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AUG 6 1963
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A NOTE ON AN APPROXIMATION TO
THE POST-TAX RATE-OF-RETURN*

26-63

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*The author gratefully acknowledges the financial support provided by the Sloan Research Fund and by the Ford Foundation Grant to the School of Industrial Management, M.I.T., for Research in Business and Finance.

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June, 1963

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The main purpose of the following remarks is to indicate the conditions under which it is reasonably valid to approximate the rate-of-return on post-tax cash flows by a simple transformation of the rate-of-return on pre-tax cash flows. Before turning to that problem, however, a few remarks aimed at indicating the general nature of the problem are offered.

The increasing volume of research in the area of business management has revealed considerable discrepancies between business practice and theory. For example, managerial economists have found a plethora of rules, conventions, and procedures which do not fit neatly into established theoretical classifications. Specifically practices such as cost-plus pricing, break-even analysis, and investment decision rules which employ payback periods do not fit well into the normative theories of economists and indeed, in some cases, seem contrary to the dictates of theoretical rules.

One result of the discrepancy between practice and theory has taken the form of an attempted reformulation of theory. Closer analysis of business rules-of-thumb and conventions frequently indicate that they are in fact surprisingly rational. This conclusion may follow if consideration is taken of organization variables and concepts such as the cost of search or

information.¹

A rule-of-thumb frequently used in capital budgeting decisions is that the rate-of-return on post-tax cash flows (k) bears a simple relationship to the rate-of-return based on pre-tax cash flows(r). Specifically,²

$$(1) \quad k = (1-T)r$$

where T is the corporate tax-rate.

While some users of equation (1) do realize that it is an approximation little effort appears to have been made to inquire into the conditions in which it is a reasonable approximation. A typical examination of this problem and reaction to the discovery that equation (1) is in fact an approximation is the following:³

1

An early general expression of this view may be found in "The Analysis of the Firm: Rationalism, Conventionalism and Behaviorism," The Journal of Business, Vol. XXXI (July, 1958), pp. 187-99. In the present context see also Vernon L. Smith's Investment and Production: A study in the Theory of the Capital-Using Enterprise (Cambridge, Massachusetts: Harvard University Press, 1961): "Certain aspects of the pay-off period rules of business practice are consistent with the requirements of rational investment theory", p. 220. For a synthesis see A Behavioral Theory of the Firm (Englewood Cliffs, N. J.: Prentice-Hall, 1963) by Richard M. Cyert and James G. March, Chapter 2.

2

It is not often clear whether or not all users of this rule appreciate the fact that it is indeed an approximation. For recent evidence of the use of pre-tax flows in capital budgeting computations see Donald F. Istvan's Capital Expenditure Decisions: How they are made in large Corporations, Bureau of Business Research, Graduate School of Business, Indiana University, Report No. 33, 1961, p. 91. The same rule also crops up in bond yield calculations.

3

This is the research of the Philadelphia Society of Business Budgeting reported by Horace G. Hill Jr., "A New Method of Computing Rate of Return on Capital Expenditures," reprinted in The Management of Corporate Capital (Glencoe, Illinois: The Free Press, 1959) edited by Ezra Solomon, p. 37.

The Committee reviewed some experiments which were made in order to determine whether the inclusion or exclusion of income tax, as a charge and a subsequent payment in the basic data, would change the relative attractiveness of a random assortment of capital investments. In the great majority of projects it made no appreciable difference whether they were evaluated before tax and the computed rate of return halved before using, or computed after including the tax as one of the basic annual expenses, but the safest course would be to treat income tax as just another expense from the beginning.

That equation (1) is not exact can be seen by writing the equations which define r and k . If an asset costing C dollars produces a pre-tax and pre-depreciation cash flow of $R(t)$ dollars at time t then the rate-of-return (r) for this asset, assuming zero salvage value and employing continuous discounting, is defined by:

$$(2) \quad C = \int_0^n R(t)e^{-rt} dt,$$

where n is the life of the asset. The corresponding expression for the post-tax rate-of-return (k) for the same asset is given by:

$$(3) \quad C = \int_0^n (1-T)R(t)e^{-kt} dt + TC \int_0^n d(t, n)e^{-kt} dt$$

where $d(t, n)$ is the depreciation, per dollar of depreciation base, at time t for an asset whose life is n years.

By inspection it is clearly unlikely that the solutions to equations (2) and (3) give values of r and k such that $k = (1-T)r$. The obvious case, of course, is where there are no taxes paid and T equals

zero. In this case the pre- and post-tax rates are equivalent.

What would be the conditions in which the approximation given by equation (1) is valid? The following analysis does not pretend to offer a general answer to this question. A partial answer may be had, however, by examining the case for which $R(t)$ is a constant (R) and the depreciation function is such that $d(t, n) = n^{-1}$ for all values of t , that is, straight-line. Under these conditions equation (2) becomes:⁴

$$(4) \quad C = \frac{R}{r} (1 - e^{-rn})$$

If, in addition, it can be assumed that the product rn is small then expanding e^{-rn} by Taylor's series up to a quadratic term leads to:

$$(5) \quad r = \frac{2}{n^2} (n - p),$$

where p is the pre-tax payback period⁵ (C/R).

4

The assumption that $R(t)$ is a **constant** is probably stronger than is necessary. Analysis similar to the following could be made where $R(t)$ is any reasonable function. In this connection note that the present value of $R(t)$ is the definition of a Laplace transform and hence any $R(t)$ which has a transform can be easily managed. See Sakari T. Jutila, "A Note on the Evaluation of the Marginal Efficiency of Capital", Econometrica, Vol. 30, No. 2 (April, 1962), pp. 332-335.

5

Relationships connecting the payback period and the rate-of-return have been developed by Myron J. Gordon, See his "The Payoff Period and the Rate of Profit", Journal of Business, October 1955. His approach, however, rests on the assumption of n being large. The approach here is to consider rn as small.

The corresponding expression for k is:

$$(6) \quad k = \frac{2[(1-T + pTn^{-1})n - p]}{n^2(1-T + pTn^{-1})}$$

Hence the ratio of the pre- and post-tax rates-of-return is:

$$(7) \quad \frac{k}{r} = (1-T)[1 - T(1 - pn^{-1})]^{-1}$$

Clearly the approximation given by equation (1) will hold if the second term in the square brackets is close to zero, that is, if either, or both, of the factors T and $(1-pn^{-1})$ are close to zero. Thus the lower the tax-rate the better is the approximation. But even if the tax-rate is large the approximation will be good if the pre-tax payback period (p) is equal to the life of the asset (n).

An important point concerning the use of the approximation $k = (1-T)r$ in the circumstances described above is that it is a biased approximation. From economic considerations investment projects which are interesting must have $p \leq n$. Hence the term in square brackets in equation (7) is always less than unity. From this it follows that the approximation $k = (1-T)r$ systematically overstates the post-tax rate-of-return. The approximation is clearly unsatisfactory if the tax-rate is high and the ratio of the asset's pre-tax payback to its life n is small.

From the argument presented above it would seem that the rough approximation for the post-tax rate-of-return is sensible if the pre-tax cash flows are constant and the asset has a payback period which is approximately equal to its life. These conditions are probably valid for a fairly wide range of equipment replacement projects. The analysis presented above suggests that these kinds of practices can lead to decisions comparable to those resulting from the application of correct criteria if they are applied to projects which have the appropriate characteristics. It is dangerous, therefore, to appraise the quality of management's investment decisions from research data which indicate the criteria and rules-of-thumb used--such as payback periods, approximations to post-tax rates-of-return and so on--but which ignore the characteristics of the projects to which these rules are applied and the subtle cross-checking of investment profitability employed by businessmen through the use of various criteria used in tandem.⁶

⁶ In 48 firms surveyed by Istvan 34 use the payback period as a measure of acceptability. "In 13 of these firms it is the measure upon which the fate of all proposals depends. In the rest of the firms, it is used to supplement a rate-of-return calculation", (Capital Expenditure Decisions, p. 91).

