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MANPOWER FLOW PROBLEMS AND GOAL PROGRAMMING SOLUTIONS

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Introduction and Purpose[2]

General

A thoroughly familiar homily is that human resources are the most valuable and most difficult to manage assets of any organization. The implication is usually that the words "resources" and "assets" tend to camouflage the moral, ethical and political issues which impinge upon manpower and personnel problems and that any manager who ignores these issues can expect extremely hazardous consequences. Therefore, any attempt to manage human resources must be based upon a sound understanding of the structural, policy and behavioral variables of the organization being considered.[3]

Structural variables.--Human resources are managed, or reacted to, within some sort of formal organizational structure, and this structure is important because it represents the boundary conditions or limits within which this management or reaction occurs.[4] Examples of boundary conditions

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1. The model presented in this paper is the result of a manpower planning study conducted in the Personnel Studies Division, Directorate of Personnel Studies and Research, Office of the Deputy Chief of Staff for Personnel, Department of the Army. The officers, enlisted men and civilians who, in one way or another, contributed to this paper are gratefully acknowledged.

2. Mason Haire discusses many of the issues related to personnel flow problems in his article "Approach to an Integrated Personnel Policy," Industrial Relations, VII (February, 1968), 107-117.

3. See Marcus Alexis and Charles Z. Wilson, Organizational Decision Making (Englewood Cliffs, N. J.: Prentice-Hall, 1967), pp. 3-13.

4. This statement may at first appear to be tautological. However, it is simply a statement of the typical interactive types of phenomena that one encounters when studying human behavior. In other words, managers affect structure as well as being affected by it.

are the total number of people in the organization, the classifications applicable to this total number, and the constraints applicable to each classification. Structural constraints are often the result of experience, intuition and judgment, but often the lack of such constraints may be the most difficult management problem. Suppose, for example, that a manager's experience and judgment indicate that he should have M types of people and that the size of each group should be N_1 through N_M respectively. One of his problems then is to attain N_1 through N_M individuals for each of the types; but, more than this, he must also maintain this structure over time.

Policy variables.--The maintenance of a structure over time, however, requires that some attention be paid to procuring, training, selecting, promoting, retiring and, in total, sustaining the people who comprise this structure. Pay, fringe benefits and incentive problems become significant and, in general, the manager is faced with personnel policy problems. That the manager is faced with policy problems is simply another way of saying that he is faced with the problem of setting organizational goals or objectives.[1] He must somehow come to grips with problems like how many people to procure from each of several sources; when to promote people; how many people are to be promoted; what sort of variances from his "ideal" structure can be tolerated; and how many people must he retain in each of his personnel categories in order to maintain a relatively smooth flow of people into, through and out of his organization. Furthermore, he must solve the structural and policy problems in a fashion that induces the required behavior from the

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1. See Richard M. Cyert and James G. March, A Behavioral Theory of the Firm (Englewood Cliffs, N. J.: Prentice-Hall, 1963), pp.27-36

participants in his organization. If this sort of solution is not possible, the manager would at least like to know what sort of behavior is required. The required behaviors then become management goals also, i.e., conditions that must be attained and maintained if anything resembling a smooth flow of people is to be achieved.

Behavioral variables.--Two of the problems mentioned in the preceding paragraph--procurement and retention--are easily classified as behavioral variables. In order to survive, an organization must be able to induce the initial and continued participation of potential and current organizational members.[1] This requires "joining" and "staying" behavior on the part of the participants, and such behavior is obviously influenced by structural and policy variables. Any organization that fails to provide the necessary inducement--because of structural and policy problems--will, at best, experience high turnover rates and general personnel flow turbulence. This, in turn, leads to serious cost and effectiveness problems. Cost and effectiveness may not be as easily measured as turnover and turbulence at any point (or interval) in time, but the longer run effects will be all too obvious.

Procurement and retention are only a small subset of the relevant behavioral variables. Aspirations and expectations, for example, are significantly affected by structural and policy considerations, and they, in turn, affect participation and performance.[2] Interpersonal

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1. James G. March and Herbert A. Simon, Organizations (New York: Wiley, 1958), pp. 83-111.

2. Mathematical formulation of aspiration level theory can be found in the author's Ph.D. dissertation "Aspirations, External Goals and Performance" (unpublished Ph.D. dissertation, Graduate School of Industrial Administration, Carnegie Institute of Technology, 1967), pp.6-42, and in Andrew C. Stedry's book Budget Control and Cost Behavior (Englewood Cliffs, N. J.: Prentice-Hall, 1960), pp. 17-42.

relationships cannot be ignored, and the list could be extended indefinitely. But, the list need not be so extended in order to note that structural, policy and behavioral variables are intimately related and that the problems associated with personnel flows are in many ways no different from any other organizational problem. The study of human behavior in organizations simply has many paths covered with the same concepts, relationships and pitfalls.

Purpose

The purpose of this paper, then, is twofold. First, a method of solving for the conditions necessary to attain and maintain a steady state personnel flow through an admittedly highly structured but not highly unusual organization will be presented. This method will be shown to have the desirable characteristic of being able to simultaneously consider a large number of management goals and some of the structural, policy and behavioral variables related to these goals. And, second, it will be shown that analyses of the type to be presented are extremely useful in that:

1. They focus attention on critical organizational problems.

2. They are constructively self-destructive in that they highlight their own shortcomings and point the way to more satisfactory solutions.

3. They provide a means for integrating personnel policy problems.[1]

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1. Haire, loc. cit.

Basic Structural Considerations

Initial assumptions and considerations

Suppose that a decision has been made to have M levels (grades, ranks or types) or individuals in a given organization and that the normal or expected carrer of each individual is Y periods. (A period may be a year, six-months, a quarter or any other similar definition.) Denote the levels as K_1 through K_M respectively and provide an ordering such that K_1 is a lower level than K_2, K_2 is a lower level than K_3 and so on until K_{M-1} is a lower level than K_M. Furthermore, suppose that the number of individuals within each level is N_1 through N_M and that $N_1 < N_2 \ldots < N_M$. This is the usual pyramidal type of structure oftentimes found in government bureaucracies, large corporations and other common types of organizations. And, assume that an individual can advance from one level to the next under a promotion policy that is yet to be defined. This sort of system can be diagrammed as shown in Figure 1. The typical entry in Figure 1 listed as X_{i,j,K_1} is meant to indicate that there are X_{i,j,K_1} individuals in our typical organization who have spent i periods in the organization and j periods in level K_1. And, according to the above definitions

(1) $$\sum_i \sum_j X_{i,j,K_1} = N_1$$

Or more generally

(2) $$\sum_i \sum_j X_{i,j,K_k} = \qquad (k = 1, \ldots, M)$$

One of the obvious conditions attendant upon the diagram in

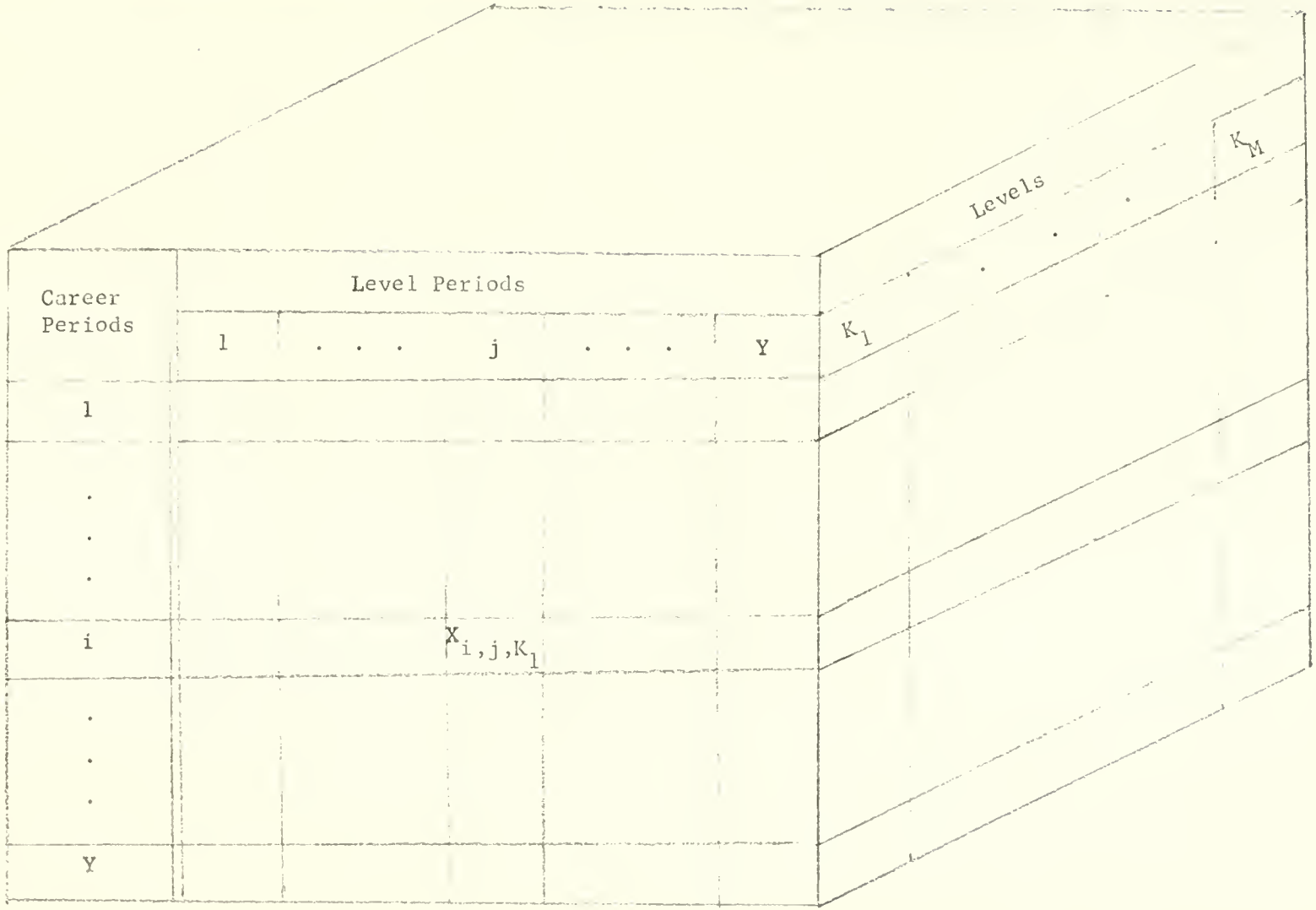
Career Periods
Level Periods
1
. . .
j
. . .
Y
1
i
Y
X_{i,j,K_1}
Levels
K_1
K_M

Figure 1 is that a particular group of individuals, say X_{i,j,K_k}, cannot have more periods in a given level (grade or rank) than they have career periods. That is, $X_{i,j,K_k} = 0$ for all $j > i$ $(i = 1, \ldots, Y)$ in each of the K_1 through K_M. If demotions as well as promotions are permitted in this system, it would be possible to have as $X_{i,j,K_k} > 0$ for some $j < i$; and if part of a particular group is promoted at one point in their careers and the other part of the same group is promoted at a different point, the condition that $X_{i,j,K_k} > 0$ for $j < i$ will hold for all levels. Thus, within each level of the diagram shown in Figure 1, all cells above the main diagonal are empty and all cells on or below the diagonal may or may not have a value. The values in each cell, of course, are the number of individuals in the organization that fit that particular classification. These considerations will be elaborated in more detail in the paragraphs which follow.

Entry and promotion conditions

The diagram in Figure 1 leaves two basic points open to question, namely:

1. At what point or points may individuals enter the structure?

2. How are individuals promoted (or demoted) from one level to the next?

In other words, the entry conditons and the promotion mechanism must be specified in order to change the static picture in Figure 1 to one that considers a flow of people through a structure.

Entry.--Clearly, an individual would always begin in the first career period and the first level period, but what about entry at different levels? Assuming that entry takes place at the beginning of every time

period, there would be M groups, say $X_{0,0,K_k}$ for K = 1, . . .,M, with zero career periods, zero level periods and ready for entry at level K_k. For purposes of this paper, however, it is assumed that $X_{0,0,K_k} = 0$ for k = 2, . . ., M, i.e., all entry into the system is at the first level. This assumption is somewhat constraining, but it is not a particularly atypical situation. Furthermore, in organizations that permit entry at other than the lowest level, the number of individuals entering at a higher level is frequently insignificant when compared to the total number in that level.

Promotion.--On the other hand, promotion at all levels is a most significant item. Promotion policies are oftentimes the most controversial and misunderstood of all personnel policies, and many so-called "philosophical" personnel discussions revolve around this issue. These discussions usually raise the types of questions concerned with whether or not a promotion is a reward for past performance, whether or not promotion should be based upon an expectation of future performance, whether or not past performance and future expectations are intimately related, and just what the criteria for promotion should be. After these, another set of questions arises concerning the correlation between promotion (or performance) criteria at two different levels in an organization and whether or not these correlations have a significant effect on the quality of the promoted individuals.[1] More will be said about these problems towards the end of this paper, but, for now, the focus will be on two types of promotion policies which will be called promotion points and promotion rates.

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1. Haire, op. cit., pp. 110-112.

For purposes of this paper, a promotion point or a set of promotion points must be defined for every level in the organization. This is probably best illustrated with the use of examples. Suppose that the ten periods in level K_{k-1} are required for promotion to level K_k. The quantity of time--ten periods--is then a promotion point with respect to level K_{k-1}. Ten periods could be considered a "normal" promotion point, however, and it may be desirable to promote some individuals with only eight periods in level K_{k-1} and others with as many as twelve periods in the same level. These time quantities--eight periods and twelve periods--would also be promotion points with respect to level K_{k-1}. Eight periods could be considered an "early" promotion point and twelve periods a "late" promotion point. Obviously, a larger set of promotion points could be defined for any level, but three--an early, a normal and a late--will be the maximum number considered here.[1] Once the promotion points have been defined for all levels, the static diagram in Figure 1 can be changed to the flow diagram shown in Figure 2.[2] Figure 2 has been modified somewhat from Figure 1 in that the organizational structure is now shown in two dimensions rather than three. The subscripts on the "X" variables, however, retain the same meaning; and the condition that all entry into the structure be at the lowest level usually obviates the need to have more than one entry per cell.

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1. The problem of setting promotion points is clearly one of the knottier ones in any organization. Furthermore, the setting of promotion points automatically implies a management decision with respect to the requisite amount of experience in one level to be qualified for promotion to the next.

2. An additional assumption in this paper is that an individual cannot skip levels. He can advance to a given level only by proceeding to one of the promotion points in all the lower levels.

Career Periods	Level Periods 1	2	3	4	5	6	7	8	9	10
1	$X_{1,1,1}$									
2		$X_{2,2,1}$								
3			$X_{3,3,1}$							
4	$X_{4,1,2}$			$X_{4,4,1}$						
5	$X_{5,1,2}$	$X_{5,2,2}$								
6		$X_{6,2,2}$	$X_{6,3,2}$							
7			$X_{7,3,2}$	$X_{7,4,2}$						
8				$X_{8,4,2}$	$X_{8,5,2}$					
9	$X_{9,1,3}$				$X_{9,5,2}$	$X_{9,6,2}$				
10	$X_{10,1,3}$	$X_{10,2,3}$				$X_{10,6,2}$				
11	$X_{11,1,3}$	$X_{11,2,3}$	$X_{11,3,3}$							
12										
13										
14										
15										
16										

Now, Figure 2 illustrates a number of the effects that promotion points have on a flow of people through a structure. First, any group of people, say $X_{0,0,K_1}$, procured for entry into the structure at the first level will split into two groups upon reaching a promotion point. The individuals not promoted at a particular point may proceed on to another promotion point, but unless all individuals in the group are eventually promoted some will remain after all promotion points in a given level have been passed. The problem is then to define some sort of "continuation" policy for the individuals who simply fail to be promoted. Should they be involuntarily separated from the organization or should they be allowed to continue in the organization for some specified period of time? This problem is illustrated in Figure 2 with the groups indicated at $X_{1,1,1}$, $X_{2,2,1}$, $X_{3,3,1}$ and $X_{4,4,1}$. The darkened cell at $X_{3,3,1}$ indicates a promotion point to level two. But if it is assumed that not all individuals are promoted, the $X_{3,3,1}$ groups splits into $X_{4,1,2}$ and $X_{4,4,1}$. The group indicated as $X_{4,1,2}$ ends their fourth career period with one level period in level two, and the group indicated as $X_{4,4,1}$ ends their fourth career period with four level periods in level one. Some of the individuals in $X_{4,4,1}$ may receive a promotion--as indicated by the darkened cell--and end their fifth career period with one level period in level two. But what happens to the individuals who fail to receive a promotion at either point? Any continuation decision obviously affects future decisions and, thus, cannot be made in an independent fashion.[1]

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1. Since a group $X_{5,5,1}$ does not appear in Figure 2, it may be assumed that all individuals who fail to be promoted after four level periods in level one are separated from the organization. Although this is true in the specific case illustrated, it need not ever be true in practice.

A second problem that is evident in Figure 2 is that group splitting at promotion points causes problems at later promotion points. Consider, for example, the groups labelled $X_{8,5,2}$, $X_{9,6,2}$, $X_{9,5,2}$ and $X_{10,6,2}$. The individuals belonging to groups $X_{9,5,2}$ and $X_{9,6,2}$ and who are promoted after five and six level periods respectively in level two, combine to form group $X_{10,1,3}$. Thus, any personnel flow model must be able to account for group forming as well as group splitting at the promotion points. Furthermore, the existence of multiple groups in a given level poses another problem at the promotion points. Assume that all individuals in level two who are not promoted are simply separated from the organization. Then, those not selected from $X_{9,6,2}$ and $X_{10,6,2}$ will be separated. But those separated from $X_{9,6,2}$ only have nine career periods as opposed to ten for those separated from $X_{10,6,2}$. And this problem gets much worse at those levels containing a larger number of groups due to group splitting at the promotion points in lower levels. What sort of continuation policies should be applied to this problem?

The answers to these promotions point problems are, of course, partially found in considerations that focus on structural or policy or behavioral considerations separately, but some method of integrating these factors would be most desirable. The problems being considered here are a bit more complicated, however, and some further issues must be raised before attempting to answer a single isolated set of questions. These issues, including promotion rates, are discussed in the next section.

Other Structural Considerations

Promotion rates

So far it has been assumed that some promotion rate $p_{i,j,k}$, where $0.0 \leq p_{i,j,k} \leq 1.0$, is associated with each cell in Figure 2 that is indicated to be a promotion point. At this point, however, the next obvious step is to consider ways of associating a specific promotion rate with each promotion point. One might, for example, conceive of flow models that maximize promotion rates subject to various flow conditions and structural constraints or models that limit promotion rates to different ranges for different levels in the organization subject to a similar set of constraints. But one of the striking observations that can be made in many organizations is that personnel managers[1] have fairly definite ideas about promotion points and promotion rates and feel that these two problems should be considered as policy matters and not as flow optimization dimensions. The arguments raised in support of this position can be very convincing and tend to force one to the conclusion that promotion points and promotion rates should be one of the most flexible aspects of a personnel flow model. In this way, various promotion point and promotion rate policies can be tested for their effects on the flow of people into, through and out of a particular structure. The manager can then decide which policy set to choose based upon some compromise between flow consequences and his experience and judgment concerning the effects that promotion rates have

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1. The term "personnel managers" is used here in the general sense and refers to individuals responsible for formulating and/or executing various implicit and explicit personnel policies.

on turnover and procurement problems. High promotion rates, for example may tend to keep people in an organization and solve various recruiting problems; low promotion rates may have the opposite effect. But the choice of a mechanism for determining promotion point and promotion rate policies must ultimately be determined on the basis of how these policies interact with various structural and behavioral variables.

The selection of appropriate promotion rates becomes even more difficult because of the group splitting at promotion points. Referring again to Figure 2, it is noted that only one group is present at each of the level one promotion points but that two groups are present at each of the level two promotion points. Three groups will be present at each of the level three promotion points, and this progression will be amplified depending upon the number of levels and the number of promotion points per level. Thus, a qualitative question arises which focuses on the "how to promote" problem as well as the "how many to promote" or the promotion rate problem. Should the groups who reach the promotion points with fewer career periods have higher promotion rates than the groups with more career periods at the same promotion point? In Figure 2, for example, should the promotion rate applied to group $X_{8,5,2}$ be higher than the one for group $X_{9,5,2}$? If this were the case and if it is assumed that the "best" people are always promoted first, the effect of such a promotion policy would be to advance the best people faster than anyone else. And, indeed, an implied policy to advance the best people faster than anyone else would seem to embody one of the standard values shared among most personnel managers.

<u>An additional variable and an example</u>.--Suppose that at some

higher organizational level, say k, that there are eleven groups of people, group A through group K.[1] The eleven groups have resulted from earlier group splitting and group forming at the promotion points with the result that some groups have been promoted to level k much earlier in their careers than other groups. Furthermore, suppose that there are three promotion points at level k for promotion to level k+1. This situation is diagrammed in Figure 3. The career periods have been arbitrarily designated to by y_1 through y_{14}, and the level periods have been simply listed as 1, 2, 3 and 4. Level period 2 would be the early promotion point, level period 3 the normal, and level period 4 the late promotion point. Finally, suppose that the middle group, group F, is the "normal group." A normal group is one composed of some individuals who have always received normal promotions and others who have received some combination of early, normal and late promotions such that the end result is equivalent to that of all normal promotions.

Now, at the end of level period 1, group F is of size F_1. At the end of level period 2--and prior to the promotions to level k+1--this group is of size F_2. Similarly, at the end of the subsequent periods and prior to the promotions in these periods, the group is of sizes F_3 and F_4. One fairly reasonable assumption is that $F_1>F_2>F_3>F_4$ and that the losses from one period to the next can be separated into two categories:

1. Losses due to people <u>voluntarily</u> leaving the organization for other organizations--retention losses.

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1. The change in notation has been used in this example only for purposes of simplicity.

CAREER PERIODS	LEVEL PERIODS 1	2	3	4
y_1	A_1			
y_2	B_1	A_2		
.	C_1	B_2	A_3	
.	D_1	C_2	B_3	A_4
.	E_1	D_2	C_3	B_4
.	F_1	E_2	D_3	C_4
.	G_1	F_2	E_3	D_4
.	H_1	G_2	F_3	E_4
.	I_1	H_2	G_3	F_4
.	J_1	I_2	H_3	G_4
.	K_1	J_2	I_3	H_4
.		K_2	J_3	I_4
.			K_3	J_4
y_{14}				K_4

Figure 3-A Promotion Rate Example

2. Losses due to people involuntarily leaving the organization--attrition.[1]

Retention losses will be a most significant item later in this discussion, but they will be ignored now in order to more closely examine promotion rates in terms of the group sizes F_1, F_2, F_3 and F_4. The problem is this: Suppose that a personnel manager is willing to specify two things in addition to the promotion points, viz.:

1. The normal promotion rates for the normal groups in all organizational levels.

2. The desired ratio between the number of early (or late) promotions and the number of normal promotions.

Then, how can the early and late promotion rates for the normal groups be established when the group size is changing due to attrition and promotion? This problem can be solved as follows.

Let p_1 = early promotion rate for group F

p_2 = normal promotion rate for group F

p_3 = late promotion rate for group F

f = the ratio between the number of early (or late) promotions and the number of normal promotions

Then, according to the above conditions

(3) $$\frac{p_1 F_2}{p_2 F_3} = \frac{1}{f}$$

and

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1. In organizations like an Army, for example, the causes of attrition are often categorized as death, disability, hardship, and discharge.

(4) $$\frac{p_3F_4}{p_2F_3} = \frac{1}{f}$$

Now, working with equation (3)

(5) $$p_1 = \frac{p_2F_3}{fF_2}$$

where p_2 and f are known. However, prior to the establishment of a steady state flow of people through an organization, F_2 and F_3 are both unknown. It is true, however, that in the absence of attrition

(6) $$F_3 = F_2(1 - p_1)$$

Therefore, with attrition $F_3 < F_2(1 - p_1)$. And, if a constant attrition rate can be assumed (at least for each level in the organization), then

(7) $$F_3 = \frac{F_2(1 - p_1)}{(1 + a)}$$

where a, $0.0 \leq a \leq 1.0$, is the attrition rate. Substitiuting equation (7) in equation (5) yields

(8) $$p_1 = \frac{p_2(1 - p_1)}{f(1 + a)}$$

And, the solution for p_1 is

(9) $$p_1 = \frac{p_2}{f(1 + a) + p_2}$$

Now, had the early and normal promotion points been separated by more than one level period, say n periods, then equation (7) would have been

$$F_3 = \frac{F_2(1 - p_1)}{(1 + a)^n} \tag{10}$$

and equation (9) would have been

$$p_1 = \frac{p_2}{f(1 + a)^n + p_2} \tag{11}$$

Note that both equations (9) and (11) insure that the condition $0.0 \leq p_1 \leq 1.0$ will hold for the early promotion rate for the normal group. Unfortunately, the same result does not hold for p_3. The same development for equation (4) yields

$$p_3 = \frac{p_2 F_3}{f F_4} \tag{12}$$

and

$$F_4 = \frac{F_3(1 - p_2)}{(1 + a)^n} \tag{13}$$

thus

$$p_3 = \frac{p_2(1 + a)^n}{f(1 - p_2)} \tag{14}$$

Obviously certain values of p_2, a, n and f will cause $p_2(1 + a)^n$ to be greater than $f(1 - p_2)$. A proof is not offered here, but simply try

the values $p_2 = 0.9$, $a = 0.01$, $n = 2$ and $f = 8$. The solution is $p_3 = 1.15$, clearly an unacceptable result. But what this result does imply is that the stipulation of normal promotion rates for the normal groups along with the desired ratio between the number of early (or late) promotions and the number of normal promotions is not enough. The personnel manager is also required to set a limit on the late promotion rate(s) for the normal group(s), say p*, such that

$$p_3 = \min\left[\frac{p_2(1 + a)^n}{f(1 - p_2)}, p^*\right] \tag{15}$$

Again, p* is not given to the manager but it is a matter of policy that must be decided.

Now, what has been accomplished by considering this example and what about the early, normal and late promotion rates for the other groups in Figure 3? First, it has been shown that promotion rates can be set <u>independent</u> of group sizes and that the addition of another variable, attrition, does not hinder this rate setting process. Second, the early, normal and late promotion rated for the normal group, group F can be used as bases to compute the early, normal and late promotion rates for all other groups. Presumably, the early, normal and late promotion rates for groups A through E will be higher than p_1, p_2 and p_3 respectively; and, similarly, the early, normal and late promotion rates for groups G through K will be lower than p_1, p_2 and p_3. The only policy issue to decide is how much higher and how much lower for each of these groups. Note that by assuming higher promotion rates for the groups "above" the normal group--the early groups--and lower

promotion rates for the groups "below" the normal group--the late groups--the implied is to advance the best people faster than anyone else. This policy has two effects:

1. The groups present at higher organizational levels will be predominantly early groups, i.e., groups who have received some combination of early, normal and late promotions such that the prevailing effect is that of early promotions.

2. Continuation policies will be more difficult to formulate at higher organizational levels. Pressures will develop to retain those individuals not promoted at higher levels because these people are supposedly the high quality people in the organization. But, in order to maintain reasonably high promotion rates at lower organizational levels and not stagnate the upward flow, these individuals must be released on order to create vacancies.

Thus, the promotion rates at the highest organizational level are really movement rates that specify the rate of movement out of the organization, and this movement must be maintained in order not to have a stagnation effect that multiplies as it proceeds down the organization's structure.

Attrition.--Equations (11) and (14) above both contain the attrition factor, a, in order to account for losses due to people involuntarily leaving the organization. Now, involuntary losses may be virtually nonexistent in any specific organization, or (contrary to the assumption made above) they may not be constant across level periods for each level. Setting a = 0 does not significantly affect either equation (11) or equation (14); but, of a is a variable, these two equations must be modified somewhat. Nevertheless, in the absence of evidence to

the contrary, it will be assumed that a constant attrition rate a_k ($k = 1, \ldots, M$) can be applied to all level periods within each level for organizations with structures similar to the one diagrammed in Figure 1. Each a_k, of course, must be empirically estimated.

A last look back

It may not be obvious yet, but this discussion has now reached the point where some extensive analysis is possible with the use of a relatively well known mathematical technique, viz., the goal programming version ot the standard linear programming model.[1] The application of this technique and the introduction of voluntary losses (retention losses) will be presented in the next section. There are, however, two rather important asides that should be considered at this juncture. First, the framework that has been developed is highly dependent upon a large number of assumptions and other conditions. The obvious consideration is whether or not these assumptions and conditions are crucial. And, second, can anything be said about behavioral variables, variables that were postulated to be important at the outset of this paper.

The first item can be placed in perspective by simply listing the assumptions and conditions imposed so far. The main ones are:

1. That an organization can be structured in a pyramidal fashion along three dimensions--career periods, level periods and levels.

2. That all entry into the organization is at the first level.

3. That a set of promotion points can be defined for every

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1. Cf. A. Charnes and W. W. Cooper, Management Models and Industrial Applications of Linear Programming (2 vols.; New York: Wiley, 1961), I pp. 215-221, and Yuji Ijiri, Management Goals and Accounting for Control (Chicago: Rand McNally, 1965), pp. 38-50.

level in the organization.

4. That a set of continuation policies can be specified for the individuals who fail to be promoted at each level.

5. That a normal promotion rate can be established for the normal group at the normal promotion point in each level.

6. That a desired ratio between the number of early (or late) promotions and the number of normal promotions can be established for the normal group at each level.

7. That an upper limit can be set on the late promotion rate for the normal group at each level.

8. That the early, normal and late promotion rates for the early and late groups can be based upon the early, normal and late promotion rates for the normal group at each level.

Now, condition 2--all entry into the organization is at the first level--can possibly be eliminated in models that are more complicated than the one to be presented, and even some of the promotion point and promotion rate conditions may be similarly relaxed. But these problems must be faced in one manner or another, and their striking feature is that they are probably no worse than any other set of assumptions and conditions that are necessary to make other kinds of management decisions. Marketing decisions, production decisions, investment decisions, and inventory decisions are all based upon a myriad of conditions, the total number being dependent upon the type of analysis used. Generally speaking, the more quantitative the analysis, the greater the number of assumptions and other side conditions that must be imposed. In fact, the conditions imposed above for purposes of personnel decisions seem to be less binding than the ones imposed for other kinds of decisions. Certainly they are

crucial, but no more so than in any other decision making area. And, if managers can outline and model personnel flow problems using conditions like those listed above, it seems obvious that the efficacy of any particular formulation should be based upon the implications of that formulation and not the formulation process itself.

The implications of this or any other formulation, however, lead quite naturally to the area of human behavior in an organization. What are the implications for behavior? In this author's opinion, the implications are not so obvious, but they do have a most significant meaning for the study of organization and the study of human behavior in organizations. Consider, for example, the following quote from Simon concerning the meaning of the term organization:

> The term organization refers to the complex pattern of communications and other relations in a group of human beings. This pattern provides to each member of the group much of the information, assumptions, goals, and attitudes that enter into his decisions, and provides him with a set of stable and comprehensible expectations as to what the other members of the group are doing and how they will react to what he says and does.[1]

Simon's definition flows automatically from his considerations of the limits on human rationality. He argues most convincingly that humans do not possess anything near the onmiscient rationality posited in classic economic theories,[2] but that humans behave according to a much more limited rationality which consists of :

1. Discovering a small subset of all the behavioral strategies

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1. Herbert A. Simon, Administrative Behavior: A Study of Decision-Making Processes in Administrative Organization (2d ed.; New York: Macmillan, 1961), p. xvi.

2. Cf. the theory of consumer behavior and the theory of the firm in James M. Henderson and Richard E. Quandt's book, Microeconomic Theory: A Mathematical Approach (New York: McGraw-Hill, 1958), pp. 6-84. Cyert and March, op. cit., pp. 8-15, provide a consensus of the criticism directed at the theory of the firm which can be extended without much difficulty to the theory of consumer behavior.

available in a particular situation.

2. Forming expectations regarding some of the consequences which will follow the choice of any strategy in this small subset.

3. Choosing a particular strategy based upon an evaluation of those consequences in terms of the values (or expected values) held by the individual.

The point of these arguments, for purposes of this paper, is that the structure provided by an organization limits the set of behavioral alternatives available to its participants, and the policies (personnel policies in this case) provide the information with which to form expectations about the consequences of those alternatives. Furthermore, the extent to which the organization (the one in question or any group of organizations including the one in question) provides the individuals with a set of values to judge these consequences will determine the extent to which specific behavioral strategies are chosen by the members of that organization. Thus, questions about organizational structure or organizational design resolve to questions concerning what limits the structure of design places on alternative behavioral strategies and what possible behavioral strategies the designer or manager must relinquish or forego when he chooses a particular structure. Choosing a particular structure along with a set of policies with respect to that structure immediately determines the alternative behavioral strategies available to the participants in that structure and provides them with the information necessary to form expectations concerning the consequences of these strategies. In this way, an organization provides the environment necessary for individuals to behave in a rational, i.e., a limited

rational fashion. Simon carries this point somewhat further by stating:

> The rational individual is, and must be, an organized and institutionalized individual. If the severe limits imposed by human psychology upon deliberation are to be relaxed, the individual must in his decisions be subject to the influence of the organized group in which he participates.[1]

Thus, as stated at the beginning of this paper, the structural and policy variables related to a personnel flow are extremely important because they set the stage for the behavior that will take place within this flow through the structure.[2]

The Model

Group flow and promotion points

The assumption stated earlier that all entry into the structure (diagrammed in Figure 1 and Figure 2) is at the first level leads to a fairly simple procedure for determining group sizes at each of the career period, level period and level combination. This can be shown by following the group procured for entry into the organization at level 1. By letting procurement (at level 1) = $X_{0,0,1} = X_1$, the number of individuals with i career periods and j level periods is given by

$$X_{i,j,1} = \frac{X_1}{(1+a_1)^n} \qquad (i,j = 1,2,\ldots) \quad (n = j) \tag{16}$$

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1. Simon, op. cit., p. 102.

2. As another example of the effect of structure and policy on behavior, consider the effects that parents have on the behavioral development of a newborn. The structure of the sleeping and feeding schedules along with the reward and punishment policies greatly limit the behavioral alternatives available to the child. Later, as the child developes and the number of behavioral alternatives increases, the system of values imposed by the parents greatly influences the choice among alternatives.

for each career (or level) period up to and including the first promotion point. In equation (16)

X_1 = procurement (at level 1)

a_1 = attrition rate (at level 1)

$X_{i,j,1}$ = number of individuals with i career periods and j level periods (at level 1)

After the first promotion point, however, the single group in level 1 splits into two groups -- one remaining at level 1 and the other proceeding to level 2. Assuming that the first promotion point occurs after n_1 level periods, the sizes of the two groups are given by

$$X_{n_1+1,\ 1,\ 2} = \frac{X_1 \ p_{1,1,1}}{(1+a_1)^{n_1} \ (1+a_2)} \tag{17}$$

and

$$X_{n_1+1,\ n_1+1,\ 1} = \frac{X_1 \ (1 - p_{1,1,1})}{(1+a_1)^{n_1+1}} \tag{18}$$

for the period immediately following the promotion point, and

a_2 = attrition rate (at level 2)

$p_{1,1,1}$ = the promotion rate at the first promotion point for group number 1 in level 1.

Generally, $p_{r,s,t}$ is the promotion rate applicable to the rth promotion point for group number s in level t. Thus, in Figure 3, the ranges on r, s and t would be (r = 1,2,3), (s = 1,2,...,11) and (t = t). At level 1 there is only one group, therefore, s = 1 and t = 1.

Now, equations (17) and (18) can be simply modified to indicate the sizes of these two groups n_2 periods after the first level 1 promotion point, viz.:

$$X_{n_1+n_2,\ n_2,\ 2} = \frac{X_1 p_{1,1,1}}{(1+a_1)^{n_1}\ (1+a_2)^{n_2}} \tag{19}$$

and

$$X_{n_1+n_2,\ n_1+n_2,\ 1} = \frac{X_1(1 - p_{1,1,1})}{(1+a_1)^{n_1+n_2}}$$

Assuming that the second level 1 promotion point occurs after (n_1+n_2) level periods, the sizes of the two groups are given by

$$X_{n_1+n_2+1,\ n_2+1,\ 2} = \frac{X_1 p_{1,1,1}}{(1+a_1)^{n_1}\ (1+a_2)^{n_2+1}} \tag{21}$$

and

$$X_{n_1+n_2+1,\ 1,2} = \frac{X_1(1 - p_{1,1,1})\ p_{2,1,1}}{(1+a_1)^{n_1+n_2}\ (1 + a_2)} \tag{22}$$

for the period immediately following the second level 1 promotion point. If the group of individuals who failed to be promoted at either promotion point were not separated from the organization, they would constitute a group of size

$$X_{n_1+n_2+1,\ n_1+n_2+1,\ 1} = \frac{X_1(1 - p_{1,1,1})\ (1 - p_{2,1,1})}{(1 + a_1)^{n_1+n_2+1}} \tag{23}$$

Equation (21), of course, tentatively assumes that no individuals in the group indicated by this equation have been promoted to level 3. And, following the same development used for equation (19) and (20), n_3 periods after the second level 1 promotion point the three group sizes would be

$$(24) \qquad X_{n_1+n_2+n_3,\, n_2+n_3,\, 2} = \frac{X_1 p_{1,1,1}}{(1+a_1)^{n_1}\,(1+a_2)^{n_1+n_2}}$$

$$(25) \qquad X_{n_1+n_2+n_3,\, n_3,\, 2} = \frac{X_1(1-p_{1,1,1})\,p_{2,1,1}}{(1+a_1)^{n_1+n_2}\,(1+a_2)^{n_3}}$$

and

$$(26) \qquad X_{n_1+n_1+n_3,\, n_1+n_2+n_3,\, 1} = \frac{X_1(1-p_{1,1,1})\,(1-p_{2,1,1})}{(1+a_1)^{n_1+n_2+n_3}}$$

The last three equations, equations (24), (25) and (26), show that in the absence of voluntary losses -- retention losses -- group sizes for each cell in Figure 1 can easily be obtained. All that is necessary is to sum over groups of persons who have reached the same level with the same number of career periods and level periods. And, the size of each group in this sum is given as the quotient of two products:

1. The numerator of the quotient is the product of procurement (at level 1) and the applicable promotion rates -- $(p_{r,s,t})$'s and $(1 - p_{r,s,t})$'s.

2. The demoninator of the quotient is the product of the attrition factors $(1 + a_k)$ at each level, each attrition factor first

being raised to the appropriate power which is simply the number of level periods in each level.

Thus, a sort of general equation for the size of any group would be

$$N_G = \sum_{g \in G} \frac{X_1 \; p_{r,s,t} \; (1 - p_{r',s',t'}) \; - - -}{(1 + a_1)^{n_1} \; (1 + a_2)^{n_2} \; - - -} \tag{27}$$

where

N_G = size of group G

g = all groups whose past promotion history has led them to be a member of G

Now, the computation of the sum indicated in equation (27) is greatly facilitated with the use of a rather simple device. This device (or procedure) is illustrated in Tables 1 and 2. Table 1 lists a hypothetical set of promotion points through the level 3 to level 4 promotion. An "x" in Table 1 indicates the exclusion of that particular promotion point. Thus, the hypothetical data in Table 1 indicates two types of promotions (normal and late) from level 1 to level 2, two types (normal and late) from level 2 to level 3, and three types (early, normal and late) from level 3 to level 4. There are then 2 x 2 x 3 = 12 paths from level 1 to level 4, and these paths are indicated in Table 2. Each path in Table 2 is outlined according to the number of level periods spent in each level prior to level 4 and the sum of these durations which is, of course, the total number of career periods prior to entering level 4. A table similar to Table 2 can be constructed for each level in the organization, the only difference being that the number of paths to the given level increases

Table 1

Hypothetical Promotion Points

Promotion		Level Periods Required for Promotion		
From Level	To Level	Early	Normal	Late
1	2	x	3	4
2	3	x	5	6
3	4	8	10	12
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
M-1	M	.	.	.

Table 2

Paths to Level 4

Path Number	Periods in			Total Periods
	Level 1	Level 2	Level 3	
1	3	5	8	16
2	3	6	8	17
3	4	5	8	17
4	4	6	8	18
5	3	5	10	18
6	3	6	10	19
7	4	5	10	19
8	4	6	10	20
9	3	5	12	20
10	3	6	12	21
11	4	5	12	21
12	4	6	12	22

as the level number under consideration increases. Note, however, that the paths in Table 2 have been arranged in a peculiar fashion. All the early promotions from level 3 (8 level periods), all the normal promotions from level 3 (10 level periods), and all the late promotions from level 3 (12 level periods) have been grouped together such that the total periods for each subgroup form a nondecreasing sequence. Thus, each level in an organization can be considered in a three dimensional array. The first dimension is career periods, the second is level periods, and the third is type of promotion. Such an array is shown in Figure 4 for the above hypothetical level 4. The career periods in Figure 4 begin at 17 since this is one more than the fewest number necessary for promotion to level 4 as indicated in the last column of Table 2.

The combination of equation (27), Table 2 and Figure 4 now form the basis for the stead state solution for a personnel flow through the structure being considered in this paper. For, exclusive of procurement at level 1, X_1 and retention, a factor can be entered into each cell of the array in Figure 4 using the data in Table 2 and equation (27). And, the method of entering the factors in arrays similar to Figure 4 completely eliminates concern about groups splitting and group forming at the promotion points.

Consider again the data in Table 2 for the group following the first path. This group spent 3 periods in level 1, 5 periods in level 2, and 8 periods in level 3. The denominator of the factor entered in cell (17, 1 Early) in Figure must then be $(1 + a_1)^3 (1 + a_2)^5 (1 + a_3)^8 (1 + a_4)$ according to the equation (27). The numerator must be the product of the

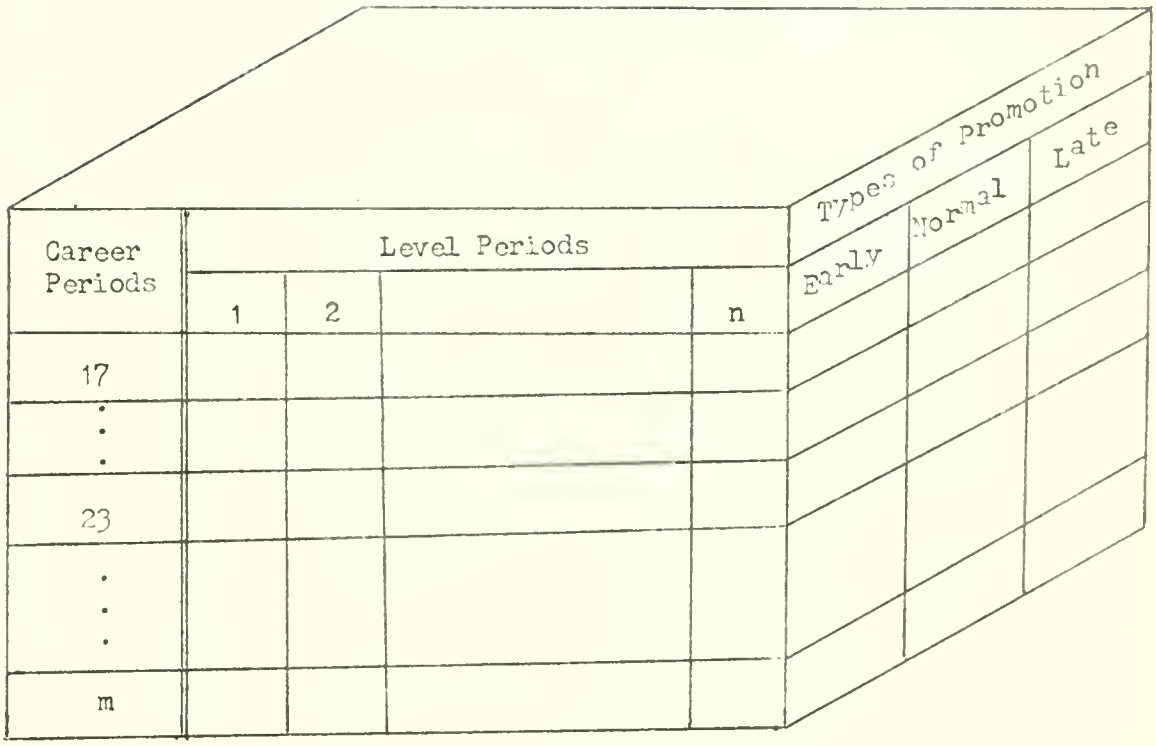

Fig. 4.--The Level Array for Level 4

$(p_{r,s,t})$'s and the $(1 - p_{r,s,t})$'s that applied to this group.[1] Since no other group received an early promotion to level 4 after 16 career periods, this is the only path to be considered for this cell. The entry for cell (18, 1, Early), however, must be the sum of the factors obtained for the two groups who followed path number 2 and path number 3 respectively. Both these groups received an early promotion to level 4 after 17 career periods. Cell (19,1, Early) would contain the factor computed for the group that followed path number 4. And, finally, the first column of the normal and late layers of the array shown in Figure 4 would be filled using the same procedures for the other groups indicated in Table 2.

All remaining cells in the array can be filled as follows:

$$X^*_{m,n,o} = \frac{X^*_{m-1,\, n-1,\, o}}{(1 + a_k)} \qquad \text{where} \tag{28}$$

$$X^*_{m,n,o} = X_{m,n,o}/X_1. \tag{29}$$

This last equation simply emphasizes the point that the factor entered into each cell does not include procurement at level 1. Furthermore, the cells that follow 1,2, . . . career periods and level periods after a promotion point should be multiplied by the factor $(1 - p_{r,s,t})$ where $p_{r,s,t})$ is the promotion rate applicable to the group reaching a specific promotion point. Lastly, any cells beyond all the promotion points that have

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1. The ways of determining the product are many and varied and depend only on the motivation and skill of the individual analyst. In fact, the efficiency obtained in the computerization of this model depends upon the same two factors.

been assigned a value greater than zero should be reset to zero if the existence of individuals in these categories violates the continuation policies that apply to the level being considered.

Thus, if the voluntary losses at each career period were nil and if procurement at level 1, X_1, were known, the number of individuals in each career period, level period, and type of promotion combination at each level could be determined simply by multiplying the appropriate factor by X_1. However, voluntary losses usually do exist and procurement at level 1 is one of the factors to be determined. The solutions for these problems are handled in the next section.

Voluntary losses (Retention)

Since the entries in each cell of the level arrays (arrays similar to Figure 4) are factors that must be multiplied by procurement and retention rates in order to obtain the number of individuals in each possible combination, it is obvious that these rates could be treated as either unknowns or givens. Treating them as givens permits the structure developed above to be analyzed in a simple accounting fashion; treating them as variables, however, permits the use of the goal programming optimization technique. In the development which follows, retention rates for any group, $R_{i,j,k}$, are assumed to be equal to (1.0 - Voluntary Loss Rate) for that same group after adjusting for involuntary losses.

Now, a retention rate $R_{i,j,k}$ could be assumed for every career period, level period and level combination. On the other hand, a more restrictive assumption would be to assume a constant retention rate across all level periods and levels for each career period, i.e., R_i, (i = 1, . . .,Y). The argument for choosing this second alternative

is that only a few (perhaps one) levels are represented at any career period, and the retention rates across the groups considered at any career period probably will not vary greatly. Extending this argument further, it may be even more reasonable to assume that there are critical career periods at which individuals make "stay" and "leave" decisions regardless of their level or periods in a level. In some organizations these critical periods are enhanced by the use of various contractual arrangements, e.g., eligibility for retirement and prior commitments as to the required length of service. The choice between this third case and the second clearly depends upon the organization being considered, and increasing the number of retention rates considered is really not a major problem if linear programming models are being used to analyze personnel flows.[1]

To continue with the model development, assume that the level arrays have been completed for each of M levels and that, for simplicity, M' critical career periods, periods when the retention rates become significant, have been chosen in the set of Y periods. Let this set of career periods be $y_1, y_2, \ldots, y_{M'}$ where $M' \geq M - 1$.[2] And, associate a retention rate R_i, $(i = 1, \ldots, M')$, with each such career period. Thus, every cell in every level array that corresponds to a career period which is greater than y_1 should be multiplied by R_1. Every cell that corresponds to a career period greater than y_2 should be multiplied by R_2, and so on.

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1. Linear programming models deal with underdetermined systems of equations. Cf. Saul I. Gass, Linear Programming: Methods and Applications (2d ed.; New York: McGraw-Hill, 1964), p. 5

2. This condition, $M' \geq M - 1$, will insure an underdetermined system of equations in the full model.

Now, define the following set of variables:

$$X_2 = R_1 X_1$$

$$X_3 = R_1 R_2 X_1$$

$$\vdots$$

$$X_{M'+1} = R_1 R_2 \ldots R_{M'} X_1$$

such that each $X_{i'}$, $(i' = 2, \ldots, M'+1)$, is the product of all R_i for $i < i'$ and X_1, i.e., $X_{i'} = (\prod_{i=1}^{i'-1} R_i) X_1$. And, let $a_{i,j}$, $(i = 1, \ldots, M;$ $j = 1, \ldots, M'+1)$ be the sum of all the factors in level array i which lie between career periods y_j and y_{j-1}, i.e., $y_{j-1} < Y \leq y_j$. There will be $M(M'+1)$ such sums in order to include the career periods that exceed $y_{M'}$. This set of sums is illustrated in Figure 5. Of course, when $j = 1$, $y_{j-1} = y_0 = 0$ career periods.

Level	Career Period Intervals				
	$y_0 < Y \leq y_1$	$y_1 < Y \leq y_2$	...	$y_{M'-1} < Y \leq y_{M'}$	$y_{M'} < Y$
1	$a_{1,1}$	$a_{1,2}$	...	$a_{1,M'}$	$a_{1,M'+1}$
2	$a_{2,1}$	$a_{2,2}$	...	$a_{2,M'}$	$a_{2,M'+1}$
.	.	.	...	.	.
.	.	.	...	.	.
.	.	.	...	.	.
M-1	$a_{M-1,1}$	$a_{M-1,2}$	...	$a_{M-1,M'}$	$a_{M-1,M'+1}$
M	$a_{M,1}$	$a_{M,2}$	...	$a_{M,M'}$	$a_{M,M'+1}$

Figure 5. Partial Sums from the Level Arrays

The array shown in Figure 5 is the basic array in the linear programming model. For, if each element on column j, ($j = 1, \ldots, M'+1$), were multiplied by its corresponding X_j, the result would be the number of individuals in level i with between y_{j-1} and y_j career periods. Remember that each $a_{i,j}$ was formed as a sum of factors across level periods and types of promotions to level i. Thus, multiplying $a_{i,j}$ by X_j has the same effect as multiplying each one of these factors individually by X_j before forming the sum $a_{i,j}$. Moreover, the number of individuals in each career period, level period, and type of promotion to current level (time in previous level) combination at each level can be obtained simply by regenerating the factors in each level array and multiplying them by the appropriate X_j once these values have been determined.

Now, let the array in Figure 5 be represented by the matrix A and let the set of unknowns be represented by the column vector x. Here A is ($M \times M'+1$) and x is ($M'+1 \times 1$). Thus, the obvious equation is

$$Ax = n \tag{30}$$

where $x = \begin{pmatrix} X_1 \\ \cdot \\ \cdot \\ \cdot \\ X_j \\ \cdot \\ \cdot \\ \cdot \\ X_{M'+1} \end{pmatrix}$ and $n = \begin{pmatrix} N_1 \\ \cdot \\ \cdot \\ \cdot \\ N_k \\ \cdot \\ \cdot \\ \cdot \\ N_M \end{pmatrix}$ is the column vector of management goals with respect to the number of people, N_k, desired for each level in the organization. If $M' = M - 1$, equation (30) can be solved by determining the inverse of A, i.e.,

(31) $$x = A^{-1}n$$

Such a solution, however, would not guarantee the non-negativity of the X_j or the proper ranges for the R_i. Each R_i must satisfy the inequalities

(32) $$0.0 \leq R_i \leq 1.0 \qquad (i = 1, \ldots, M')$$

And, the above definitions for the X_j lead to the following considerations for each R_i:

(33) $$0.0 \leq R_j = \frac{X_{j'}}{X_j} \leq 1.0 \qquad (j = 1, \ldots, M') \quad (j' = j + 1)$$

Thus, there will be 2M' inequalities of the form

(34) $$X_{j'} \geq 0.0 \qquad (j' = 2, \ldots, M'+1)$$

and

(35) $$-X_j + X_{j'} \leq 1.0 \qquad (j = 1, \ldots, M')$$

in addition to the obvious constraint that $X_1 \geq 0.0$. The constraints in the inequalities (34) and $X_1 \geq 0.0$ are simply the necessary non-negativity requirements, and the constraints in the inequalities (35) are simply a restatement of the inequalities (32). By adding non-negative slack variables to the inequalities (35), the inequalities become equalities and the flow problem could be restated as

$$
(36)\quad
\begin{array}{lllll}
a_{1,1}X_1 + \cdots + a_{1,M'+1}X_{M'+1} & & & & = N_1 \\
\vdots & & & & \vdots \\
a_{M,1}X_1 + \cdots + a_{M,M'+1}X_{M'+1} & & & & = N_M \\
-X_1 + X_2 & & + X_{M'+2} & & = 0.0 \\
\quad\ddots & & \ddots & & \vdots \\
 & -X_{M'} + X_{M'+1} & & + X_{2M'+1} & = 0.0 \\
 & & & \text{all } X_j & \geq 0.0
\end{array}
$$

and, except for a linear functional, the manpower flow problem has resolved to a linear programming problem.

But the manpower manager is not attempting to optimize with respect to the X_j, $(j = 1, \ldots, 2M'+1)$. Presumably, he is attempting to reach his goals, N_k, "as closely as possible" given his promotion points, promotion rates, attrition rates and continuation policies. Said in another way, the manager is attempting to obtain a steady state flow of people through his organization in such a fashion that the amounts by which the actual numbers of people in all levels differ from their respective N_k's are minimized. Fortunately, with the use of the goal programming technique, the system (36) can be augmented to handle this problem.

Consider the first equation in the system (36), i.e., $a_{1,1}X_1 + \cdots + a_{1,M'+1}X_{M'+1} = N_1$. Let y_1^+ be the amount by which the left hand side of this equation would have to exceed N_1 in order to permit a steady state flow through all levels given a set of promotion and continuation policies and a set of attrition rates. Similarly, let y_1^- be the amount by which the left hand side of this equation would have to be less than N_1 in order to obtain the same set of conditions.

Then, rewrite this equation as

$$a_{1,1}X_1 + \ldots + a_{1,M'+1}X_{M'+1} - y_1^+ \; y_1^- = N_1 \tag{37}$$

and perform a similar operation for each of the first M equations in the system (36). The result would be a system

$$\begin{array}{lllll}
a_{1,1}X_1+\ldots+a_{1,M'+1}X_{M'+1} & & & -y_1^+ + y_1^- & = N_1 \\
\vdots & & & \ddots & \vdots \\
a_{M,1}X_1+\ldots+a_{M,M'+1}X_{M'+1} & & & & -y_M^+ + y_M^- = N_M \\
-X_1+X_2 & & +X_{M'+2} & & = 0.0 \\
\vdots & & \vdots & & \vdots \\
 & -X_{M'+1}+X_{M'+1} & +X_{2M'+1} & & = 0.0 \\
 & & & \text{all } X_j,\ y_i^+,\ y_i^- & \geq 0.0
\end{array} \tag{38}$$

Now, since the column vector of coefficients pertaining to y_1^+ has a -1 in the first row and the rest zeros and the column vector of coefficients pertaining to y_1^- has a +1 in the first row and the rest zeros, these two vectors are linearly dependent. Thus, both y_1^+ and y_1^- cannot both have values in a linear programming solution.[1] The same argument holds for all other y_i^+ and y_i^-. Thus, objective of the nampower manager is to minimize a linear functional

$$c_1^+y_1^+ + c_1^-y_1^- + \ldots + c_M^+y_M^+ + c_M^-y_M^- \tag{39}$$

of the overages and underages--the y_i^+ and y_i^- values--subject to the constraints of the system (38). He can order the relative importance

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1. Cf. Gass, op. cit., pp. 46-53.

of his goals--N_1 through N_M --by weighting the y_i^+ and the y_i^- in the objective function (39) with appropriate cost coefficients c_i^+ and c_i^-.[1] The formulation of manpower flow problems in terms of goal programming problems is now complete and can be restated as

$$\begin{aligned} &\text{minimize} && c'y \\ (40) \qquad &\text{subject to} && A^*x^* + By = n^* \\ &\text{and} && x^*, y \geq 0.0 \end{aligned}$$

where

A^* = the augmented matrix A

B = the matrix of coefficients for the y_i^+ and the y_i^- variables

x^* = the vector x augmented to include the slack variables

y = the vector of overages and underages

n^* = the augmented goal vector

c = the vector of cost coefficients

The solution to the problem (40), x^* and y, would then be the procurement rate, the retention rates, and the overages or underages by level necessary to maintain a steady state flow of people through a specific structure given the set of promotion policies, the set of continuation policies, and the set of attrition rates.

Finally, two other points can be made in connection with this solution. First, if a manager has a specific reason for preselecting some retention rates, e.g., $R_j = r_j$ and $R_{j'} = r_{j'}$, he may do so simply by multiplying the coefficients in columns j and j' of the system (38) by r_j and $r_{j'}$ respectively. These columns should then be combined with columns (j-1) and (j'-1) respectively, and the two constraints

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1. Cf. Ijiri, op.cit., pp. 45-50 for a complete discussion of the weighting and ordering of goals.

$$-X_j + X_{j+1} + X_{M'+j+1} = 0.0$$

and

$$-X_{j'} + X_{j'+1} + X_{M'+j'+1} = 0.0$$

should be removed before solving the goal programming problem. And, second, additional constraints of the form

$$y_i^+ - y_{i,1}^+ + y_{i,1}^- = b_i$$

and

$$y_{i'}^- - y_{i',1}^+ + y_{i',1}^- = b_{i'}$$

may be added to the system (38) in order to state any further desirable conditions with respect to the overages and underages for any level. For example, it may be desired that any necessary overage in level M not exceed 5% of the goal for level M. The relevant constraint would then be

$$y_M^+ - y_{M,1}^+ + y_{M,1}^- = .05N_M$$

Conclusions

The system (40) does then provide a method for simultaneously considering a large number of management goals and some of the structural, policy and behavioral variables related to these goals. It focuses attention on promotion, continuation, retention and attrition problems and forces some explicit formulations necessary for the resolution of these problems. Furthermore, the very fact that this kind of analysis can be performed is evidence that the design of integrated personnel policies can be something more than a pompous discussion. And, even more important, analyses such as this one clearly state just what decisions will be important for personnel managers by differentiating between model inputs and outputs. This statement, though rather trivial for individuals who concern themselves daily with model design and analysis, is anything but trivial to a manager who must constantly attend to the various demands by, for and about the people in his organization. Where should he begin, what consequences will his decisions have, and how should he choose among alternatives are questions that can lead him down many dead end paths.

On the other hand, the model presented above does have some serious limitations. Its most obvious shortcoming is that it describes a steady state flow. It does not permit management goals to change over time, particularly goals pertaining to the number of people in each level. Presumably this problem can be solved by expanding the linear model to a more complex linear or dynamic programming framework,[1] and, indeed, initial

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1. Cf. G. Hadley, Nonlinear and Dynamic Programming (Reading, Mass.: Addison-Wesley, 1964).

efforts in this direction have begun. Secondly, it does not contain any of the usual cost considerations, i.e., estimates of the amount of money it will take to sustain a steady flow of people in addition to the necessary procurement rates, retention rates, overages and underages. And, third, the determination of necessary procurement and retention rates does provide a manager with information concerning the behaviors that will be required of the individuals in his organization, but it does not suggest what that behavior, in fact, will be. The manager must still rely on various forms on intuition, persuasion and suggestion in order to attain and maintain the required behaviors. Thus, it is obvious that a second area for additional research in this field is to begin constructing the predictive links between behavior and various structural and policy variables. This sort of analysis obviously pertains to the ancient problem of organizational design and the choice among alternative structure and policy combinations in terms of what behavior the designer or manager must relinquish or forego when he chooses among alternative combinations.

These areas of further study--dynamic personnel models and predictive personnel models--are but two of the necessary developments. Management scientists must somehow also be able to cope with situations that simply cannot be structured in the neat and orderly fashions presented in this paper. Oftentimes sheer expediency as well as moral and ethical arguments will not permit an analyst to structure an organization in a fashion that resembles the block diagrams in this paper. Two very competent personnel managers in the same organization will not be able to agree on the categories necessary to provide a personnel structure for their organization; and a

third may argue that serious attempts at categorization are simply contrary to all the human values commonly shared by the participants in the organization. But, all three managers will agree that personnel flows are something that should be managed and not left to chance. Admittedly, a certain amount of bargaining will occur before a final structure emerges, but this structure may not be amenable to analysis using the deterministic optimizing type of model presented in this paper. Various kinds of probabilistic models, e.g., a Markov process,[1] may be more appropriate in these situations in order to predict the probable outcomes of various personnel actions. In choosing this path, however, the analyst must be aware of the fact that he will probably sacrifice a measure of direct control over personnel actions in order to solve a somewhat more loosely defined structure.[2] Nevertheless, this is exactly the type of result desired and required in many situations and the presence of these demands cannot be simply ignored. For if the analysis of personnel flows is within the realms of possibility and feasibility, the economic and social consequences of not performing these studies impede the ability of any personnel manager to compete with his environment, particularly with organizations similar to his own.

Finally, the implications that these analyses have for organization theory in general are far too important to be overlooked. As suggested above, the effects that organization structure and personnel policies

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1. Cf. Emanuel Parzen, Stochastic Processes (San Francisco: Holden-Day, 1962), pp. 187-306.

2. Kalman J. Cohen and Richard M. Cyert provide an excellent discussion of this problem in their book Theory of the Firm: Resource Allocation in a Market Economy (Englewood Cliffs, N. J.: Prentice-Hall, 1965), pp. 22-27.

have on the determination of behavioral strategies, the formation of expectations and the execution of choices (decision making itself) are at the heart of any study of organization. Certainly, any serious attempt to specify precise models that interrelate these factors will be effort well spent. All too often organization theory has been approached from a rather grandiose or global point of view with the details of actual decision making behavior being dismissed as relatively unimportant features that can be ignored without any serious concern. Unfortunately, human decision making behavior is a prime concern of organization theory and its description can escape even the most complex forms of analysis.

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