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NONTRADING, MARKET-MAKING, AND ESTIMATES OF STOCK PRICE VOLATILITY

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Working Paper #1549-83 Revised: July 1985

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Interest Rate Arbitrage

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ABSTRACT

We examine the effects of market making and intermittent trading on estimates of stock price volatility. When observed price changes are correctly tied to a stock's true price dynamics, it is found that nontrading *per se* causes a loss of efficiency but no bias in traditional volatility estimates. Nontrading induces substantial inefficiency in the extreme value estimator of volatility which it biases downward. Market making's effects add to the nontrading-induced inefficiency in the traditional estimator, while information trading imparts a downward bias, and liquidity trading a potentially removable upward bias, in that estimator.

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1. Introduction

Estimates of the volatility of stock prices or stock returns play an important role in a number of areas of finance. These include the pricing of financial claims whose payoffs are contingent on stock prices [e.g., Black and Scholes (1973)], event studies, and variance-bounds tests of stock market rationality [e.g., LeRoy and Porter (1981) and Shiller (1981)]. In the most commonly used geometric Brownian motion model of stock price dynamics, logarithmic price changes have a constant volatility, and the properties of both traditional and extreme value estimates of that volatility are well-known [e.g. Thorpe (1976), Parkinson (1980), Garman and Klass (1980)]. In this paper, we discuss how these volatility estimates are affected by nontrading and market making "noise" in observed stock prices. It is shown that, in general, the efficiency of the estimates is considerably reduced and thus, for example, significance levels for tests of hypotheses in event studies can be mis-stated even when those levels appear to have been properly computed from observed prices.[1]

We investigate the extent of inefficiency and bias in volatility estimates induced by nontrading and market making. We show that, contrary to Gottlieb and Kalay's (1983) conclusion, traditional volatility estimates are not biased by nontrading *per se*. More importantly, however, nontrading *does* cause a loss of efficiency in the traditional volatility estimator which we show can be severe for lower price stocks.

Market making has two effects on point estimates of volatility computed from observed prices. On the one hand, liquidity trading results in a tendency for observed prices to bounce between a market maker's bid and ask prices, and it is well known that this induces a potentially-removable upward bias in traditional volatility estimates [e.g. Working (1954), Roll (1984)].

Unfortunately, as we show, removal of the bias will often decrease, not increase, estimation efficiency. On the other hand, information-motivated trades with market makers tend to generate prices which will lead to a downward bias in traditional volatility estimates. In addition to causing these biases, we show that the impact of information trading and liquidity trading on observed prices exacerbates the loss of efficiency in traditional volatility estimators which is attributable to nontrading per se.

The efficiency of the extreme-value estimator of volatility, which Parkinson (1980) and Garman and Klass (1980) showed is substantially greater than that of the traditional estimator under "ideal" conditions, is more adversely affected by nontrading than is the traditional estimator. In addition, unlike the traditional estimator, it is downward biased by nontrading. As an example of the severity of nontrading's effects, our computations suggest that, in the best possible case, a stock must have a price of at least $30.00 and a volatility of percentage price changes of 50% or more per annum before nontrading's effect on the efficiency of the extreme value estimator of volatility becomes insignificant enough for it to regain parity with the traditional estimator.

An interesting feature of our analysis is the way in which nontrading-induced noise is assumed to affect observed prices. Many models of nontrading assume that the stochastic process directing trades is independent of stock price dynamics [e.g. Clark (1973), Praetz (1972), Blattberg and Gonedes (1974), and Epps and Epps (1974)]. In our paper, the stochastic process directing trades is tied down to the true stock price dynamics. This seems more appropriate if, in the presence of transaction costs, trading occurs only when stock prices move beyond predetermined stopping boundaries [e.g. Miller and Orr (1967), Magill and Constantinides (1976)].

In the following section, we explain how the effects of nontrading result in a tied-down distribution of observed prices. We then determine the extent of nontrading-induced inefficiency in traditional estimates of volatility which are computed from observed prices drawn from this distribution. In Section 3, we investigate the extent of nontrading-induced inefficiency in extreme value estimates of volatility. In Section 4, we briefly describe how market making activity affects the traditional volatility estimator.

2. The Effect of Nontrading on the Traditional Estimator of Volatility

In this section, we investigate how the properties of traditional estimates of volatility, which are computed from sequences of changes in a stock's closing prices, are affected by nontrading. We assume that the minimum price movement required for a trade to occur in a stock is an eighth of a dollar, as is the case for exchange traded stocks. Our analysis could be applied to more general specifications of this minimum required price movement which are consistent with the reduced form assumption(s) made about equilibrium stock price dynamics, e.g., specifications which would arise in rational expectations equilibria which incorporate transactions models like those of Miller and Orr (1967) and Magill and Constantinides (1976).[2]

While the eighths restriction is intended only as an example, there is no doubt that this restriction could *per se* induce considerable discontinuity in daily stock price movements, particularly for low priced stocks.[3] For example, a stock price change from (say) $5.00 to $5-1/8 in one day corresponds to roughly a 625% annualized return. Alternatively, in terms of the implied nontrading, simulations using our specification of the eighths-induced nontrading which is discussed below indicate that a stock with

an initial price of $5.00, an expected price change of 12% per year, and a standard deviation of 30% per year, does not trade for at least a day approximately 20% of the time. Such rough computations, then, might suggest to some that there is potential for considerable bias in volatility estimates.[4] Gottlieb and Kalay (1985) have recently added support to this conjecture by reporting that volatility estimates can be biased upward by some 898% for a stock with a price of $1.00.

To see the effect of the minimum-price-move regulation on observed prices, assume as in Figure 1 that the true stock price $S(t_0)$ at time t_0 is $2.50, and that a random draw of the true price from a lognormal (geometric Brownian motion) distribution at $t_0 + \delta$, defined to be $S(t_0 + \delta)$, equals $3.07. If we assume that stock prices must move by an eighth to trigger transactions, then the stock will not trade at $t_0 + \delta$ because the true price is not equal to an eighth multiple. Assuming that the stock trades at every eighth, the price <u>observed</u> at $t_0 + \delta$ will be either $3.00 or $3-1/8, depending upon which of those two prices the last trade occurred at. The probability distribution of the price of the last trade is the probability distribution of the last passage of the price across one of the two "barriers" $3.00 or $3-1/8, conditional upon the true price being $2.50 at the beginning of the interval, t_0, and $3.07 at the end of the interval, $t_0 + \delta$. If the continuous sample path of prices over the interval δ happened to be the one labelled A in Figure 1, the stock price observed at $t_0 + \delta$ would be $3.00. If, on the other hand, the stock price had followed path B, the price observed at $t_0 + \delta$ would be $3-1/8.[5]

To obtain a distribution for the price of the last trade over a given interval, we have made a <u>time reversal</u> in the so-called "tied down Brownian

motion" or "Brownian bridge" process in Figure 1 (not in the unconditional Brownian motion process), so that the problem of finding a last exit time distribution is transformed to one of finding a first passage time distribution in "reverse time." Conditional upon $S(t_0)$ and the end-of-interval true price $S(t_0 + \delta)$, this first passage probability distribution for the end-of-interval observed price is obtained by a relatively straightforward adaptation of Anderson (1960, Theorem 4.2). With the frequency of trading specified in terms of a first passage time distribution, the determination of observed prices follows completely from the stochastic process assumed to describe true stock prices.

Imagine now a sequence of intervals, each of length δ, like the one depicted in Figure 1. The traditional estimator of stock price volatility is the average squared change between the closing price generated by the last transaction in any interval and the closing price in the next consecutive interval, centered about the mean change if it is non-zero. If a closing price for any given interval is "stale," in the sense that the stock did not trade for a long time prior to the end of the interval, then on average the change from that stale observed closing price to the the next interval's observed closing price will be larger. Thus, if the sequence of a stock's closing prices is used to compute volatility, as in the case of the traditional estimator, it is intuitive that the volatility rate will be measured without asymptotic bias, even if there is nontrading. We now state this slightly more formally.

Denote the sequence of consecutive discrete intervals, each like the one in Figure 1, as $t_1 = t_0 + \delta$, $t_2 = t_0 + 2\delta, \ldots, t_T = t_0 + T\delta$, and the corresponding random length of nontrading at the end of each of the intervals as $h_1, h_2, \ldots, h_T$. Then it can easily be shown [e.g.,

Scholes and Williams (1977, Appendix)] that the stock price variance computed from the observed price sequence $S(t_1)$, $S(t_2)$,...,$S(t_T)$, which we denote σ_S^2 [the subscript S stands for successive observations], will be:

$$\sigma_S^2 = \sigma^2 + \text{var}(\tilde{h}_j - \tilde{h}_{j-1})\mu^2$$

where $\mu = \alpha - 1/2\,\sigma^2$ is the stock's logarithmic mean rate of return. If the logarithmic mean return μ [not the drift parameter α], is zero,[6] then there will be no eighths-induced bias in the estimate $\hat{\sigma}_S$ of σ over a "long run" of observations.

This contention that the traditional estimator of volatility is not biased by nontrading differs from that of Gottlieb and Kalay (1985). The reason is that they infer the effects of nontrading on traditional volatility estimates from a distribution of observed closing prices which is conditioned on a given beginning-of-day (or equivalently, end-of-previous-day) price. In terms of Figure 1, their distribution of closing prices would consist of repeated draws of $S(t_0 + \delta)$ for a given $S(t_0)$.

If the traditional volatility estimate is computed using Gottlieb and Kalay's approach with observed end-of-day prices generated by the tied-down Brownian motion, it will appear to be biased downward. The intuitive explanation for the apparent downward bias is straightforward. While the choppiness in price changes, and hence price changes squared, averages out over repeated draws [ignoring drift for simplicity], trading will, on average, span less than a day. Since the variance of a geometric Brownian motion process is proportional to the length of the time interval over which it is measured, the variance measure based on an average of less than a day's trading will be biased downward.

Because the traditional estimate of volatility is not computed in practice by making repeated draws from a single interval's probability distribution, we claim that it is simply incorrect to perform such an "experiment" to infer a nontrading-induced bias in the traditional estimator. As we discuss in Section 3, the reasoning underlying this incorrect experiment does make sense for the extreme value estimator of volatility.

Although the correct analysis leads to the conclusion that nontrading does not bias the traditional estimator of volatility, it does reduce its efficiency. This is particularly unfortunate for those uses, such as the pricing of contingent claims, in which efficiency rather than unbiasedness per se, is important [e.g. Boyle and Ananthanarayanan (1977)]. To determine the extent of the efficiency loss, we made five hundred draws of a sequence of two hundred and fifty daily observed price changes from the tied-down geometric Brownian motion described above, with a zero logarithmic mean and each of four volatility rates, 10%, 20%, 30%, and 40% per annum. The mean-squared-error[7] of the traditional estimator $\tilde{\sigma}_S$ over the five hundred replications was then expressed as a ratio of the mean-squared-error of the traditional estimator $\tilde{\sigma}$ with no nontrading. The ratios are reported in Table 1. The inefficiency in the traditional volatility estimate is substantial for $1.00 and $2.00 stocks with volatilities below 20% or 30% per year. The most substantial inefficiency occurs for a $1.00 stock with a volatility of 10% per year, where the mean-squared-error of $\hat{\sigma}_S$ is about 154 times what it would be if there were no nontrading. However, the nontrading-induced inefficiency tails off for higher priced stocks, becoming negligible for stocks with prices of $10.00 or more, so long as it is assumed that stocks trade every time (a potentially infinite number of times) that

their price hits the boundary for minimum price moves--in this case, an eighth of a dollar.

3. The Effect of Nontrading on Extreme-Value Estimates of Volatility

Extreme value estimates of volatility are based on the range between maximum and minimum prices realized over each interval, as opposed to the changes in successive closing prices. Intuitively, intermittent trading will tend to result in a lower observed maximum and a higher observed minimum price in any given interval. Since the range observed in each interval is, in effect, independent of the nontrading-induced error in the range observed in the previous (or next) interval, the observations for the range can be thought of as independent draws from a single interval. The result is that estimators based on this range will tend to be biased downward.

The effect of nontrading on the variance and bias of the extreme-value estimator is reported in Table 2. The first of the two numbers in each cell of that table is the mean-squared-error of the extreme value estimator in the presence of nontrading, expressed as a ratio of what it would be if there were no nontrading. The second number in each cell, in brackets, is (the absolute value of) the bias in the extreme value estimator, expressed as a ratio of the square root of the mean-squared-error of the extreme value estimator in the absence of nontrading.

It can be seen that, even for stocks with quite high prices and volatilities, the mean-squared-error of the extreme value estimator increases sharply when there is nontrading, in part because of the nontrivial bias which results from the nontrading. The reason is that observed prices follow the tied-down Brownian motion process described above when there is nontrading,

not the geometric Brownian motion process, and extreme-value estimators are very sensitive to this type of change in the probability distribution of prices. Given that the mean-squared-error of the extreme-value estimator is theoretically less than that of the traditional estimator by a factor of five when there are no errors in price observations, it can be seen from Table 2 that a stock must have a price of at least $30.00 and a volatility of about 50% before the efficiency of the extreme value estimator "catches up" with the efficiency of the traditional estimator when there is nontrading of the type discussed here.

4. Market Makers

The preceding analysis is now briefly extended to incorporate the effects of market making. A structural specification for liquidity-motivated and information-motivated trades with the market maker is, _inter alia_, already implicit in the reduced form behavior which is assumed to describe true bid and ask prices. We assume that, in this structural specification, trades with the market maker tend to be triggered by price movements. As before, we will examine the polar case in which eighth movements in price are sufficient to induce transactions.

To the extent that intermittent trades with the market maker are liquidity motivated, the prices they generate are true prices at the time the trades occur. If these transactions could be labelled as market maker sales or purchases, the volatility of ask prices could be estimated from the prices generated by the sales, and/or the volatility of bid prices from the purchases. For the reasons discussed in Section 2, these estimates would be inefficient but unbiased.

However, transactions with the market maker are not labelled as sales or purchases in price records. The result is that "the" observed price tends to jiggle between the market maker bid and ask over time. Working (1954), Niederhoffer and Osborne (1966), West and Tinic (1971), and Roll (1984) have all pointed out that the jiggling in observed prices causes an upward bias in volatilities--here, the volatility of bid or ask prices. However, the jiggling also induces a negative serial dependence in price changes, and it is easily shown that, for a constant one-eighth bid-ask spread, the upward bias in the volatility estimate can be eliminated by adding twice the (negative) autocovariance of the price changes to the estimate [e.g. Roll (1984)].

The effects of market maker transactions with information traders are more insidious. The market maker's ask (bid) will be hit by traders who have inside information that the true price is higher (lower). It is easy to see that, in absolute value, the true price change will be above the one registered in the trade with the market maker, and thus that the volatility of observed price changes will understate the volatility of true prices to the extent that the former are generated by market maker transactions with informed traders. There is no obvious way to adjust for the downward bias.

To investigate the extent of the upward bias induced by liquidity trades, we assumed, as before, that the minimum price move required to trigger a transaction is one-eighth. We assumed a constant bid-ask spread of an eighth, and that once a trade was triggered, there was a fifty-fifty chance that it occurred at the bid or the ask. To incorporate information trading, we assumed that ten percent of the time, information traders learn of eighth movements in prices before the liquidity traders or market makers. As soon as the information trades occur, market makers revise their bid and ask quotes fully.[8]

The results are presented in Table 3. Each cell in that table contains three numbers in the following order: (i) the mean-squared-error of the traditional volatility estimator measured from observed prices, expressed as a ratio of the mean-squared-error of the traditional estimator of volatility of the geometric Brownian motion process to which the observed prices are subordinate; (ii) the mean squared-squared-error of the bias-corrected traditional volatility estimator measured from observed prices, again expressed as a ratio of the mean-squared-error of the traditional estimator for the volatility of the geometric Brownian motion to which observed prices are subordinate; and (iii) the uncorrected point estimate of the volatility of observed price changes, as a ratio of the volatility of the geometric Brownian motion to which observed prices are subordinate.

As can be seen from Table 3, the point estimates of volatility computed from observed prices are close to unbiased for stocks with prices above $20.00. The inefficiency in the traditional volatility estimator persists for reasonably high price stocks. For example, for a $10.00 stock with a volatility of 50% per annum, the contamination in observed prices still causes the mean-squared-error of the volatility estimator to be almost three times what it would be in the absence of nontrading and market making.

It can be seen from Table 3 that the market making-induced bias in volatility estimates is always upward. The upward bias implies that the effect of liquidity trading on observed prices dominates that of of the information trading allowed here. As long as traders never have access to information about true price changes bigger than an eighth, variations in the frequency of their arrival relative to liquidity trades have little effect. For example, for the $20.00 stock with a return volatility of 20% per annum, reduction of the arrival rate from the ten percent assumed in Table 3 to one

percent increases the bias (i.e. reduces the offset to the positive bias induced by liquidity trading) from 1.06 to 1.07.

The mean-squared-error of the bias-corrected volatility estimator is below that of the uncorrected estimator for stocks with prices below $8.00 and for higher priced stocks with low volatilities. However, for high priced stocks with high volatilities, the bias-corrected volatility estimator is less efficient than the uncorrected estimator. Moreover, the behavior of the bias-corrected estimator's efficiency for high priced stocks is not monotonic in their volatility rates. For example, as the volatility increases from 10% to 20% per annum on a $20.00 stock, the corrected estimator's mean-squared-error first decreases from 4.74 to 2.91 times what it would be in the absence of nontrading and market making, but then increases again to 3.48 if volatility is increased further to 30%. What happens, of course, is that for the initial increase in volatility, the autocovariance correction is better estimated, but for higher volatilities for which the bias itself becomes less important, the loss in having to estimate two parameters rather than one is relatively more important.

The results in Table 3 concerning the efficiency of the bias-corrected volatility estimator do not bode well for attempts to adjust daily stock price changes for bid-ask bounce, to estimate effective bid-ask spreads or adjust for differences between quoted and effective bid-ask spreads, etc.[9] Better estimates of the correction factor(s) can be obtained from transaction-to-transaction prices, though the need for the correction is itself commensurately higher when this data is used to estimate volatilities. Data on daily trading volume and cross-sectional stock prices may also be helpful.

5. Summary and Discussion

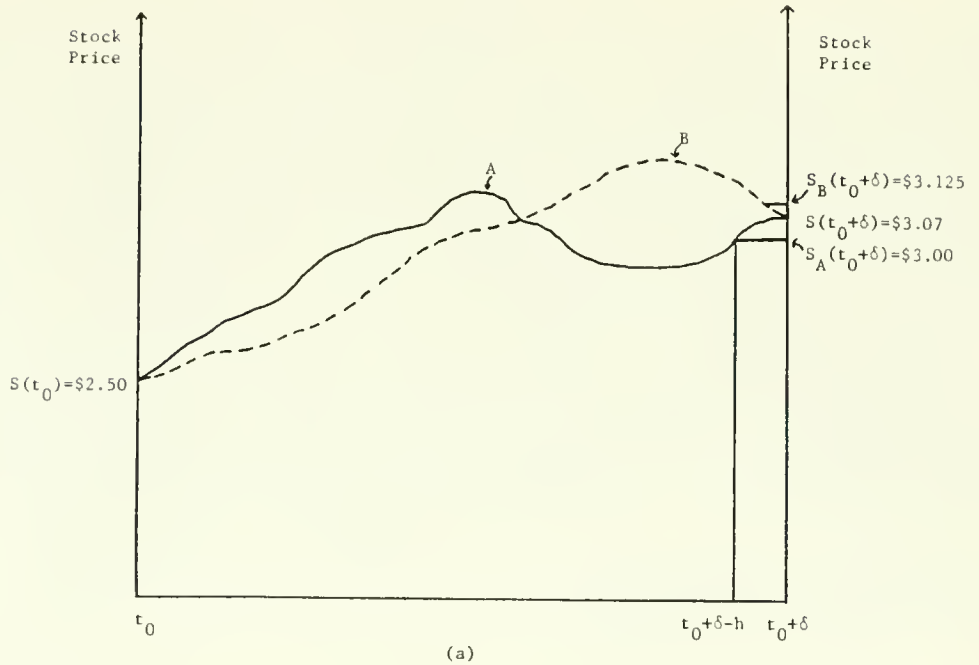
We have discussed three potential sources of distortion in estimates of stock price volatility, and illustrated their minimum effects. The first potential problem discussed concerns intermittent transactions in a stock. The concomitant nontrading can result in "stale" end-of-day prices, the distribution of which we showed to be "tied down" to the stochastic process for true prices. In our analysis, nontrading is assumed to depend upon the magnitude of price movements, as it does in transactions models like those of Miller and Orr (1967) and Magill and Constantinides (1976). We showed that such nontrading, which is obviously not independent of a stock's price dynamics, does not bias the traditional estimator of volatility, but reduces its efficiency, and that it both biases and reduces the efficiency of the extreme value estimator. Second, we argued that if price observations are generated by market maker trades with information-motivated investors, then the traditional estimator of volatility will be biased downward as well. Third, prices generated by market maker trades with liquidity-motivated individuals tend to impart an upward bias to volatility estimates, but that element of bias can be eliminated.

Our assessment of the effects of nontrading and market making on volatility estimates are specific to the geometric Brownian motion model which we employed, but our methodology holds generally for martingale models of prices in which variance rates per unit time increase as the length of the observation interval increases. This robustness is important since our approach consists of deducing as many implications concerning volatility estimates as possible from the interaction between the salient features of market microstructure and stock price dynamics, rather than simply injecting "noise" to account for these features.

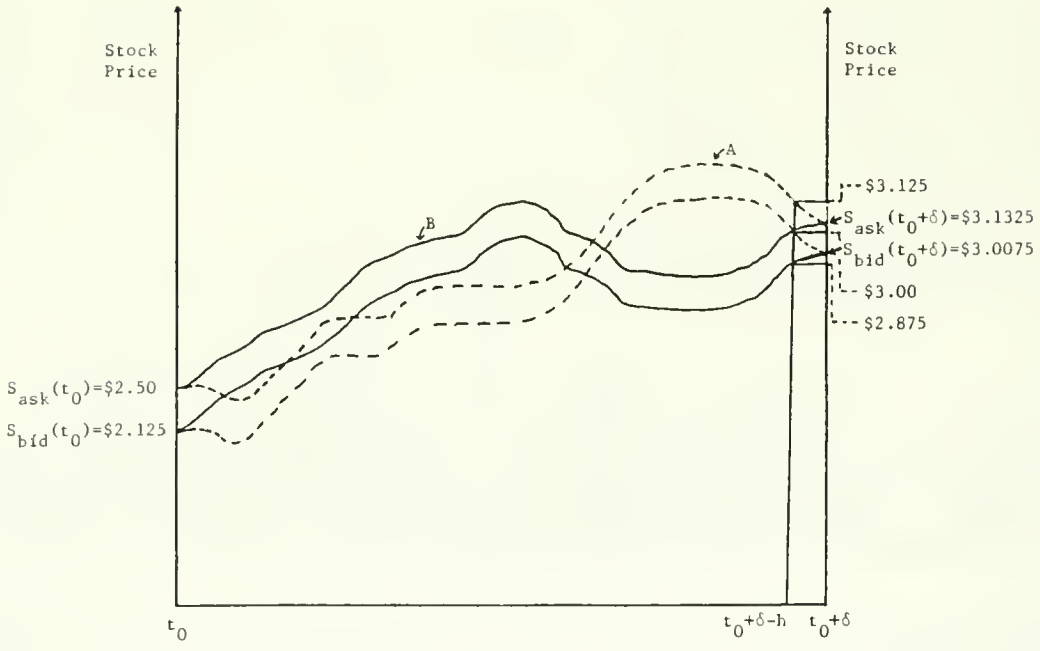
We have measured the effects of nontrading and market making on estimates of daily volatilities. The problems we have discussed become more severe as the observation interval shrinks. At the transactions level, for example, eighths-induced discontinuities and bid-ask bounce in prices are particularly severe.

Figure 1

Plot of a hypothetical realized path of prices showing how the distribution of stock prices observed at the end of an interval can be determined endogenously from the stock price dynamics to which observed prices are subordinate. For illustrative purposes, it is assumed that the stock trades each and every time its price passes through an eighth multiple.



(a)



(b)

In Fig. 1(a), a stock's observed price at time $t_0+\delta$ is $S_A(t+\delta)=\$3.00$ if there is no market maker and the geometric Brownian motion for its price followed path A over the interval $(t_0, t_0+\delta)$, conditional on a beginning-of-interval price of $S(t_0)=\$2.50$. If the geometric Brownian motion had followed path B, the observed price would instead be $S_B(t_0+\delta)=\$3.125$.

In Fig. 1(b), a market maker is assumed to be present, and price paths A and B each consist of bid and ask prices with an assumed constant one-eighth spread. If the bid and ask prices followed path A, the observed closing price would be \$3.125 if the last transaction at time $t_0+\delta-h$ was a market maker sale, or \$3.00 if it was a market maker purchase. If the bid-ask price pair had followed path B, the observed closing price would instead be \$3.00 for a market maker sale and \$2.875 for a market maker purchase.

FOOTNOTES

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1 The effects of price data contamination caused by nontrading and various features of market microstructure will be magnified in typical event study methodologies in which measures of pre-event daily return volatilities are used to estimate standard errors in order to test the significance of cumulative average returns over a multi-day period following the event.

2 However, the parameters of some expressions for this minimum price movement would then have to be estimated simultaneously with the volatility parameter.

3 We investigate the efficiency of, and bias in, volatility estimates induced by nontrading. Obviously other problems may be caused by intermittence in observed prices. For example, the discontinuous sample path for observed prices could lead to an incorrect conclusion that transaction-to-transaction prices are generated by jump processes [e.g., Oldfield, Rogalski, and Jarrow (1977)] when the "true" process has a continuous sample path.

4 Note that the estimation issues discussed here will not be avoided by computing stock price volatilities implied by option prices as long as these option prices and/or the prices of the underlying stocks are generated by (nonsynchronous) intermittent trades.

5 The analysis here can be followed mutatis mutandis if there is a constant bid-ask spread, as illustrated in Fig. 1(b), where the spread is one-eighth. By analogy with Fig. 1(a), the stock's true ask price at the end of interval δ is assumed to be \$3.1325, so its true bid price is one-eighth lower at \$3.0075. If, as before, the stock trades (at most) every time its true bid or ask price passes through an eighth multiple of

a dollar, then the two possible (parallel) paths of bid and ask prices depicted in Fig. 1(b) could lead to the transaction which generates the price observed at the end of interval δ. If the realized path of bid and ask prices is that labelled A, the last trade will occur at the market maker's ask of \$3.25 and/or the market maker's bid of \$3.125; in the path labelled B, the last trade will occur at an ask price of \$3.125 or a bid of \$3.00. As before, the interval of nontrading will be the time between the last passage of the bid and ask prices through the eighth multiples and the end of interval δ. If the analysis in the text is followed literally, then there will always be a simultaneous sale to, or purchase from, the market maker at his or her bid and ask prices at the instant of their last passage through the eighth multiple. Provided that both simultaneous prices are reported, no problem would arise in distinguishing bid and ask prices. So long as the bid-ask spread is constant, volatility could be estimated from either the sequence of bid prices or the sequence of ask prices, and both sequences would follow a "tied down" stochastic process such as that described in the text.

6 Given the typical daily volatilies of stock price changes relative to their mean, this assumption is of trivial consequence.

7 Throughout this paper, we assume that the mean-squared-errors of volatility estimates computed from the simulations are "true" values, i.e., not sample estimates of the mean-squared-error of sample estimates of volatility. We verified that the five hundred replications did indeed yield a mean-squared-error for the noise-free geometric Brownian motion which was equal to its analytic asymptotic value.

8 If information traders are uncertain about the extent of their monopoly access to information, they will trade immediately on the information, and thereby reveal it to the market maker--Gammill (1985) shows that the market maker has an incentive to structure the market so this occurs. The market maker can only make an ex post adjustment to take the information into account, and thus expects to lose to the information traders: in essence, the liquidity traders pay the market maker, who in turn pays the information traders to collect the information.

9 We have assumed a constant bid-ask spread in our analysis. If the bid-ask spread is itself being estimated, it is hard to see how a model for the equilibrium behavior of that spread over time can be avoided. Crucial ingredients would involve specifications for information and liquidity trading, and the technology for placing limit orders.

TABLE 1

Efficiency[1] of traditional estimates of volatility measured from observed prices subordinated to a geometric Brownian motion process through eighths-induced nontrading, relative to what it would be if there were no nontrading.[2]

	Annualized True Volatility			
Stock Price	0.1	0.2	0.3	0.5
1	154.75	37.58	15.60	6.45
2	52.27	7.13	3.61	1.72
4	6.66	1.93	1.28	1.23
8	1.79	1.21	1.04	1.02
10	1.53	1.12	1.02	1.01
20	1.04	1.00	1.00	1.00
30	1.00	1.00	1.00	1.00

[1]The measure of relative efficiency is defined, as usual, as the ratio of mean-squared-errors of the estimators.

[2]For each level of volatility and initial stock price, ratios are computed from 500 replications of a 250-day sequence of true stock price changes drawn from a geometric Brownian motion process, and the implied 250 day sequence of observed stock price changes drawn from the tied down Brownian motion process.

TABLE 2

Efficiency[1] and bias[2] of extreme-value estimates of volatility measured from observed prices subordinated to a geometric Brownian motion through eighths-induced nontrading, relative to what they would be if there were no nontrading.[3]

	Annualized True Volatility			
Stock Price	0.10	0.20	0.30	0.50
1	611.57[4] [14.84]	480.49 [12.55]	413.30 [12.20]	355.69 [11.24]
2	523.04 [13.72]	437.65 [12.54]	353.82 [11.26]	270.60 [9.76]
4	374.42 [11.61]	354.95 [11.27]	211.99 [8.69]	103.63 [5.99]
8	306.60 [10.48]	156.25 [7.47]	88.17 [5.59]	40.07 [3.70]
10	241.49 [9.24]	116.42 [6.44]	64.64 [4.78]	28.94 [3.14]
20	99.80 [5.98]	43.03 [3.91]	22.56 [2.81]	10.56 [1.90]
30	57.30 [4.53]	23.04 [2.86]	12.32 [2.08]	6.20 [1.45]

[1]The measure of relative efficiency is defined, as usual, as the ratio of mean-squared-errors of the estimators.

[2]The (absolute value of) the bias of the extreme-value estimator of volatility with nontrading is expressed as the ratio of the square-root of the mean-squared error of this estimator of volatility with no nontrading.

[3]The numbers in the table are computed from 500 replications of a 250-day sequence of observed prices. The maximum and minimum prices observed on each of the 250 days are obtained by partitioning each of the 250 days into 200 "approximately infinitesimal" subintervals.

[4]The first figure in each cell is the relative efficiency. The second figure, in brackets, is the bias relative to (the square root of) the mean-squared-error.

TABLE 3

Properties of volatility estimates when observed prices are affected by both nontrading and market making.[1] The three numbers[2] in each cell of the table are, in order: (a) the efficiency of traditional estimates of volatility computed from observed prices relative to what it would be if there were no nontrading and no market maker;[3] (b) the efficiency of the traditional estimator of volatility, corrected for bias[4] induced by liquidity trading, relative to what it would be if there were no nontrading and no market maker; and (c) the bias in the traditional estimator of volatility resulting from market maker transactions with information motivated and liquidity motivated traders.

	Annualized True Volatility			
Stock Price	0.10	0.20	0.30	0.50
	6769.89 [5/]	3959.14	2056.46	1103.86
1	2658.02	879.65	511.87	302.87
	5.91	3.73	2.99	2.34
	2936.12	1167.47	490.25	240.50
2	609.55	302.34	123.27	40.64
	3.63	2.45	2.02	1.66
	956.44	304.57	135.83	38.06
4	231.99	56.07	16.41	5.93
	2.38	1.77	1.50	1.26
	295.42	66.42	16.64	5.16
8	48.98	6.63	3.93	3.60
	1.73	1.32	1.16	1.08
	168.39	27.83	8.18	2.91
10	18.38	4.41	4.09	3.48
	1.58	1.22	1.11	1.00
	28.90	2.70	1.58	1.16
20	4.74	2.91	3.48	3.25
	1.22	1.06	1.03	1.00
	6.06	1.36	1.17	1.03
30	3.28	2.89	3.08	3.09
	1.09	1.02	1.01	1.00

TABLE 3 (Continued)

[1]The results in the table assume that information-motivated traders gain inside knowledge of eighth movements in price ten percent of the time. In that (random) ten percent of cases, information traders hit the market maker's last bid if prices have fallen, and the market maker's last ask if prices have risen. The information trade is assumed to fully reveal the inside information. In the ninety percent of trades which are liquidity-motivated, it is assumed that sales to, or purchases from, the market maker are equally likely.

[2]For each level of volatility and initial stock price, ratios are computed from 500 replications of a 250-day sequence of true stock price changes drawn from a geometric Brownian motion process, and the implied 250 day sequence of observed stock price changes drawn from the tied down Brownian motion process.

[3]The measure of relative efficiency is defined, as usual, as the ratio of mean-squared-errors of the estimators.

[4]The upward bias in the traditional estimator of the volatility of daily stock returns, induced by market maker transactions with liquidity-motivated traders, is corrected here by adding twice the autocovariance of those stock returns to that estimator.

[5]The first number in each cell is the mean-squared-error of the traditional estimator of volatility, expressed as a ratio of the mean-squared-error of the volatility in the absence of nontrading and market making; the second number is the mean-squared-error of the traditional estimator "corrected" for upward bias, also as a ratio of what it would be in the absence of nontrading and market making; and the third number is the average volatility estimate computed from observed prices, as a ratio of the volatility of prices in the absence of nontrading and market making.

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