A Method for Synchronizing

Traffic Lights

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This is a working paper and is not yet considered to be in final form. The authors welcome comments.

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ABSTRACT

A method is given for synchronizing traffic lights along a main street. It is an embellishment of the conventional traffic engineering method of maximizing bandwidth. (By bandwidth is meant the fraction of a light cycle during which a car could start at one end of the street and, by traveling at a preassigned speed, go to the other end without stopping for a red light.) For the case of equal traffic flows in each direction, the method produces maximum equal bandwidths for each direction. If more than one synchronization does this, a secondary measure of effectiveness is maximized. For the case of unequal flows, the direction with the greater flow is given greater bandwidth in a manner depending on the ratio of the flows. The method has been programmed for a digital computer and successfully applied to a street in Cleveland.

Introduction

It is proposed to study a method for synchronizing traffic lights for a main artery in an urban area. On such a street the through traffic at any one point is generally much greater in volume than the turn traffic. Furthermore, in an urban area, traffic lights tend to be close together so that the input to a downstream light is very similar to the output from the adjacent upstream light and thus may be considered as a controlled input. Along most urban arteries the traffic pattern varies with the time of day. For example, main arteries frequently carry heavy rush hour traffic in one direction twice a day, and lighter but roughly equal volumes for the rest of the time. It is unlikely that the synchronization appropriate to any particular time would be ideal all the time. Consequently, we wish to be able to find an appropriate synchronization for any given traffic conditions. We shall study only the synchronization problem, that is, it will be assumed that the lights, their locations, their splits between red and green, their cycle length, and the speed of cars are given, and it is wished to know how to set their relative phasing. However, our method can be used to test the sensitivity of the results to the various input parameters.

What is meant by optimal synchronization of traffic lights? To answer this question the ideal approach might be to find and define a measure of effectiveness of the traffic system and relate the various control variables to it. Unfortunately, this is a rather formidable task and has not yet been done. Some of the difficulties are: What

does the driver really want — how does he dislike stopping as compared to delay at signals, or total delay on his journey? Then, how do drivers actually behave — how do they adapt their driving to the lights? Finally, what does the controller want and how is this to be related to what the driver wants.

A number of starts on the problem have been made[1,2,3,4]. An interesting one is that of Newell[2], for it takes into consideration the dynamics of individual cars. His approach, however, may overemphasize the importance of reducing delay. In his method, the first few drivers of the platoon stop at every light, and so, in two or three miles of urban driving, a car could stop at 20 consecutive lights. This might be considered objectionable, even if the total delay at these lights were minimized.

The approach here is closer to the usual one of traffic engineers than to an ideal one. The measures of effectiveness are motivated by a simplified model of driver behavior and by intuition about what the driver might consider to be desirable characteristics of signal settings. As a starting point we use the goal of maximizing bandwidth. However, we take a calculation which is traditionally done graphically[5,6] and reduce it to a few seconds on a digital computer. Certain embellishments have been added, in particular, a systematic way to favor the heavy flow direction over the light flow direction. There is some evidence that, in many cities, rather obvious traffic signal improvements are not being made for lack of engineering manpower. We hope that

a readily available computer program for setting traffic lights, even one using a criterion that is by no means ultimate, will help get some needed improvement.

DEVELOPMENT OF THE METHOD

The idealized model that motivates the bandwidth criterion is that vehicles travel down the street at some constant speed, V, unless halted by a red light. Vehicles so halted form a queue and, when the light turns green, leave in a compact platoon travelling at speed V.

We define the following terms:

Cycle length or period, C, of a signal is the time between successive reds or greens at the signal.

Green split, (g,r), at a signal is the ordered pair: (proportion of green time, proportion of 'not green time') along a given road.

Volume or flow of traffic, F, is the number of vehicles passing a point per cycle.

Platoon passage time for a flow F, t_F, is the average time required by a compact platoon of F vehicles to pass a point. We shall assume a constant time interval between such cars and denote it by h. Thus $t_F = hF$.

Green band, in one direction along a length of road, is the time interval in a cycle of the light during which a single vehicle leaving a light may travel at speed V and pass through all the lights without stopping at a red light.

Bandwidth is the width of the green band in a given direction. Let b = bandwidth, expressed as a fraction of a period.

Consider a flow F through the sequence of signals in one direction. Whatever the arrival pattern of the F vehicles, if the platoon passage time, hF, is less than the green band, bC, then the average delay at red lights for a vehicle passing through the system will be less than one period: all vehicles will emerge from the system in or before the succeeding green band. If the traffic input is fairly random there will be a certain amount of delay, if only to vehicles arriving at the initial red, but we shall consider an average delay of less than one period as not serious. Hence, if $hF < bC$ in each direction and b is the same in each direction, we shall be satisfied with the synchronization.

Our initial mathematical goal therefore will be to maximize the green bands in each direction while keeping them equal. If more than one setting of the lights does this, there is more flexibility. We shall use this to maximize a further criterion related to the bandwidth through adjacent lights.

Now, if $hF > bC$ for one direction or the other, the platoon will be broken up as it passes through the lights and the computation of delay and the number of stops made by a car is likely to become quite complicated, even for the simple driver behavior postulated. We shall sidestep this complication and adopt a procedure which simply favors one direction over the other in a manner which has some intuitive appeal. The green band for one direction will be increased at the expense of the other according to a parameter, k, which expresses the degree of favoritism. At the same time, the general evenness of the synchronization is increased in the favored direction. The case k = 1 corresponds to

equal green bands and in synchronism, whereas k = 0 (or reciprocally, $k = \infty$) assigns to the favored direction an unobstructed green band constrained only by the width of the smallest green in the signal sequence. The parameter, k, can be set in any way desired, but here it is taken to be the ratio of the two flows.

The method for synchronizing the lights will be developed by starting with simple situations and building up to the general case. Let

t = time, in units of a period.

$L_1, \ldots, L_n$ = the sequence of traffic lights down the road, all assumed to have the same period.

(g_i, r_i) = the green split at L_i

f_{ij} = traffic flow from L_i to L_j

b_{ij} = the bandwidth in the direction L_i to L_j considering these two lights alone.

ϕ_{ij} = the relative phasing of L_i and L_j = the time from the center of a red at L_i to the next (in time) center of red at L_j as a fraction of a period.

x_i = the position of L_i on the street with respect to some fixed origin. We assume the lights are indexed such that $x_j > x_i$.

The notation $(y)^*$ will be used to denote the mantissa of y as obtained by removing the integral part of y and, if the result is negative, adding unity. Thus, $(5.2)^* = .2$, $(-.2)^* = .8$ and, in general, $0 \leq (y)^* < 1$.

Now, let

$X_{ij} = ([y_j - y_i]/VC)^*$ = the separation between L_i and L_j in the outbound direction.

$\overline{X}_{ij} = ([y_i - y_j]/VC)^*$ = the separation between L_i and L_j in the inbound direction.

Figure 1 illustrates the meaning of X_{ij} and $\overline{X}_{ij}$. Consider a car which is outbound on the street and is not stopped anywhere. If it passes L_i at the start of a cycle of L_i, it passes L_j at a time X_{ij} after the start of a cycle of L_j. The separation, $\overline{X}_{ij}$, has the same meaning for a car inbound along the street. The quantities are dimensionless, having been expressed as fractions of a period. From the definitions or the figure, we see that $\overline{X}_{ij} = X_{ji}$ and $X_{ij} + \overline{X}_{ij} = 1$ so that $X_{ij} + X_{ji} = 1$. (The sums to 1 will actually be zero when $X_{ij} = 0$.)

We shall be mostly interested in X_{ij}. It makes possible a convenient representation of lights L_i and L_j on the X,t plane. For example, see Figure 2. Let $X = X_{ii} = 0$ represent the position of L_i. The trajectory of an unimpeded car outbound can be represented by a 45° line. A car passing L_i at t passes L_j at $t + X_{ij}$. A -45° line represents inbound travel: a car passing L_i at t passes L_j at $t - X_{ij}$.

The analysis will start out with the simple case of two traffic lights with (1/2,1/2) green splits and then build up to the general case. For the two light case, explicit results can be given.

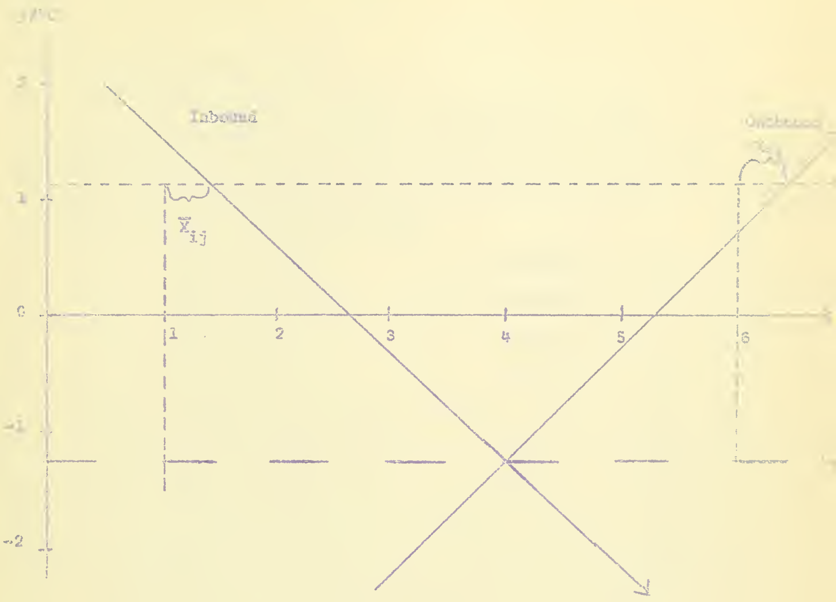


Figure 1. Space-time diagram showing X_{ij} and $\overline{X}_{ij}$. Heavy lines at L_i indicate the red intervals of L_i^j.

A. Two signals, equal flows

Theorem 1: Consider two signals L_1 and L_2, both with green splits (1/2,1/2), and having an arbitrary separation $X_{12} = X$. They will be phased for the maximum equal bandwith, b(i.e., $b = b_{12} = b_{21}$ and b is maximum) when $\pi_{12} = \pi$ is set as follows

(i) for $0 \leq X \leq 1/4$, $\pi = 0$ and then $b = 1/2 - X$

(ii) for $1/4 \leq X \leq 1/2$, $\pi = 1/2$ and then $b = X$

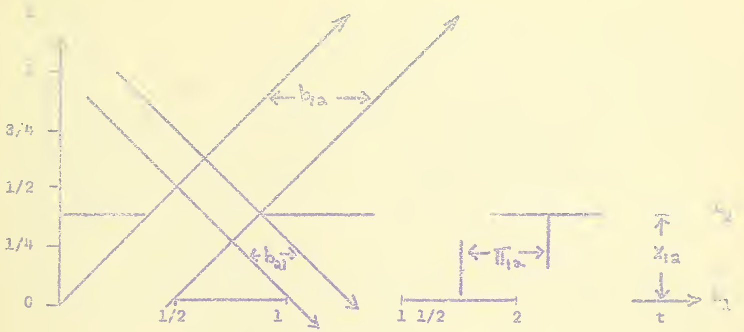
(iii) for $1/2 \leq X \leq 3/4$, $\pi = 1/2$ and then $b = 1 - X$

(iv) for $3/4 \leq X \leq 1$, $\pi = 0$ and then $b = X - 1/2$

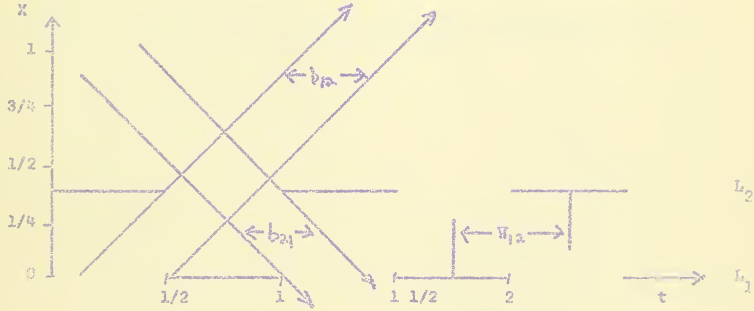
Proof. We consider the four cases separately. In each case, let $t = 0$ correspond to the start of green for L_1. We proceed by examining all possible phasings as follows: First set L_2 to give $b_{12} = 1/2$ by setting $\pi = X$. Figure 2a shows this for case (ii). Then move π through its possible values until $b_{12} = b_{21} = b$ and b has its maximum value. The result can be obtained by inspection. Note that when a small increase in b_{12} causes a decrease in b_{21} (or vice versa) the loss for one equals the gain for the other. The final phasing is shown in Figure 2b. The other cases may be worked out similarly.

Notice that in the final phasing the lights are either exactly in phase ($\pi = 0$) or diametrically out of place ($\pi = 1/2$). We shall call this situation in-or-out phasing.

Proceeding in a like manner for each case, the theorem is established. The results can be summarized in the phase-time diagram of Figure 3. For any $X_{12} = X$ the proper timing for L_2 is found by drawing a horizontal



a) $\pi_{12} = x_{12}$



b) $\pi_{12} = 1/2$

Figure 2. Finding phasing for maximum equal bandwidths.

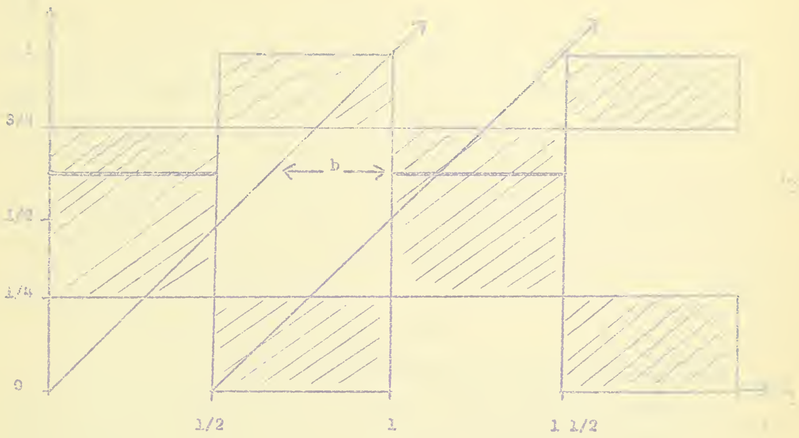


Figure 3. Maximum equal bandwidths for an L_2 having separation X are obtained by placing L_2's red time in the shaded areas as shown.

line crossed at X. The red time for L_2 is given by the region where the horizontal line passes through the shaded areas. An outbound car leaving L_1 at $t = 0$ travels along the 45° line leaving from 0 and reaches L_2 at $t = X$. Similarly a car leaving at $t = 1/2$ travels along the other 45° line and reaches L_2 at $t = X + 1/2$. Thus the bandwidth at X is given by the length of the horizontal line segment which lies between the two diagonal lines in an unshaded region.

Next we consider more general green splits.

Theorem 2: <u>If L_1 and L_2 have a separation $X_{12} = X$ and green splits (g_1, r_1) and (g_2, r_2) with $r_1 \geq r_2$, the lights can be phased for maximum equal bandwidth, b, (i.e. $b_{12} = b_{21} = b$ and b is maximum) by setting $\pi_{12} = \pi$ as follows:</u>

(i) for $0 \leq X \leq 1/4$, $\pi = 0$ and then $b = g_1 - \text{Max}\{0, X - 1/2(r_1 - r_2)\}$

(ii) for $1/4 \leq X \leq 1/2$, $\pi = 1/2$ and then $b = g_1 - \text{Max}\{0, 1/2 - X - 1/2(r_1 - r_2)\}$

(iii) for $1/2 \leq X \leq 3/4$, $\pi = 1/2$ and then $b = g_1 - \text{Max}\{0, X - 1/2 - 1/2(r_1 - r_2)\}$

(iv) for $3/4 \leq X \leq 1$, $\pi = 0$ and then $b = g_1 - \text{Max}\{0, 1 - X - 1/2(r_1 - r_2)\}$

Note that the phasing is the same as in Theorem 1. The proof can be made by the same procedures as for theorem 1. Or, consider Figure [illegible] which shows the final results. In this figure, which is analogous to Figure 3, dashed lines leading out of the edges of green at the bottom represent trajectories for outbound cars; lines leading out of edges of green at the top represent inbound cars. Now, for arbitrary X, draw a heavy horizontal

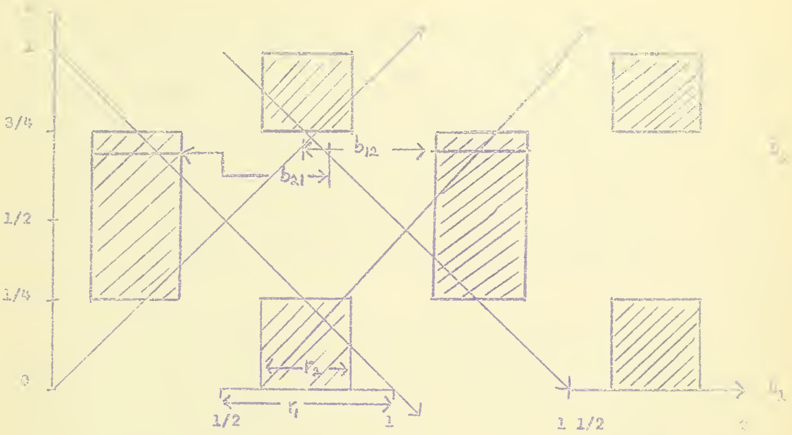


Figure 4. Maximum equal bandwidth phasing for L_2 at X when $r_1 > r_2$.

line through the shaded areas. The segments represent the red times for L_2 if L_2 is located at X. It may be seen that these lines are placed horizontally so as to make $b_{12} = b_{21}$. Furthermore, if the row of lines is moved horizontally through a full cycle in time, it will be seen that no other position will give greater equal bandwidths. As in Figure 3, the bandwidth when L_2 is at X is given by the length of a horizontal line which lies within a pair of the parallel diagonal lines but outside the shaded area.

If X is in any of the intervals, $(0,[r_1-r_2]/2)$;$([1/2]-[r_1-r_2]/2, [1/2]+[r_1-r_2]/2)$; or $(1-[r_1-r_2]/2,1)$ the π which yields maximum equal bandwidths is not unique but any value in certain range would work. Here the bandwidth equals g_1, the shorter green, and a reduction in g_2 would not affect bandwidth. The choice of the symmetric solution is made for convenience.

Corollary: <u>If the conditions of theorem 2 hold except $r_2 > r_1$, the results hold with r_2 interchanged with r_1 and g_2 with g_1.</u>

<u>Proof:</u> Interchange L_1 and L_2 and apply theorem 2, recalling that $X_{21} = 1 - X_{12}$.

B. <u>Two Lights, Unequal Flows</u>

Consider lights L_1 and L_2 separated by X and handling flows F_{12} and F_{21}. We seek a way to favor the heavier of the two flows. Let $k = F_{12}/F_{21}$, i.e., the ratio of outbound flow to inbound flow. Let $= \beta$ the loss of green band in the direction of F_{12} because of the presence of L_2, i.e., $\beta = g_1 - b_{12}$. We shall choose β so that $k\beta$ is the loss in

the direction of F_{21} because of L_2. Thus $k\beta = g_1 - b_{21}$. We shall take Case 1 maximum equal bandwidth solution and adjust π_{12} by that amount α. Since the loss to one direction is an equal gain to the other, the sum of the losses stays constant and

$$\beta + k\beta = 2g_1 - (b_{12} + b_{21}) = 2(g_1 - b)$$

Furthermore, the loss or gain is the amount of shift in π and so

$$\alpha = (g_1 - b) - \beta$$

$$\alpha = (g_1 - b)\,(k-1)/(k+1)$$

The shift α is added to or subtracted from π_{12}, whichever will favor the direction of F_{12} when $\alpha > 0$. The results in the (1/2,1/2) case are displayed in Figure 5. Explicitly

(i) For $0 \le X \le 1/4$, $\beta = 2X/(k+1)$ $\quad \pi_{12} = X(k-1)/(k+1)$

(ii) For $1/4 \le X \le 1/2$, $\beta = (1-2X)/(k+1)$ $\quad \pi_{12} = X(k-1)/(k+1)+1/(k+1)$

(iii) For $1/2 \le X \le 3/4$, $\beta = (2X-1)/(k+1)$ $\quad \pi_{12} = X(k-1)/(k+1)+1/(k+1)$

(iv) For $3/4 \le X \le 1$, $\beta = (2-2X)/(k+1)$ $\quad \pi_{12} = X(k-1)/(k+1)+2/(k+1)$

2. A sequence of signals

Consider signals $L_1,\ldots,L_n$ with green splits $(g_1,r_1),\ldots,(g_n,r_n)$ for flows $F_{1,n}$ going from L_1 to L_n and $F_{n,1}$ going from L_n to L_1. The traffic lights are to be synchronized in the following steps:

1. Select the signal settings to give maximum equal bandwidth.

2. If more than one setting gives maximum equal bandwidth, select the best from among these according to a secondary criterion of maximizing the total of the bandwidths of adjacent pairs of lights.

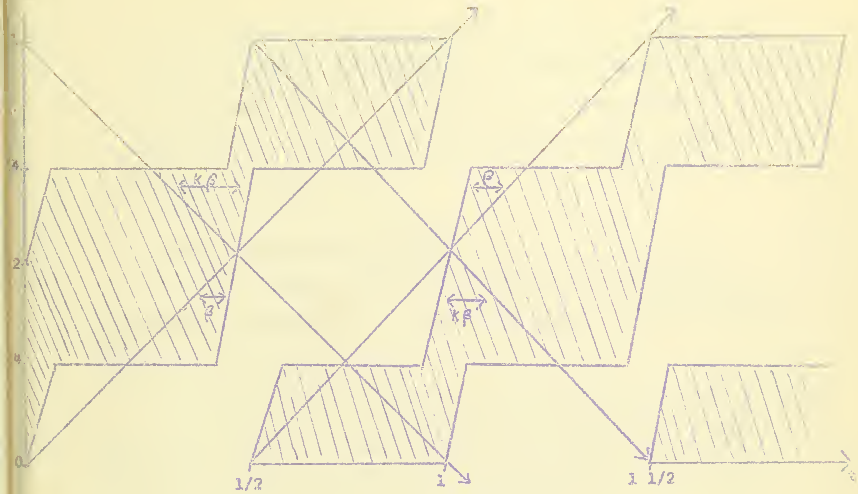


Figure 5. Favoring the outbound flow.

3. If both platoon passage times $hF_{1,n}$ and $hF_{n,1}$ are less than b, then we have the required setting. Otherwise one or both platoon passage times are greater than b, and those signals which restrict b to less than $\mathrm{Max}(hF_{1,n}, hF_{n,1})$ will be adjusted to favor the heavier flow.

C. 1. Finding Maximum Equal Bandwidth

Theorem 3. Suppose that the lights have been set to give the maximum total bandwidth, $b_{1n} + b_{n1}$, such that each direction has a bandwidth greater than zero.* Then:

a) If one side of the red for L_i limits bandwidth in one direction, then the other side limits bandwidth in the other direction. Furthermore, in one direction the front edge (earlier in time) of the green band is limited, and in the other the rear edge.

b) Either a single light limits both edges of both green bands, or else there is a pair of lights, say L_i and L_j, such that L_i limits the front and L_j the rear of one green band; then L_j limits the front and L_i the rear of the other green band.

c) There exists an in-or-out phasing which yields the maximum total bandwidth and splits it equally between the two directions.

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* The reason for this restriction is that, under some circumstances, total bandwidth, $b_{1n} + b_{n1}$, can be greater than twice the maximum equal bandwidth, 2b. If $\mathrm{Min}\, g_i > 2b$, we can make a complete green wave along one direction to achieve $b_{1n} > 2b$.

Part (a) is illustrated, for example, by L_1 or L_2 in Figure 7A. The first sentence of (a) must be true, for otherwise the position of red for L_i could be shifted, increasing bandwidth in one direction without hurting bandwidth in the other. The second sentence is obvious from examination of L_1 or L_2 in Figure 7A.

Part (b) follows from (a). Suppose L_i limits the front edge of one green band and L_k the front edge of the other. By (a) the near edges are limited too. Either $L_i = L_k$ or $L_i \neq L_k = L_j$. (It is not excluded that more than two lights might limit bandwidth.)

For (c) we first show that, if three lights serve to limit bandwidth, at least two of them will have in-or-out phasing with respect to each other. (Figure 7b would contain a good example if L_6 had a slightly larger red.) Of the three lights, some two must touch the bands on the same edges (L_2 and L_6 in the cited situation). For each light these edges form the equal sides of an isosoceles triangle having as a base the line of reds and greens of the light. The two triangles are similar and can be drawn to have a common apex. The perpendicular bisector of their bases is then common. By symmetry, however, the center of each base is either the center of a green or of a red. Therefore the two lights are either exactly in or diametrically out of phase, as claimed. Extending the argument, all lights which limit bandwidth can be broken into two groups such that within each group all lights will have in-or-out phasing with respect to each other. Call these, groups 1 and 2.

Now, starting with a setting which yields maximum total bandwidth, we shall move reds without loss of total bandwidth, until in-or-out phasing is achieved. The procedure is as follows: Hold group 1 lights fixed. Move the group 2 reds all together in the direction which will tend to equalize bandwidths. The total bandwidth stays constant since the loss to the large band equals the gain to the small band. If other lights start to obstruct bandwidth, move them, too. They will only touch a green band on one side as they move. Eventually groups 1 and 2 will have in-or-out phasing with regard to each other.

Consider then any one of the remaining lights, say L_i. Its red can be moved somewhat without interfering with either green band. Define a light L_i' which has a red which contains that of L_i and which is large enough just to touch a side of each green band at L_i. Now make the center of red for L_i coincide with the center of red for L_i. But L_i' limits bandwidth and so is either in group 1 or 2 and has the desired type of phasing. In this way all lights can be given in-or-out phasing and maximum equal bandwidth is established.

Theorem 4. <u>All in-or-out phasings yield equal bandwidths in each direction.</u>

Under in-or-out phasing, the inbound green band in a space-time diagram is (except for directions of flow) the mirror image of the outbound band as reflected about a vertical line through the center of any red. Bandwidth are therefore equal. See Figure 7a.

Because of Theorems 4 and 3c we need only maximize bandwidth under

in-or-out phasing and in one direction. Let

b_i = the greatest outbound bandwidth that can be obtained by having L_i the light whose red limits the rear (later in time) of the outbound green band under in-or-out phasing.

$\overline{b}_i$ = Same, but limiting the front.

Then, by part (b) of theorem 3, maximum equal bandwidth is

$$b = \max_i \left\{ \mathrm{Max}[b_i, \overline{b}_i] \right\}$$

However, because some light always limits the front when L_i limits the rear, it is sufficient to write:

$$b = \max_i \left\{ b_i \right\} \qquad (1)$$

Next we wish to calculate b_i. In Figure 6 we illustrate L_i and three other lights having in-or-out phasing with respect to L_i. Note that L_i limits the rear of the outbound band, i.e., no red interval intersects the outbound arrow marked r. Furthermore, the bandwidth is the distance between that arrow and the furthest encroachment of red into the region between the outbound arrows f and r.

Perhaps the easiest way to calculate the outbound bandwidth is to project the edges of red for each light diagonally back to the time interval [0,1] at L_i. Let u_{ij} (π) be the position in [0,1) of the left hand side of the projection of L_j's red under the phasing π. Let the zero of the time scale be the left hand side of L_i's red. It may be seen that

$$u_{ij}(\pi) = [(r_i/2) + \pi - (r_j/2) - X_{ij}]^* \qquad (2)$$

Then $u_{ij} + r_j$ is the position of the right hand edge of L_j's red. Under in-or-out phasing, π = 0 or 1/2. Of the pair, we want the one which

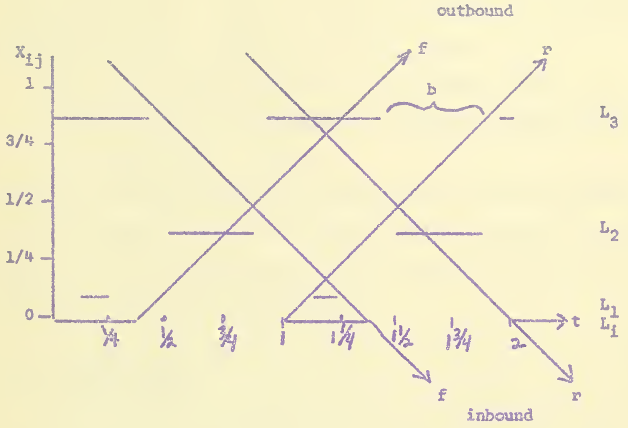


Figure 6. Four lights under in-or-out phasing: Lights L_i, L_1 have a common center for red. The center of red for L_2 differs in phase from the others by 1/2.

brings the right hand edge nearest zero, provided that L_j's red does not then limit the rear of the band instead of L_i's red. Now, the way we have calculated $u_{ij} + r_j$, it will be in (0,1] if L_j does not interfere and in (1,2] if it does. There, we can find b_i from

$$b_i = 1 - \underset{j}{\text{Max}} \left\{1, \text{Min} \left[u_{ij}(0) + r_j,\ u_{ij}(1/2) = r_j\right]\right\} \quad (3)$$

As b_i and then b are calculated, we record the phasing vectors

$$\pi_i = (\pi_{i1}, \pi_{i2}, \ldots, \pi_{in})$$

which are developed by the Min choice in (3). Also, it is necessary to save the index, say y(i) of the light which most constrains the bandwidth, i.e., the one that causes the Max in (3). We need only save these for i such that b_i is maximized.

C.2. Breaking Ties

Let B be the set of i for which $b_i = b$, the maximum bandwidth. Frequently B will contain more than one index. It seems desirable to use the resulting flexibility to maximize some further measure of effectiveness. We pick a simple one: the sum of the bandwidths between adjacent signals. In phasing a number of signals for maximum bandwidth through all lights, it is likely that bandwidths through some pairs of adjacent lights are decreased in order to increase bandwidth through all. By breaking ties in favor of a large sum of bandwidths between adjacent lights, we tend to keep bandwidth wide where the principal constriction on the street is not ruling.

Under the phasing π_i, $i \in B$, adjacent lights L_k and L_{k+1} have the phasing $\left| \pi_{ik} - \pi_{i,k+1} \right|$.

Using (2) we can define

$$u^{(i)}_{k,k+1} = ((1/2)r_k + |\pi_{ik} - \pi_{i,k+1}| + (1/2)r_{k+1} - X_{k,k+1})^* \quad (4)$$

which will give a corresponding $b^{(i)}_{k,k+1}$ as follows:

If $u^{(i)}_{k,k+1} + r_{k+1} \leq 1$,

$$b^{(i)}_{k,k+1} = 1 - \text{Max}\,[r_k, u^{(i)}_{k,k+1} + r_{k+1}] + \text{Max}\,[0, u^{(i)}_{k,k+1} - r_k] \quad (5a)$$

If $u^{(i)}_{k,k+1} + r_{k+1} > 1$,

$$b^{(i)}_{k,k+1} = \text{Max}\left\{0,\ u^{(i)}_{k,k+1} - \text{Max}[r_k, u_{k,k+1} + r_{k+1} - 1]\right\} \quad (5b)$$

Then we choose $i \in B$ to maximize

$$\sum_{k=1}^{n-1} b^{(i)}_{k,k+1} \quad (6)$$

Let $i = z$ pick the light that does this. Then $\pi_z = [\pi_{z1}, \ldots, \pi_{zn}]$ is chosen for phasing the lights for maximum equal bandwidth. Further ties are settled arbitrarily.

C.3 Adjustment for Unequal Flows

It remains to determine which lights need to be adjusted to favor the heavier stream. If

$$t_f = \text{Max}\,(hF_{1n}, hF_{n1}) \leq b \quad (7)$$

no adjustment will be made and the final phasing is π_z. If, however, $t_f > b$, signals with obstructing red will be adjusted to favor the heavier stream. The reason for adjusting only the obstructing signals is that the adjustment of an non-obstructing signal yields no net gain in bandwidth to the heavier stream but may inadvertently obstruct the lighter stream.

Suppose first that traffic is heavier in the outbound direction or is equal in both directions. Then $k = F_{1n}/F_{n1} \geqslant 1$. The amount of obstruction to the outbound direction by L_j over that caused by L_z alone is $\mathrm{Max}\ \{0, u_{zj}(\pi_{zj}) + r_j - r_z\}$. Proceeding as in Section B.

$$\alpha_j = [(k-1)/(k+1)]\ \mathrm{Max}\ \{0, u_{zj}(\pi_{zj}) + r_j - r_z\} \qquad (8)$$

Since, under π_z, the obstruction is always on the front edge of the outbound band, we have as the final phasing, say π_z^0,

$$\pi_{zj}^0 = \begin{cases} \pi_{zj} \text{ if } t_F \leqslant 1 - u_{zj}(\pi_{zj}) - r_j \\ \pi_{zj} - \alpha_j \text{ otherwise} \end{cases} \qquad (9)$$

Notice that as the outbound flow $F_{1n} \to \infty$, $k \to \infty$ and the right hand sides of most reds will be lined up with the front edge of the outbound band. There may be a few non-obstructing lights which are not affected.

Next suppose traffic is heavier in the inbound direction so that $k < 1$. To maintain symmetry of procedure the reference light will be switched from L_z to L_y where $y = y(z)$ is the index of a light which constrains the rear edge of the inbound band. The phasing of all lights relative to L_y is

$$\pi_y = [\pi_{y1}, \ldots, \pi_{yn}] \text{ where } \pi_{yi} = \left| \pi_{zy} - \pi_{zi} \right|$$

Project the edges of red for each light diagonally along the inbound direction to the time interval [0,1] at L_y. Let $v_{ij}(\pi)$ be the position in [0,1) of the left side of the projection of L_j's red under the phasing π. Let the zero of the time scale be the left side of L_y's red. Then

$$v_{yj}(\pi) = ([r_{y/2}] + \pi + X_{yj} - [r_{j/2}])^* \qquad (10)$$

The obstruction of L_j to the inbound band over that of L_y alone is Max $\left\{0, v_{yj}(\pi_{yj}) + r_j - r_y\right\}$. The adjustment to favor the inbound flow will be chosen to be

$$\alpha_j = [(k-1)/(k+1)] \text{ Max } \left\{0, v_{yj}(\pi_{yj}) + r_j - r_y\right\} \tag{11}$$

and the final phasing

$$\pi_{yj}^0 = \begin{cases} \pi_{yj} & \text{if } t_F \leq 1 - v_{yj}(\pi_{yj}) - r_j \\ \pi_{yj} - \alpha_j & \text{otherwise} \end{cases} \tag{12}$$

To summarize the method: For each i and j calculate x_{ij} and the u_{ij} from (2). Next determine b_i from (3) and b from (1), recording the vectors π_i for each i which makes b_i = b and also y(i) = the index of a light constraining the other side of the outbound band from i. For these i calculate (6) using (4) and (5). Let z be the i which maximizes (6). If $k \geq 1$, compute α_j from (8) and the final phasing from (9). If $k < 1$, the reference light is switched to y = y(z) and (10), (11) and (12) are used.

D. Discussion

A simple generalization can be made by letting the speed differ between lights. Let $V_{k,k+1}$ be the speed between two adjacent lights. Then

$$x_{ij} = \left(\sum_{k=i}^{j-1} (y_{k+1} - y_k)/V_{k,k+1}C\right)^*$$

No mention has been made of the fact that on most main roads vehicles are spread across a number of lanes. However, we can define an $h_{1,n}$ such that $h_{1,n} F_{1,n}$ is the time length of platoon going from L_1 to L_n and

similarly choose an $h_{n,l}$. Substitution of these for h above adapts the method to multilane roads.

The method has been programmed for a computer and, with the co-operation of the City of Cleveland Traffic Engineering Department, has been applied to a section of Euclid Avenue in Cleveland. The settings have been judged qualitatively to be working very satisfactorily. The road section contained ten signals and the computation time to produce a synchronization was about three seconds on a Burroughs 220. Three representative synchronizations are shown in Figure 7.

(a) TIMING FOR $F_{1,10} = F_{10,1}$

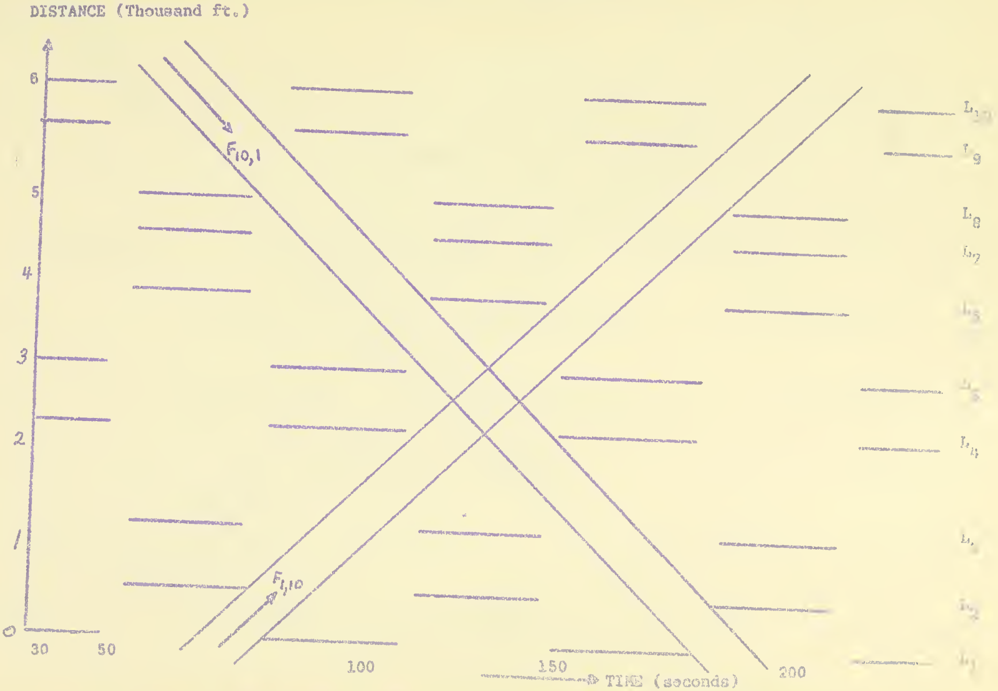
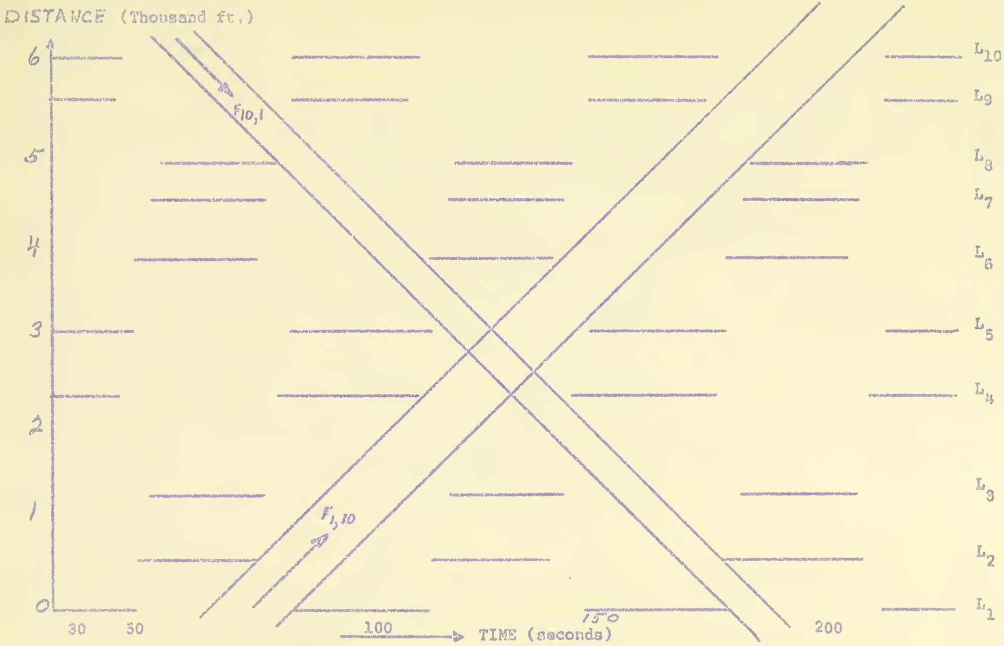
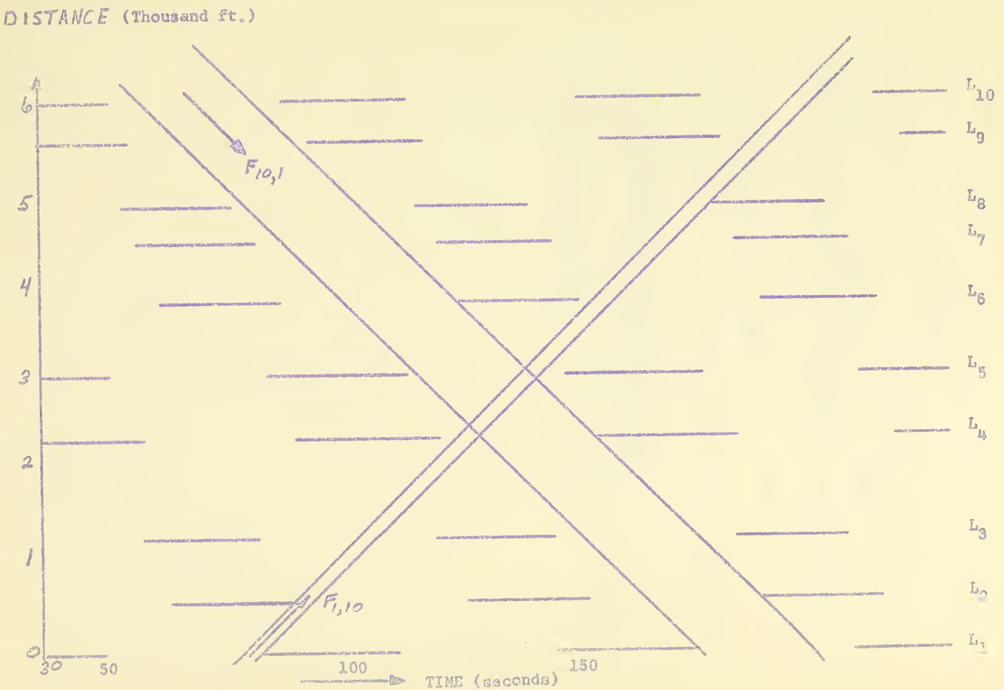


Figure 7. Space-time diagram for ten traffic lights on Euclid Avenue in Cleveland.

(b) TIMING FOR $F_{1,10} = 1.5 F_{10,1}$



(c) TIMING FOR $F_{1,10} = 1/3\ F_{10,1}$



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