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THE MARKET MODEL APPLIED TO EUROPEAN COMMON STOCKS: SOME EMPIRICAL RESULTS

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I. Introduction

The stock price literature abounds with applications of the Markowitz [1] - Sharpe [15] market model to American stock price data.[2] There is a lack of corresponding studies for non-American securities, due primarily to the absence of generally available machine readable data bases (see however [13] and [16]). The purpose of this paper is to present the results of some initial tests of the market model for a broad cross-section of the European common stocks. Our data base consists of daily price and dividend data for 228 stocks from seven European countries.[3] In addition, for comparison purposes we have included a sample of 65 American securities.

Assuming the market model to be the stochastic process generating security returns, regression analysis was used to estimate the model's parameters for various return measurement intervals and test periods. The analysis focuses on the measures of systematic risk (beta), proportion of variation explained by market movements (R^2), excess return (alpha) and the statistical significance of the excess return measures (T-alpha). The estimated parameters were tested for robustness to changes in measurement interval and stability over time. Finally, the results were examined for their implications for relative efficiency of the various markets studied.

The paper is organized as follows: part II presents a brief review of the market model, part III describes the data base, part IV the methodology, part V the empirical results and part VI the implications for market efficiency.

II. The Market Model

The model is based on the hypothesis that the risk premium on security j during interval t, $(\tilde{R}_{jt})$, is a linear function of the market risk premium, $(\tilde{R}_{Mt})$. The risk premiums are formed by subtracting the riskless rate from the respective security and market returns. The relationship is given by

$$\tilde{R}_{jt} = \alpha_j + \beta_j \tilde{R}_{Mt} + \tilde{\varepsilon}_{jt}, \tag{1}$$

where α_j and β_j are parameters[4] and $\tilde{\varepsilon}_{jt}$ is the non-market related component of security risk premium. The $\tilde{\varepsilon}_{jt}$ variables are usually assumed to have the following properties: zero expected values; uncorrelated with the market return;[5] pairwise[6] and serially uncorrelated; finite variance,[7] σ_j^2.

Using these assumptions, the beta parameter is given by

$$\beta_j = \frac{\mathrm{Cov}(\tilde{R}_{jt}, \tilde{R}_{Mt})}{\mathrm{Var}(\tilde{R}_{Mt})} \tag{2}$$

The usual interpretation of β_j is as a measure of the systematic risk of security j relative to that of the market index; that is, the numerator of the right hand side of equation 2 represents the systematic or non-diversifiable security risk, the denominator the systematic risk of the market index.

To gain insight into the nature of the α_j parameter, we rely on the equilibrium predictions of the Sharpe [14] - Lintner [9] Capital Asset Pricing Model (CAPM). The CAPM relates the expected security risk premiums to their systematic risk coefficients, β_j. That is,

$$E(\tilde{R}_{jt}) = \beta_j \cdot E(\tilde{R}_{Mt}) \qquad (3)$$

where $E(\tilde{R}_{jt})$ and $E(\tilde{R}_{Mt})$ are the expected security and market risk premiums. Comparing equations (1) and (3), α_j is seen to be a measure of the return on security j in excess of that predicted by the CAPM. Under CAPM assumptions the expected value of α_j is equal to zero. Thus, realized values, while not necessarily zero should tend to be small and serially uncorrelated through time.

III. The Data Base

The data base consists of daily prices and dividend data for 228 common stocks of seven European countries. The time period covered is from March 1966 to April 1971. In addition, a sample of 65 American stocks was used for comparison purposes. The American data covered the same period and was taken from the Standard and Poor's I.S.L. tape of New York Exchange securities.

The distribution of the sample by country is shown in table 1. Within each European country, the companies in our sample tend to be the largest in terms of market value of shares outstanding. The 30 Italian stocks, for example, comprise about three-fourths of the market value of all listed Italian shares. For the United Kingdom, France and Germany, the number is not as high but still in excess of 50 percent in each case. Fifty of the 65 American stocks were randomly selected from the population of all NYSE stocks in existence as of March 1966. The remainder of the sample was composed of 15 corporations among those with the largest total equity market value listed on the NYSE.

Security risk premiums were computed on a daily, weekly, bi-weekly and monthly basis, as follows:

$$R_t = \frac{P_t + d_t - P_{t-1}}{P_{t-1}} - R_{Ft} \qquad (4)$$

where R_t = the risk premium during interval t

R_{Ft} = risk free rate during interval t

P_t = the stock price at the end of period t

P_{t-1} = the stock price at the end of the previous period

d_t = dividends paid during the interval (assuming payments on ex-dividend dates)

The stock price and dividend data were corrected for all capital adjustments (splits, rights, etc.). This feature can be very important since firms pay out most of their earnings this way in some countries. Dividend data were not readily available for two of the countries, the Netherlands, and Switzerland; thus risk premiums are measured in these cases by proportionate change in stock price.

For each country, the risk premiums on a market index were computed on a comparable basis. For the six countries (including the USA) for which stock dividend data were available, the risk premiums on the index include dividends. For the reamining two where security dividend data was not available, dividends were not included in the index. The names of the market indexes used are given in Exhibit 1.

It was not practical to attempt to collect short-term rates for each return interval for each country. As an approximation we used the same short-term rate for each return interval (adjusted to duration of interval). It is not felt that this approximation will introduce any significant distortions in the results. For all of the European countries, with the exception of

the United Kingdom, we used the short-term prime bank rate. The names of the risk-free rates used are given in Exhibit 1.

IV. Methodological Issues

The purpose of our empirical tests is to measure the relative magnitude and stability of various market model parameters during the five years covered by our data base.

A principal question involves the choice of an interval for measurement of security and market returns. Since daily data is available, it would perhaps seem logical to use daily returns, as opposed to, say, weekly or monthly values. However, this question is complicated by difficulties resulting from measurement errors and adjustment lags in the rates of return. We will briefly discuss each of these issues in turn.

(i) Estimator Efficiency

As shown in Appendix A, any grouping of the daily data into longer intervals will result in an increase in the standard errors associated with the estimated market model parameters. Thus, if no other considerations were present, the daily returns would provide the most efficient estimates of the coefficients.

(ii) Measurement Errors

Errors in reporting security prices will produce noise in the security return measures. This will result in a higher variance of the residual term in equation (1) than when correct returns are available. The effect will be a reduction in the R^2 between the security and market returns. However, there will be no expected attenuation in the estimated beta coefficient as long as the measurement errors and not correlated with the market returns

Exhibit 1

SUMMARY OF DATA BASE USED

Country	Number of Stocks in Sample	Market Index Used	Risk Free Rate Used
France	65	I.N.S.E.E.	Short term prime bank rate
Italy	30	24 ORE	" " " " "
United Kingdom	40	Financial Times, Industrial Ordinary	31 day treasury notes
Germany	35	Herstatt Index	Short term prime bank rate
Netherlands	24	ANP/CBS	" " " " "
Switzerland	17	Schweizerische Kreditanstalt	" " " " "
Belgium	17	Indice de la Bourse de Bruxelles	" " " " "
United States	65	Standard & Poor's 500 Stock Composite Index	30 day U.S. Government Treasury Bills

(a plausible assumption).

Other than simple reporting errors or lags, a common source of error results from the rounding of prices to fractional values (e.g. quarters). This small source of error can be important for short return intervals. It should also be recognized that the European Markets do not have the same information gathering and dissemination facilities as the major United States markets. Thus, short term price changes are much more likely to be in error than their American counterparts.

The effect of measurement errors will diminish as the length of the return interval increases. Thus, measurement errors alone would suggest the use of an interval of maximum length.

(iii) Adjustment Lags

Adjustment lags result from the failure of stock prices to fully adjust to market changes during the trading day. Instead the adjustment may be spread over two or more days. This can be caused either by the lack of trading in the stock, or failure to comprehend market conditions when delays in index reporting exist. The result is that the price will "catch up" for previous market activity during later days. This phenomenon would imply a distributed lag model for explaining security returns; for example,

$$R_t = \alpha + \beta_o R_{Mt} + \beta_1 R_{M,t-1} + \beta_2 R_{M,t-2} \cdot \cdot + \varepsilon_t \tag{5}$$

where $R_{Mt}, R_{M,t-1} \ldots$ are the market returns on the current and previous days.

When daily returns are used to estimate the market model parameters

(equation 1) the price lag effect will result in a reduction of the percentage of security return variation explained by the market and the estimated beta $\hat{\beta}$ will be a downward biased estimate of the true beta, β. This problem would also tend to disappear as the measurement interval increased. The adjustment lags will be more important in the European markets, where trading volume is typically much lower than for U.S. stocks, and where reliable market information is more difficult to obtain.

An additional problem results from errors in the market index itself. In a number of European markets the market index is not computed on daily closing prices, but on prices at some convenient time during the day. The result is that the stock returns might well lead rather than lag the index. This would imply that we should add a term $\beta_{-1}R_{t+1}$ to equation (5) to reflect this effect. Any index lag would also result in a lower R^2 and attenuation of β as in the stock price lag case. The importance of this effect will diminish as the interval is increased.

In summary, the above discussion indicates that optimal interval length for measuring returns is by no means obvious. Consequently, we have presented summary results in part V for each of four intervals; daily, weekly, bi-weekly and monthly. The results for the different intervals are compared in part VI for their market efficiency implications.

* * *

A second major emphasis of the paper is to examine the stability of model parameters over time. This was accomplished by subdividing the five years and measuring model parameters for each sub-period. The first period extends from March 1966 to November 1968, the second from December 1968 to

April 1971. To test for stability of the parameters, parameter estimates for the second period were regressed on values for the first period. The regression equation is given by:

$$\hat{P}_{2j} = \gamma_o + \gamma_1 \hat{P}_{1j} + \eta_j \qquad j=1, \ldots, M \tag{6}$$

where $\hat{p}_{1j}$ and $\hat{p}_{2j}$ are the subperiod parameter estimates for security j, γ_0 and γ_1 are regression coefficients and M the number of securities.

The parameters chosen for testing were alpha, the t statistic for alpha, beta and the coefficient of determination. Beta was chosen since it represents the systematic risk. Alpha and t-alpha were chosen to examine the predictability and significance of the excess returns earned and R^2 measures the percentage of security variation explained by market movements.

Using cross-sectional correlation coefficients to measure parameter stability is complicated by measurement errors; the estimates $\hat{p}_{1j}$ and $\hat{p}_{2j}$ are not the true parameter values, p_{1j} and p_{2j}, but are only estimated values. As shown in appendix B, these errors will cause the estimated cross-section correlation coefficient $\hat{\rho}$ and regression coefficient $\hat{\gamma}_1$ to be attenuated. The degree to which the estimates $\hat{\rho}$ and $\hat{\gamma}_1$ understate the true values ρ and γ_1 depends on the size of the measure error variation relative to the dispersion of the true parameters P_1 and P_2. This attenuation bias must be kept in mind when evaluating the empirical results in part V. For example, it would not be unexpected to find higher cross-sectional correlations for parameters based on daily returns since their measurement errors (as given by the standard errors of the estimates) would be the smallest.

V. Empirical Results

The empirical results are summarized in tables 1 through 6. In some cases parameter estimates are given for each of the four measurement intervals; however, for brevity, when all observation frequencies lead to similar conclusions, only the bi-weekly results are given. The robustness tests, which examine the sensitivity of parameter estimates to changes in the return measurement interval, are based on the total five-year period; the stability tests are based on the two 2 1/2 year subperiods.

A. Robustness of Market Model Parameters

Table 1 gives the average security return and standard deviation for each country sample. Corresponding data are given for the market indexes used.[8] The security averages are unweighted, and thus can differ substantially from those of the market weighted indexes. The average returns vary from 0.015 percent for the U.S. to 0.466 percent per two-week period for Germany. It is interesting to note that the U.S. stocks had the highest average standard deviations. However, this may reflect the broader composition of the U.S. sample rather than inter-market differences.

Table 2 presents average estimated beta, t-beta and R^2 for each country sample. The results are presented for each of the four observation frequencies. The most striking observation is the extent to which the average parameter values depend on return interval.

The estimated betas are lowest for daily returns, highest for monthly returns. As discussed in part IV, this effect most likely results from lags in the adjustment of stock prices to changes in market levels. As such, the effect would be expected to diminish for longer return intervals.

In an efficient market, where such lags were absent, the expected value of beta should be invariant to the return measurement interval. Thus, differences in average betas between daily and monthly returns can be used to measure the relative importance of adjustment lags in the various markets. The slower the price adjustment speed, the greater the range between beta values based on daily and monthly returns. The following gives the ratios of average betas based on these return intervals.

Exhibit 2

Effect of Lags in Stock Price Adjustment

Country	$\mu = \frac{\bar{\beta}\ \text{monthly}}{\bar{\beta}\ \text{daily}}$
United States	1.08
Great Britain	1.33
France	1.43
Germany	1.60
Italy	1.39
Belgium	3.20
The Netherlands	1.50
Switzerland	2.23

The European countries have larger ratios in all cases than the U.S. Further, the smaller European countries tend to have the largest values. These results will be discussed further in part VI.

The average t statistics for beta confirm the statistical efficiency argument developed in part IV. The largest t statistics (smallest standard errors) result from daily calculations. Thus, the daily beta estimates are most efficient (minimum variance) but biased due to price adjustment lags.

The average t values are typically reduced by half between daily and monthly observations.

Finally, the average R^2 figures show the percentage of variation in stock returns explained by market movements in the various countries. The numbers display the same general pattern as the beta estimates, rising from a low for daily to a high for monthly return observations. However, as shown below the spreads between daily and monthly R^2 are larger than for beta.

Exhibit 3

Effect of Lags and Measurement Errors

Country	$\mu = \frac{\bar{R}^2 \text{ monthly}}{\bar{R}^2 \text{ daily}}$
United States	2.90
Great Britain	1.66
France	2.66
Germany	2.33
Italy	2.08
Belgium	8.38
The Netherlands	3.65
Switzerland	3.72

The reason for the greater spreads is related to the fact that R^2 are affected by stock price measurement errors, as well as price adjustment lags. The measurement errors would be expected to diminish in importance as the return interval increases.[9]

Among the Big Five (U.S. plus four largest European Countries) the U.S. has the lowest average R^2. For monthly observations, the sample values range from 0.244 for the U. S. to 0.558 for Germany.

The average R^2 for the Small Three (Belgium, the Netherlands and Switzerland), while typically smaller for daily returns, have values similar to the other European countries for monthly returns.[10]

Table 3 presents data on the bi-weekly excess returns achieved in each country; that is average alpha values. Under the CAPM assumptions, the expected average alpha value is zero for each market. The table shows the mean alphas and t-alphas for each country. The t statistics to a first approximation allow us to identify significant individual alpha values. The table gives the frequency distribution for t_α for each country. The range of t_α values is larger for all but one of the European countries than for the U.S. For the U.S., 6.1 percent of the t_α values exceed 1.5 in absolute value; for Europe the percentage are typically higher, ranging from a low of 5.9 (Switzerland) to a high of 46.6 percent (Italy).

To summarize, the estimated beta and R^2 parameters are sensitive to the return interval used. The effect on beta is smallest in the U.S; the effect on R^2 is similar in the U.S. and the four larger European countries, and larger in the smaller three.

B. Parameter Stability

In this section we will focus on cross-sectional analysis of market model parameters estimated in two sub-periods -- March 1966 to November 1968 and December 1968 to March 1971. Our purpose is to compare parameter stability across the eight markets, and thus gain insight into the degree of predictability of key parameters. The cross-sectional regressions were run for parameters obtained for each of the four return intervals.

Table 4 presents average bi-weekly rates of return for each subperiod. The data illustrate the low degree of intermarket correlation re-

ported by several previous authors. Additionally, for each country the subperiod returns show little correlation as well, while average standard deviations are much more predictable.

Table 5 presents average subperiod parameter estimates for beta, R^2, α and t_α. The statistics are based on bi-weekly data. The beta results show that for seven of the eight countries the average value was larger in the second period. The R^2 results do not present such a consistent pattern. The average R^2 increased for four countries (including the U.S.) and declined for four. Similarly, the alpha and t alpha statistics do not appear to change in any systematic way. The sample standard deviations associated with the averages in the table show the range of estimates differs considerably among markets.

Table 6 presents the results of interperiod cross-sectional regressions. The coefficient of cross-sectional determination measures the magnitude of parameter correlation (i.e. the portion of variation in $\hat{P}_{2j}$ explained by $\hat{P}_{1j}$). The significance of the relationship is indicated by the t statistic associated with the "slope" parameter ($t_{\gamma 1}$). The regression results are presented for all four sets of subperiod parameters.[11]

Cross-sectional comparisons are complicated by two factors, differing sample sizes and dispersion of sub-period parameters. For example, the subperiod German betas are relatively tightly grouped, and thus (as predicted in part IV) the inter-period correlations will be more sensitive to parameter measurement error.

The intra-country results for standard deviation, beta and R^2 show

a consistent pattern for most countries; correlation decreases as the return observation frequency decreases. Daily estimates have the highest degree of predictability, but as discussed above the estimates are biased. As predicted the correlations are most quickly attenuated as the return interval increases for those countries with low parameter dispersions. In the case of Germany, for example, the cross sectional R^2 for beta decreases from 0.718 for daily observations to 0.00 for bi-weekly intervals.[12]

When inter-country comparisons are made, the three parameters are seen to be more stable in several of the European countries (e.g. Great Gritain, France, Italy) than in the U.S. market. This result is most evident for daily observations, less so for monthly data. The results for the small Three show a lower correlation of daily parameters than the Big Five; however, the results are not significantly different for monthly sub-parameters. Interestingly, the cross-sectional correlation of β and R^2 actually increase with interval size for one of the smaller countries (The Netherlands).[13]

As for the alpha excess return measures, little evidence for their predictability can be found. Exceptions perhaps are Great Britain and Italy. For Great Britain the correlation between sub-period alpha values is significant at the five percent level for all intervals. For Italy the relationship between the subperiod t_α values is significant at the five percent level

VI. Implications for Market Efficiency

Our final task is to review our results for their implications regarding relative market efficiency. While our tests are not extensive enough to permit definitive conclusions, they permit a number of interesting observations.

The first evidence comes from the sensitivity of the average beta values to the length of the return interval. In an efficient market, beta would not be sensitive to the interval size. This, however, was not the case. As reported in part V, average monthly betas exceeded average daily betas in all eight countries, the ratios ranging from 1.08 for the U.S. to 3.20 for Belgium. Thus, price adjustment lags appear least significant in the U.S. market. On the whole, the four major European countries had lower ratios than the three smaller countries, indicating a somewhat shorter adjustment period for the larger markets.

The next issue involves the extent to which measures of total and systematic risk are predictable. If securities are to be properly priced, investors must be able to predict their future riskiness. Our subperiod correlation analysis sheds some light on this question. Both standard deviations and beta values were found to be as predictable in the European markets as in the U.S.

Under the CAPM assumptions applied to each market, the average alpha values for each country should average to zero and be uncorrelated across time. The results in table 6 support this hypothesis in most of the countries, with the possible exception of Great Britain where there is some evidence of significant inter-period correlation. Similarly, the t statistics for alpha should be uncorrelated between subperiods. This hypothesis is again generally true, with the exception of Great Britain and Italy. In these countries it would have been possible to earn subperiod returns relative to the CAPM standard by selection stocks with significant first period t-alpha values.

The distribution of t_α statistics show the European sample to have larger proportions of outlier values. This result is particularly striking for Italy where nearly 1/2 the t_α values exceeded 1.5 in absolute value. However, given our lack of knowledge concerning the degree of statistical dependence between intra country t_α values, it is difficult to draw very strong conclusions from these results.

On the whole our evidence does not show substantial differences between the U.S. and four major European markets. Some cases can be made for the three smaller markets being less efficient. Our conclusions regarding efficiency must be viewed as tentative in nature. We have dealt with only a five-year period, using pre-selected security samples. More definitive conclusions must await more extensive and statistically powerful tests.

Appendix A: Effect of Grouping Procedure on Estimator Efficiency

Consider the loss of efficiency associated with grouping k daily returns into H N-day intervals ($K = NH$). The market model for the k daily returns is given by

$$\tilde{R}_{\tau t} = \alpha + \beta\tilde{R}_{M\tau t} + \tilde{\varepsilon}_{\tau t} \qquad \text{(A 1)}$$

where $\tilde{R}_{\tau t}$. $\tilde{R}_{M\tau t}$ and $\tilde{\varepsilon}_{\tau t}$ are the return on day t ($t=1, \ldots, N$) within interval τ ($\tau=1, \ldots, H$).

Now group the security and market returns into the N day values by averaging the N daily returns in each of the H intervals. Equation (A 1) can be restated for the N day returns

$$\tilde{R}_{\tau} = \alpha + \beta\tilde{R}_{M\tau} + \tilde{\varepsilon}_{\tau} \qquad \tau=1, \ldots, H \qquad \text{(A 2)}$$

where $\tilde{R}_{\tau}$, $\tilde{R}_{M\tau}$ and ε_{τ} are averages of $R_{\tau t}$, $R_{M\tau t}$ and $\varepsilon_{\tau t}$ ($t=1, \ldots, N$) respectively. Let α^* and β^* be the regression estimates based on daily returns and $\hat{\alpha}$, $\hat{\beta}$ the estimates based on N day return. If the least squared assumptions hold for equation (A 1), they also hold for equation (A 2). Thus α^* and β^* are unbiased estimators, as are $\hat{\alpha}$ and $\hat{\beta}$. The variances of α^* and $\hat{\alpha}$ are given by[14]

$$\text{Var}(\alpha^*) = \frac{\text{Var}(\varepsilon_{\tau t})}{\sum_{\tau=1}^{H}\sum_{t=1}^{N}(R_{\tau t} - \bar{R}_{\tau})^2} \qquad \text{(A 3)}$$

$$\text{Var}(\hat{\alpha}) = \frac{\text{Var}(\varepsilon_{\tau})}{\sum_{\tau=1}^{H}(R_{\tau} - \bar{R}_{\tau})^2} \qquad \text{(A 4)}$$

where $\bar{R}_\tau$ is the average of the H R_τ values (note that $\bar{R}_\tau$ is also equal to the average of the K $R_{\tau t}$ returns). The variance of ε_τ is equal to the variance of $\varepsilon_{\tau t}$ divided by N. Therefore,

$$\mathrm{Var}(\hat{\alpha}) = \frac{\mathrm{Var}(\varepsilon_{\tau t})}{N \cdot \sum_{\tau=1}^{H} (R_\tau - \bar{R}_\tau)^2} \qquad \text{(A 5)}$$

Comparison of the two variance is equivalent to comparison of denominators. Expanding the denominator of $\mathrm{Var}(\alpha^*)$,

$$\sum_{\tau=1}^{H} \sum_{t=1}^{N} (R_{\tau t} - \bar{R}_\tau)^2 = \sum_\tau \sum_t (R_{\tau t} - R_\tau + R_\tau - \bar{R}_\tau)^2$$

$$= \sum_\tau \sum_t (R_{\tau t} - R_\tau)^2 + N \sum_\tau (R_\tau - \bar{R}_\tau)^2 \qquad \text{(A 6)}$$

Thus, the denominator of $\mathrm{Var}(\alpha^*)$ is always greater than that of $\mathrm{Var}(\hat{\alpha})$. The ratio of $\sum_\tau \sum_t (R_{\tau t} - R_\tau)^2$ to $N \sum_\tau (R_\tau - \bar{R}_\tau)^2$ is a measure of the loss of precision resulting from the grouping. The results apply equally to the variances of β^* and $\hat{\beta}$.

Appendix B. Attenuation of Cross-Sectional Correlation Coefficients

Assume the M estimated parameter values $\hat{P}_{1j}$ and $\hat{P}_{2j}$ are related to the true values P_{1j} and P_{2j} as follows

$$\hat{P}_{1j} = P_{1j} + u_j \qquad j=1, \ldots, M$$

$$\hat{P}_{2j} = P_{2j} + v_j \tag{B 1}$$

The measurement errors u_j and v_j are assumed to be distributed independently from each other and from the true parameter values.

The measured correlation coefficient between $\hat{P}_{1j}$ and $\hat{P}_{2j}$ (j=1, . . ., M) (designated $\hat{\rho}_M$) will approach a limiting value, given by[15]

$$\text{plim } \hat{\rho}_M = \frac{\rho}{\left\{(1 + \frac{\sigma_u^2}{\sigma_{P_1}^2})(1 + \frac{\sigma_v^2}{\sigma_{P_2}^2})\right\}^{1/2}} \tag{B 2}$$

where ρ is the true correlation coefficient, σ_u^2 and σ_v^2 are the variance of u and v, and $\sigma_{P_1}^2$ and $\sigma_{P_2}^2$ are the cross-sectional variances of P_1 and P_2. Thus, $\hat{\rho}_M$ is not an unbiased estimate of ρ, but will understate the true correlation between the parameters by an amount depending on the size of the measurement error variation relative to the dispersion of the true parameters. Similarly, the estimated regression coefficient $\hat{\gamma}_1$ in equation (6) will be downward biased. The estimated values of γ_1 will tend to a limit which is less than the true value. The limit is given by[16]

$$\text{plim } (\hat{\gamma}_{1M}) = \frac{\gamma_1}{(1 + \frac{\sigma_u^2}{\sigma_{P_1}^2})} \tag{B 4}$$

Thus, if the variance of the error term for $\hat{P}_1$ is 10 percent of the variance of the true values (as estimated by $\hat{\sigma}_{P_1}$), then least squares would underestimate $\hat{\gamma}_1$ by about 10 percent, even for very large sample sizes.

FOOTNOTES

1. Respectively, Associate Professor of Finance, Sloan School of Management, M.I.T. and Assistant Professor of Finance, Graduate School of Business, Stanford University.

2. See, for example, Blume [1,2], Cohen and Pogue [3], Fama [4], King [7].

3. We wish to express our appreciation to the Research Department of Eurofinance, Paris, for providing the data on which the study is based.

4. The parameter values α_j and β_j do not have time subscripts since they are assumed to remain stationary through time. Of course, one of our main concerns in this paper is to test the validity of this assumption.

5. This assumption can only be precisely true in the limit as the proportion of security j in the market portfolio approaches zero.

6. This assumption rules out industry effects. However, these effects (see King [7]) typically account for only about ten percent of variation in security returns, so that to a first approximation they can be be ignored. This assumption has no effect on the estimation of the α_j and β_j parameters, but makes cross-sectional comparisons more difficult.

7. This assumption presupposes the existence of variances for the $\tilde{\varepsilon}_{jt}$ variables. The work of Mandelbrot [11] and Fama [4] suggests, however, that time series values of $\tilde{\varepsilon}_{jt}$ conform more closely to a non-normal stable paretian distribution for which the variance does not exist than to the usually assumed normal distirubtion. Nonetheless, in this paper we will make the more common assumption of the existence of these statistics since Fama [5] has shown that insights into the effects of diversification

on dispersion of returns that are derived from the mean-standard deviation model remain valid when the model is generalized to include the entire stable family.

8. The market returns shown in the table are averages, computed in the same way as the stock return averages. The figures equal the calendar returns only for those countries for which the all-stock return series were complete over the March 1966-March 1971 period. This includes the U.S. and the major European countries. For countries where some of the series began after March 1966, the market return reflects the average market return faced by the sample of stocks; that is

$$\bar{R}_M = \frac{1}{M} \sum_{j=1}^{M} \bar{R}_{Mj}$$

where M = number stocks in sample

$\bar{R}_{Mj}$ = average market return during period of complete data for stock j.

9. The measurement error tends to be submerged with longer return intervals. For example, using monthly returns, only two of the approximately 22 daily returns in the monthly figure would be affected by measurement error. For daily observations, on the other hand, both the beginning and ending prices of the interval would be subject to error.

10. A previous study by King [7] examined the relationship between market and security returns for NYSE stocks. Sixty-three common stocks were analyzed, with returns computed on a monthly basis from June 1927 to December 1960. The average R^2 value decreased monitorically, from 0.584 for the 1927-1935 period to 0.307 for the 1965-1960 interval. More recently Blume [1] replicated these tests using a larger sample of stocks and found results consistent with those of King. The mean proportion of return variation explained by the market was 0.280 for the period 1961-1968.

11. For reasons of brevity we have not presented the details of the cross-sectional regression results. However, for bi-weekly intervals the estimated $\hat{\gamma}_0$ and $\hat{\gamma}_1$ coefficients can be computed by combining results from tables 5 and 6.

$$\hat{\gamma}_1 = \hat{\rho} \cdot \frac{\hat{\sigma}_{p1}}{\hat{\sigma}_{p2}}$$

where $(\hat{\rho})^2$ is given in table 6, $\hat{\sigma}_{p1}$ and $\hat{\sigma}_{p2}$ in table 5. Therefore,

$$\hat{\gamma}_0 = \overline{\hat{P}}_2 - \hat{\gamma}_1 \overline{\hat{P}}_1$$

where the average subperiod estimates, ($\overline{\hat{P}}_1$ and $\overline{\hat{P}}_2$), are given in table 5.

12. The stability of common stock beta parameters has been studied by Blume [1] and Levy [8]. Using monthly returns and seven year estimation periods, Blume found an average product moment correlation between stock betas of 0.618 ($R^2 = 0.37$). More recently Robert Levy replicated these tests using weekly returns and various short estimation periods (e.g. 26, 52 weeks). For the period from 1962 through 1970 the average correlation between annual beta values was 0.486 ($R^2 = 0.25$).

13. An additional factor which complicates comparisons between U.S. and European results is the potential lack of compatibility between data samples. The U.S. sample, due to its random content, is more representative of the complete cross-section of NYSE stocks. The European samples were pre-selected, and mainly represent the major stocks in their respective markets. Further, the U.S. daily price tapes used for the study may well have a higher error rate than the carefully screemed European data file.

14. See Malinvaud [10], pp. 282-283.

15. the $\rho\text{lim}(\hat{\rho}_M)$ expression is derived in an analogous manner to that for $\text{plim}(\hat{\gamma}_{1M})$. See equation B4 and footnote 16.

16. See Johnston [6], equation (9-43), page 282.

Table 1

Average Total Period Returns and Standard Deviations for Securities and Market Index

-- Bi-Weekly Return Intervals (all figures in percentages)

Country	Securities		Market Indexes		Riskless Rate
	Return	St. Dev. of Return	Returns	St. Dev. of Return	All Periods
America {65}	0.015 (0.523)	7.896 (2.840)	0.154	3.135	0.220
Great Britain {40}	0.430 (0.530)	5.547 (1.133)	0.274	3.341	0.295
France {65}	0.330 (0.345)	4.901 (1.072)	0.301	3.292	0.235
Germany {36}	0.466 (0.257)	4.430 (0.587)	0.353	2.806	0.190
Italy {30}	0.105 (0.902)	6.115 (2.461)	0.062	2.672	0.165
Belguim {17}	0.306 (0.388)	3.198 (0.997)	0.269	1.724	0.200
Netherlands {24}	0.336 (0.431)	4.422 (0.969)	0.379	2.637	0.208
Switzerland {17}	0.389 (0.367)	4.609 (0.814)	0.464	2.851	0.136

{ } Number stocks
( ) Sample standard deviations

Table 2

Average Total Period Estimated Parameters for Various Return Measurement Intervals

untry	Beta				t Statistic for β				Coefficient of Determination (R²			
	Day	Week	2 Week	Month	Day	Week	2 Week	Month	Day	Week	2 Week	Mont
nerica {65}	1.037 (0.506)	1.116 (0.485)	1.129 (0.472)	1.218 (0.525)	9.82 (5.23)	7.52 (3.04)	5.94 (2.14)	4.36 (1.79)	0.084 (0.072)	0.194 (0.108)	0.228 (0.110)	0.2 (0.1
r. Br {40}	0.787 (0.183)	0.956 (0.198)	0974 (0253)	1.046 (0.311)	18.67 (4.75)	11.22 (2.50)	7.93 (2.13)	6.06 (1.58)	0.248 (0.090)	0.369 (0.098)	0.368 (0.116)	0.4 (0.1
rance {65}	0.586 (0.242)	0.769 (0.245)	0.816 (0.258)	0.840 (0.283)	12.90 (6.39)	9.03 (3.51)	7.38 (2.74)	5.64 (2.03)	0.127 (0.094)	0.248 (0.126)	0.302 (0.137)	0.3 (0.1
ermany {36}	0.0639 (0.183)	0.858 (0.182)	1.048 (0.187)	1.025 (0.157)	18.31 (6.58)	12.22 (3.77)	10.37 (3.07)	8.14 (2.38)	0.215 (0.106)	0.363 (0.132)	0.446 (0.141	0.5 (0.1
taly {30	0.724 (0.195)	0.921 (0.216)	0.950 (0.244)	1.005 (0.304)	21.73 (8.54)	14.59 (5.56)	11.00 (4.32)	9.46 (3.41)	0.268 (0.139)	0.434 (0.174)	0.462 (0.178)	0.5 (0.1
elguim {17}	0.349 (0.141)	0.693 (0.325)	0.732 (0.250)	1.116 (0.403)	7.13 (2.30)	6.11 (2.21)	4.79 (1.71)	5.86 (1.81)	0.044 (0.022)	0.142 (0.076)	0.166 (0.085)	0.3 (0.1
ether {24}	0.535 (0.245)	0.757 (0.231)	0.773 (0.217)	0.800 (0.276)	9.71 (4.37)	7.23 (2.26)	5.74 (1.75)	4.86 (1.70)	0.080 (0.062)	0.186 (0.090)	0.222 (0.101)	0.2 (0.1
Switzer. {17}	0.456 (0.118)	0.898 (0.220)	0.924 (0.254)	1.015 (0.271)	10.71 2.69	8.32 2.30	6.54 1.92	6.04 1.97	0.126 0.052	0.299 0.112	0.344 0.128	0. 0.

{ } Number of Stocks in Sample
( ) Standard leviation of Sample

Table 3

Average Total Period Alpha and t-alpha for Bi-weekly Return Intervals

Country	Alpha	t-Alpha	Distribution of t Statistics										% $t_\alpha \geq \lvert 1.5 \rvert$
			<-2	-2 -1.5	-1.5 -1.0	-1.0 -0.5	-0.5 0	0 0.5	0.5 1.0	1.0 1.5	1.5 2.0	>2	
America {65}	-0.131 (0.533)	-0.278 (0.743)	0	2	8	20	13	12	8	0	2	0	6.1
Breat Britain {40}	0.155 (0.544)	0.360 (1.112)	0	2	3	4	5	8	6	8	1	3	15.0
France {65}	0.041 (0.339)	-0.315 (3.410)	0	2	6	11	13	14	7	7	3	1	9.2
Germany {36}	0.105 (0.240)	0.351 (0.877)	0	0	4	3	6	9	5	5	4	0	11.1
Italy {30}	-0.035 (0.423)	-0.175 (1.626)	3	5	3	3	4	1	3	2	4	2	46.6
Belgium {17}	0.055 (0.376)	0.138 (1.253)	0	1	3	2	2	1	5	1	0	2	17.6
Netherlands {24}	-0.005 (0.433)	0.099 (1.179)	0	1	2	5	4	5	2	2	1	2	16.6
Switzerland {17}	-0.050 (0.330)	-0.112 (0.844)	0	1	1	4	3	5	0	3	0	0	5.9

{ } Number of stocks
( ) Sample standard deviation

Table 4

Average Sub-Period Returns and Standard Deviations for Securities and Market Index

-- Bi-weekly Return Intervals (all figures in %)

Country	Securities				Market Indexes			
	Rate of Return		Std. Dev. of Return		Rate of Return		Std. Dev. of Return	
	First Period	Second Period	First Period	Second Period	First Period	Second Period	First Period	Second Period
America	0.325 (0.818)	-0.308 (0.645)	7.438 (3.062)	8.002 (3.432)	0.298	0.026	2.990	3.492
Gr. Britain	1.410 (0.983)	-0.273 (0.447)	5.242 (1.231)	5.587 (1.309)	1.069	-0.307	3.101	3.432
France	0.176 (0.643)	0.530 (0.420)	5.037 (1.202)	4.686 (1.053)	0.042	0.591	3.511	2.924
Germany	0.634 (0.372)	0.280 (0.391)	4.074 (0.815)	4.662 (0.827)	0.604	0.091	2.974	2.595
Italy	-0.051 (0.606)	0.113 (0.475)	3.464 (1.399)	4.429 (1.814)	-0.060	0.193	2.452	2.934
Belgium	0.272 (0.526)	0.347 (0.507)	3.273 (1.019)	2.996 (1.049)	0.259	0.322	1.621	1.739
Netherlands	0.677 (0.533)	-0.010 (0.565)	3.905 (1.116)	4.708 (1.110)	0.598	0.186	2.303	2.953
Switzerland	1.174 (0.743)	0.118 (0.402)	4.164 (0.787)	4.742 (0.926)	1.248	0.172	2.510	3.106

{ } Number of stocks in sample
( ) Sample standard deviations

Table 5

Average Sub-Period Parameter Estimates -- Bi-Weekly Return Intervals

Country	Beta		R^2		Alpha		t Alpha	
	First Period	Second Period	First Period	Second Period	First Period	Second Period	First Period	Second Period
America	0.999 (0.558)	1.200 (0.529)	0.168 (0.121)	0.325 (0.143)	0.036 (0.813)	-0.295 (0.666)	-0.054 (0.823)	-0.284 (0.696)
Gr. Britain	0.926 (0.315)	0.986 (0.272)	0.343 (0.143)	0.384 (0.130)	0.398 (0.931)	0.026 (0.431)	0.398 (1.163)	0.086 (0.748)
France	0.803 (0.302)	0.826 (0.265)	0.335 (0.177)	0.272 (0.119)	0.096 (0.598)	0.001 (0.406)	-0.196 (2.717)	-0.306 (2.415)
Germany	0.981 (0.242)	1.144 (0.252)	0.505 (0.169)	0.418 (0.131)	0.038 (0.348)	0.204 (0.392)	0.200 (0.938)	0.383 (0.840)
Italy	0.980 (0.310)	0.918 (0.272)	0.523 (0.188)	0.433 (0.202)	0.004 (0.597)	-0.078 (0.476)	0.020 (1.444)	-0.255 (1.248)
Belgium	1.052 (0.362)	0.544 (0.301)	0.262 (0.123)	0.115 (0.076)	0.011 (0.526)	0.081 (0.499)	0.065 1.285	0.117 (1.099)
Netherlands	0.681 (0.293)	0.817 (0.252)	0.149 (0.099)	0.280 (0.118)	0.204 (0.584)	-0.200 (0.566)	0.527 (1.156)	-0.338 (1.113)
Switzerland	0.768 (0.305)	0.952 (0.254)	0.249 (0.143)	0.379 (0.124)	0.184 (0.606)	-0.053 (0.401)	0.230 (1.052)	-0.132 (0.809)

{ } Sample size
( ) Sample standard deviations

Table 6

Correlation Between Sub-Period Parameter Estimates $(\hat{\rho}_{1j}, \hat{\rho}_{2j})$ for Various Return Measurement Intervals

Regression Equation $\hat{\rho}_{2j} = \gamma_0 + \gamma_1 \hat{\rho}_{1j} + n_j$

Country	Standard Deviation of Return				Beta				R^2			
	Day	Week	2 Week	Month	Day	Week	2 Week	Month	Day	Week	2 Week	Month
America	* 0.001 **(-0.249)	0.051 (1.839)	0.082 (2.367)	0.064 (2.083)	0.514 (8.163)	0.467 (7.431)	0.255 (4.648)	0.271 (4.844)	0.075 (2.252)	0.094 (2.563)	0.029 (1.368)	0.029 (1.387)
Gr. Britain	0.509 (6.272)	0.311 (4.146)	0.246 (3.521)	0.253 (3.585)	0.630 (8.046)	0.326 (4.290)	0.310 (4.138)	0.297 (4.006)	0.432 (5.381)	0.169 (2.780)	0.216 (3.232)	0.098 (2.032)
France	0.579 (9.301)	0.525 (8.345)	0.499 (7.93)	0.384 (6.262)	0.590 (9.53)	0.469 (7.463)	0.480 (7.625)	0.230 (4.332)	0.661 (11.076)	0.420 (6.755)	0.388 (6.321)	0.149 (3.318)
Germany	0.311 (3.919)	0.071 (1.606)	0.001 (-0.188)	0.015 (0.719)	0.718 (9.297)	0.317 (3.972)	0.002 (0.290)	0.002 (0.116)	0.817 (2.307)	0.613 (7.335)	0.268 (3.535)	0.118 (2.133)
Italy	0.627 (6.867)	0.573 (6.125)	0.578 (6.189)	0.503 (5.323)	0.525 (5.566)	0.248 (3.035)	0.253 (3.077)	0.303 (3.490)	0.798 (10.516)	0.559 (5.956)	0.461 (4.898)	0.179 (2.473)
Belgium	0.754 (6.775)	0.649 (5.267)	0.629 (5.049)	0.735 (6.448)	0.402 (3.172)	0.6332 (4.776)	0.053 (0.192)	0.306 (2.573)	0.153 (1.643)	0.415 (3.262)	0.049 (0.875)	0.024 (0.092)
Netherlands	0.198 (2.329)	1.09 (1.622)	0.144 (1.921)	0.131 (1.825)	0.138 (1.879)	0.192 (2.290)	0.149 (1.962)	0.201 (2.350)	0.209 (2.418)	0.245 (2.670)	0.206 (2.388)	0.224 (2.518)
Switzerland	0.324 (2.683)	0.316 (2.635)	0.374 (2.994)	0.369 (2.962)	0.469 (3.641)	0.242 (2.191)	0.376 (3.009)	0.292 (2.487)	0.668 (5.499)	0.600 (4.741)	0.514 (3.980)	0.426 (3.333)

* Coefficient of Determination

** t Statistic for γ_1

Table 6 (continued)

Correlation Between Sub-Period Parameter Estimates (ρ_{1j}, ρ_{1j}) for Various Return Measurement Intervals

Regression Equation $\rho_{2j} = \gamma_0 + \gamma_1 \rho_{1j} + n_j$

Country	Alpha				t Statistic for Alpha			
	Day	Week	2 Week	Month	Day	Week	2 Week	Month
America	0.144 (3.249)	0.001 (0.269)	0.006 (0.598)	0.000 -0.055	0.016 (1.023)	0.002 (-0.336)	0.002 (0.356)	0.000 (-0.170)
Gr. Britain	0.192 (3.002)	0.182 (2.912)	0.113 (2.189)	0.195 (3.035)	0.125 (2.326)	0.103 (2.090)	0.048 (1.377)	0.071 (1.781)
France	0.002 (-0.357)	0.000 (-0.148)	0.001 (-0.214)	0.009 (-0.792)	0.005 (-0.538)	0.000 (0.160)	0.758 14.056	0.777 14.811
Germany	0.006 (-0.438)	0.004 (-0.346)	0.016 (-0.742)	0.001 (0.140)	0.024 (0.925)	0.008 (0.508)	0.012 (0.638)	0.046 (1.279)
Italy	0.067 (1.420)	0.052 (1.279)	0.060 (1.333)	0.072 (1.473)	0.228 (2.873)	0.248 (3.035)	0.289 (3.377)	0.328 (3.694)
Belgium	0.001 (-0.112)	0.001 (-0.137)	0.000 (-0.031)	0.027 (-0.563)	0.002 (0.180)	0.015 (0.481)	0.025 (0.587)	0.009 (0.364)
Netherlands	0.044 (1.000)	0.026 (0.772)	0.025 (0.753)	0.022 (0.707)	0.053 (1.101)	0.027 (0.730)	0.025 (0.746)	0.036 (0.903)
Switzerland	0.145 (1.589)	0.191 (1.882)	0.140 (1.560)	0.173 (1.768)	0.157 (1.673)	0.236 (2.150)	0.183 (1.835)	0.214 (2.023)

REFERENCES

[1] Blume, M., "On the Assessment of Risk," Journal of Finance, March 1971, pp. 1-10.

[2] ___ ___, "Portfolio Theory: A Step Toward Its Practical Application," Journal of Business, April 1970, pp. 152-173.

[3] Cohen, K., and G. Pogue, "An Empirical Evaluation of Alternative Portfolio-Selection Models," Journal of Business, April 1967, pp. 166-193.

[4] Fama, E., "The Behavior of Stock Market Prices," Journal of Business, January 1965, pp. 34-104.

[5] ________, "Risk Return and Equilibrium," Journal of Political Economy, January-February 1971, pp. 30-55.

[6] Johnston, J., Econometric Methods, Second Edition, McGraw-Hill, New York, 1972.

[7] King, B. F., "Market and Industry Factors in Stock Price Behavior," Journal of Business, January 1966, pp. 139-190.

[8] Levy, R. A., "Stationarity of Beta Coefficients," Financial Analysts Journal, November-December 1971, pp. 55-63.

[9] Lintner, "Security Prices, Risk and Maximal Gains from Diversification," Journal of Finance, December 1965, pp. 387-615.

[10] Malinvaud, E., Statistical Methods of Econometrics, Second edition North-Holland Publishers, 1970.

[11] Mandelbrot, B., "The Variation of Certain Speculative Prices," Journal of Business, October 1963, pp. 394-419.

[12] Markowitz, H., Portfolio Selection, Efficient Diversification of Investments, Wiley, New York, 1959

[13] Modigliani, F., G. A. Pogue, M. Scholes and B. Solnik, "Efficiency of European Capital Markets and a Comparison with the American Market," Presented at First World Congress on the Stock Exchange, Milan, Italy, March 1972. Published in Conference Proceedings.

[14] Sharpe, W. F., "A Simplified Model for Portfolio Analysis," Management Science, January 1962, pp. 277-293.

[15] Sharpe, W. F., "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," Journal of Finance, September 1964, pp. 425-442.

[16] Solnik, B. H., "European Capital Markets: Towards a Theory of an International Capital Market," unpublished Ph.D. Thesis, M.I.T. June 1972.