Introduction

The models examined in the last chapter raise two basic issues that we must deal with before proceeding. First, can the approach of explicitly considering all markets be usefully applied to real economies? The answer is clearly no, since any actual economy has hundreds of thousands of markets, and the amount of information that would be needed to model all of them and their interactions is little short of infinite.

Macroeconomics generally proceeds by means of aggregation. We group goods and services into aggregate quantities, and we speak, for instance, of the "market" for consumer goods, when no such market really exists. We thus consider the economy as composed of a small number of interdependent "markets" for economic aggregates. It is by no means obvious a priori what level of aggregation should be employed for any particular problem. Choosing the right level is often the key to getting useable answers.

A second observation based on Chapter III is the following. Both GE1 and GE2 always had full employment of both factors of production. This is a characteristic of competitive economies with no rigidities, even when allowance is made for the presence of monetary and non-monetary

wealth. But real economies do experience involuntary unemployment, for at least two reasons. First of all, most real markets are not perfectly competitive. Second, there are rigidities in the system. Since Keynes, economists have placed great emphasis on rigidities in the labor market, on the "stickiness" of wages. These two elements are not really independent, of course, since rigid prices are characteristic of imperfectly competitive markets.

In this chapter, we combine the notion of multi-market equilibrium with the ideas of aggregation and rigidities to produce two static macroeconomic models of the usual textbook variety. We call them Hicksian models, since they follow from J. R. Hicks' classic interpretation and generalization of Keynes.[1] Both models take into account the government's budget constraint, recently emphasized, via an interesting simulation model, by C. F. Christ.[2] Models of the sort presented here are usually called IS-LM models in textbooks, hence our names for them.

ISLM1 is a model of aggregate demand in a closed economy in which the prices of all goods and factors are rigid. The implicit assumption is that any demanded level of output can be produced with no increase in unit costs, so that ISML1 might best be thought of as a deep depression

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[1] J. R. Hicks, "Mr. Keynes and the 'Classics'; A Suggested Interpretation," Econometrica, 5 (April, 1937), 147-159.

[2] C. F. Christ, "A Short-Run Aggregate-Demand Model of the Interdependence and Effects of Monetary and Fiscal Policies with Keynesian and Classical Interest Elasticities," American Economic Review, 57 (May, 1967), 434-443.

model of a closed economy. In ISLM2, an aggregate supply schedule has been added. Also, changes in the price level, income, and interest rates affect the balance of payments.[1]

Both models are static; they describe only equilibrium situations. Models of the IS-LM sort are called "short-period" equilibrium models. That is, equilibrium is assumed to be attained quickly enough that technology, labor force, and capital stock are unaltered.

Table IV.1 presents the notation used in the two models, while Tables IV.2 and IV.3 give the equations they are composed of. (More details may be found in Appendices A and B.) In the next section, we shall examine the general form of IS-LM models with a government budget constraint. We then examine the structure of ISLM1 and ISLM2 in some detail. The chapter concludes with a brief discussion of the sort of exercises that can be performed with these systems.

Short-Period Macro Models

Models of this sort consider the economy as composed of three aggregate markets: the market for goods and services, the market for money, and the market for public and private securities. (Securities

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[1]ISLM2 is thus more general than ISLM1, but it neglects the so-called wealth effects (or Pigou effects) mentioned in the theoretical literature. That is, it does not allow for the effects of the price level on households' real wealth and the impact of changes in real wealth on consumer spending. Econometric investigations of such effects have found them to be unimportant, and we gain simplicity by ignoring them.

are stocks and bonds.) There are three demanders in the market for goods and services: households, businesses, and governments. The sum of households' demand for consumption, businesses' demands for investment, and governments' demand for goods and services is equal to Gross National Product. Government demand is taken as a policy parameter. Business demand depends both on the level of GNP that must be produced and on the rate of interest in the economy. Household demand depends on after-tax income, which in turn depends on GNP, tax rates, and business saving behavior. Recall from national income accounting that GNP is equal to household after-tax income plus net taxes, capital consumption allowances, business savings, and a few small items.

For given values of tax rates and government spending, the condition for equilibrium in the final product market may be written as

$$(4.1) \quad \begin{aligned} Y &= C(YD) + I(Y,r) + G, \text{ where} \\ YD &= Y - NTX(Y) - OLK(Y) \end{aligned}$$

Here Y is Gross National Product, C is consumption, I is gross investment, G is government demand, and r is the interest rate. The functions $C(\cdot)$ and $I(\cdot)$ are to be interpreted as giving desired consumption and investment. The variable YD is Disposable Personal Income, and

OLK is other "leakages" between GNP and disposable income. As mentioned above, OLK is mainly capital consumption allowances and business savings. Equations (4.1) can be solved for r as a function of Y, and a graph of this function is usually termed an IS curve. Under the

usual assumptions, (which generally hold in ISLM1 and ISLM2) the derivative of r with respect to Y is negative along this curve. This makes intuitive sense: lowering the interest rate increases investment at each level of income and, via the multiplier, increases equilibrium income.

The two basic unknowns in these simple systems are the rate of interest and the level of income. The IS curve provides one equation involving these two quantities. In most textbooks, the other equation is the so-called LM curve that describes equilibrium in the money market. Households can hold their wealth either in the form of money or securities. If the money market is in equilibrium, and if the government does not buy or sell securities, the market for securities must also be in equilibrium. The reason is the one outlined in the last chapter: given M-1 demands, the Mth is identically determined by the budget constraint. So most texts explicitly consider only the money market, assuming the securities market in equilibrium in the background.

The government can determine the level of high-powered money, consisting of currency and deposits with the Federal Reserve. The term "high-powered" is used because this money can be used as bank reserves and hence permits the banking system to expand deposits and loans. Let M be the money supply, determined by the amount of high-powered money available, the public's currency holdings, and the banking multiplier. Money is used primarily for transactions purposes and, given the price level, the amount of transactions made in the economy will vary directly with the level of GNP. Doubling prices will double the money value of all transactions, so we may sensibly relate the real money supply, M/P, to the level of real

GNP, Y. The opportunity cost of holding cash is given by the interest rate, r. Given the money supply, we can write the condition for equilibrium in the money market as

$$(4.2) \quad M/P = M(r,Y).$$

When Y is increased, the demand for money for transactions purposes will rise. If the market is to be in equilibrium with fixed prices and the given money supply, the interest rate must increase. Thus r varies directly with Y along the LM curve.

For given P, equations (4.1) and (4.2) can be solved for (short-period) equilibrium r and Y.

We must now consider the government's budget constraint, which forces us to explicitly include the securities market in our model. If government spending is in excess of net tax receipts, the government must finance the deficit. It can do this by printing money to pay its bills; this adds to the stock of high-powered money. Or, it can borrow the money from the public by selling bonds. This adds to the supply of securities on the market and will act to lower the price of securities and thus raise the interest rate. Yields must be increased if the private sector is to voluntarily increase its holding of government bonds. Thus a complete model that takes into account the government's budget constraint must explicitly consider the bond market: see the paper by Christ cited above. We attempt to capture the impact of government bond transactions in a more casual way in ISLM1 and ISLM2.

The general IS-LM model, represented here by ISLM2, allows the price level to vary and considers the balance of payments. The first equation in (4.1) must then be modified to read

$$(4.3) \quad Y = C(YD) + I(Y,r) + G + BTR(Y,P),$$

where BTR is the balance of trade, equal to exports minus imports. This quantity will depend on the level of income and the relation of domestic to foreign prices, where the latter are usually assumed fixed.

The price level, in turn, will be affected by how closely the economy's aggregate demand approaches its productive capacity. The closer demand is to capacity, the tighter will be labor and product markets, and the more rapid will be the rate of increase of wages and prices.

It is worth a few lines to examine the differences between the models just discussed and the other models presented in this text. In the GE models, relative prices were of prime importance, while here the level of income is paramount. This is a feature of the dynamic models of the next Part also, and of most of macroeconomics. The assumption is made that relative prices are quite sticky and that income changes are more important than movements in relative prices. There were explicit production functions in Chapter III, but not here. This is a characteristic only of simple short-period models; we shall see production functions again in later chapters. And even here there is an implicit production function present in ISLM2 from which we determine the level of unemployment. Money

was not really present in the GE's, but it is important here. The first three dynamic models do not have money, and it is present in a very superficial way in DYNEC4. The theory of dynamic economies with money is not well developed. The general equilibrium models of Chapter III made no mention of interest rates, investment, or capital stock. The ISLM models are also static, but they do [illegible] interest rates and investment. The capital stock is assumed constant here, while changes in the capital stock are of central importance in Chapters VI - IX. Thus in several ways the IS-LM models serve as a link between the GE's and the DYNEC's.

The Structure of ISLM1

We shall examine the equations presented in Table IV.1. Block I contains the equations of the IS curve. The consumption function employed is fairly standard. Gross investment has an autonomous component, replacement, and a component that responds to the levels of income and of capital cost, net investment. As an approximation, we have taken the cost of funds to the typical firm to be two and a half times the yield on corporate bonds. This formulation reflects the fact that most funds are obtained from retained earnings and new stock issues. Monies obtained from those sources must be valued at the cost of equity. We have approximated this cost by examining the dividend yield on common stocks and the rate of growth of aggregate dividend payments. The cost of equity in recent years has averaged about 3.28 times the average corporate bond

yield. Taking the weighted average of the costs of debt and equity, weighting by the fractions of total funds raised from the two sources, we obtian the approximation that the cost of funds is 2.5 times the corporate bond rate.

We further assume that all capital deteriorates at a rate of 10% per year. Then the total opportunity cost of investing a dollar in new plant or equipment is the cost of raising that dollar (2.5R) plus the cost of the deterioration that will take place in the new capital each year (.10). The sensitivity of investment to this cost is measured by the structural parameter J, equal to minus the elasticity of net investment (non-autonomous investment) with respect to opportunity cost. To make sense, J must be positive. The constant inside the square brackets ensures that changing J will not affect the equilibrium obtained if all other parameters are maintained at their pre-set values. Changes in J serve to rotate the IS curve around the initial solution point.

In reality, gross private domestic investment includes investment in housing and inventories as well as gross additions to business capital stock. It should be obvious, however, that changes in income and interest rates act to alter home-building in the same directions as business investment. While it is not clear that inventories respond to the level of interest rates, total sales (which relate to GNP) clearly do matter. In sum, our investment equation formally treats housing and inventories exactly like investment in plant and equipment, but there is no reason

to suppose that this simplification seriously compromises analysis at this level of aggregation.

The income identity assumes we are modeling a closed economy, one without foreign trade. The fourth equation is simply the second of equations (4.1). Both the variables OLK and NTX and the equations that determine them are relatively straightforward. The final equation in block I computes the government deficit. DEF must be financed by some combination of borrowing from the public and printing money.

Government policy parameters appearing in block I are G, TRF, and MTR; the other parameters depict structural characteristics of the economy modeled.

The first two equations in block II determine the net increases in high-powered money and government bonds. The policy parameters here are DMFR and FDB, which represent Federal Reserve open market operations and government financing policy, respectively. The third equation determines the money supply from the stock of high-powered money and a money multiplier. This multiplier can be considered a policy parameter, as it will be affected by such things as bank reserve requirements and the rediscount rate.

Equation 4 in block II is designed to simulate both the impact of changes in the money supply and changes in the stock of government bonds. (The parameters GDBI and HPMI are best thought of as initial conditions.) We assume first that the demand for real cash balances is given by an equation of the form

(4.4) $MRS = a\ Y\ R^{-L}$,

where a and L are positive constants. The income velocity of money is an increasing function of the interest rate; the larger is L the more responsive money demand is to changes in R. Solving for (4.4) we obtain

(4.5) R = (aY/MRS)**(1/L).

To incorporate the bond market via the back door, we multiply equation (4.5) by an expression that rises as the supply of government bonds is increased. The expression used is

(4.6) (GDBI + DGB)**(1/EB).

The constant EB can be thought of as the net interest elasticity of demand for government bonds. If EB is very large, changes in the supply of government bonds will not affect the interest rate. (Note that the government budget constraint does not really matter in this case, as the amount of bond financing will not affect any real parameters.) On the other hand, if EB is small, changes in government debt may cause sizeable changes in interest rates.

As in the investment equation, the constants in the square brackets in the equation for R ensure that changes in L and EB will not affect the system's initial equilibrium. Changing these elasticities will merely rotate the LM curve around the initial equilibrium.

In both IS-LM models, the solutions obtained using the pre-set parameter values correspond roughly to the magnitudes that characterized the U. S. economy in the fourth quarter of 1965. Arbitrary constants have been given difinite values to bring this about. Consequently, the models are somewhat less general and somewhat more realistic than they would have been if all constants had been made parameters.

For the ISLM1 model, parameters are pre-set to the following values:

G = 143.3	TRF = 50.		DMFR = 0.
J = .60	MTR = .275	L = .20	MMULT = 2.81
MI = .0401	MPC = .90	LFR = .1062	FDB = 0.
GDBI = 300.	EB = .50		HPMI = 59.45

The corresponding values of the endogenous variables are

C = 447.4	R = .0472
I = 113.2	OLK = 74.75
Y = 703.9	NTX = 143.6
DEF = -.27	YD = 485.6
	MRS = 166.3
	DHPM = -.27
	DGB = 0.0

Changes is J, EB, or L, will not alter these values.

The Structure of ISLM2

In ISLM2, the IS and LM curves are determined by the same basic functions encountered in ISLM1. The demand for money is now a demand for real cash balances. The nominal money supply (the number of dollars) is still a parameter, but the real money supply falls as the price level rises. If the price level were to double, for instance, people would require twice as many dollar bills to engage in the same real transactions as before. (The price level doubles when PD = 1.0) Also, the balance of trade, exports minus imports, is now included in the definition of GNP. The various constants have been adjusted to take these changes into account. Since the price level can change, all GNP components should be understood as being in constant dollars. That is, they are valued at the price level prevailing the fourth quarter of 1965. We are thus assuming that all demands are placed in real terms.

The balance of trade depends on income and the rate of inflation. The latter is a function of the rate of unemployment, via a simple Phillips Curve. The rate of unemployment is calculated by an equation based on "Okun's Law" and on the assumption that full employment involves a measured unemployment rate of 3%.

Block III determines the rate of inflation. The unemployment rate is calculated from income and "Okun's Law". This "Law" states that when income moves 3% closer to full-employment income, the unemployment rate drops by 1%. Given the income and rate of unemployment prevailing

in the fourth quarter of 1965, and making the assumption that a 3% rate of unemployment corresponds to full employment, we compute full-employment income as $736.3 billion in that quarter. From that figure and the "Law" that underlies it, we obtain the equation for the unemployment rate.

The second equation in Block III is a Phillips Curve, relating the rate of inflation to the rate of unemployment. The parameter PDF measures the severity of this problem; the larger is PDF, the greater the changes in PD caused by any given change in the unemployment rate, U. The function is set up so that changes in PDF rotate the Phillips Curve around the initial equilibrium point where U = .042 and PD = .0288.

The last Block in ISLM2 determines the balance of trade and the overall balance of payments. The balance of trade depends in a fairly arbitrary way on the level of income and the rate of inflation. Net capital inflows, BCF, depend positively on the level of interest rates. The constant .20 is a proxy for potential earnings abroad. The sensitivity of capital flows to interest rate differentials is governed by the elasticity K. The larger is K, the more impact a given change in R will have on capital flows. As before, the constant in the bracketed term ensures that K may be changed without altering the initial equilibrium.

Preset parameter values for ISLM2 are as follows:

G	=	143.3	TRF	=	50	DMFR	=	0.	FDB	=	0.
J	=	.60	MTR	=	.273	L	=	.20	MMULT	=	2.81
MI	=	.0401	MPC	=	.90	PDF	=	.06	HPMI	=	59.71
			LFR	=	.10489	K	=	.80	GDBI	=	300.00
									EB	=	.50

As compared to model ISLM1, ISLM2 has two more structural parameters, PDF and K.. Corresponding to the pre-set parameter values is the following solution:

C = 447.5	R = .0472	PD = .0290	MRS = 161.6
I = 113.2	U = .0419	BCF = -7.4	DHPM = -.54
BTR = 6.0	NTX = 143.8	BOP = -1.4	DSB = 0.0
Y = 710.0	DEF = -.54		
YD = 491.7	OLK = 74.47		

Changing only parameters in the set J, L, PDF, EB, and K will leave these values unaltered. As before, these quantities correspond roughly to the magnitudes that characterized the U. S. economy in the fourth quarter of 1965. The difference between these numbers and those in the initial solution to ISLM1 arise because the balance of trade is set to zero in ISLM1.

Computer Analysis and Exercises

Both ISLM1 and ISLM2 are static models. Solutions on TROLL should begin with year 2. The solution found for year N + 1 will depend only on the parameters prevailing in that year, not on the solution in the year N.

If for any quarter R becomes equal to .0001, you should ignore that solution. Such a value means that equilibrium could not exist with positive

R given the parameters you specified. You should try another run, changing the parameters so as to increase equilibrium R.

TROLL should encounter no difficulty in finding a solution to the system, provided you have "tuned", as described in Chapter II. If problems arise, change the parameters, moving them nearer the pre-set values.

It is helpful to analyze these models in terms of targets and instruments. Government policy in ISLM1 would presumably be concerned with the level of income, Y. In ISLM2, there are two obvious target variables, the unemployment rate and the balance of payments. In both economies, policy-makers might well want to keep the level of investment high in order to promote growth. Both models have six policy instruments: TRF, MTR, G, MMULT, DMFR, and FDB. Changes in the structural parameters of the systems will alter the impact of the instruments on the target variables.

In ISLM1, it is instructive to consider fixed changes in one or more of the policy parameters. It is then possible to vary one or more of the structural parameters and see how this affects the response of the system to the given policy changes. How does L, for instance, affect the impact of fiscal policy on Y? The effects should be explained both intuitively and in terms of the IS and LM curves. Be careful in using the parameters that change the initial equilibrium of the system, and remember that the problem is to relate changes in the structural parameters to changes in the impact of policy parameters. All changes should be at least 10% of the pre-set values.

The same analysis can be carried out in ISLM2, but there are more targets and more structural parameters to be considered. Remember that changes in the rate of inflation will shift the LM curve, and changes in the balance of trade will shift the IS curve. ISLM2 lends itself to analysis of policy choices: pick values for the target variables and see what values of the policy parameters will bring them about.

Special interest attaches to the influence of the government budget constraint. The classical economists and, more recently, Milton Friedman have argued that debt-financed fiscal policy will have no impact on income. Are there parameter values in ISLM1 or ISLM2 for which this is true?

Finally, there is an error in ISLM2, which we have retained to simplify the equations. Can you find it? (Hint: Prices are involved.) It does not affect the qualitative properties of the model.

Table IV.1

Notation Used in ISLM Models*

I. Parameters

MPC	Marginal propensity to consume out of disposable income (fraction)
MI	Income coefficient in the investment equation (fraction)
J	The negative of the capital cost elasticity in the investment equation (pure number)
G	Government expenditures for goods and services
TRF	Net autonomous government transfer payments
MTR	Marginal rate of taxation (fraction)
LFR	Average rate of other leakages between GNP and disposable income (fraction)
DMFR	Net Federal Reserve purchases of government bonds
FDB	Fraction of the government deficit to be bond-financed (fraction)
MMULT	Ratio of the money supply to high-powered money (pure number)
HPMI	High-powered money at the start of the period
GDBI	Government bonds in the hands of the public at the start of the period
L	The negative of the interest elasticity of demand for money (pure number)
EB	The "net" interest elasticity of demand for government bonds (pure number)
PDF	Rate of increase of prices when U = .02 [ISLM2 only] (fraction, must be greater than .0288)
K	Elasticity of capital flows with respect to interest rate differentials [ISLM2 only] (pure number)

II. Endogenous Variables: Both Models

C	Consumption expenditures
I	Gross private domestic investment
Y	Gross National Product
YD	Disposable personal income
NTX	Net government tax receipts
OLK	Other leakages between GNP and disposable income
DEF	Government deficit on national income accounts
DHPM	Net increase in high-powered money
DGB	Net increase in publicly-held government bonds
MRS	Real money supply
R	Rate of interest on corporate bonds (fraction)

(Table IV.1, Continued)

III. Endogenous Variables: ISLM2 only

U	Unemployment rate (fraction)
PD	Rate of price increase (fraction)
BTR	Balance of trade
BCF	Balance on capital account
BOP	Balance of payments net surplus

*Unless otherwise indicated, all variables are measured in billions of 1965IV dollars. Flows are at annual rates.

Table IV.2

ISLM1: Basic Closed IS-LM Model

I. IS Curve

1. C = 10.4 + MPC*YD
2. I = 42.8 + MI*Y/(.400934* [(2.5*R + .10)/.218] **J)
3. Y = C + I + G
4. YD = Y - NTX - OLK
5. NTX = -TRF + MTR*Y
6. OLK = LFR*Y
7. DEF = G - NTX

II. LM Curve

1. DHPM = DMFR + (1. - FDB)*DEF
2. DGB = -DMFR + FDB*DEF
3. MRS = MMULT*(HPMI + DHPM)
4. R = .0472* [.23626*Y/MRS] **(1/L)* [(GDBI + DGB)/300.] **(1/EB)

Table IV.3

ISLM2: A Complete IS-LM Model

I. IS Curve

1. C = 5.0 + MPC*YD
2. I = 42.2 + MI*Y/(.401* [(2.5*R + .10)/.218] **J)
3. Y = C + I + G + BTR
4. YD = Y - NTX - OLK
5. NTX = -TRF + MTR*Y
6. OLK = LFR*Y
7. DEF = G - NTX

II. LM Curve

1. DHPM = DMFR + (1. - FDB)*DEF
2. DGB = -DMFR + FDB*DEF
3. MRS = MMULT*(HPMI + DHPM)/(1. + PD)
4. R = .0472* [.2276*Y/MRS] **(1/L)* [(GDBI + DGB)/300.] **(1/EB)

III. Unemployment and Inflation

1. U = .03 + (1 - Y/736.3)/3
2. PD = .105*(PDF - .0288)/U + (.1008 - 2.5*PDF)

(Table IV.3, Continued)

IV. Foreign Trade and Investment

1. BTR = 99.44 - .1243*Y*(1 + PD)**2

2. BCF = 5.1 - 12.5* [(.20 - R)/.1528]**K

3. BOP = BTR + BCF

Part III

Dynamic Models

CHAPTER V

Economic Dynamics and Distributed Lags

Introduction

This chapter will deal with the analysis of dynamic systems of the sort often encountered in economics. There is no one precise and concise distinction between static and dynamic models that all economists would subscribe to. Yet we all know intuitively what the difference is. Dynamic models are models of changing situations. More than that, they are models in which the process of change is basic. Analysis of such models is concerned with the way in which change takes place. Another way of looking at the distinction is that static models are concerned with a particular moment of time, while dynamic models focus on the relations between variables at different points in time, or, in the simplest case, on one variable at different points in time.

In much of economics, dynamic systems are represented by means of difference equations. These are equations relating values of variables at different periods of time. A general first-order difference equation is

$$(5.1) \qquad Y(t) = F[Y(t-1),t].$$

That is, the value of Y in period t is determined by the value Y took on in period t-1 and the period number, t. A special case of equation (5.1) will occupy us in the next section, the first-order linear difference equation with constant coefficient:

$$(5.2) \qquad Y(t) = a\,Y(t-1) + f(t),$$

where a is a constant. Difference equations of N^{th} order involve N lagged values of the variable involved; we shall consider the general N^{th} order linear difference equation briefly in the third section of this chapter.

There are two main ways that difference equations arise in economics. The first is by relating stocks and flows. The flow of net investment changes the value of the capital stock, so that today's capital stock depends identically on yesterday's capital stock. The cash in my pocket this evening is identically determined by the amount that was present this morning, my income today, and my spending during the day. The second source of difference equations is lags in economic behavior. The behavior of any economic unit today is affected by yesterday's events, as well as those farther in the past. There is a lag between the time that income is earned and the time that it is received. Increases in sales do not lead firms to immediately place orders for new plant and equipment. Further, it takes time to fill such orders. The last two sections of this chapter are concerned with the formal analysis of lags in economic behavior.

Many real-world relations between past and present values of economic variables can be expected to be non-linear. General analytical results are difficult to obtain for non-linear equations, however. We will deal in this chapter with analytical solutions only for linear difference equations with constant coefficients. Hopefully, this will provide some insight into the behavior of the non-linear systems dealt with in Chapters VI - IX. In any case, linear systems appear often enough in economic theory that the material is useful in its own right. Throughout, our aim is insight, not rigor.

The fourth section of this chapter deals explicitly with the relation between linear and non-linear systems.

Before studying the first-order linear difference equation with constant coefficient, a final point must be discussed. Much of the theory of economic dynamics is written in terms of differential equations, rather than difference equations. While difference equations relate to finite changes in variables, differential equations are concerned with instaneous rates of change, with the derivatives of the quantities involved with respect to time. While difference equations treat time as composed of discrete periods, differential equation models treat time as continuous. Two points should be made. First, difference equations are often used to approximate differential equations when the latter are to be solved numerically. Sometimes the reverse procedure is employed: differential equations are used to approximate difference equations for analytical simplicity. The second point is that the theory of linear differential equations with constant coefficients very closely resembles the theory we shall discuss in the nest two sections.[1]

The First-Order Linear Difference Equation

Let us consider a very simple dynamic economy. Consumption is a constant fraction, m, of last period's income. Income is made up of

[1]We refer the reader to William Baumol's excellent text, Economic Dynamics (Macmillan), for the relations between difference and differential equations, as well as for a clear treatment of both types of dynamic models.

consumption and government spending. The equations that describe this economy are

$$(5.3)\qquad \begin{aligned} C(t) &= m\,Y(t-1) \\ Y(t) &= C(t) + G(t) \end{aligned}$$

Suppose we are given <u>initial conditions</u> $G(0) = G_0$ and $Y(0) = Y_0$. What will be the path of income over time in this economy?

Combine equations (5.3) to yeild

$$(5.4)\qquad Y(t) = m\,Y(t-1) + G(t)$$

Assume that $G(t) = G_0$ for all t, for simplicity. We can then substitute into (5.4) to find the values taken on by Y:

$$Y(1) = m\,Y_0 + G_0$$

$$Y(2) = m[mY_0 + G_0] + G_0 = m^2Y_0 + G_0[1+m]$$

$$Y(3) = m[m^2Y_0 + G_0\,(1+m)] + G_0 = m^3Y_0 + G_0[1+m+m^2]$$

Continuing this procedure, we obtain the general form of the solution:

$$(5.5)\qquad \begin{aligned} Y(t) &= m^tY_0 + G_0(1-m^t)/(1-m) \\ &= G_0/(1-m) + m^t[Y_0 - G_0/(1-m)], \end{aligned}$$

where we have summed the geometric series in m.

We normally assume that m, the marginal propensity to consume, is positive and less than one. In this case, the second term above vanished as t becomes very large, and income converges to its equilibrium level, $G_0/(1-m)$. Note that this is the solution predicted by static theory, but how rapidly it is attained depends on the initial disequilibrium (the term in brackets) and the value of m. Government spending has the same multiplier effect as in static models, but the impact of G takes time to be felt.

Equation (5.4) need not represent an economy; it is a general first-order linear difference equation with constant coefficient. Let us now think of Y and G as arbitrary variables and of m as an arbitrary constant. Assume that $G(t) = G_0$ for all t, as above. What can we say about solutions to (5.4)? Write $Y_e = G_0/(1-m)$ for simplicity. We then have the general solution to (5.4) as

$$(5.6) \qquad Y(t) = Y_e + m^t[Y_0 - Y_e],$$

from the discussion above and equation (5.5).

If m is between zero and +1, the second term in (5.6) will vanish as t becomes large, and the system will move steadily closer to Y_e, regardless of the value of Y_0. We say that the system is stable. If m is larger than +1, however, the second term in (5.6) will not approach zero. If Y_0 is greater than Y_e, Y will grow without limit. If initial Y is less than equilibrium Y, on the other hand, Y will race towards

minus infinity. The system is unstable; there is unchecked motion away from equilibrium. (If m=1, the general solution to (5.4) is $Y(t) = Y_0 + tG_0$, and the system is unstable unless $G_0 = 0$.)

Under some circumstances, the coefficient m could be negative. Examining (5.6), it is clear that Y(t) will then oscillate around Y_e. If Y(t) is less than Y_e, Y(t+1) will be greater than Y_e, and vice versa. As above, though, if the absolute value of m is less than one, the second term in (5.6) will eventually vanish, and the system will be stable. We would observe damped oscillations. If m is less than minus one, the oscillations explode; Y(t) moves farther and farther away from Y_e. The system is unstable. When m = -1, the oscillations neither die away nor expand indefinitely. The system is unstable in the sense that there is no tendency for Y(t) to approach Y_e.

We can thus characterize the type of solution to (5.4) as a function of the constant m:

value of m	type of solution
$m \leq -1$	unstable, oscillations
$-1 < m < 0$	stable, oscillations
$0 < m < +1$	stable, monotone
$+1 \leq m$	unstable, monotone

Throughout this discussion, we have been assuming that G(t) was constant. Would relaxing this assumption affect the table above? We shall now show that the answer to this question is no.

Write (5.4) as follows:

$$(5.4') \qquad Y(t) - m\,Y(t-1) = G(t).$$

We define a homogeneous difference equation as one in which the driving function, $G(t)$, is identically zero for all t. A homogeneous equation thus involves only past and present values of Y. The homogeneous equation corresponding to (5.4') is simply $Y(t) - m\,Y(t-1) = 0$; we call this the reduced equation. The general solution to this reduced equation is clearly $Y_h(t) = Km^t$, where K may be any constant. (Verify this by substitution.) The value of K will be determined so that the solution satisfies whatever initial condition is specified; that is, so that $Y/0) = Y_0$. We next search for a particular solution to (5.4'), a solution to the original equation with the given function $G(t)$. The general solution to (5.4') (and to (5.4)) is the sum of the solution to the reduced equation and the particular solution:

$$(5.7) \qquad \begin{aligned} Y(t) &= Y_h(t) + Y_p(t) \\ &= Km^t + Y_p(t) \end{aligned}$$

The constant K is chosen so that $Y(0) = Y_0$. The key fact is that $Y_h(t)$ will vanish if and only if the absolute value of m is less than one. Then and only then will $Y(t)$ approach $Y_p(t)$, regardless of the form of $Y_p(t)$. We are now in a position to formulate a broader (informal) definition of stability: $Y(t)$ is stable if it approaches $Y_p(t)$. And we can state that unless the absolute value of m is less than one, the system is unstable. If m is negative, there will be oscillations around $Y_p(t)$, while positive

m implies that $Y(t) - Y_p(t)$ will be monotone increasing or decreasing. Thus stability depends entirely on the form of the solution to the reduced equation, not on the driving function, $G(t)$.

Two forms of the driving function are of particular interest to us. We showed above that if $G(t) = G_0$, the particular solution is $Y_p(t) = G_0/(1-m)$. This can be verified by substitution. The other case of interest is where $G(t) = G_0 k^t$, with G_0 and k constants. We will <u>guess</u> a solution of the form $Y_p(t) = Nk^t$, with N a constant. Substituting into (5.3'), we have

$$(5.8) \qquad \begin{aligned} Nk^t - mNk^{t-1} &= G_0 k^t, \text{ or} \\ k^{t-1}[Nk-mN-G_0 k] &= 0. \end{aligned}$$

If k is not equal to m, equation (5.8) will be satisfied for all t if and only if $N = G_0 k/(k-m)$. So the particular solution is $Y(t) = G_0 k^{t+1}/(k-m)$. (If k=m), it can be shown that a particular solution of the form $Y_p(t) = Ntk^t$ exists, with $N = G_0$.)

Given the driving function, we can characterize the behavior of the first-order linear difference equation with constant coefficient by the table given above. But now stability must be interpreted in terms of approaching the particular solution of the equation, and oscillations are oscillations around the particular solution. In terms of the model which motivated this section's discussion, we have the following result: If the marginal propensity to consume is less than one, the level of income will approach a value that is a function only of government spending,

not of initial conditions. If government spending is growing according to $G(t) = G_0k^t$, $Y(t)$ will approach the growing equilibrium level $G(t)k/(k-m)$.

Linear Difference Equations of Higher Order

This section is fairly long, and it is a bit more formal than the rest of the chapter. We shall summarize the results here, so that the mathematics may be skimmed on a first reading. The section considers linear difference equations with two or more lagged values of the endogenous (dependent) variable, though we formally analyze only the second-order constant-coefficients case. We find that such equations may be stable or unstable, and solutions may be monotone or oscillatory, as for the first-order case. The new feature of higher-order systems is the possibility of sinusoidal solutions. When such a solution is graphed against time, the result is a damped, explosive, or constant-amplitude sine wave. The last case can occur only when parameter values exactly satisfy certain equations.

Let us begin with an economic example. Suppose that consumption is, as before, a constant fraction, m, of last period's income. Government spending is exogenously determined. The new element is investment. We incorporate a simple accelerator theory of investment by letting investment equal a constant, v, times the change in income lagged one period. (The accelerator is considered at more length in the next chapter.) Our

model in structural form is now

$$(5.9)\quad \begin{aligned} C(t) &= m\,Y(t-1) \\ I(t) &= v[Y(t-1) - Y(y-2)] \\ Y(t) &= C(t) + I(t) + G(t) \\ &(G(t) \text{ exogenous}) \end{aligned}$$

The reduced form of this system would involve each of the endogenous variables [C(t), I(t), and Y(t)] as functions only of exogenous and predetermined quantities. The only equation that needs to be altered is the one giving Y(t), since the other equations are of the correct form. Substituting, we find

$$(5.9)\qquad Y(t) = (m+v)\,Y(t-1) - v\,Y(t-2) + G(t),$$

a linear <u>second-order</u> difference equation with constant coefficients and driving function G(t). If we can solve (5.9) for the time-path of Y(t), we can easily substitute back into (5.8) and obtain the behavior of C(t) and I(t).

In general, any linear dynamic discrete-time system with constant coefficients, such as (5.8), can be reduced to one key difference equation, such as (5.9), the solution to which can be used to characterize the behavior of all the endogenous variables. The relation between the order of the difference equations in the structural form of the model and the order of the key equation depends on the exact structure of the model. The order of the key equation will be at least as great as the greatest order of any equation in the system. A system composed of two second-order

difference equations will solve for a key equation of at least second order and generally of higher order.[1]

We shall employ the approach used in the last part of the last section to analyze the solutions to equations (5.9). Re-write (5.9) to yield

$$(5.9') \quad Y(t) - (m+v)\,Y(t-1) + v\,Y(t-2) = G(t).$$

We first consider particular solutions. Suppose $G(t) = G_0$ for all t. Then we look for a particular solution of the form $Y(t) = Y_e$, a constant for all t. Substituting into (5.9'), we obtain immediately

$$(5.10) \quad Y_e = G_0/(1-m).$$

Since Y is constant, the corresponding particular solution for investment must be $I(t) = 0$. From the consumption equation, $C(t) = mG_0/(1-m)$.

Now suppose that government spending is growing according to $G(t) = G_0k^t$. As before, we look for a particular solution of the form $Y(t) = Nk^t$. Substituting into (5.9'), we have

$$(5.11) \quad \begin{aligned} Nk^t - (m+v)Nk^{t-1} + vNk^{t-2} &= G_0k^t, \text{ or} \\ N[k^2 - (m+v)k+v] &= G_0k^2. \end{aligned}$$

[1] We shall return briefly to this point below. The reader is again referred to Baumol's Economic Dynamics.

Assuming that the expression in brackets is non-zero, we have our particular solution:

$$(5.12) \qquad Y_p(t) = k^t \, [G_o k^2]/[k(k-m)+v(1-k)].$$

Now we turn to an examination of stability; we want to know if the system will approach these (or any other) particular solutions. As before, stability depends on the solution to the reduced equation corresponding to (5.9'). We shall guess that a solution of the form X^t exists. Note that if X^t is a solution to the reduced equation, so is KX^t, for any constant K. So the critical thing is finding X; K will come from the initial conditions. Substituting $Y(t) = X^t$ into the reduced equation corresponding to equation (5.9'), we obtain

$$(5.13) \qquad X^t - (m+v)\,X^{t-1} + vX^{t-2} = 0, \text{ or}$$

$$X^{t-2}[X^2 - (m+v)\,X + v\,] = 0.$$

If X is non-zero, the expression in brackets must equal zero for $Y(t) = X^t$ to be a solution to the reduced equation of (5.9'). Setting the expression in brackets equal to zero and using the well-known formula for the solution of a quadratic equation, we obtain the two roots as follows:

$$(5.14) \qquad \begin{aligned} X_1 &= \frac{(m+v) + \sqrt{(m+v)^2 - 4v}}{2} \\ X_2 &= \frac{(m+v) - \sqrt{(m+v)^2 - 4v}}{2} \end{aligned}$$

Two problems arise immediately. Suppose that $X_1 = X_2$. That is, suppose $4v = (m+v)^2$. Then it can be shown that there is a solution of the form tX^t, where X is the solution to (5.14). We shall verify this below. The other problem arises if 4v is greater than $(m+v)^2$. Then the quantity under the square root sign is negative. The square root of a negative number is an imaginary number; what does this mean?

To examine these two problems, it will be easiest to work with the general second-order homogeneous difference equation with constant coefficients:

$$(5.15) \qquad Y(t) + a\,Y(t-1) + b\,Y(t-2) = 0.$$

Substituting $Y(t) = X^t$ as above, we are led to examine the solutions to the quadratic equation

$$(5.16) \qquad X^2 + a\,X + b = 0.$$

The solutions are given by

$$(5.17) \qquad X_1 = \frac{-a + \sqrt{a^2 - 4b}}{2} \qquad X_2 = \frac{-2 - \sqrt{a^2 - 4b}}{2}$$

If a^2 is greater than 4b, the quantity under the square root sign in (5.17) will be positive, and both X_1 and X_2 will be real numbers. Both must be less than one is absolute value if the system is to be stable.

Since X_1 is greater than X_2, the conditions for stability are

$$(5.18) \quad \begin{array}{l} X_1 < +1, \text{ or } a+b > -1 \\ X_2 > -1, \text{ or } a-b < +1 \end{array}$$

We thus have a sufficient condition for stability: if $a^2 > 4b$ and $-(b+1) < a < (b+1)$, the system is stable. Note that if a is positive, X_2 will be less than zero, and the solution will involve oscillations of the sort encountered in the last section. The other root, X_1, may also be less than zero, of course. If $a^2 > 4b$, it is easy to show that both X_1 and X_2 will be positive and no oscillations will occur if a is negative and b is positive.

We noted above that if $a^2 = 4b$, $X_1 = X_2 = X$, and a solution of the form tX^t exists. This is easy to verify. Substitute $Y(t) = tX^t$ into (5.15). This yields

$$(5.19) \quad tX^2 + a(t-1)X + B(t-2) = 0.$$

If $a^2 = 4b$, the solution to (5.17) is $X = -a/2$. This condition can be written as $b = a^2/4$. Substituting according to these last two expressions into equation (5.19), we have

$$(5.20) \quad \begin{array}{l} ta^2/4 - (t-1)a^2/2 + (t-2)a^2/4 = 0, \text{ or} \\ t[a^2/4 - a^2/2 + a^2/4] + [a^2/2 - a^2/2] = 0, \end{array}$$

which is identically satisfied for any a.

What of stability in this case? If the absolute value of X is less than one, tX^t will go to zero eventually, and if the absolute value of X is greater than one, this term will grow without limit as t becomes large. So if $a^2 = 4b$, we must have $-1 < X < +1$, or $-2 < a < +2$ for stability. We have another sufficient condition for stability.

We now come to the case where a^2 is less than 4b. Define j as the square root of minus one. The two solutions to (5.17) may be written in the following form:

$$\text{(5.21)} \quad \begin{matrix} X_1 = c + jd \\ X_2 = c - jd \end{matrix} \quad \text{with} \quad \begin{matrix} c = -a/2 \\ d = \sqrt{4b - a^2}/2 \end{matrix}$$

These are called complex numbers. More precisely, they are complex conjugates because they have the same real part (c) and the same imaginary part (jd), but with different sign. Let R be the angle for which the cosine is $c/\sqrt{c^2 + d^2}$ and for which the sine is $d/\sqrt{c^2 + d^2}$. We can re-write X_1 and X_2 identically as

$$\text{(5.21')} \quad \begin{matrix} X_1 = \sqrt{c^2 + d^2}\,[\cos(R) + j\sin(R)] \\ X_2 = \sqrt{c^2 + d^2}\,[\cos(R) - j\sin(R)]. \end{matrix}$$

Let $D = \sqrt{c^2 + d^2}$. This quantity is called the modulus of $c + jd$ and of $c - jd$. We now recall DeMoivre's Theorem: if $X = a[\cos(Q) + j\sin(Q)]$, for n an integer, X^n is equal to $a^n[\cos(nQ) + j\sin(nQ)]$. Using this, we

can write the solution to equation (5.15) in the case $a^2 < 4b$ as follows:

$$Y(t) = K_0 X_1^t + K_1 X_2^t$$

$$= K_0 D^t \cos(Rt) + K_0 j D^t \sin(Rt) + K_1 D^t \cos(Rt) - K_1 j D^t \sin(Rt), \text{ or}$$

$$(5.22) \quad Y(t) = D^t [k_0 \cos(Rt) + k_1 \sin(Rt)], \text{ where}$$

$$k_0 = K_0 + K_1, \text{ and } k_1 = j(K_0 - K_1).$$

The constants k_0 and k_1 are, as usual, chosen to satisfy the initial conditions. We now have a solution exhibiting sinusoidal fluctuations. The fluctuations will be damped and vanish for large t if D is less than one; they will explode if D is greater than one. The fluctuations will repeat forever with no change in amplitude if D is just equal to one. If the angle R is expressed in degrees, elementary trigonometry shows that the cycle will repeat itself every 360/R periods.

Let us summarize the foregoing development. If a^2 is less than 4b, the solution will involve sinusoidal fluctuations and will be of the form of (5.22). These fluctuations will die away if and only if D is less than one. From (5.21), $D = \sqrt{b}$, so b must be less than one for stable (damped) fluctuations. If b is exactly one, the fluctuations will neither die away nor explode. Since, from (5.21), R is the angle whose cosine is $-a/2\sqrt{b}$ and whose sine is $\sqrt{4b - a^2} \; / \; 2\sqrt{b}$, we can find the periodicity of the fluctuations easily.

Conbining our analyses of the three possible cases, we have sufficient conditions for stability:

$$a+b > -1 \qquad -2 < a < +2$$

$$a-b < +1 \qquad b < +1$$

It is easy to show that the condition on a alone will be fulfilled if the other three conditions hold. We can thus state that the general second-order linear difference equation with constant coefficients will be stable if b is less than +1 and if $-(1+b) < a < (1+b)$. Sinusoidal fluctuations will occur if $a^2 < 4b$. No oscillations of any kind will occur if X_2 is real and positive; this requires $a^2 > 4b$ and $a < 0$, $b > 0$.

In terms of the multiplier - accelerator model with which we began this section, the stability conditions for equation (5.9) are v less than one and $-(1+2v) < m < +1$. The latter condition will generally hold, but the former will fail in many situations. Whenever v is greater than one, the system will be unstable. Income will undergo sinusoidal fluctuations if $v(2-v) > m^2$. If m is between zero and one, there are values of v between one and two that satisfy this inequality. (In particular, $v = 1$ satisfies it for $m < 1$.) Thus if there are sinusoidal fluctuations, they will be unstable unless v is exactly one, in which case they will persist unchanged forever.

What about linear difference equations of order greater than two? The procedure for solution is just the same as employed above. It is first necessary to find the particular solution, to see what the system's stationary or moving equilibrium is. One next finds the solution to the reduced

equation by assuming a solution of the form X^t, substituting, and examining the resulting polynomial equation in X. The roots of that equation will be the values of X such that X^t is a solution. The particular solution and the solutions to the reduced equation are then added to give the general solution. Arbitrary constants are specified by initial conditions. General conditions for stability and for the absence of sinusoidal fluctuations can be formulated, but they will not concern us here.

To find the solution to a linear difference equation involving, say, six lagged values, a sixth-degree polynomial must be solved. This can be a rather complicated process. No general formulae exist, and the solutions must be found numerically, usually by an iterative method. Once the solutions have been found, the behavior of the system is known precisely. But it may be more instructive to simulate the system for particular parameter values of interest, to specify the initial conditions and see how the system actually evolves over time. Simulation will be the main tool used to analyze the difference equation systems presented in the next four chapters.

So far, we have dealt only with linear difference equation systems. We have discussed stability, the question of whether (and how rapidly) the system converges to its equilibrium solution. We have spoken of sinusoidal fluctuations, stable and unstable. These modes of behavior also characterize non-linear systems, though, as one might guess, more can go on in the non-linear case.

The Analysis of Non-Linear Systems

In general, non-linear systems can generate the four basic types of behavior observed in the last two sections:

1. damped, no oscillations
2. explosive, no oscillations
3. damped, oscillations
4. explosive, oscillations

One basic difference is that non-linear difference equation systems may change the type of behavior over time. Broadly speaking, it may be helpful to distinguish two sorts of non-linearities: corners and curves. A difference equation with a curve non-linearity is

$$(5.23) \quad Y(t) = a[Y(t-1)]^2 + b\,Y(t-2) + G(t).$$

With some luck, the solution to this equation can probably be approximated fairly well over some range by the solution to a linear difference equation. A difference equation system with a corner is

(5.24) $YS(t) = a\,Y(t-1) + b\,Y(t-2) + G(t)$

$Y(t) =$ IF Y(t) LESS THAN $\overline{Y}$ THEN Y(t) ELSE $\overline{Y}$.

That is, Y(t) cannot rise above $\overline{Y}$ in model (5.24). When Y(t) is below $\overline{Y}$, the system is just a linear difference equation in Y. But at the corner, the behavior of the system changes sharply.

Another basic difference between linear and non-linear systems is that the latter are more likely to display sustained regular fluctuations. Suppose there were also a lower bound on income in (5.24). Then it would be possible for the system to bounce back and forth between the upper and lower bounds forever, for a wide variety of values of a and b. This is what is called a limit cycle; you will see one of these in the next chapter. Such behavior is possible for linear systems only for certain precise parameter values. Since such razor's edge situations are unlikely ever to be observed, business cycle theorists have turned to non-linear models when attempting to explain regular fluctuations in economic activity.

The distinction between corners and curves is intuitively useful, but it should not be over-emphasized. Sharp curves are essentially equivalent to corners. These concepts should help in deciding when a non-linear system can be approximated by a linear system. The models presented in the next four chapters are all non-linear, and they have two very important corners. Away from the corners, they still have curve-type non-linearities, but these are not really sharp. Away from the corners, the output from these models can be approximately analyzed by thinking of it as the output of a high-order linear system.

When any variable is approaching an equilibrium level that is known to be constant over time, the history of that variable can be used to describe its evolution in terms of a few basic parameters. These descriptions are exact for first- and second-order linear systems, and they will be acceptable approximations for complex linear systems and for

non-linear systems in the absence of sharp non-linearities. The key parameters are the equilibrium level and the rate of approach to equilibrium. If a solution is explosive, this will be clear after a casual inspection. If not, it is of interest to see how fast the solution approaches equilibrium. Much of the analysis of the dynamic models will be concerned with how changes in structural parameters affect stability.

In case 1 above, if the equilibrium level of the variable is constant, the solution will be of the form

$$(5.25) \quad Y(t) = A + B X^{t}$$

to a first approximation, where A and B are constants. For stability, X must be less than one. We shall write this solution as

$$(5.25') \quad Y(t) = EQ + K e^{-DR\ t}, \text{ where } DR = -\log_e(X),$$

and EQ is the equilibrium value of Y. We refer to DR as the <u>damping rate</u> of the solution; the larger is DR, the more rapidly equilibrium is being approached. In case 3 above, the peaks and troughs of the fluctuations will be connected by a curve of this sort: see equation (5.22). The constant K may be either positive or negative; it measures the initial disequilibrium.

Still assuming that EQ is a constant, suppose we have three observations on Y. These may be either three peaks, three troughs, or three points on a non-oscillatory path. Let DT be the (constant) time separating adjacent observations. Substituting into the equation above, we have

$$\text{(5.26)}\quad \begin{aligned} YA &= EQ + K && \text{first observation} \\ YB &= EQ + K\,e^{-DR\,DT} && \text{second observation} \\ YC &= EQ + K\,e^{-2\,DR\,DT} && \text{third observation} \end{aligned}$$

These are three equations in three unknowns: K, EQ, and DR. The latter two quantities are, of course, of greatest interest.

Equations (5.26 can be easily solved for the damping rate:

$$\text{(5.27)}\quad DR = -(1/DT)\,\log_e\,[\,(YA-YB)/(YB-YC)\,]$$

If we define $g = e^{-DR\,DT}$, the other unknowns in (5.26) can be computed from

$$\text{(5.28)}\quad \begin{aligned} K &= (YA-YB)/(1-g) \\ EQ &= (YB - gYA)/(1-g) \end{aligned}$$

It must be emphasized that these solutions merely summarize the history of a variable over a certain range. The model may well behave quite differently in other time-periods. There is no reason to suppose that an estimate of DR from, say, periods 5 - 10 will be the same as the estimate from, say, periods 15 - 20. Indeed, the change in DR may be of considerable interest. Presumably, the best estimates of EQ will come when the system has almost attained equilibrium, when convergence has almost occurred.

Distributed Lags: Introduction

Most economic theory is static; time does not enter in an essential way. Static theory does not supply all the information needed to model the real world in most cases. Suppose that the static theory says that some variable, Y, should depend on another variable, X. If the quantity Y represents the outcome of a decision process, as total consumption and total investment do, it is unlikely that changes in X will be immediately reflected in Y. It is often quite important, especially for policy decisions, to be able to characterize the lag involved, to determine how long it takes Y to respond to changes in X.

Individual decision-makers will probably respond to changes in X some time after they occur, but not all people will wait the same length of time to act. If they did, changes in Y would lag changes in X by some fixed length of time. If individuals' lags differ, the aggregate response to changes in X will be spread over more than one period of time. Such lags are called distributed lags. These may exist at the individual level as well as in the aggregate, if individuals consider more than one lagged value of X in making decisions. Distributed lags at the household level are suggested by the Permanent Income theory, for instance.

If Y is determined by X through a distributed lag, this means that Y depends on more than one value of X. If we assume a linear model, we can write the general distributed lag relation as

$$(5.29) \quad Y(t) = a[\, w_0X(t) + w_1X(t-1) + w_2X(t-2) + \cdots \,].$$

The w's add to one, and they are usually all assumed to be positive. There may or may not be an infinite number of w's. (The difference between an infinite series that drops off quickly and a finite series is difficult to detect in practice, and the former is usually simpler to work with.) The constant a represents the eventual impact of a maintained unit change in X.

In the finite world we inhabit, there is never enough data to permit statistical estimation of an infinite number of w's directly. Some assumptions about the shape of the sequence of w's must be made; the sequence must be expressed in terms of a few parameters so the parameters can be estimated from data. Theoretical work can then proceed in terms of the parameters of the sequence of w's.

One approach is to assume that the sequence of w's can be adequately approximated by a polynomial function of the lag involved. If you then specify the degree of the polynomial, and, if you wish, place constraints on its behavior at the end-points (which you must specify), you can estimate the coefficients of the polynomial. For instance, you might assume that

$$(5.30) \quad w_i = a + b\,i + c\,i^2, \text{ for } i=0, 1, \ldots, 8,$$

and that $w_i=0$ for i greater than or equal to nine. Using a computational method proposed by S. Almon, the coefficients a, b, and c may be estimated statistically.

This approach has the drawback that you must specify the number of non-zero w's and the degree of the polynomial. One ends up searching at

some length for the "best" combination. Also, if seven lagged X's are assumed to influence Y, one must begin estimation with the eighth observation on Y. With long lags and few available observations, a good deal of information may be lost this way.

The second and more common approach is to assume that there are an infinite number of non-zero w's. For this to make any sense, we need w_i to fall rapidly to zero as i becomes large. The simplest assumption of this sort is

$$(5.31) \quad w_i = (1-k)k^i, \text{ for } i=0,1,2,\ldots$$

Here k must be a constant between zero and plus one, in order for the w_i to sum to one. This imposes the condition that all the w_i are positive.[1] This assumption on the w's was first proposed and explored by Koyck, and we speak of this as a <u>first-order or geometric or Koyck distributed lag</u>.

The beauty of this lag structure and more complicated variants of it is that equation (5.29) can be re-written so as to involve X(t) and a few lagged values of Y. In fact, if there are N parameters like k in (5.31) that determine the lag structure, (5.29) can be re-written to involve N lagged values of Y. We shall return to this general point in the next section. Let us now examine the implications of (5.13) in some detail.

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[1]This restriction is reasonable in almost all situations; the matter is fully discussed elsewhere.

Substituting (5.31) into (5.29), we obtain

$$(5.32) \quad Y(t) = a(1-k) \sum_{i=0}^{\infty} k^i X(t-i)$$

$$= a(1-k)X(t) + a(1-k) \sum_{i=1}^{\infty} k^i X(t-i).$$

Notice that the smaller is k, the more rapidly the influence of past X's decays. Lagging (5.32) by one period and multiplying by k, we have

$$(5.33) \quad kY(t-1) = a(1-k)k \sum_{i=0}^{\infty} k^i X(t-1-i)$$

$$= a(1-k) \sum_{i=1}^{\infty} k^i X(t-i).$$

Subtracting (5.33) from (5.32), we have

$$(5.34) \quad Y(t) = a(1-k)\, X(t) + K\, Y(t-1), \text{ or}$$

$$Y(t) - k\, Y(t-1) = a(1-k)\, X(t).$$

This is a first-order linear difference equation with constant coefficient and driving function $a(1-k)X(t)$. It will be stable if k is less than one. From the section on first-order equations, it should be clear that the smaller is k, the more rapidly equilibrium is approached, the more rapidly Y responds to X. (See equations (5.4) - (5.6).) It is easy to verify that an increase in X of one unit will raise equilibrium Y by a units.

We shall now examine parameters used to summarize lag distributions, and we shall evaluate these quantities for the Koyck lag structure. Clearly this structure is easily summarized by the parameter k, but the summary parameters we shall consider and (especially) the way we shall find them will be useful in the consideration of more complex lag structures. Also, these parameters will provide more insight than a statement like "k= .8".

We first consider the <u>median lag</u>. In equation (5.34), suppose that $Y(0) = aX(0)$. That is, assume that the system is in equilibrium in period zero. Suppose $X(1) = X(0) + 1$, and that this value of X is maintained thereafter. Then $Y(1) = a(1-k)[X(0)+1] + kY(0) = Y(0) + a(1-k)$. Substituting further, we find

$$
\begin{aligned}
Y(2) &= (1-k)Y(0) + a(1-k) + k[Y(0) + a(1-k)] \\
&= Y(0) + a(1-k)(1+k), \\
Y(3) &= (1-k)Y(0) + a(1-k) + k[Y(0) + a(1-k)(1+k)] \\
&= Y(0) + a(1-k)(1+k+k^2),
\end{aligned}
$$

and in general, summing the geometric series in k,

$$(5.35) \quad Y(t) = Y(0) + a(1-k^t).$$

The new equilibrium value of Y will be $Y_e = Y(0) + a$. The fraction of the adjustment to this new equilibrium completed after t periods is simply

$$\frac{Y(t) - Y(0)}{Y_e - Y(0)} = \frac{a(1-k^t)}{a} = (1-k^t).$$

The median lag, T_{md}, is simply that value of t for which the fraction of adjustment completed equals one half. Thus we have

$$(5.36) \quad .5 = 1 - k^{T_{md}}, \text{ or } T_{md} = \log(.5)/\log(k).$$

Note that as k goes to zero, so does the median lag, as one might expect.

In complicated lag structures, the median lag may be hard to compute. In its place, we use the <u>mean lag</u>, T_m, to measure the speed of response. The mean lag is defined by

$$(5.37) \quad T_m = \sum_{i=0}^{\infty} i\, w_i .$$

Before computing the mean lag for the Koyck case, it will be useful to introduce two concepts of broad application. The first is the <u>lag operator</u>, which we shall write as L. This operator is defined by the following identity, where V is any time-series variable:

$$(5.38) \quad L^k\, V(t) = V(t-k),$$

for k a non-negative integer. We can re-write the general distributed lag equation (5.29) in this notation as

$$(5.39) \quad Y(t) = [\sum_{i=0}^{\infty} w_i\, L^i\,]\, a\, X(t) = P(L)\, a\, X(t).$$

The quantity in brackets is called a <u>lag polynomial</u>. It is formally clear that the mean lag can be found by differentiating the lag polynomial with respect to L, treating L as an ordinary variable, and setting L=1; compare (5.37). This is more useful than it appears, as we shall see.

From (5.31), equation (5.29) in the Koyck case becomes

$$(5.40) \quad Y(t) = [\ (1-k) \sum_{i=0} k^i L^i\ ]\ a\ X(t)$$

$$= \frac{1-k}{1-kL} \quad a\ X(t).$$

To obtain the second line, we treated L like an ordinary constant between zero and one, and we expressed the sum of the geometric series in closed form. Notice that if we multiply both sides of (5.40) by (1-kL) and substitute Y(t-1) for L Y(t), we obtain equation (5.34). Differentiating the lag polynomial in the second line with respect to L and setting L equal to one, we obtain

$$(5.41) \quad T_m = P'(1) = \frac{k}{1-k}.$$

As with the median lag, when k goes to zero, the mean lag does also.

Occasionally, people speak of the <u>variance</u> of the lag distribution, V_L. This quantity is defined by

$$(5.42) \quad V_l = \sum_{i=0} w_i\ [\ i - T_m\ ]^2 = \sum_{i=0} i^2 w_i - T_m^2.$$

An examination of (5.39) should make it clear that the first term is found by differentiating the lag polynomial twice respect to L, setting L equal to one, and adding the mean lag. In the Koyck case

$$(5.43)\quad \begin{aligned} V_1 &= P''(1) + P'(1) - [\,P'(1)\,]^2 \\ &= \frac{2k^2}{(1-k)^2} + \frac{k}{(1-k)} - \frac{k^2}{(1-k)^2} = \frac{k}{(1-k)^2} \end{aligned}$$

The lag polynomial has one other important use. Notice that the Nth derivative of the lag polynomial in (5.39) with respect to L is given by

$$(5.44)\quad \frac{d^N P(L)}{dL^N} = \sum_{i=N}^{\infty} \frac{i!}{(i-N)!} w_i L^{i-N},$$

since the terms corresponding to i less than N vanish identically. (Recall that $N! = N(N-1)(N-2)\cdots 2\ 1.$) Setting $L=0$, all terms with i greater than N vanish. Since 0! is identically equal to one, we have the result

$$(5.45)\quad \left.\frac{d^N P(L)}{dL^N}\right|_{L=0} = N!\, w_N.$$

We can thus go back uniquely from the lag polynomial to the w's. This relation is easy to verify in the Koyck case using (5.40), and it is useful in higher-order structures.

Before examining such structures, it will be useful to illustrate the application of the tools we have developed. Suppose we have estimated a distributed lag relation between Y and X and have obtained

$$(5.46)\quad Y(t) = .30\, X(t) + .80\, Y(t-1).$$

The short-run impact of X upon Y is simply .30. To obtain more information, compare equation (5.46) to equation (5.34). It is clear that k=.80 and

that a, the long-run impact of X on Y, is equal to .30/(1 - .80) = 1.50. Using equation (5.36) we can compute the median lag:

$$T_{md} = \log(.5)/\log(.8) = (-.693)/(-.223) = 3.11 \text{ periods.}$$

From equation (5.41), the mean lag is simply .80/(1.80) = 4 periods. Similarly, equation (5.42) could be used to compute the variance of the lag distribution, and equation (5.31) could be employed to compute the individual lag weights.

Now suppose that the estimated relation between Y and X had been

$$(5.47) \quad Y(t) = .20\, X(t) + 1.10\, Y(t-1).$$

Can we compute similar statistics for this equation? No, since the implied value of k, 1.10 is not consistent with the Koyck lag scheme. It may be possible to make sense of equation (5.47), but it cannot be interpreted as an estimate of a geometric distributed lag function.

Distributed Lags: General Analysis

We shall begin this brief discussion of more complex lag schemes with a second-order example. Suppose that the quantity X in (5.20) represents an observed data series, but that Y is not observable. For instance, in an investment study, X might be sales and Y might be decisions to purchase new capital goods. No data on Y is available, but it is desired to explain investment spending, Z. We assume that Z is observable and that it is related to Y according to

$$(5.48) \quad Z(t) = \left[\frac{1-m}{1-mL} \right] b \, Y(t),$$

where L is the lag operator, as before, and m is a constant between zero and one. The mean lag of (5.48) is clearly m/(1-m). In the investment study, this would represent the mean lag between decisions and deliveries. It is easy to show that we can write Z(t) as a function only of the observable variable X(t) as follows:

$$(5.49) \quad Z(t) = \left[\frac{(1-m)(1-k)}{(1-mL)(1-kL)} \right] a \, b \, X(t).$$

By differentiating the lag polynomial in brackets (5.49) with respect to L and setting L equal to one, it can be shown that the mean lag in (5.40) is equal to [m/(1-m)] + [k/(1-k)]. The mean lags add. The variance of the lag can also be computed, and equation (5.45) can be used to compute the lag weights, the w's.

Multiplying (5.49) through by (1-mL)(1-kL) and re-writing, we obtain

$$(5.50) \quad Z(t) = [\ (1-m)(1-k)ab\ ]\ X(t) \ + \ (k+m)\ X(t-1) \ - \ km\ Z(t-2).$$

This is a second-order linear difference equation with constant coefficients and driving function [(1-m)(1-k)ab]X(t). Call this quantity D(t), and re-write (5.50) as

$$(5.50') \quad Z(t) \ - \ (k+m)\ Z(t-1) \ + \ km\ Z(t-2) \ = \ D(t).$$

The behavior of this system is determined by the solutions to the reduced equation, with $D(t) = 0$. We can apply the conditions we derived earlier for second-order difference equations. The solutions will not involve sinusoidal fluctuations if $(k+m)^2 > 4km$; this condition is always satisfied. Both real roots will be positive, since $-(k+m) < 0$ and $km > 0$. Sufficient conditions for stability are $km < +1$ and $-(1+km) < -(m+k) < (1+km)$. These conditions are always satisfied for $0 < k, m < +1$. Thus changes in X will result in Z(t) steadily approaching the new equilibrium. This corresponds precisely to what we mean by a distributed lag.

Suppose we estimate the coefficients in (5.50) from time-series data on Z(t) and X(t). The question naturally arises whether the estimated coefficients make sense as having come from the structure we have just described. One could solve for the estimated values of k and m and see if they are both positive and less than one. Alternatively, the conditions for non-oscillatory, stable solutions to the reduced equation could be applied directly to the estimated coefficients. If these conditions are not met, the estimated equation does not represent a distributed lag of the usual form.

In the general case, distributed lag equations may involve more than two lagged Z's, and there may be lagged X's as well. The restrictions that estimated coefficients must satisfy in order to be sensible may be quite complex; they will not concern us here.

To examine the general case, we define

$$F(L) = a_0 + a_1 L + a_2 L^2 + \dots + a_m L^m$$

$$G(L) = 1 - b_1 L - b_2 L^2 - \dots - b_n L^n$$

The general rational distributed lag difference equation may then be written as

$$(5.51) \quad Z(t) = \frac{F(L)}{G(L)} X(t).$$

This is called a rational lag since the lag polynomial P(L) may be written as the ratio of two polynomials in L. This term and much of the theory of such lags are due to D. Jorgenson. From equation (5.23), it should be clear that unless G(1) is positive, the equation makes no sense as a distributed lag. It can be shown that $G(1) > 0$ is a necessary condition for the stability of the difference equation. Notice that the long-run impact of X(t) on Z(t) is given by F(1)/G(1). (This is the change in equilibrium Z brought about by a unit change in X.) We can write (5.51) in the same form as (5.49):

$$(5.52) \quad Z(t) = \left[\frac{G(1)\ F(L)}{F(1)\ G(L)} \right] \frac{F(1)}{G(1)} X(t).$$

The quantity in brackets is now a lag polynomial, as we have been using the term. It can be differentiated with respect to L to find the mean lag, the variance of the lag distribution, and the individual lag weights, the w's.

We conclude this chapter with an illustration of the use of the tools developed here. Consider the following estimated equation:

$$(5.53) \quad Y(t) = 1.0\ X(t) + 2.0\ X(t-1) + 1.10\ Y(t-1) - .20\ Y(t-2).$$

It can be easily be shown that the estimated k and m are both between zero and one, so the w_i are all positive. The initial impact of X on Y is simply 1.0, while the long-run effect of a change in X is given by (1.0 + 2.0)/(1.0 - 1.10 + .20) = 30.

To obtain further results, we re-write (5.53) in the form of (5.52). Here we have

$$F(L) = 1.0 + 2.0\,L;\ F(1) = 3.0$$

$$G(L) + 1.0 - 1.10\,L + .20\,L^2;\ G(1) = .10$$

Hence (5.53) may be re-written as

$$(5.54)\quad Y(t) = \left[\frac{(.10)(1 + 2L)}{(3)(1 - 1.1L + .2L^2} \right] 30.\ X(t)$$

Differentiating the lag polynomial in brackets with respect to L and evaluating the derivative at L=1, we obtain the mean lag:

$$T_m = [\ .10/3.0\ ]\ [\ (.2 + 2.1)/.01\ ] = 7.7 \text{ periods.}$$

We could compute the variance of the lag distribution similarly, and equation (5.45) could be used to obtain the lag weights.

There is an easier way to obtain the first few lag weights in this case, however, and we now illustrate it. The lag polynomial can be re-written in the following form after some trivial re-arrangement:

$$\left[\frac{(1+2L)}{30} \right] \left[\frac{1}{[\ 1 - (1.1L - .2L^2)\ ]} \right]$$

The second term is simply the sum of a geometric series. Writing the series out, we obtain

$$\frac{(1+2L)}{30} \sum_{i=0} (1.1L - .2L^2)^i$$

$$= (1/30)\,[\,1 + 2L\,][\,1 + 1.1L - .2L^2 + 1.21\,L^2 - .22L^3 + .04L^4 \ldots\,]$$

$$= (1/30)\,[\,1 + 3.1L + 3.21\,L^2 + \ldots\,]$$

Thus we have $w_0 = (1/30)$, $w_1 = (3.1/30)$, and $w_2 = (3.21/30)$.

CHAPTER VI

The Simplest Multiplier-Accelerator Interaction

Introduction

In this chapter, we shall discuss model DYNEC1, the first of four dynamic macroeconomic models presented in this text. These models are nested, in the sense that the model presented in Chapter VII, DYNEC2, is an elaboration of DYNEC1, Chapter VIII's model, DYNEC3, is an elaboration of DYNEC2, and the model in Chapter IX, DYNEC4, is an elaboration of DYNEC3. It is thus of some importance that the reader understand the material presented in one chapter before going on to the next one. In all of these models, the pre-set parameter values and initial conditions bear a broad resemblance to magnitudes characterizing the U.S. economy in the late 1960's. Each time period is designed to correspond to a quarter (three months) of calendar time in a real economy.

In contrast with the models presented in Part II, we no longer deal only with equilibrium situations. Further, we do not assume that the capital stock is essentially unchanged as the economy moves towards equilibrium. Also, basic economic variables such as technology and the labor force may change over time. It is quite possible for the DYNEC models never to reach an equilibrium position; our interest is mainly in the path they follow towards (or away from) equilibrium. Unemployment situations and unemployment equilibria are quite possible here, since there are rigidities. In particular, we assume that the real wage rate is

determined outside the model.

The demand for investment is not directly assumed to be a function of income or of the change in income. Rather, we begin with firms' fundamental demand for capital goods. The discrepancy between actual and desired capital stock generates demand for gross investment. As in the IS-LM models, we do not explicitly consider investment in housing. Inventory investment is added to the model in Chapter VIII.

A number of important simplifying assumptions are made in this model. First, foreign trade and net foreign investment are assumed zero. All the DYNEC's are models of closed economies. In DYNEC1, no lags are present. This assumption of instantaneous perception and reaction on the part of economic agents is relaxed in DYNEC2. In DYNEC1 and DYNEC2, no inventories are present, and production is always equal to sales. These assumptions are relaxed in DYNEC3. Finally, the rate of interest is assumed to be exogenously determined until DYNEC4, when the market for real cash balances is explicitly introduced.

The structure of DYNEC1 is spelled out in Tables VI.1 and VI.2 and in more detail in Appendix A. (Remember as you read these that X**A means X raised to the A power, while X*A means X times A.) The DYNEC models all use Archive DYNEC, presented in Appendix B.

Even with all the simplifying assumptions that have gone into its construction, DYNEC1 is a rather complicated model. Compare Table VI.2 to any of the Tables in Chapters III and IV. Instead of proceeding directly to an equation-by-equation discussion of the model, we shall present a simplified linearized version of DYNEC1 in the next section.

Once the basic outline of the model is clear, we can proceed with a detailed examination of its structure. The final section describes the model's behavior and discusses the sort of experiments that can be performed with it.

The Basic Model

Making all equations linear and explaining only the most important variables, we can write a simplified version of DYNEC1 as follows:

(6.1)
a. YDI = k*YP

b. CD = m*YDI

c. ID = 4*[v*YP - (1-RD/4)*K(-1)]

d. K = ID/4 + (1-RD/4)*K(-1)

e. YP = CD + ID + GD

The variable K is the economy's capital stock in billions of constant dollars. The following flow variables are measured in billions of constant dollars at annual rates:

YDI = Personal Disposable Income

CD = Consumption Demand

ID = Investment Demand

GD = Government Demand

YP = Gross National Product

Constants appearing in equations (6.1) are the following:

k = Ratio of YDI to YP

m = Marginal (and Average) Propensity to
Consume out of YDI

v = Desired Ratio of Capital to Output

RD = Annual Rate of Depreciation of Capital Stock

The notation here is a bit different from that used in the last chapter. Basically, we've suppressed the t's. Thus YP here would be YP(t) in Chapter V, and K(-1) here would be K(t-1) there. This change will simplify our equations and introduce notation more consistent with computer output; it should cause no confusion.

All the income variables are quarterly totals expressed at annual rates, as are all Department of Commerce quarterly national income and product series. Thus YP = 800 means that 200 billion dollars (neglecting seasonal adjustments) of final goods and services were produced during the quarter. This means that when ID = 100, only $25 billion in gross investment actually took place during the quarter. Capital stock, on the other hand, has no associated time dimension. It thus requires some care to correctly relate measured gross investment to changes in the capital stock.

Let us examine equations (6.1) one at a time. Equation (6.1.a) makes disposable personal income a constant fraction of gross national product. As tax rates rise, this fraction falls. Consumption spending is determined in (6.1.b) as a constant fraction of disposable income. We can combine these two equations to obtain

(6.1.b') CD = k*m*YP.

Thus km is the marginal (and average) propensity to consume out of GNP.

In equation (6.1.c), v is the desired ratio of capital stock to GNP,

where GNP is measured at annual rates. Thus v*YP is the desired capital stock. If capital stock depreciates at a rate of RD per year, a fraction RD/4 will effectively vanish each quarter. Thus (1-RD/4)*K(-1) is the amount of capital on hand if no investment is undertaken. The quantity in brackets in (6.1.c) is thus the amount by which desired capital exceeds capital on hand, the amount of gross investment firms will seek to make. Since ID/4 is the amount of investment actually made, the bracketed expression is multiplied by 4 to obtain ID. Equation (6.1.d) is the identity that gives the capital stock as a function of the past stock, depreciation, and gross investment. Note the use of the constant 4 to convert gross investment at annual rates into actual gross investment and to convert the annual rate of depreciation into the quarterly rate.

The last equation in (6.1) is simply the accounting identity found in all macroeconomic models; GD is assumed exogenously determined. Substituting equations (6.1.a) - (6.1.c) into (6.1.e) we obtain

$$(6.2) \qquad YP = \frac{1}{1-mk-4v} GD - \frac{4d}{1-mk-4v} K(-1),$$

where d = (1-RD/4) for simplicity. Substituting this into (6.1.c) yields

$$(6.3) \qquad ID = \frac{4v}{1-mk-4v} GD - \frac{4d(1-mk)}{1-mk-4v} K(-1).$$

Finally, we substitute (6.3) into (6.1.d) to obtain a difference equation in K:

$$(6.4) \qquad K = \frac{-4vd}{1-mk-4v} K(-1) + \frac{v}{1-mk-4v} GD.$$

This is a first-order linear difference equation with constant coefficient and driving function vGD/(1-mk-4v). As we saw in the last chapter, the interesting

properties of solutions of this type of equation depend only on the coefficient of the lagged variable. Let us therefore examine the coefficient of K(-1) in some detail.

First of all, v is normally found to be well above .25 in empirical work. If v takes on such a value, as it does in the pre-set parameters to DYNEC1, the coefficient of K(-1) must be positive, since mk is positive. We thus know that the solution to the reduced equation of (6.4) involves no oscillations. This solution will be stable if and only if the coefficient of K(-1) is less than plus one. After a little algebra, it can be shown that this reduces to the condition

$$1 - mk < v\ RD.$$

At the pre-set parameter values, RD is .10, mk is about .66, and v is about .60. The inequality does not hold, since .06 (=vRD) is a good deal less than .34 (=1-mk). The basic linear model (6.1) therefore yields a solution in which K either grows or decays without limit, depending on initial conditions. From (6.2), YP will also steadily diverge from equilibrium. (Consider GD constant. If K grows, so does K(-1), and so does YP.) Similarly, both CD and ID will head for minus or plus infinity, depending on initial conditions.

This result is patently unrealistic. One basic way in which the linear model (6.1) violates reality is by ignoring two important barriers present in any real economy. First of all, gross investment cannot be negative for the economy as a whole. Second, there is an upper limit to production, YP, which depends on available capital, labor, and technology.

To take these two barriers into account, the DYNEC models all have

two corner-type nonlinearities. We use equation (6.1.c) to calculate desired investment, IDS, and we add the equation

(6.5) ID = IF IDS GRT 0.0 THEN IDS ELSE 0.0.

The term "GRT" is shorthand for "greater than". Similarly, we use equation (6.1.e) to compute aggregate demand, YD, and we add the equation

(6.6) YP = IF YFE GRT THAN YD THEN YD ELSE YFE,

where YFE, full-employment production, is the maximum amount the economy can produce. (In reality, YFE is never a fixed number. We gain some simplicity, however, by inserting this non-linearity as a corner rather than as a sharp curve.) Thus income is bounded by two values: the level of YD corresponding to zero gross investment, and full-employment production.

Along with these two inequalities, an equation must be added to determine actual gross investment when YD is above YP. When this is done, the modified system is still unstable, but motion away from equilibrium will be checked by the built-in barriers. In fact, the economy described here will bounce back and forth between these two limiting levels, in what we called a <u>limit cycle</u> in the last chapter.

It is easy to show that it must do this when full-employment income is constant over time. In this case, once YP arrives at the full-employment level, it can rise no further. Investment demand will continue only until the desired capital stock corresponding to YFE is attained. Then IDS will fall, and it will carry income down with it. At the new, lower level of income, there is excess capital. IDS will be negative, and ID will be zero.

There will be zero gross investment until depreciation has reduced the capital stock below the KS corresponding to the lower bound on income. When businesses (finally) find themselves with too low a capital stock, they will undertake investment. This increases income and, since the system is unstable, eventually sends it all the way to YFE.

Before going into the details of DYNEC1, we should mention the role of economic growth in this system. If technology and the labor force grow, YFE will grow also. This means that the economy has a higher potential real GNP. Whether this potential is realized or not depends on the structure of the economy and the policies undertaken by government. In the simple model (6.1), GD must grow as rapidly as YFE if successive recessions are not to involve higher and higher rates of unemployment. All the DYNEC models can be used to simulate growing economies; they include both cyclical and growth elements. But again, all the DYNEC models (and especially DYNEC1) are extremely simplified portraits of a very complex reality.

The Structure of DYNEC1

In this section we shall go through the equations listed in Table VI.2 in the order they are presented. See Table VI.1 for notation.

Block I gives the relations between Gross National Product and Disposable Personal Income. The quantity P is the ratio of the price level in the current quarter to the price level in the base period, which is taken as the quarter before the simulation begins. Thus P*YP is GNP in billions of current dollars. We assume that Personal Income, YPI, is a constant fraction, (1-LFR), of Gross National Product. This is a not unreasonable

approximation to a rather long list of national income accounting relations that are not really important for an understanding of business fluctuations.

The second equation in this block subtracts personal income taxes from personal income and adds current dollar transfer payments, TRP, to yield Disposable Income. Personal taxes are assumed to be given by the following equation:

$$(6.7) \qquad \text{Personal Taxes} = L\ TRA\ (YPI/L)^{TRB}.$$

Since L is the labor force in billions of persons, and since we assume that L is a constant fraction of the population, YPI/L is proportional to personal income per capita. At pre-set values, TRB is greater than one, so that we are modeling a progressive tax structure; taxes are an increasing fraction of personal income as personal income per capita rises. The rate level parameter, TRA, can be made to grow or decline, as we shall discuss shortly.

In the second block, the first and last equations are essentially the same as their counterparts in model (6.1). Consumption is a constant fraction of constant dollar disposable income, and aggregate demand is the sum of consumption, investment, and government demands. The equations determining K, IDS, and ID are also as above. The only difference is that desired capital stock, KS, appears explicitly, and it is determined by a rather forbidding equation of its own instead of being simply v*YP.

To explain the equation giving KS, we need to begin with the economy's Cobb-Douglas aggregate production function:

$$(6.8) \qquad YP = A\ MH^{.85}\ K^{.15},$$

where MH refers to man-hours worked. The exponents of this function imply constant returns to scale; they are consistent with recent econometric work. Since YP is expressed at annual rates, MH must be man-hours at annual rates. Also, MH must be in billions, since YP and K are in billions. We obtain MH by multiplying E, employment in billions of persons, by 2000, the assumed average number of man-hours per man-year.

We obtain KS, the desired capital stock, as follows. Assume that YP is given to the firms in the economy. The cost of a man-hour of labor is the real wage rate, W. The cost of a dollar of capital stock held for a year is (2.5*R+RD), as in Chapter IV. The equation given for KS, except for the parameter CUS, is the solution to the problem of choosing K and MH so as to minimize the cost of producing YP. Notice that the higher the wage rate or the lower the opportunity cost of capital, the larger will be the capital stock desired to produce a given output. Similarly, the larger is A, the lower will be KS. The parameter CUS is present in case it is desired to model a situation in which businesses wish to hold excess capital. If CUS is .90, for instance, KS will be approximately 10% larger than the profit-maximizing capital stock. Block II is really the critical section of this model. It should be well understood before simulations are performed.

We next consider the third block. Full-employment production, YFE, is determined from the production function and from the definition that full employment corresponds to a measured unemployment rate of 2%. The second equation in this block is the upper barrier on income, and the third equation expresses the equality of production and final sales. In DYNEC3 and DYNEC4, inventories are present, and this equality no longer holds.

In block VI, the price level is determined. The rate of inflation is related to the rate of unemployment by a Phillips Curve of the following form:

$$(6.9) \qquad PD = a(1/U) + b,$$

where a and b are constants. These constants are parameterized, so the user can specify the PD corresponding to 2% unemployment, MPD, and the rate of unemployment corresponding to zero inflation, UZ. Since PD is the annual rate of inflation, it must be divided by four in the identity that gives the new price level as a function of the old level and the rate of inflation. Remember that P is the ratio of the current price level to the price level prevailing in the quarter before the simulation begins.

Block V yields the variables that describe the economy's utilization of capital and labor. Employment in billions of persons, E, is computed by solving the production function. The unemployment rate is then obtained from an identity. This rate does not correspond to the unemployment rate measured by the Bureau of Labor Statistics, since we make the unrealistic assumption that the labor force is a constant fraction of the population. In real life, the labor force rises relative to the working age population in prosperity, and declines in recessions. Most of this variability in participation comes in the categories of older workers, teenagers, and women. These persons are not always participants in the job market, whereas the fraction of prime-aged males in the labor force is essentially constant. Our U can be most sensibly interpreted as the rate of unemployment of the full-employment labor force. It makes sense as a measure of resource utilization, but its dynamics will not be the same as those of the measured unemployment rate. In particular, it will rise more than the measured rate for any given fall in YP.

The third equation in this block yields a new measure of the utilization of the economy's capital stock. The capacity utilization rate, CU, is defined as "optimal capital in use" divided by the existing capital stock. "Optimal capital in use" is in turn defined as the number of man-hours used (at annual rates) times the cost-minimizing ratio of capital to man-hours. This ratio is equal to KS, optimal capital, divided by the optimal number of man-hours given YP. It is a constant equal to .15W/[.85(2.5R+RD)], as can be easily demonstrated. When CU is above CUS, businesses will want to add to their capital; when CU is below CUS, they will desire less capital than they have, so CU is sensible in this regard.

Block VI describes what happens when YD is above YFE. (Note that if YD is less than YFE, RSD is equal to one.) We assume that the same fraction of all demands are satisfied when this occurs. This fraction is equal to YS/YD = YFE/YD = RSD. We multiply CD by RSD to obtain the real demands for consumer goods and services that are satisfied. Similarly, RSD* GD is actual government purchases of goods and services, and RSD*ID is actual satisfied investment demand of firms.[1]

Finally we come to block VII. Here we determine government demand, the labor force, the level of technology, the real wage rate, and the level of personal tax rates. These are completely exogenous to the rest of the

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[1]Many interesting questions of just how excess demand is actually rationed among sectors are largely unanswered.

model, since they depend only on initial conditions and on parameters supplied by the user. The equations permit both trends and stepwise changes in these variables for the user's convenience. Consider, for instance, the equations for A. If it is desired to leave A unchanged for a particular run, set GA = DA = 0. If A is to grow, leave DA=0 and make GA positive. If A is to be increased once and for all, set GA=0 and make DA positive. Finally, by using both GA and DA it is possible to investigate stepwise changes relative to a basic trend. For instance, two runs could be made with GA=.001, one with DA=0 and one with DA=5. A comparison of the output would indicate the impact of a different level of A with the same trend. Using the equations in block VII, it is possible to simulate a wide variety of changing environments.

It must be emphasized that the only reason for block VII is to permit the user to conveniently vary GD, L, A, W, and TRA from period to period during a simulation. The just-named variables are exogenous quantities that affect the economy.

Computer Analysis and Exercises

All the DYNEC models are dynamic. Solutions should begin with year 3. Each "year" corresponds to three months of calendar time. The solution for year N+1 does depend on the solution found for year N. To observe the impact of a parameter change, it will usually be necessary to simulate at least thirty "years". Graphical output will almost always aid in understanding what is going on. All the DYNEC models use Archive DYNEC. None of these models present any difficulties of non-convergence on the TROLL

system. As long as parameter changes are not too drastic, no problems of this sort should occur.

The pre-set values of the parameters are as follows:

LFR .185	TRB 1.21
	TRP 0.0
APC .94	RD .10
R .045	CUS 1.0
MPD .10	UZ .10
GGD 0.0	DGD 0.0
GL 0.0	DL 0.0
GA 0.0	DA 0.0
GW 0.0	DW 0.0
GTRA 0.0	DTRA 0.0

Also, initial conditions must be provided for those variables whose lagged values are present in the model. These are as follows:

K(2)	400.	P(2)	1.0
GDTR(2)	220.	ATR(2)	4.67
LTR (2)	.0756	WTR(2)	4.02
TRAT(2)	.021		

With these parameter values and initial conditions, the model's behavior is well described by the basic model presented above. The economy

rockets from one barrier to the other. It remains a long time on the lower barrier once it arrives there, since a great deal of capital stock must be depreciated. One question that may be investigated is whether there are any reasonable parameter values that will stabilize the system. Would a larger TRB, for instance, make for a noticeable increase in stability?

Another set of questions have to do with the pattern of growth in this economy. The impact of various factors on the trend rate of growth can be analyzed, along with the response of the economy to shocks around trends.

Let g(X) be the percentage rate of growth of the quantity X, defined by

$$(6.10) \qquad g(X) = \frac{dX}{dt} \frac{1}{X} .$$

Since the rate of growth of man-hours is equal to the rate of growth of employment, we can differentiate the aggregate production function and obtain

$$(6.11) \qquad g(YFE) = .15\ g(K) + .85\ g(L) + g(A).$$

In equilibrium, with R and RD constant, it is easy to show that

$$(6.12) \qquad g(W) + g(L) = g(K).$$

Substituting into the equation just above, we have

$$(6.13) \qquad g(YFE) = g(L) + g(A) + .15g(W).$$

If growth is to be balanced on average, it should be the case that g(GD) = g(YFE). A set of parameters that satisfies this and is reasonably compatible with post-war U.S. experience is the following:

GA = .00595 (2,38% per year)

GL = .00125 (.5% per year)

GW = .0071 (2.84% per year)

GGD = .00825 (3.3% per year)

With these parameters and all other parameters and initial conditions as above, the economy settles into oscillatory growth with about 10% of the full-employment labor force unemployed on average. DYNEC1 still hits the barriers, but now the barriers are growing.

However, this is not a true equilibrium. As income per capita increases, a progressive tax structure (TRB greater than one) leads to an even more rapid rise in taxes per capita. This is the phenomenon of fiscal drag; it will eventually slow the rate of growth. Assuming LFR, TRB, and TRP constant, the condition that real personal taxes grow as rapidly as YP is obtained from equation (6.14).

$$(6.14) \qquad g(TRA) = (TRB - 1)[g(L) - g(P) - g(YP)]$$

In terms of the model, this means that real personal disposable income will be a constant fraction of GNP if

$$(6.15) \qquad GTRA = (TRB - 1)[GL - PD/4 - g(YP)]$$

where g(YP) is the per-quarter rate of growth of YP. If prices are rising, the tax rate must fall if equilibrium is to be maintained at the given rate of price increase. Otherwise, rising prices will increase taxes more than income and slow growth. It if is desired to obtain balanced growth in a situation in which the quantity in brackets in (6.15) will not be zero,

the rate of growth of TRA must be non-zero as well. Unless the rate of growth of per-capita income or of prices are quite large, this can be expected to be a second-order effect for reasonable values of TRB.

In a growth situation, what is the impact of parameter changes on stability? What effect do parameter changes have on the equilibrium rate of growth?

In experimenting with the DYNEC models, one must be very careful to distinguish between the impact of initial conditions and the mode of behavior generated by the model. It is possible for a given parameter change to increase the initial dis-equilibrium and yet to stabilize the system. The first few fluctuations will then be larger than before the change, but they will damp out faster. And it is the rate of damping that is of interest, along with the economy's equilibrium position.

Fundamentally, DYNEC1's behavior is not very rich. For most parameter values, it will race from one barrier to the other indefinitely in a limit cycle. The basic elements of DYNEC1 are, however, the fundamental ingredients of most models of cycles based on swings in plant and equipment spending, though the need for modifications is clear. No real economy is as unstable as DYNEC1. The main purpose of this chapter has been to present the structure of DYNEC1, since the more interesting models of the next three chapters are modifications of DYNEC1 that include additional elements of reality.

Table VI.1

Notation Used in DYNEC1*

I.a. Consumer Income: Parameters

LFR	Ratio of "leakages" between GNP and Personal Income to GNP (fraction)
TRB	Tax Rate B: degree of progression (pure number)
TRP	Government transfer payments to persons (billions, curr.$)

I.b. Consumer Income: Variables

YPI	Personal Income (billions, curr.$)
YDI	Disposable Personal Income (billions, curr. $)

II.a. Aggregate Demand: Parameters

APC	Average and marginal propensity to consume out of YDI (fraction)
R	Rate of interest (fraction)
RD	Annual rate of depreciation of capital stock (fraction)
CUS	Desired Rate of capacity utilization (fraction)

II.b. Aggregate Demand: Variables

CD	Consumption demand (billions, cnst.$)
K	Capital stock (billions, cnst. $)
KS	Desired capital stock (billions, cnst.$)
IDS	Desired investment (billions, cnst.$)
ID	Investment demand (billions, cnst.$)
YD	Aggregate demand (billions, cnst.$)

III.a. Production and Sales: Parameters

none

III.b. Production and Sales: Variables

YFE	Full-employment GNP (billions, cnst.$)
YS	Final sales (billions, cnst.$)
YP	Production (billions, cnst.$)

(Table VI.1, Continued)

IV.a. Prices: Parameters

MPD	Annual rate of inflation at full employment (fraction)
UZ	Unemployment rate for which inflation is zero (fraction)

IV.b. Prices: Variables

PD	Annual rate of inflation (fraction)
P	Price level relative to base period (pure number)

V.a. Resource Utilization: Parameters

none

V.b. Resource Utilization: Variables

E	Employment (billions, persons)
U	Unemployment rate (fraction)
CU	Rate of capacity utilization (fraction)

VI.b. Rationing: Parameters

none

VI.b. Rationing: Variables

RSD	Fraction of demands satisfied (fraction)
C	Satisfied consumption demand (billions, cnst.$)
I	Satisfied investment demand (billions, cnst.$)
G	Satisfied government demand (billions, cnst.$)

VII.a. Exogenous Influences: Parameters

GGD	Trend rate of growth of government demand per quarter (fraction)
GL	Trend rate of growth of labor force per quarter (fraction)
GA	Trend rate of growth of productivity per quarter (fraction)
GW	Trend rate of growth of real wage rate per quarter (fraction)
GTRA	Trend rate of growth of personal tax rates per quarter (fraction)
DGD	Deviation of government demand from trend (billions, cnst.$)
DL	Deviation of labor force from trend (billions, persons)
DA	Deviation of productivity from trend (pure number)
DW	Deviation of real wage rate from trend (cnst.$ per manhour)
DTRA	Deviation of personal tax rates from trend (fraction)

(Table VI.1, Continued)

VII.b. Exogenous Influences: Variables

GDTR	Trend level of government demand (billions, cnst.$)
LTR	Trend level of labor force (billions, persons)
ATR	Trend level of productivity (pure number)
WTR	Trend level of real wage rate (cnst.$ per manhour)
TRAT	Trend level of personal tax rates (fraction)
GD	Government demand (billions, cnst.$)
L	Labor Force (billions, persons)
A	Productivity (pure number)
W	Real wage rate (cnst.$ per manhour)
TRA	Level of personal tax rates (fraction)

* The organization of this table corresponds to that of Table IV.2. Parameters are defined when they first appear; variables are defined when they appear on the left of an equation. Fractions and pure numbers are unit-free. The following abreviations are used:

cnst.$ = constant (base period) dollars
curr.$ = current dollars

All flow variables are measured as seasonally-adjusted quarterly totals at annual rates.

Table VI.2

DYNEC1: Basic Multiplier-Accelerator Model

I. Consumer Income

```
YPI = (1.-LFR)*YP*P
YDI = YPI - L*TRA*(YPI/L)**TRB + TRP
```

II. Aggregate Demand

```
CD = APC*YDI/P
K = I/4. + (1.-RD/4.)*K(-1)
KS = [[.15*W/(.85*(2.5*R+RD))]**.85]*[YP/(A*CUS)]
IDS = 4.*[KS - (1.-RD/4.)*K(-1)]
ID = IF IDS GRT 0.0 THEN IDS ELSE 0.0
YD = CD + ID + GD
```

III. Production and Sales

```
YFE = A*(.98*L*2000.)**.85*K**.15
YS = IF YFE GRT YD THEN YD ELSE YFE
YP = YS
```

IV. Prices

```
PD = [(MPD*.02*UZ)/(UZ-.02)]*(1./U) - [MPD*.02/(UZ-.02)]
P = (1.+PD/4.)*P(-1)
```

V. Resource Utilization

```
E = (1./2000.)*[(YP/A)**(1./.85)]*[K**(-.15/.85)]
U = 1. - E/L
CU = .15*W*E*2000./(.85*(2.5*R+RD*K)
```

VI. Rationing when Demand Exceeds Capacity

```
RSD = YS/YD
C = RSD*CD
I = RSD*ID
G = RSD*GD
```

(Table VI.2, Continued)

VII. Exogenous Influences

GDTR = (1.+GGD)*GDTR(-1)

GD = GDTR + DG

LTR = (1.+GL)*LTR(-1)

L = LTR + DL

ATR = (1.+GA)*ATR(-1)

A = ATR + DA

WTR = (1.+GW)*WTR(-1)

W = WTR + DW

TRAT = (1.+GTRA)*TRAT(-1)

TRA = TRAT + DTRA

CHAPTER VII

A Model with Distributed Lags

Introduction

The model presented in the last chapter, DYNEC1, was quite unstable. We now discuss a very stable model, DYNEC2. The two sets of equations are quite similar, except that DYNEC2 considers lags in economic behavior. At several key points, DYNEC2 has distributed lags of the sort considered in Chapter V. These serve to smooth the response of the economic agents being modeled to changes in their environment, and this in turn stabilizes the system.

Since DYNEC2 is quite similar to DYNEC1, we can proceed at once to a detailed examination of the differences in the structures of the two models. We then discuss experiments with DYNEC2. Tables VII.1 and VII.2 contain the notation used in DYNEC2 and the equations that compose it. Elements not present in DYNEC1 are indicated. See the Appendices for more details..

The Structure of DYNEC2

All elementary textbooks speak of the circular flow of income. Factor earnings result in spending, which leads to production, which in turn gives rise to factor incomes. We can distinguish three sorts of lags in this process:[1]

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[1] See R.G.D. Allen, Macro-Economic Theory (London: MacMillan, 1968), Chapters 2, 5, and 9.

1. The Robertsonian lag of aggregate demand behind its determinants.

2. The Lundbergian lag of aggregate production behind aggregate demand.

3. The Output-Income lag of income payments behind aggregate production.

In DYNEC2, lags of the first and third type are present. To consider Lundbergian lags, we need to have inventories present, and we shall introduce inventory holding in the next chapter. Also, DYNEC2 allows for lags in the process of wage and price determination.

The output-income lag appears in block I in the Tables. It is an econometric fact that personal income is smoother over time than GNP. There are at least two reasons for this. The first is that businesses are reluctant to hire and fire workers in the face of training and severance costs. Also, the demand for non-production workers is not responsive to short-term sales changes. Wage payments are thus smoother than production. Another reason for this lag is corporate dividend policy. Dividends are not adjusted as rapidly as profits change. Rather, dividends are smoothed and retained earnings (after-tax profits minus dividends) absorb fluctuations in profits. For these and other reasons, it is reasonable to expect personal income, YPI, to adjust gradually to changes in production, YP. (Recall that YP is equal to Gross National Product.)

We use a first-order lag scheme to take the factors smoothing YPI into account. The equation which in DYNEC1 determined YPI now gives YPIE, equilibrium personal income. Personal income is determined from a first-order distributed lag equation of the form

(7.1) $\quad YPI = k\ YPIE + (1 - k)\ YPI(-1).$

In Chapter V, we showed that the mean lag of (7.1) was (1-k)/k periods. Writing this mean lag as LYP, we can solve for k and obtain

$$(7.2) \qquad k = 1/(1 + LYP)$$

Substituting (7.2) into (7.1), we have the equation shown for YPI in block I. The larger is LYP, the more slowly personal income responds to changes in production. Setting LYP = 0, we have the equation used in DYNEC1.

In block II, there are two Robertsonian lags: one in the determination of consumer demand, and one in the determination of investment spending. Both lags have theoretical and empirical support. The lag in consumer spending behind disposable income was first discussed by Friedman as the Permanent Income Theory. This theory holds that consumption spending is not determined by current disposable income, but rather by what consumers regard as their permanent or normal disposable income. Changes in disposable income that are felt to be transitory alter savings, not consumption spending. In our version of the theory, we let permanent income be a weighted sum of past real disposable incomes, with the weights declining geometrically over time:

$$(7.3) \qquad CD = APC\ (1-w) \sum_{i=0}^{\infty} w^{i} YDI(-i)/P(-i).$$

In Chapter V, we showed that this leads to the following equation:

$$(7.4) \qquad CD = k[APC\ YDI/P] + (1-k)\ CD(-1)$$

In block II of DYNEC2, we call the quantity in brackets CDE. We then write the constant k as a function of the mean lag in consumer demand, LCD, as before.

The lag structure determining investment demand is a good deal more complicated. Equilibrium desired capital stock, KSE, is given by the same equation that gave KS, desired capital stock, in DYNEC1. Thus, KSE is the instantaneously-optimal capital stock. But businesses do not determine their desired capital on the basis of current sales, wage rates, and interest rates alone. The history of these quantities also has an impact. To allow for this, we smooth KSE via a Koyck lag with mean lag LID to obtain KS. Thus even if sales today are high, capital stock demand may or may not be high, depending on what past sales have been. It takes some time before a change in sales is considered permanent enough to warrant placing orders for new equipment.

Given KS, the quantity

$$\text{(7.5)} \qquad KS - (1. - RD/4.)*K(-1)$$

is the discrepancy between desired and carry-over capital stock. We assume that it takes some time for orders for new capital equipment to be filled. We can express this relation as

$$\text{(7.6)} \qquad IDS = W(L)\ IDSE,$$

where W(L) is a lag polynomial, IDSE is new orders, and IDS is investment demands. We assume that businesses choose IDSE so that the sum of all unfilled orders, including current IDSE, equals the discrepancy between carry-over and desired capital stock.

Current new orders plus the backlog of unfilled past orders is equal to

$$IDSE + IDSE(-1) - IDS(-1) + IDSE(-2) - IDS(-2) + \ldots$$
$$= [IDSE - IDS(-1)][1+L+L^2+L^3+\ldots]$$
$$= [IDSE - IDS(-1)]/[1 - L],$$

where L is, as usual, the lag operator. Setting this equal to four times the discrepancy between actual and desired capital stock (as in DYNEC1) and multiplying through by (1 - L), we have

$$(7.7) \qquad IDSE - IDS(-1) = 4[KS - KS(-1)] - (1 - RD/4)K(-1) + (1-RD/4)K(-2) .$$

Using the identity

$$K = I/4 + (1-RD/4)K(-1),$$

this becomes

$$(7.8) \qquad IDSE = 4[KS-KS(-1)] + RD\ K(-1) + [IDS(-1)-I(-1)],$$

the equation used in DYNEC2 to determine new orders for capital goods.

Note that if IDS was negative in the last period, I was zero, and the third term in equation (7.8) is negative. That is, if a desire to lower capital was frustrated by the fact that gross investment must be non-negative for the economy as a whole, new orders are lower today than they otherwise would have been. Similarly, if demands exceeded capacity in the last quarter (YD greater than YFE), the third term in (7.8) would be positive. If rationing resulted in the frustrating of some investment demands, new orders are higher than they would have been otherwise. Most of the time, the economy is not on either the upper or lower barrier to income, and the third term in (7.8) is zero.

Given IDSE, we use equation (7.6) to obtain IDS. We assume that W(L) generates a second-order lag with equal roots, a so-called Pascal lag, with mean lag LIP. Under this assumption, we can write (7.6) as

$$(7.9) \qquad IDS = \left[\frac{(1-k)}{(1-kL)}\right]^2 IDSE.$$

The mean lag of (7.7) is equal to 2k/(1-k): see Chapter V. Setting this equal to LIP and substituting, we obtain the equation for IDS shown in Table VII.2.

The final difference between DYNEC2 and DYNEC1 appears in block VI, where the rate of inflation is determined. There we allow for the fact that prices and wages do not respond immediately to changes in the economic environment. The Phillips Curve equation used in DYNEC1 now determines the equilibrium rate of inflation, PDE. The actual rate of inflation, PD, is determined by putting PDE through a Koyck lag with mean lag LPD.

In DYNEC2, we have used only the simplest lag structures, the Koyck and the Pascal. This is not because we feel that reality is made up only of these two forms, but rather because of a desire to keep matters simple. Both structures employed have the advantage that they are completely determined by one parameter, the mean lag.

Computer Analysis and Exercises

DYNEC2 is quite clearly a dynamic model; the solution for period N+1 does depend on the solution found for period N. As before, simulations should begin with year 3, where each "year" corresponds to three months of calendar time. Archive DYNEC is used with this model.

The pre-set values of the parameters are as follows:

LFR	.185	TRP	0.0
TRB	1.21	LYP	1.8
APC	.94	RD	.10
LCD	1.0	CUS	1.0
R	.045	LID	1.5
		LIP	1.5
MPD	.10	LPD	4.0
UZ	.10		
GGD	0.0	DGD	0.0
GL	0.0	DL	0.0
GA	0.0	DA	0.0
GW	0.0	DW	0.0
GTRA	0.0	DTRA	0.0

Also, initial conditions must be provided for those variables whose lagged values are present in the model. These are as follows:

YPI(2)	540.		
K(2)	400.	KS(2)	400.
IDS(1)	50.	CD(2)	514.
IDS(2)	50.	I(2)	50.
P(2)	1.0	PD(2)	.02
GDTR(2)	220.	ATR(2)	4.67
LTR(2)	.0756	WTR(2)	4.02
		TRA(2)	.021

With these parameter values and initial conditions, the economy is quite stable. It first undergoes a boom because the initial capital stock is below equilibrium. It then moves rather quickly to equilibrium with considerable unemployment. One question that may be asked of this model is which lags are critical for stability. Setting LYP=LCD=LID=LIP=LPD=0 makes DYNEC2 into DYNEC1. Will small reductions in any of the lags de-stabilize the system?

Another set of experiments involves the dynamics of the economy's response to shocks. A control solution should be computed with the pre-set parameter values. Other time-paths can then be calculated after changing policy and structural parameters, and the differences between these paths and the control solution can be analyzed. (This is mechanically easy with 360 TROLL; see Chapter II.) What are the multipliers? How important is the interest rate? Which structural parameters are critical in determining the multipliers? Which determine how rapidly the economy reacts to policy changes?

One feature of this economy's behavior deserves mention at this point. The only true equilibrium involves U=UZ, unless MPD is set to zero or TRB is set to unity. This is because of the progressive personal income tax structure. If U is above UZ, prices are rising, personal income per capita in current dollars is rising, and taxes are rising more rapidly than income. This tends to lower income. The same process operates in reverse when U is below UZ and prices are falling. This process takes a long time, however, and should not be a cause of concern except in growth situations. It can be off-set by having TRA change over time, as discussed in the last chapter.

All of the experiments just described can be performed in growth situations. We examined the economics of balanced growth in the DYNEC models in Chapter VI; the reader should refer to that discussion. A reasonable base set of parameters for a growth situation are the following:

GTRA = .0

GA = .00595 (2.38% per year)

GL = .00125 (.5% per year)

GW = .0071 (2.84% per year)

GGD = .00825 (3.3% per year)

With these parameters and all other parameters and initial conditions as above, the economy settles into steady growth with about 10% of the full-employment labor force unemployed. Since UZ = .10 in the pre-set parameters, this is (approximately) an equilibrium path. Using this path as a control, a variety of changes can be made in the system and the results compared to those of this path.

With the pre-set parameters and initial conditions, the economy spends some time along the YP=YFE ceiling. Expansionary policies cannot have any real impact here. Policy parameters should not be changed until the system has left the ceiling in the control solution, unless they are changes designed to prevent the ceiling's being reached.

A final note. One must be very careful to distinguish between the impact of initial conditions and the mode of behavior generated by the system. Most parameter changes will affect the initial dis-equilibrium.

Some changes will stabilize the system, others will tend to destabilize it. The two effects on the economy's evolution must be carefully distinguished for the purposes of analysis.

Table VII.1

Notation Used in DYNEC2*

I.a. Consumer Income: Parameters

	LFR	Equilibrium ratio of "leakages" between GNP and Personal Income to GNP (fraction)
(N)	LYP	Mean lag in personal income (pure number)
	TRB	Tax Rate B: degree of progression (pure number)
	TRP	Goverment transfer payments to persons (billions, curr.$)

I.b. Consumer Income: Variables

(N)	YPIE	Equilibrium Personal Income (billions, curr. $)
	YPI	Personal Income (billions, curr.$)
	YDI	Disposable Personal Income (billions, curr.$)

II.a. Aggregate Demand: Parameters

	APC	Long-run average and marginal propensity to consume out of YDI (fraction)
(N)	LCD	Mean lag in consumer demand (pure number)
	R	Rate of interest (fraction)
	RD	Annual rate of depreciation of capital stock (fraction)
	CUS	Desired Rate of capacity utilization (fraction)
(N)	LID	Mean lag in investment demand (pure number)
(N)	LIP	Mean lag in production of investment goods (pure number)

II.b. Aggregate Demand: Variables

(N)	CDE	Equilibrium consumption demand (billions, cnst.$.$)
	CD	Consumption demand (billions, cnst.$)
	K	Capital stock (billions, cnst.$)
(N)	KSE	Equilibrium desired capital stock (billions, cnst.$)

(Table VII.1, Continued)

	KS	Desired capital stock (billions, cnst.$)
(N)	IDSE	Equilibrium desired investment, new orders (billions, const.$)
	IDS	Desired investment (billions, cnst.$)
	ID	Investment demand (billions, cnst.$)
	YD	Aggregate demand (billions, cnst.$)

III.a. Production and Sales: Parameters

none

III.b. Production and Sales: Variables

	YFE	Full-employment GNP (billions, cnst.$)
	YS	Final sales (billions, cnst.$)
	YP	Production (billions, cnst.$)

IV.a. Prices: Parameters

	MPD	Equilibrium annual rate of inflation at full employment (fraction)
(N)	LPD	Mean lag in price changes (pure number)
	UZ	Unemployment rate for which inflation is zero (fraction)

IV.b. Prices: Variables

(N)	PDE	Equilibrium annual rate of inflation (fraction)
	PD	Annual rate of inflation (fraction)
	P	Price level relative to base period (pure number)

V.a. Resource Utilization: Parameters

none

V.b. Resource Utilization: Variables

	E	Employment (billions, persons)
	U	Unemployment rate (fraction)
	CU	Rate of capacity utilization (fraction)

(Table VII.1, Continued)

VI.a. Rationing: Parameters

none

VI.b. Rationing: Variables

RSD	Fraction of demands satisfied (fraction)
C	Satisfied consumption demand (billions, cnst.$)
I	Satisfied investment demand (billions, cnst.$)
G	Satisfied government demand (billions, cnst.$)

VII.a. Exogenous Influences: Parameters

GGD	Trend rate of growth of government demand per quarter (fraction)
GL	Trend rate of growth of labor force per quarter (fraction)
GA	Trend rate of growth of productivity per quarter (fraction)
GW	Trend rate of growth of real wage rate per quarter (fraction)
GTRA	Trend rate of growth of personal tax rates per quarter (fraction)
DGD	Deviation of government demand from trend (billions, cnst.$)
DL	Deviation of labor force from trend (billions, persons)
DA	Deviation of productivity from trend (pure number)
DW	Deviation of real wage rate from trend (cnst. $ per manhour)
DTRA	Deviation of personal tax rates from trend (pure number)

VII.b. Exogenous Influences: Variables

GDTR	Trend level of government demand (billions, cnst.$)
LTR	Trend level of labor force (billions, persons)
ATR	Trend level of productivity (pure number)
WTR	Trend level of real wage rate (cnst. $ per manhour)
TRAT	Trend level of personal tax rates (pure number)
GD	Government demand (billions, cnst.$)

(Table VII.1, Continued)

L	Labor force (billions, persons)
A	Productivity (pure number)
W	Real wage rate (cnst. $ per manhour)
TRA	Level of personal tax rates (pure number)

* The organization of this table corresponds to that of Table VII.2. Parameters are defined when they first appear; variables are defined when they appear on the left of an equation. Fractions and pure numbers are unit-free. The following abreviations are used:

cnst.$ = constant (base period) dollars
curr.$ - current dollars

(N) - A quantity not present in DYNEC1

All flow variables are measured as seasonally-adjusted quarterly totals at annual rates.

Table VII.2

DYNEC2: A Model with Distributed Lags

I. Consumer Income

```
(N) YPIE = (1. - LFR) *YP*P
(N) YPI  = (1./(1. + LYP))*YPIE + (LYP/(1. + LYP))*YPI(-1)
    YDI  = YPE - L*TRA*(YPI/L)**TRB + TRP
```

II. Aggregate Demand

```
(N) CDE = APC*YDI/P
(N)  CD = (1./(1. + LCD))*CDE + (LCD/(1. + LCD))*CD(-1)
      K = I/4. + (1. - RD/4.)*K(-1)
(N) KSE = [[.15*W/(.85*(2.5*R + RD))]**.85]*[YP/(A*CUS)]
(N)  KS = (1./(1. + LID))*KSE + (LID/(1. + LID))*KS(-1)
(N)IDSE = 4.*(KS - KS(-1)) + RD*K(-1) + (IDS(-1)- I (-1))
(N) IDS = (1./(1. + LIP)**2)*IDSE + (.2*LIP/(1. + LIP))*IDS(-1)
          - ((LIP/(1. + LIP))**2)*IDS(-2)
     ID = IF IDS GRT 0.0 THEN IDS ELSE 0.0
     YD = CD + ID + GD
```

III. Production and Sales

```
    YFE = A*(.98*L*2000.)**.85*K**.15
     YS = IF YFE GRT YD ELSE YFE
     YP = YS
```

IV. Prices

```
(N) PDE = [(MPD*.02*UZ)/(UZ - .02)]*(1./U) - [MPD*.02/(UZ - .02)]
(N) PD  = (1./(1. + LPD))*PDE + (LPD/(1. + LPD))*PD(-1)
     P  = (1. + PD/4.)*P(-1)
```

(Table VII.2 , Continued)

V. Resource Utilization

E = (1./2000.)*[(YP/A)**(1./.85)]*[K**(-.15/.85)]

U = 1. - E/L

CU = .15*W*E*2000./(.85*(2.5*R+RD*K))

VI. Rationing when Demand Exceeds Capacity

RSD = YS/YD

C = RSD*CD

I = RSD*ID

G = RSD*GD

VII. Exogenous Influences

GDTR = (1. + GGD)*GDTR(-1)

GD = GDTR + DG

LTR = (1. + GL)*LTR(-1)

L = LTR + DL

ATR = (1. + GA)*ATR(-1)

A = ATR + DA

WTR = (1. + GW)*WTR(-1)

W = WTR + DW

TRAT = (1. + GTRA)*TRAT(-1)

TRA = TRAT + DTRA

(N) - An equation not present in DYNEC1.

CHAPTER VIII

Sales Expectations and the Production Decision

Introduction

This model is very much like DYNEC 2, discussed in the last chapter. DYNEC3 adds inventories to DYNEC2, thereby considering a mechanism that has been very important in the post-war period. Our development is based on Metzler's pioneering theoretical work on inventory cycles.[1]

The next section outlines the inventory-sales-production relations in DYNEC3. We then discuss the parameters and initial conditions employed, and we examine the sorts of experiments that can be performed with DYNEC3. Tables VIII.1 and VIII.2 present the notation and equations that make up this model; see the Appendices for more details.

The Structure of DYNEC3

The mechanism built into this model is most appropriate for final goods inventories. We do not explicitly consider mechanisms relating to raw materials and goods-in-process inventories, and we ignore completely the role of new and unfilled orders. The basic motivation for holding finished goods inventories is that they provide a buffer against unexpected changes

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[1]See especially L. A. Metzler, "The Nature and Stability of Inventory Cycles," Review of Economic Statistics, 23(August, 1941), 113 - 129.

in sales. Such buffers are desired because it is costly to change the rate of production rapidly. In any particular case, the optimal inventory stock of a firm or industry will depend in a complicated way on a number of parameters. We make the common simplifying assumption that there is a constant target ratio between aggregate final sales and aggregate inventory holdings. Letting H be end-of-period inventory holdings, we express this assumption as

$$(8.1) \quad H^* = HSR\ YS,$$

where HSR is the target ratio.

At the start of a period, when the production decision is made, firms rarely know exactly what their sales will be. (Indeed, in the aggregate the production decisions of firms will to some extent determine their sales.) If we let YSX be expected sales, equation (8.1) must be re-written as

$$(8.1a) \quad H^* = HSR\ YSX$$

We now must explain the determination of YSX. Metzler proposed the following equation:

$$(8.2) \quad YSX = YS(-1) + MS\ [\ YS(-1) - YS(-2)\ ].$$

If MS is equal to zero, firms do not extrapolate past changes. If MS is equal to one, firms assume that past trends will continue. If MS is equal

to minus one, firms assume that the change in sales they most recently observed was temporary. Sensible values of MS thus range from minus one to plus one.

In econometric work, many writers have made the assumption that sales forecasts are correct on average. This permits using YS in place of YSX in estimated equations, on the grounds that YS is on average equal to YSX. It is sensible that current sales would affect current production decisions, since it is hard to believe that equation (8.2) adequately describes business forecasting and that businesses cannot alter production decisions made at the start of each quarter. To the extent that production decisions can be re-examined during the quarter, it is YS that will govern that decision, rather than YSX as defined above.

DYNEC3 incorporates both the notion that YS should affect current production and equation (8.2) in its equation for expected sales. Our approach is somewhat novel, but it permits greater flexibility in modeling an aspect of behavior about which little is actually known. We let LS be a fraction which indicates the extent to which expectations about sales can be revised within each quarter. Denoting the forecast given by (8.2) as YSX_m, we then write our equation for the expected sales variable that influences production decisions as

$$(8.3) \quad YSX = LS*YS + (1-LS)*YSX_m$$

$$= LS*YS + (1+MS)*YS(-1) - (1-LS)*MS*YS(-2)$$

Given the expected sales, we can write down equilibrium production:

(8.4) YPE = YSX + (HSR*YSX - H(-1)).

Equation (8.4) says that equilibrium production is equal to expected sales plus the difference between desired inventories and the inventories on hand at the start of the quarter. Except for the difference between our YSX equation and Metzler's the mechanism so far is essentially his.

In Metzler's formulation, actual production would equal YPE. In DYNEC3, however, we explicitly incorporate the idea that it is generally costly to change the level of production. Thus YPS, desired production, is obtained by putting YPE through a Koyck lag equation with mean lag LPR. This is broadly consistent with the work on optimal production decision rules under conditions of quadratic costs. Actual production, YP, is equal to YPS unless the latter is above the full-employment ceiling, in which case full-employment output is produced.

Sales are equal to aggregate demand unless demand cannot be met. In this case, sales are equal to capacity output plus inventory stocks on hand. Given sales and production, inventory change, DH, is computed from the identity

(8.5) DH = YP - YS,

which of course assumes no deterioration of inventory stocks. Intended inventory change, DHP, is equal to expected sales minus desired production, YPS. Both these variables are under firms' control. Unintended inventory change

can come about because YPS was above YFE or, more usually, because expected sales did not equal actual sales.

All the equations we have discussed are indicated in Table VIII.2. Other than these new relations, DYNEC2 and DYNEC3 are identical.

Computer Analysis and Exercises

As usual, we shall first exhibit the pre-set parameter values and initial conditions for this model. All initial conditions are, as usual, contained in Archive DYNEC. The parameter values are as follows:

LFR	.185	TRP	0.0
TRB	1.21	LYP	1.8
APC	.94	RD	.10
LCD	1.0	CUS	1.0
R	.045	LID	1.5
		LIP	1.5
LS	.10	HSR	.25
MS	.90	LPR	.50
MPD	.10	LPD	4.0
UZ	.10		

GGD	0.0	DGD	0.0
GL	0.0	DL	0.0
GA	0.0	DA	0.0
GW	0.0	DW	0.0
GTRA	0.0	DTRA	0.0

Initial conditions for those variables whose lagged values are present in the model are as follows:

YPI(2) 540.

K(2)	330.	KS(3)	330.
IDS(1)	0.	CD(2)	514.
IDS(2)	26.	I(2)	26.

H(2)	175.	YS(2)	700.
YS(2)	700.	YP(2)	700.

P(2)	1.0	PD(2)	.02

GDTR(2)	220.	ATR(2)	4.67
LTR(2)	.0756	WTR(2)	4.02
		TRAT(2)	.021

Metzler, in the article cited above, has derived analytical results for a model containing a production-inventory system much like that presented

here. In fact, with LPR=0 and LS=0, the systems are identical. But Metzler's model takes fixed investment as exogenous, and it has a very rudimentary consumption function. It is of some interest to see if Metzler's conclusions about the parameter values necessary for stability hold in the more complex economy of DYNEC3.

In a more general vein, all of the experiments that can be performed on DYNEC2 can also be done here. One can examine the impact of changing one or more of the structural parameters, either in the production-inventory system or elsewhere, on the dynamic response of the system to changes in the exogenous influences. In particular, growth situations are of considerable interest. Do Metzler's conclusions, for instance, hold when the economy is undergoing balanced growth?

As discussed in the last chapter, the best approach to these and other problems involves computing a control solution, varying structural and policy parameters, and examining the difference between the two paths.

It should be noted that with the given parameters and initial conditions, DYNEC3 rises rapidly to full employment and stays there for some time. The decline from full employment is quite rapid. Until the economy leaves the full employment ceiling, there is no room for any expansionary changes to have impact.

Table VIII.1

Notation Used in DYNEC3*

I.a. Consumer Income: Parameters

LFR	Equilibrium ratio of "leakages" between GNP and Personal Income to GNP (fraction)
LYP	Mean lag in personal income (pure number)
TRB	Tax Rate B: degree of progression (pure number)
TRP	Government transfer payments to persons (billions, curr.$)

I.b. Consumer Income: Variables

YPIE	Equilibrium Personal Income (billions, curr. $)
YPI	Personal Income (billions, curr.$)
YDI	Disposable Personal Income (billions, curr. $)

II. a. Aggregate Demand: Parameters

APC	Long-run average and marginal propensity to consume out of YDI (fraction)
LCD	Mean lag in consumer demand (pure number)
R	Rate of interest (fraction)
RD	Annual rate of depreciation of capital stock (fraction)
CUS	Desired Rate of capacity utilization (fraction)
LID	Mean lag in investment demand (pure number)
LIP	Mean lag in production of investment goods (pure number)

II. b. Aggregate Demand: Variables

CDE	Equilibrium consumption demand (billions, cnst.$.$)
CD	Consumption demand (billions, cnst.$)
K	Capital stock (billions, cnst.$)
KSE	Equilibrium desired capital stock (billions, cnst. $)

(Table VIII.1, Continued)

KS Desired capital stock (billions, cnst.$)

IDSE Equilibrium desired investment, new orders (billions, const. $)

IDS Desired investment (billions, cnst.$)

ID Investment demand (billions, cnst.$)

YD Aggregate demand (billions, cnst.$)

III.a. Production and Sales: Parameters

(N) LS Lovell coefficient of sales expectations (fraction)

(N) MS Metzler coefficient of sales expectations (pure number)

(N) HSR Desired inventory-sales ratio (pure number)

(N) LPR Mean lag in production change (pure number)

III.b. Production and Sales: Variables

YFE Full-employment GNP (billions, cnst.$)

YS Final sales (billions, cnst.$)

(N) YSX Expected final sales (billions, cnst.$)

(N) YPE Equilibrium production (billions, cnst.$)

(N) YPS Desired production (billions, cnst.$)

YP Production (billions, cnst.$)

(N) DH Inventory investment (billions, cnst. $)

(N) H End-of-period inventory stock (billions, cnst.$)

(N) DHP Planned inventory investment (billions, cnst.$)

(N) DHU Unplanned inventory investment (billions, cnst.$)

IV. a. Prices Parameters

MPD Equilibrium annual rate of inflation at full employment (fraction)

LPD Mean lag in price changes (pure number)

UZ Unemployment rate for which inflation is zero (fraction)

(Table VIII.1, Continued)

IV.b. Prices: Variables

PDE Equilibrium annual rate of inflation (fraction)

PD Annual rate of inflation (fraction)

P Price level relative to base period (pure number)

V.a. Resource Utilization: Parameters

none

V.b. Resource Utilization: Variables

E Employment (billions, persons)

U Unemployment rate (fraction)

CU Rate of capacity utilization (fraction)

VI.a. Rationing: Parameters

none

VI.b. Rationing: Variables

RSD Fraction of demands satisfied (fraction)

C Satisfied consumption demand (billions, cnst.$)

I Satisfied investment demand (billions, cnst.$)

G Satisfied government demand (billions, cnst.$)

VII.a. Exogenous Influences: Parameters

GGD Trend rate of growth of government demand per quarter (fraction)

GL Trend rate of growth of labor force per quarter (fraction)

GA Trend rate of growth of productivity per quarter (fraction)

GW Trend rate of growth of real wage rate per quarter (fraction)

GTRA Trend rate of growth of personal tax rates per quarter (fraction)

DGD Deviation of government demand from trend (billions, cnst.$)

DL Deviation of labor force from trend (billions, persons)

DA Deviation of productivity from trend (pure number)

DW Deviation of real wage rate from trend (cnst. $ per manhour)

DTRA Deviation of personal tax rates from trend (pure number)

(Table VIII.1, Continued)

VII.b. Exogenous Influences: Variables

GDTR Trend level of government demand (billions, cnst.$)

LTR Trend level of labor force (billions, persons)

ATR Trend level of productivity (pure number)

WTR Trend level of real wage rate (cnst. $ per manhour)

TRAT Trend level of personal tax rates (pure number)

GD Government demand (billions, cnst. $)

L Labor force (billions, persons)

A Productivity (pure number)

W Real wage rate (cnst. $ per manhour)

TRA Level of personal tax rates (pure number)

*The organization of this table corresponds to that of Table VIII.2. Paramters are defined when they first appear; variables are defined when they appear on the left of an equation. Fractions and pure numbers are unit-free. The following abreviations are used:

cnst. $ = constant (base period) dollars
Curr. $ - current dollars
(N) - A quantity not present in DYNEC2.

All flow variables are measured as seasonally-adjusted quarterly totals at annual rates.

Table VIII.2

DYNEC3: A Production-Inventory System Added

I. Consumer Income

```
YPIE = (1. - LFR) *YP*P

YPI  = (1./(1. + LYP))*YPIE + (LYP/(1. + LYP))*YPI(-1)

YDI  = YPE - L*TRA*(YPI/L)**TRB + TRP
```

II. Aggregate Demand

```
CDE  = APC*YDI/P

CD   = (1./(1. + LCD))*CDE + (LCD/(1. + LCD))*CD(-1)

K    = I/4. + (1. - RD/4.)*K(-1)

KSE  = [[.15*W/(.85*(2.5*R + RD))]**.85]*[YP/(A*CUS)]

KS   = (1./(1. + LID))*KSE + (LID/(1. + LID))*KS(-1)

IDSE = 4.*(KS - KS(-1)) + RD*K(-1) + (IDS(-1)- I (-1))

IDS  = (1./(1. + LIP)**2)*IDSE + (.2*LIP/(1. + LIP))*IDS(-1)

         - ((LIP/(1. + LIP))**2)*IDS(-2)

ID   = IF IDS GRT 0.0 THEN IDS ELSE 0.0

YD   = CD + ID + GD
```

III. Production and Sales

```
   YFE  = A*(.98*L*2000.)**.85*K**.15

(N) YS  = IF (YFE + H(-1)) GRT YD THEN YD ELSE (YFE + H(-1))

(N) YSX = LS*YS + (1. - LS)*(1. + MS)*YS(-1) - (1. - LS)*MS*YS(-2)

(N) YPE = YSX + (HSR*YSX - H(-1)

(N) YPS = (1./(1. + LPR))*YPE + (LPR/(1. + LPR))*YP(-1)

(N) YP  = IF YFE GRT YPS THEN YPS ELSE YFE

(N) DH  = YP - YS

(N) H   = DH + H(-1)

(N) DHP = YPS - YSX

(N) DHU = DH - DHP
```

(Table VIII.2, Continued)

IV. Prices

PDE = [(MPD*.02*UZ)/(UZ - .02)]*(1./U) - [MPD*.02/(UZ - .02)]

PD = (1./(1. + LPD))*PDE + (LPD/(1. + LPD))*PD(-1)

P = (1. + PD/4.)*P(-1)

V. Resource Utilization

E = (1./2000.)*[(YP/A)**(1./.85)]*[K**(-.15/.85)]

U = 1. - E/L

CU = .15*W*E*2000./(.85*(2.5*R + RD*K))

VI. Rationing when Demand Exceeds Capacity

RSD = YS/YD

C = RSD*CD

I = RSD*ID

G = RSD*GD

VII. Exogenous Influences

GDTR = (1. + GGD)*GDTR(-1)

GD = GDTR + DG

LTR = (1. + GL)*LTR(-1)

L = LTR + DL

ATR = (1. + GA)*ATR(-1)

A = ATR + DA

WTR = (1. + GW)*WTR(-1)

W = WTR + DW

TRAT = (1. + GTRA)*TRAT(-1)

TRA = TRAT + DTRA

(N) - An equation not present in DYNEC2

CHAPTER IX

Endogenous Determination of the Rate of Interest

Introduction

This chapter presents DYNEC4, the most complex (and last) of the DYNEC's. In a sense, we have returned to the IS-LM models of Chapter IV, since DYNEC4 has an endogenously determined interest rate. The real sector is, of course, quite complicated, as it is identical with the DYNEC3 model. The difference between the two models is that the interest rate is endogenous to DYNEC4, the nominal money supply replacing it as an exogenous variable.

We shall first examine the differences between DYNEC3 and DYNEC4. We then discuss the behavior of DYNEC4 and the uses to which it may be put. The Tables at the end of the chapter present the equations that compose DYNEC4 and define the quantities appearing therein.

The Structure of DYNEC4

The rate of interest, R, was a parameter in DYNEC1 - DYNEC3. In DYNEC4, it is an endogenous variable. The nominal money supply, M, is now exogenous to the model. It is determined by the user through the following equations:

(9.1) MTR = (1.+GM)*MTR(-1)

M = MTR + DM.

This is the same structure used to determine GD, A, L, W, and TRA. With these equations, the user can investigate steady growth in M (DM=0, GM≠0), disturbances in M (DM≠0, GM=0), or deviations of M from steady growth (DM≠0, GM≠0).

The nominal money supply, real income, and the price level interact to determine the interest rate, much as they did in the IS-LM models. The equilibrium demand for real cash balances, MRE, is given by

$$(9.2) \qquad MRE = .121*YP*R^{-IE}.$$

The constant is simply a scaling factor, and IE is (minus) the interest elasticity of the demand for real cash balances. Recall the impact of changes in IE on the slope of the LM curve.

Equation (9.2) is quite similar to the demand for money equations in ISLM1 and ISLM2. The main difference is that DYNEC4 does not take into account the government budget constraint and hence neglects the market for securities. We have done this for the same reason that most texts neglect this constraint in static models: for simplicity. Adding a government budget **constraint** in a sensible way to DYNEC4 (the trick used in Chapter IV would not work sensibly here) would greatly complicate the model.

It is assumed that there is a Koyck lag mechanism operating in the money market, so that the actual demand for real cash balances is given by

$$(9.3) \qquad M/P = [1./(1.+LMD)]*MRE + [LMD/(1.+LMD)]*[M(-1)/P(-1)].$$

Here LMD is the mean lag in the money demand equation. The longer the lag, the more action in MRE - and thus in R - is required to make the public content to hold cash balances of M when, for instance, Y changes. Since M is determined by the user as a policy variable, this equation is solved for MRE in DYNEC4.

Aside from these few equations, DYNEC4 is exactly equivalent to DYNEC3.

Computer Analysis and Exercises

The pre-set parameter values in DYNEC4 are the following:

LFR	.185	TRP	0.0
TRB	1.21	LYP	1.8

APC	.94	RD	.10
LCD	1.8	CUS	1.0
LID	1.5	LIP	1.5

LS	.10	HSR	.25
MS	.90	LPR	.50

MPD	.10	LPD	4.0
UZ	.10		

GGD	0.0	DGD	0.0
GL	0.0	DL	0.0
GA	0.0	DW	0.0
GTRA	0.0	DTRA	0.0
GM	0.0	DM	0.0
LMD	2.0	IE	.20

Initial conditions for those variables whose lagged values are present in the model are as follows:

YPI(2)	540.

K(2)	330.	KS(2)	330.
IDS(1)	0.	CD(2)	514.
IDS(2)	26.	I(2)	26.

H(2)	175.	YS(2)	700.
YS(2)	700.	YP(2)	700.
P(2)	1.0	PD(2)	.02
GDTR(2)	220.	ATR(2)	4.67
LTR(2)	.0756	WTR(2)	4.02
MTR(2)	175.	TRAT(2)	.021

As before, with this set of parameters and initial conditions the economy rises to full employment, remains there a while, and falls away into a recession with U approximately equal to .10.

The main thrust of experimentation with DYNEC4 should involve examining the impact of different money market parameters on the economy's stability and behavior. How are multipliers altered, how do speeds of response and patterns of response vary when IE and LMD are changed?

It should be possible with this model to investigate the question of policy design, to try and devise optimal policy rules to stabilize the economy. This may require altering the model to build in various policy equations.

Many experiments will involve growth situations, so the conditions for full dynamic equilibrium should be discussed at this point. If R is to be held constant, equation (9.2) indicates that the following equation should determine the rate of growth of the nominal money supply, g(M):

$$g(M) = g(YP) + g(P). \qquad (9.4)$$

The nominal money supply must grow as rapidly as <u>current dollar</u> GNP. Unless

the unemployment rate is to be held steady at UZ, this will imply that g(M) is not equal to g(YP). In fact, g(M) will be greater or less than g(YP), according as equilibrium U is less or greater than UZ. In Chapter VI, we presented a set of parameters that give approximately equilibrium growth with U=UZ. To these should be added

GM = .00825 (3.3% per year)

for DYNEC4.

Table IX.1

Notation Used in DYNEC4

I.a. Consumer Income: Parameters

LFR	Equilibrium ratio of "leakages" between GNP and Personal Income to GNP (fraction)
LYP	Mean lag in personal income (pure number)
TRB	Tax Rate B: degree of progression (pure number)
TRP	Government transfer payments to persons (billions, curr.$)

I.b. Consumer Income: Variables

YPIE	Equilibrium Personal Income (billions, curr. $)
YPI	Personal Income (billions, curr.$)
YDI	Disposable Personal Income (billions, curr. $)

II. a. Aggregate Demand: Parameters

APC	Long-run average and marginal propensity to consume out of YDI (fraction)
LCD	Mean lag in consumer demand (pure number)
RD	Annual rate of depreciation of capital stock (fraction)
CUS	Desired Rate of capacity utilization (fraction)
LID	Mean lag in investment demand (pure number)
LIP	Mean lag in production of investment goods (pure number)

II. b. Aggregate Demand: Variables

CDE	Equilibrium consumption demand (billions, cnst.$.$)
CD	Consumption demand (billions, cnst.$)
K	Capital stock (billions, cnst.$)
KSE	Equilibrium desired capital stock (billions, cnst. $)

(Table IX.1, Continued)

KS Desired capital stock (billions, cnst.$)

IDSE Equilibrium desired investment, new orders (billions, const. $)

IDS Desired investment (billions, cnst.$)

ID Investment demand (billions, cnst.$)

YD Aggregate demand (billions, cnst.$)

III.a. Production and Sales: Parameters

LS Lovell coefficient of sales expectations (fraction)

MS Metzler coefficient of sales expectations (pure number)

HSR Desired inventory-sales ratio (pure number)

LPR Mean lag in production change (pure number)

III.b. Production and Sales: Variables

YFE Full-employment GNP (billions, cnst.$)

YS Final sales (billions, cnst.$)

YSX Expected final sales (billions, cnst.$)

YPE Equilibrium production (billions, cnst.$)

YPS Desired production (billions, cnst.$)

YP Production (billions, cnst.$)

DH Inventory investment (billions, cnst. $)

H End-of-period inventory stock (billions, cnst.$)

DHP Planned inventory investment (billions, cnst.$)

DHU Unplanned inventory investment (billions, cnst.$)

IV. a. Prices Parameters

MPD Equilibrium annual rate of inflation at full employment (fraction)

LPD Mean lag in price changes (pure number)

UZ Unemployment rate for which inflation is zero (fraction)

(Table IX.1, Continued)

IV.b. Prices: Variables

PDE Equilibrium annual rate of inflation (fraction)

PD Annual rate of inflation (fraction)

P Price level relative to base period (pure number)

V.a. Resource Utilization: Parameters

none

V.b. Resource Utilization: Variables

E Employment (billions, persons)

U Unemployment rate (fraction)

CU Rate of capacity utilization (fraction)

VI.a. Rationing: Parameters

none

VI.b. Rationing: Variables

RSD Fraction of demands satisfied (fraction)

C Satisfied consumption demand (billions, cnst.$)

I Satisfied investment demand (billions, cnst.$)

C Satisfied government demand (billions, cnst.$)

VII.a. Exogenous Influences: Parameters

GGD Trend rate of growth of government demand per quarter (fraction)

GL Trend rate of growth of labor force per quarter (fraction)

GA Trend rate of growth of productivity per quarter (fraction)

GW Trend rate of growth of real wage rate per quarter (fraction)

GTRA Trend rate of growth of personal tax rates per quarter (fraction)

(N) GM Trend rate of growth of money supply per quarter (fraction)

DGD Deviation of government demand from trend (billions, cnst.$)

DL Deviation of labor force from trend (billions, persons)

DA Deviation of productivity from trend (pure number)

DW Deviation of real wage rate from trend (cnst. $ per manhour)

DTRA Deviation of personal tax rates from trend (pure number)

(N) DM Deviation of money supply from trend (billions, curr.$)

(Table IX.1, Continued)

VII.b. Exogenous Influences: Variables

	GDTR	Trend level of government demand (billions, cnst.$)
	LTR	Trend level of labor force (billions, persons)
	ATR	Trend level of productivity (pure number)
	WTR	Trend level of real wage rate (cnst. $ per manhour)
	TRAT	Trend level of personal tax rates (pure number)
(N)	MTR	Trend level of money supply (billions, curr. $)
	GD	Government demand (billions, cnst. $)
	L	Laobr force (billions, persons)
	A	Productivity (pure number)
	W	Real wage rate (cnst. $ per manhour)
	TRA	Level of personal tax rates (pure number)
(N)	M	Money supply (billions, curr.$)

IX.a. Money Market: Parameters

(N)	LMD	Mean lag in money demand (pure number)
(N)	IE	Absolute value of interest elasticity of money demand (pure number)

IX.b. Money Market: Variables

(N)	MRE	Equilibrium money demand (billions, curr. $)
(N)	R	Rate of interest (fraction)

*The organization of this table corresponds to that of Table IX.2. Parameters are defined when they first appear; variables are defined when they appear on the left of an equation. Fractions and pure numbers are unit-free. The following abreviations are used:

cnst. $ = constant (base period) dollars
curr. $ = current dollars
(N) = A quantity not present in DYNEC3

All flow variables are measured as seasonally-adjusted quarterly totals at annual rates.

Table IX.2

DYNEC3: A Market for Real Cash Balances Added

I. Consumer Income

```
YPIE = (1. - LFR) *YP*P

YPI  = (1./(1. + LYP))*YPIE + (LYP/(1. + LYP))*YPI(-1)

YDI  = YPE - L*TRA*(YPI/L)**TRB + TRP
```

II. Aggregate Demand

```
CDE  = APC*YDI/P

CD   = (1./(1. + LCD))*CDE + (LCD/(1. + LCD))*CD(-1)

K    = I/4. + (1. - RD/4.)*K(-1)

KSE  = [[.15*W/(.85*(2.5*R + RD))]**.85]*[YP/(A*CUS)]

KS   = (1./(1. + LID))*KSE + (LID/(1. + LID))*KS(-1)

IDSE = 4.*(KS - KS(-1)) + RD*K(-1) + (IDS(-1)- I (-1))

IDS  = (1./(1. + LIP)**2)*IDSE + (.2*LIP/(1. + LIP))*IDS(-1)

          - ((LIP/(1. + LIP))**2)*IDS(-2)

ID   = IF IDS GRT 0.0 THEN IDS ELSE 0.0

YD   = CD + ID + GD
```

III. Production and Sales

```
YFE  = A*(.98*L*2000.)**.85*K**.15

YS   = IF (YFE + H(-1)) GRT YD THEN YD ELSE (YFE + H(-1))

YSX  = LS*YS + (1. - LS)*(1. + MS)*YS(-1) - (1. - LS)*MS*YS(-2)

YPE  = YSX + (HSR*YSX - H(-1)

YPS  = (1./(1. + LPR))*YPE + (LPR/(1. + LPR))*YP(-1)

YP   = IF YFE GRT YPS THEN YPS ELSE YFE

DH   = YP - YS

H    = DH + H(-1)

DHP  = YPS - YSX

DHU  = DH - DHP
```

(Table IX.2, continued)

IV. Prices

PDE = [(MPD*.02*UZ)/(UZ - .02)]*(1./U) - [MPD*.02/(UZ - .02)]

PD = (1./(1. + LPD))*PDE + (LPD/(1. + LPD))*PD(-1)

P = (1. + PD/4.)*P(-1)

V. Resource Utilization

E = (1./2000.)*[(YP/A)**(1./.85)]*[K**(-.15/.85)]

U = 1. - E/L

CU = .15*W*E*2000./(.85*(2.5*R + RD*K))

VI. Rationing when Demand Exceeds Capacity

RSD = YS/YD

C = RSD*CD

I = RSD*ID

G = RSD*GD

VII. Exogenous Influences

GDTR = (1. + GGD)*GDTR(-1)

GD = GDTR + DG

LTR = (1. + GL)*LTR(-1)

L = LTR + DL

ATR = (1. + GA)*ATR(-1)

A = ATR + DA

WTR = (1. + GW)*WTR(-1)

W = WTR + DW

TRAT = (1. + GTRA)*TRAT(-1)

TRA = TRAT + DTRA

(N) MTR = (1. + GM)*MTR(-1)

(N) M = MTR + DM

(Table IX.2, continued)

VIII. Money Market

(N) MRE = (1. + LMD)*(M/P) - LMD*(M(-1)/P(-1))

(N) R = (MRE/(.121*YP))**(-1./IE)

(N) - An equation not present in DYNEC3

Appendices

APPENDIX A

The Models in TROLL

The models discussed in the text are listed here as they appear in the TROLL system. These listings correspond closely to the descriptions given in the Tables in the text, but there are some differences. First of all, the declarations and specifications required by TROLL at the start of each model are shown here. This is presented for convenience.

A basic difference is the presence of equations not discussed in the text. These have as dependent variables quantities declared "CONSTRUCT". These equations are used only to help the system find a solution. Each is associated with an IF - THEN expression which does not affect the solution to the model. In GE1, for instance, LCS is a construct not discussed in the text. In equilibrium, though, LC=LCS, and the equation for LC given in the text holds.

The reason for declaring a variable CONSTRUCT is simple. When the user requests all * variables in the simulation output phase, constructs are not printed. Thus only the "real" endogenous variables are printed or graphed, unless constructs are specifically requested in a list of variables.

TROLL distinguishes two types of endogenous variables: ENDOGENOUS and DEFINITION. From the point of view of the user, these are equivalent. Hence we did not bring up this distinction in the text. The differences from the point of view of the model-builder are based on the fact that no data files are associated with definitions, while data for endogenous

variables must be present. We have used both types of variables in the models that follow; a particular quantity was declared DEFINITION or ENDOGENOUS depending on which was more convenient.

The remainder of this appendix lists the models presented in the text in the order they are discussed. Each model represents a separate file in the time-sharing system. Each model also has an associated parameter file. Thus, "ge1 model" is the file containing the GE1 model, and "ge1 param" is the file containing the pre-set parameter values mentioned in the text. Values for all endogenous variables are stored in Archives; these are discussed in Appendix B.

GE1 Model

```
TITLE     A TWO-GOOD, TWO-FACTOR CLOSED GENERAL
                EQUILIBRIUM MODEL //
TIMEUNIT YEARLY
MODEL
ENDOGENOUS PF, TC, TF, LC, LF, PC, W, R, QC, QF,
       WCC, WCF, RCC, RCF, WI, RI $,
PARAMETER LT, TT, AC, AF, WTC, RTC $,
CONSTRUCT LCS, TCS, PFS $,
QC = AC*(LC**.75)*(TC**.25)                                  $,
QF = AF*(LF**.30)*(TF**.70)                                  $,
LCS $=$ (.75*PC*QC*LF)/(.30*QF*PF)                               $,
LC = IF LCS GRT 0.999*LT THEN 0.999*LT ELSE
     IF LCS LES 0.001*LT THEN 0.001*LT ELSE LCS              $,
TCS $=$ (.25*PC*QC*TF)/(.70*QF*PF)                               $,
TC = IF TCS GRT 0.999*TT THEN 0.999*TT ELSE
     IF TCS LES 0.001*TT THEN 0.001*TT ELSE TCS              $,
W = .75*PC*QC/LC                                             $,
R = .25*PC*QC/TC                                             $,
LF = LT - LC                                                 $,
TF = TT - TC                                                 $,
WI = W*LT                                                    $,
RI = R*TT                                                    $,
PC = (WTC*WI + RTC*RI)/QC                                    $,
 PFS$=$2.276-1.22*PC $,
PF=IF PFS GRT .001 THEN PFS ELSE .001                        $,
WCC = WTC*WI/PC                                              $,
WCF = WI - WTC*WI                                            $,
RCC = RTC*RI/PC                                              $,
RCF = RI - RTC*RI                                            $,
END
```

GE2 Model

```
TITLE    A TWO-GOOD, TWO-FACTOR GENERAL EQUILIBRIUM
              MODEL WITH TRADE //
TIMEUNIT YEARLY
MODEL
ENDOGENOUS PC, PF, TC, TF, LC, LF, W, R, QCP, QFP,
      QCC, QFC, NXC, NXF, WCC, WCF, RCC, RCF, WI, RI $,
PARAMETER LT, TT, AC, AF, WTC, RTC, IRPCF $,
CONSTRUCT PFS, LCS, TCS $,
QCP = AC*(LC**.75)*(TC**.25)                                $,
QFP = AF*(LF**.30)*(TF**.70)                                $,
LCS $=$ (.75*PC*QCP*LF)/(.30*QFP*PF)                          $,
LC = IF LCS GRT 0.999*LT THEN 0.999*LT ELSE
     IF LCS LES 0.001*LT THEN 0.001*LT ELSE LCS             $,
TCS $=$ (.25*PC*QCP*TF)/(.70*QFP*PF)                          $,
TC = IF TCS GRT 0.999*TT THEN 0.999*TT ELSE
     IF TCS LES 0.001*TT THEN 0.001*TT ELSE TCS             $,
W  = .75*PC*QCP/LC                                          $,
R  = .25*PC*QCP/TC                                          $,
LF = LT - LC                                                $,
TF = TT - TC                                                $,
WI = W*LT                                                   $,
RI = R*TT                                                   $,
QCC = (RI*RTC + WI*WTC)/PC                                  $,
QFC = (PF*QFP + PC*(QCP-QCC)) / PF                              $,
NXC = QCP - QCC                                             $,
NXF = QFP - QFC                                             $,
PFS$=$2.276-1.22*PC                                       $,
PF=IF PFS GRT .001 THEN PFS ELSE .001                      $,
 PC=IRPCF*PF      $,
WCC = WTC*WI/PC                                             $,
WCF = WI - WTC*WI                                           $,
RCC = RTC*RI/PC                                             $,
RCF = RI - RTC*RI                                           $,
 END
```

ISLM1 Model

```
TITLE     THIS IS A BASIC CLOSED IS-LM MODEL //
TIMEUNIT YEARLY
MODEL
ENDOGENOUS C, I, R, DEF, NTX, OLK, MRS, DHPM, DGB $,
PARAMETER G, MTR, MI, J, L, LFR, TRF, DMFR, FDB, MMULT
              HPMI, GDBI, EB $,
DEFINITION Y, YD $,
CONSTRUCT RS, YS $,
YS $=$ C + I + G $,
Y $=$ IF YS LES 1.0 THEN 1.0 ELSE YS  $,
YD $=$ Y-NTX-OLK    $,
 NTX=-TRF+MTR*Y       $,
 OLK=LFR*Y            $,
 DEF=G-NTX            $,
C = 10.4 + MPC*YD      $,
I = 42.8 + MI*Y/(0.400943*((2.5*R+.1)/.218)**J)      $,
RS $=$ .0472*(.23626*Y/MRS)**(1/L)*((GDBI+DGB)/300.)**(1/EB) $,
 DHPM=DMFR+(1.-FDB)*DEF        $,
 DGB=-DMFR+FDB*DEF  $,
  MRS=MMULT*(HPMI+DHPM)      $,
R = IF RS LES 0.0001 THEN 0.0001 ELSE RS $,
END
```

IS-LM2 Model

```
TITLE      THIS IS AN OPEN IS-LM MODEL WITH UNEMPLOYMENT RATE,
    RATE OF INFLATION, AND BALANCE OF PAYMENTS ALSO DETERMINED //
TIMEUNIT YEARLY
MODEL
ENDOGENOUS C, I, BTR, DEF, NTX, OLK, MRS, DHPM, DGB, R, U,
                   PD, BCF $,
PARAMETER G, LFR, TRF, DMFR, FDB, MMULT, HPMI, GDBI, FB, MTR,
                 MPC, MI, J, L, PDF, K $,
DEFINITION Y, YD, BOP $,
CONSTRUCT RS, YS $,
YS $=$ C + I + G + BTR                                                     $,
Y $=$ IF YS LES 1.0 THEN 1.0 ELSE YS                                       $,
YD $=$ Y -NTX-OLK                        $,
NTX=-TRF+MTR*Y            $,
OLK=LFR*Y                 $,
DEF=G-NTX                 $,
C = 5.0 + MPC*YD                                                           $,
I = 42.2 + MI*Y/(0.401*((2.5*R+.1)/.218)**J)                  $,
RS $=$ .0472*(.2276*Y/MRS)**(1/L)*((GDBI+DGB)/300.)**(1/FB)          $,

DHPM=DMFR+(1.-FDB)*DEF                 $,
 DGB=-DMFR+FDB*DEF          $,
MRS=MMULT*(HPMI+DHPM)/(1.+PD)     $,
R = IF RS LES 0.0001 THEN 0.0001 ELSE RS                                   $,
U = 0.03 + (1 - Y/736.3)/3                                                 $,
PD = 0.105*(PDF - 0.0288)/U + (0.1008 - 2.5*PDF)                           $,
BTR = 99.44 - 0.1243*Y*(1 + PD)**2                                         $,
BCF = 5.1 - 12.5*((.20-R)/.1528)**K                        $,
BOP $=$ BTR + BCF                                                          $,
END
```

DYNEC1 Model

```
TITLE  MODEL 1. SIMPLE MULTIPLIER-ACCELERATOR WITH BARRIERS //
TIMEUNIT YEARLY
MODEL
DEFINITION YD,  IDS, YS, CD, ID, YFE, CU, RSD, YPI, YDI  $,
ENDOGENOUS YP, E, U, C, I, G, PD, K, L, P, A,
W, GD, GDTR, ATR, KS, LTR, WTR $,
ENDOGENOUS TRAT, TRA  $,
CONSTRUCT  YDB, KB  $,
PARAMETER GL, R, RD, APC, MPD, UZ,  GA,  GW, GGD, DTRA, GTRA $,
PARAMETER TRB, TRP, LFR,  DW, DGD, DL, DA, CUS  $,
CD $=$ APC * YDI/P  $,
KS $=$ (.15* W/(.85*(2.5*R +RD)))**.85 *(YP/(A*CUS)) $,
IDS $=$ 4. * (KS - (1. -  RD/4.) *K(-1))  $,
ID $=$ IF IDS GRT 0.0 THEN IDS ELSE 0.0  $,
GDTR = (1+ GGD) * GDTR (-1)  $,
GD = GDTR + DGD  $,
YDB $=$ CD + ID + GD  $,
YD $=$ IF YDB LES 1.0 THEN 1.0 ELSE YDB  $,
YFE $=$ A*(0.98*L*2000.)**0.85*K**0.15  $,
YS $=$ IF YFE GRT YD THEN YD ELSE YFE  $,
YP = YS  $,
YPI $=$ ( 1 -LFR) * YP * P  $,
YDI $=$ YPI - L * TRA * (YPI/L) ** TRB + TRP  $,
E =  (1./2000.) * (YP/A)**(1./.85)*K**(-.15/.85)  $,
U = 1 - E/L  $,
CU $=$ .15 * W * E * 2000./(.85 *(2.5 * R + RD) *K)  $,
KB $=$ I/4. + (1. - RD/4.) * K(-1)  $,
K = IF KB LES 1.0 THEN 1.0 ELSE KB  $,
LTR = (1 + GL) * LTR(-1) $,
L = LTR + DL  $,
ATR = (1 + GA) * ATR(-1) $,
A = ATR + DA  $,
RSD $=$ YS/YD  $,
C = RSD*CD  $,
I = RSD*ID  $,
G = RSD*GD  $,
P = (1 + PD/4.)*P(-1)  $,
PD $=$ ((MPD*0.02*UZ)/(UZ - 0.02))*(1/U) - MPD*0.02/(UZ - 0.02)   $,
WTR = (1 + GW) * WTR(-1) $,
W = WTR + DW  $,
TRAT = (1 + GTRA ) * TRAT (-1)  $,
TRA = TRAT + DTRA  $,
END
```

DYNEC2 Model

```
TITLE  MODEL 2. DISTRIBUTED LAGS IN CONSUMPTION, INVESTMENT,
          PRICE CHANGES, AND PERSONAL INCOME //
TIMEUNIT YEARLY
MODEL
DEFINITION YD, YS, YFE, CU, RSD,  YPI, ID,  KSE    $,
DEFINITION CDE, YPIE, PDE, IDSE  $,
ENDOGENOUS YP, E, U, C, I, G,  K, L, P, A,
W, GD, GDTR, ATR, KS, LTR, WTR, IDS $,
ENDOGENOUS YPI, PD, CD, TRAT, TRA $,
CONSTRUCT  YDB, KB  $,
PARAMETER GL, R, RD, APC, MPD, UZ,  GA,  GW, GGD, LCD, LIP, LYP, LPD $,
PARAMETER TRB, TRP, LER, DW, DGD, DL, DA, CUS, LIP, DTRA,  GTRA  $,
CDE $=$ APC * YPI/P  $,
CD = (1./(1. + LCD)) * CDE + (LCD/(1.+ LCD)) * CD(-1)  $,
KSE $=$ (.15* W/(.85*(2.5*R +RD)))**.85 *(YP/(A*CUS)) $,
KS = (1./(1. + LID))*KSE + (LID/(1.+LID)) * KS(-1)  $,
IDSE $=$ 4. *(KS-KS(-1)) + RD * K(-1) + (IDS(-1) - I(-1))  $,
IDS = (1./(1. + LIP)**2.)*IDSE+(2.*LIP/(1.+LIP))*
IDS(-1)-((LIP/1.+LIP))**2.)*IDS(-2)  $,
ID $=$ IF IDS GRT 0.0 THEN IDS ELSE 0.0  $,
GDTR = (1+ GGD) * GDTR (-1)  $,
GD = GDTR + DGD  $,
YDB $=$ CD + ID + GD  $,
YD $=$ IF YDB LES 1.0 THEN 1.0 ELSE YDB  $,
YFE $=$ A*(0.98*L*2000.)**0.85*K**0.15  $,
YS $=$ IF YFE GRT YD THEN YD ELSE YFE  $,
YP = YS  $,
YPIE $=$ ( 1 -LER) * YP * P  $,
YPI =(1./(1.+LYP))*YPIE + (LYP/(1.+LYP))*YPI(-1)  $,
YDI $=$ YPI - L * TRA * (YPI/L) ** TRP + TRB  $,
E =  (1./2000.) * (YP/A)**(1./.85)*K**(-.15/.85)  $,
U = 1 - E/L  $,
CU $=$ .15 * W * E * 2000./(.85 *(2.5 * R + RD) *K)  $,
KB $=$ 174. + (1. - RD/4.) * K(-1)  $,
K = IF KB LES 1.0 THEN 1.0 ELSE KB  $,
LTR = (1 + GL) * LTR(-1) $,
L = LTR + DL  $,
ATR = (1 + GA) * ATR(-1) $,
A = ATR + DA  $,
RSD $=$ YS/YD  $,
C = RSD*CD  $,
I = RSD*ID  $,
G = RSD*GD  $,
P = (1 + PD/4.)*P(-1)  $,
PDE $=$ ((MPD*0.02*UZ)/(UZ - 0.02))*(1/U) - MPD*0.02/(UZ - 0.02)  $,
PD = (1./(1. + LPD)) * PDE + (LPD/(1. + LPD)) * PD(-1)  $,
WTR = (1 + GW) * WTR(-1) $,
W = WTR + DW  $,
TRAT = ( 1 + GTRA) * TRAT (-1)  $,
TRA = TRAT + DTRA  $,
END
```

DYNEC3 Model

```
 TITLE   MODEL 3.  DISTRIBUTED LAGS, PRODUCTION AND INVENTORIES,
      AND IMPERFECT REALIZATION OF SALES FORECASTS  //
TIMEUNIT YEARLY
MODEL
DEFINITION YD,  YFE, CU, RSD,  YDI, ID,  KSE    $,
DEFINITION CDE, YPIE, PDE, IDSE, DH, DHU, DHP, YSX, YPE, YPS  $,
ENDOGENOUS YP, E, U, C, I, G,  K, L, P, A,
 W, GD, GDTR, ATR, KS, LTR, WTR, IDS, H, YS $,
ENDOGENOUS YPI, PD, CD, TRAT, TRA $,
CONSTRUCT  YDB, KB  $,
PARAMETER GL, R, RD, APC, MPD, UZ,  GA,  GW, GGD, LCD, LID, LYP, LPD $,
PARAMETER TRB, TRP, LER, DW, DGD, DL, DA, CUS, LIP, DTRA,  CTRA  $,
PARAMETER LPR, MS, LS, MSR  $,
CDE $=$ APC * YDI/P  $,
CD = (1./(1. + LCD)) * CDE + (LCD/(1.+ LCD)) * CD(-1)  $,
KSE $=$ (.15* W/(.85*(2.5*R +RD)))**.85 *(YP/(A*CUS)) $,
KS = (1./(1. + LID))*KSE + (LID/(1.+LID)) * KS(-1)  $,
IDSE $=$ 4. *(KS-KS(-1)) + RD * K(-1) + (IDS(-1) - I(-1))  $,
IDS = (1./(1. + LIP)**2.)*IDSE+(2.*LIP/(1.+LIP))*
IDS(-1)-((LIP/(1.+LIP))**2.)*IDS(-2)  $,
ID $=$ IF IDS GRT 0.0 THEN IDS ELSE 0.0  $,
GDTR = (1+ GGD) * GDTR (-1)  $,
GD = GDTR + DGD  $,
YDB $=$ CD + ID + GD  $,
YD $=$ IF YDB LES 1.0 THEN 1.0 ELSE YDB  $,
YFE $=$ A*(0.98*L*2000.)**0.85*K**0.15  $,
YS = IF (YFE +H(-1)) GRT YD THEN YD ELSE (YFE + H(-1))  $,
YSX $=$ LS * YS + (1. - LS) * (1. + MS) * YS(-1) -
(1.-LS)*MS*YS(-2) $,
YPE $=$ YSX + (MSR * YSX - H(-1))  $,
YPS $=$ (1./(1.+ LPR)) * YPE + ( LPR/ (1. + LPR )) * YP (-1)  $,
YP = IF YFE GRT YPS THEN YPS ELSE YFE  $,
DH $=$ YP - YS  $,
DHP $=$ YPS - YSX  $,
DHU $=$ DH - DHP  $,
H = DH + H(-1)  $,
YPIE $=$ ( 1 -LER) * YP * P  $,
YPI =(1./(1.+LYP))*YPIE + (LYP/(1.+LYP))*YPI(-1)  $,
YDI $=$ YPI - L * TRA * (YPI/L) ** TRB + TRP  $,
E =  (1./2000.) * (YP/A)**(1./.85)*K**(-.15/.85)  $,
U = 1 - E/L  $,
CU $=$ .15 * W * E * 2000./(.85 *(2.5 * R + RD) *K)  $,
KB $=$ I/4. + (1. - RD/4.) * K(-1)  $,
K = IF KB LES 1.0 THEN 1.0 ELSE KB  $,
LTR = (1 + GL) * LTR(-1) $,
L = LTR + DL  $,
ATR = (1 + GA) * ATR(-1) $,
A = ATR + DA  $,
RSD $=$ YS/YD  $,
```

(DYNEC3 Model, Continued)

```
C = RSD*CD  $,
I = RSD*ID  $,
G = RSD*GD  $,
P = (1 + PD/4.)*P(-1)  $,
PDE '=' ((MPD*0.02*UZ)/(UZ - 0.02))*(1/U) - MPD*[illegible]/(UZ - 0.02)   $,
PD = (1./(1. + LPD)) * PDE + (LPD/(1. + LPD)) * PD(-1)  $,
WTR = (1 + GW) * WTR(-1) $,
W = WTR + DW  $,
TRAT = ( 1 + GTRA) * TRAT (-1)  $,
TRA = TRAT + DTRA  $,
END
```

DYNEC4 Model

```
TITLE   MODEL 4.  ENDOGENOUS INTEREST RATE  //
TIMEUNIT YEARLY
MODEL
DEFINITION YD,  YEE, CU, RSD,  YDI, ID,  KSE, R, MRE   $,
DEFINITION CDE, YPIE, PDE, IDSE, DM, DMU, DMP, YSX, YPE, YPS  $,
ENDOGENOUS YP, E, U, C, I, G,  K, L, P, A, M,
GD, GDTR, ATR, KS, LTR, MTR, IDS, M, YS $,
ENDOGENOUS YPI, PD, CD, TRA, TRAT, M, MTR  $,
CONSTRUCT  YDB, KB  $,
PARAMETER GL,  RD, APC, MPD, UZ,  GA,  GM, GGD, LCD, LID, LYP, LPD $,
PARAMETER TRB, TRP, LFR, DW, DGD, DL, DA, CUS, LIP, DTRA,  GTRA  $,
PARAMETER LPR, MS, LS, MSR, DM, IE, GM, LMD  $,
MRE $=$  (1. + LMD) * (M/P) - LMD * M(-1)/ P(-1) $,
R $=$ (MRE/(.072 * YP))** (-1./IE)  $,
CDE $=$ APC * YDI/P  $,
CD = (1./(1. + LCD)) * CDE + (LCD/(1.+ LCD)) * CD(-1)  $,
KSE $=$ (.15* W/(.85*(2.5 * R +RD)))**.85 *(YP/(A*CUS)) $,
KS = (1./(1. + LID))*KSE + (LID/(1.+LID)) * KS(-1)  $,
IDSE $=$ 4. *(KS-KS(-1)) + RD * K(-1) + (IDS(-1) - I(-1))  $,
IDS = (1./(1. + LIP)**2.)*IDSE+(2.*LIP/(1.+LIP))*
IDS(-1)-((LIP/(1.+LIP))**2.)*IDS(-2)  $,
ID $=$ IF IDS GRT 0.0 THEN IDS ELSE 0.0  $,
GDTR = (1+ GGD) * GDTR (-1)  $,
GD = GDTR + DGD  $,
YDB $=$ CD + ID + GD  $,
YD $=$ IF YDB LES 1.0 THEN 1.0 ELSE YDB  $,
YEE $=$ A*(0.98*L*2000.)**0.85*K**0.15  $,
YS = IF (YEE +M(-1)) GRT YD THEN YD ELSE (YEE + M(-1))  $,
YSX $=$ LS * YS + (1. - LS) * (1. + MS) * YS(-1) -
(1.-LS)*MS*YS(-2) $,
YPE $=$ YSX + (MSR * YSX - M(-1))  $,
YPS $=$ (1./(1.+ LPR)) * YPE + ( LPR/ (1. + LPR )) * YP (-1)  $,
YP = IF YEE GRT YPS THEN YPS ELSE YEE  $,
DM $=$ YP - YS  $,
DMP $=$ YPS - YSX  $,
DMU $=$ DM - DMP  $,
M = DM + M(-1)  $,
YPIE $=$ ( 1 -LFR) * YP * P  $,
YPI =(1./(1.+LYP))*YPIE + (LYP/(1.+LYP))*YPI(-1)  $,
YDI $=$ YPI - L * TRA * (YPI/L) ** TRP + TRB  $,
E =  (1./2000.) * (YP/A)**(1./.85)*K**(-.15/.85)  $,
U = 1 - E/L  $,
CU $=$ .15 * W * E * 2000./(.85 *(2.5 * R + RD) *K)  $,
KB $=$ I/4. + (1. - RD/4.) * K(-1)  $,
K = IF KB LES 1.0 THEN 1.0 ELSE KB  $,
LTR = (1 + GL) * LTR(-1) $,
L = LTR + DL  $,
ATR = (1 + GA) * ATR(-1) $,
A = ATR + DA  $,
RSD $=$ YS/YD  $,
```

(DYNEC4 Model, Continued)

```
C = RSD*CD  $,
I = RSD*ID  $,
G = RSD*GD  $,
P = (1 + PD/4.)*P(-1)  $,
PDE $=$ ((MPD*0.02*UZ)/(UZ - 0.02))*(1/U) - MPD*0.02/(UZ - 0.02)   $,
PD = (1./(1. + LPD)) * PDE + (LPD/(1. + LPD)) * PD(-1)  $,
WTR = (1 + GW) * WTR(-1) $,
W = WTR + DW  $,
TRAT = ( 1 + GTRA) * TRAT (-1)  $,
TRA = TRAT + DTRA  $,
MTR = (1.+ GM) * MTR(-1)  $,
M = MTR + DM  $,
END
```

APPENDIX B

The Archives in TROLL

In order to use the models described in the text and listed in Appendix A, archives must be present in the system giving values for the ENDOGENOUS variables. In the static models, these numbers serve as the first guesses in the iterative solution process. In the dynamic models (the DYNEC's), they perform this function as well as providing initial conditions for the evolution of the system.

We shall not describe in detail how archives are constructed or how they "look" in the system. Instead, we shall discuss in turn each of the three archives necessary to use these simulation models, and we shall simply state what values must be entered for what variables for what years. All files mentioned must have "yearly" as their timeunit.

Archive GENEQ

The GENEQ archive provides the data necessary for the two models discussed in Chapter III, GE1 and GE2. The following values have been entered in year 1 for the following variables:

LC	762681.	R	4.046907
LF	237318.9	W	.0050017
NXC	0.	RCC	43171.93
NXF	0.	RCF	2023.454
PC	1.0469	RI	4046.908
PF	1.0	WI	5001.743

QC	108380.4	TC	314.2289
QCC	108523.2	TF	685.771
QCP	108523.2	WCC	65203.48
QF	3974.949	WCF	1945.678
QFC	3968.485		
QFP	3968.485		

Archive ISLM

This archive provides the data necessary to use the ISLM models discussed in Chapter IV. The following values have been entered for the following variables for year 1:

BCF	-7.4	DGB	0.
BTR	6.0	PD	.0288
C	447.4	R	.0472
I	113.2	NTX	143.7
DEF	0.	U	.042
OLK	74.4	DHPM	0.
		MRS	166.3

Archive DYNEC

The DYNEC models are to be simulated beginning in year 3. Thus values must be provided for year 2 for all ENDOGENOUS variables. Where only one number is listed below, it indicates the value stored for year 2. Where two numbers are given, the first is for year 1 and the second is for year 2.

The ENDOGENOUS variables and the corresponding values are as follows:

A	4.67	P	1.0
ATR	4.67	PD	.02
C	514.	PDE	.025
CD	514.	R	.04
E	.07267	TRA	.021
G	220.	TRAT	.021
GD	220.	U	.10
GDTR	220., 220.	W	4.02
H	120., 120.	WTR	4.02
I	50., 50.	YP	700., 700.
IDS	50., 50.	YPE	700., 700.
K	400.	YPI	540.
KS	400.	YS	700., 700.
L	.0756	YSX	700., 700.
LTR	.0756		
M	175., 175.		
MTR	175.		

Date Due

BINDCAT 7/18/95		
NOV 7 '73		