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THE OPTIMALITY OF A COMPETITIVE STOCK MARKET

Robert C. Merton *Robert C. Merton*
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ABSTRACT

Robert C. Merton and Marti G. Subrahmanyam

In their paper "Corporate Investment . . . Pareto Optimality in the Capital Markets," Jensen and Long purport to show that when investment decisions are formulated according to either the (centralized) criterion of maximizing society's wealth or the (decentralized) criterion of maximizing market value, the resulting allocation is not a pareto optimum. They view this result as somewhat paradoxical, and suggest the possibility of some form of externalities. While their analysis is quite interesting, it is only valid for a non-competitive system.

Our paper is both a comment on and an extension to the J-L analysis. We derive the pareto optimal level of investment for both the case of irreversible investment (which J-L implicitly assume) and the case of reversible investment. Under the usual competitive assumptions of free entry and exit (i.e., the technology is available to everyone and investment is reversible or contingent as in the \hat{T} atonnement process), an algorithm is derived which is used to prove that value-maximization by firms leads to the pareto optimal amount of investment. It is further shown that if just entry is free, then the equilibrium amount of investment using the value-maximization rule will be no less than the pareto optimal level. (This result is contrary to the J-L findings that investment is always less than the pareto optimal amount.) We discuss maximization of social wealth as a criterion and show that in this context, it corresponds to the pure monopoly solution which explains why investment by this criterion is always less than the pareto optimum level.

The Optimality of a Competitive Stock Market*

Robert C. Merton
Marti G. Subrahmanyam

I. Introduction

In their important papers on the optimality of the stock market allocation, Jensen and Long [] and Stiglitz [] use a mean-variance model of the capital market to demonstrate that the investment allocation by value-maximizing firms will not in general be pareto optimal. While both analyses are technically correct, the interpretation of their findings has led to a controversy over whether the sources of the non-optimality of value maximization are certain non-competitive assumptions about the capital market or are from some inherent externalities associated with uncertainty which do not disappear even in a competitive market. If the answer is the former, then their findings are consistent with certainty theory analysis where value or profit maximization by firms with some positive degree of monopoly power need not lead to pareto optimal allocations. If the answer is the latter, then, indeed, the findings are more fundamental. In his comment on the Jensen-Long and Stiglitz papers, Fama [] argues that because both analyses use non-competitive assumptions, they do not answer the question whether in the mean-variance model, a competitive equilibrium is a pareto optimum. Fama does perform the analysis based on what he calls the appropriate competitive assumptions and concludes that, in general, a competitive equilibrium will not be a pareto optimum due to inherent externalities.

* Aid from the National Science Foundation is gratefully acknowledged.

Our paper focuses on the issue of whether a competitive equilibrium in the mean-variance model is a pareto optimum or not, and its principal conclusion is that it is. The reason that our conclusion differs from the previous analyses is not due to a technical mistake, but rather to differences in interpretation of what the competitive market assumptions are: specifically, we assume that in a competitive market, entry into that market is free. All three of the aforementioned papers restrict entry.

If new firms are restricted from raising funds in the capital market on the same terms as already existing firms, then the capital market does not satisfy the pure competition assumptions. This restriction is implicitly built into a model which holds constant the number of (non-colluding) firms. To restrict entry into an industry by assuming that all technologies are not freely available to all firms (both existent and potential) is also a violation of the pure competition assumptions. This restriction can be built into a model by holding constant the number of firms, or more subtly, by assuming the correlation between returns on the "same" project taken by different firms is not perfect. Since in a portfolio context, projects and technologies, or for that matter, assets and firms, are defined by their probability distributions, it is misleading and inaccurate to refer to a project being considered by two (or more) firms as being the "same" project if for the same inputs, the cash flows from the project in each state of nature are not the same for each firm (i.e., if the cash flows from the same project taken by different firms are not perfectly correlated). For example, two projects or

or technologies whose cash flows or outputs are identically and independently distributed should not be considered the same project for the same reason that an investor does not treat two assets whose returns happen to be identically and independently distributed as the same asset or even as members of the same "risk class."¹

Jensen and Long are careful to acknowledge that either the assumption of holding the number of firms fixed or the assumption that the correlation between cash flows on the same project taken by different firms is not perfect may not be consistent with the competitive model. Stiglitz recognizes that by holding the number of firms fixed, he is restricting entry, but because he often assumes in his analysis that the distributions of returns for the "same" project across firms are independent, he is led to conclude that even if the number of firms allowed to enter is expanded, the pareto optimal allocation is not achieved. Fama also assumes that the number of firms is fixed and that returns on projects are not perfectly correlated across firms, and, as we show later, it is these restrictions on entry which leads to the non-pareto optimal allocation of investment in his model.

In section II, we re-examine the Jensen-Long model and show that with free entry, the equilibrium level of investment will be no smaller than the pareto optimal amount and if free exit is allowed, then it will equal the pareto optimal amount. In section III, we modify the Jensen-

¹This same error is at the root of a number of "counter-examples" to the classical Modigliani-Miller theorem, particularly with respect to firm size.

Long model to allow for reversible investment and again, show that the competitive equilibrium is a pareto optimum. In section IV, we discuss the Jensen-Long alternative criterion of social wealth maximization. Because the strict assumption of technologies being freely available to all firms (i.e., perfect correlation for the same project across firms) is not empirically descriptive, in section V, we examine the non-perfect competition case of imperfect correlation and conclude that provided new firms can enter and raise capital on the same terms as pre-existing firms, the resulting equilibrium will be a pareto optimum.

II. Jensen-Long Model

In this section, we use the same model as Jensen and Long: namely, all investors are risk-averse and characterize their decisions based on the mean and variance of end-of-period wealth; the market is perfect with all assets infinitely divisible, all investors having homogeneous expectations; transactions costs and taxes are zero. Under these conditions, they show that the equilibrium value of the firm is:

$$V_j = \frac{1}{r} [\bar{D}_j - \lambda \sigma_{jM}] \text{ for all } j \quad (1)$$

where

\bar{D}_j = the expected value of the total end-of-period cash flow to the shareholders of firm j ,

$r = 1 + i$, where i is the one-period riskless rate of interest at which every investor can borrow or lend,

$\sigma_{jM} = \sum_k \sigma_{jk} =$ covariance of the total cash flows of the firm, \tilde{D}_j , with \tilde{D}_M the total cash flows from all firms,

$$\sigma_{jk} = \text{Cov}(\tilde{D}_j, \tilde{D}_k) \quad \text{for all } j \text{ and } k,$$

$$\sigma_M^2 = \text{Var}(\tilde{D}_M),$$

$$\lambda = [D_M - rV_M] / \sigma_M^2 = \text{market price per unit of risk}$$

$$V_M = \sum_k V_k = \text{total market value of all firms.}$$

We also assume that λ is a fixed number which is consistent with constant absolute risk aversion on the part of investors. Consider a new investment opportunity or project with cash flow return per unit input, $\tilde{\rho}$, where

$$E(\tilde{\rho}) = \bar{\rho}$$

$$\text{Var}(\tilde{\rho}) = \sigma\rho^2$$

$$\text{Cov}(\tilde{\rho}, \tilde{D}_j) = \sigma_j \rho.$$

It is assumed that the distribution of $\tilde{\rho}$ is independent of which firm takes it and that there are stochastic constant returns to scale. I.e., if I_j denotes the amount of investment in the project by the j^{th} firm, then $I_j \tilde{\rho}$ is the random variable cash flow to the j^{th} firm as a result of taking the project and the random variable ρ is independent of the choice for I_j . As Jensen and Long do, we assume that the cash flows from all other assets remains fixed, independent of the amount of investment made in the new opportunity. I.e., it is assumed that firms can not change the level of investment in other projects in response to investment in the new technology. Thus, not only is investment assumed to be irreversible, but, in addition, new investment cannot be made in the "old" technologies. While such an assumption is probably not very realistic, the qualitative results of the Jensen-Long model are preserved when this assumption is weakened to allow for reversible investment as we do in section III.

As Jensen and Long have shown, the pareto optimal amount of investment in the new project is²

$$I_w^* = \max \left\{ 0, \frac{1}{\lambda \sigma_\rho^2} [\bar{\rho} - r - \lambda \sigma_{M\rho}] \right\} \quad (2)$$

which is derived from the condition that, at the pareto optimal level of investment,

$$\bar{\rho} = r + \lambda \sigma_{m,\rho} \quad (3)$$

where $\tilde{D}_{M'} = \tilde{D}_M + I_{\tilde{\rho}}$ is the total cash flow for all firms when the amount of (aggregate) investment in the new project is I and where $\sigma_{M',\rho} = \sigma_{M\rho} + I\sigma_\rho^2$ is the covariance between $\tilde{\rho}$ and the return on the market portfolio.

Jensen and Long also show that the value-maximizing level of investment in the project for the j^{th} firm is

$$I_j^* = \max \left\{ 0, \frac{1}{2\lambda \sigma_\rho^2} [\bar{\rho} - r - \lambda (\sigma_{M\rho} + \sigma_{j\rho} + I' \sigma_\rho^2)] \right\} \quad (4)$$

where I' is the aggregate amount of investment in the project by all firms other than the j^{th} firm.³

Because the value-maximizing level of investment depends upon the other projects already taken by the firm, expression (4) seems to conflict with the notion of risk independence of projects.⁴ I.e., suppose

²As discussed in section I, a project is defined by its probability distribution, and hence, the correlation of returns on the same project across firms is perfect. Hence, the aggregate level of investment for a pareto optimum is independent of the distribution of investment in the project across firms.

³Equation (4) is valid only for perfect correlation of returns on the project across firms which has been assumed.

⁴See S. C. Myers [] for a discussion and proofs of risk independence of projects.

that a project is available which will not affect the distribution of cash flows of already existing projects, then the doctrine of risk independence would imply that the firm can determine whether to take a project or not, independent of the other assets already held by the firm. Therefore, it can act as if it were a new firm with no other pre-existing assets. While most studies proving risk independence of projects have used partial equilibrium analysis with given prices or have used general equilibrium analysis with complete markets where there exists a perfect substitute for the project under consideration, our analysis shows that even in a general equilibrium model with incomplete markets, risk independence of projects obtains provided that the capital market is purely competitive or for non-competitive markets, if certain wealth effects across investors are neglected.

Theorem 1. If, at least, one new firm (i.e., a firm with no previous investment in the project, and if there exists a general equilibrium with some distribution of investment in the project across firms, and if the initial wealth of each investor is kept fixed, then, for any other distribution of investment in the project across firms with the same aggregate amount of investment, there exists another general equilibrium such that: (a) the investment opportunity set available to investor's and the returns on each investor's optimal portfolio are the same in both equilibria; (b) the aggregate market value of the project will be the same in both equilibria.

Proof: let I_j denote the amount of investment by the j^{th} firm in the project, $j = 1, 2, \dots, N, N + 1$, where there are N "old" firms and one new firm (by convention, the $(N + 1)^{\text{st}}$); let $I = \sum_1^{N+1} I_j$ denote the aggregate amount of investment in the project which, by hypothesis, is held

fixed. $\tilde{D}_j(I_j) \equiv \tilde{D}_j + I_j \tilde{\rho}$ is the random variable end-of-period cash flow for firm j where, by definition, $\tilde{D}_{N+1} = 0$. Let $V_j(I_j; I)$ denote the equilibrium market value of firm j if it invests I_j in the project when the aggregate amount of investment in the project is I . Let $\delta_j^k(I_j; I)$ denote the fraction of firm j purchased by the k^{th} investor for his optimal portfolio. Then $\delta_j^k(I_j; I)$ is his dollar demand for the j^{th} firm in equilibrium. Suppose that the distribution of investment across firms were such that $I_j = 0$, $j = 1, 2, \dots, N$ and $I_{N+1} = I$. I.e., all investment in the project is taken by the new firm. The equilibrium demand functions for the k^{th} investor would be $\delta_j^k(0; I) V_j(0; I)$, $j = 1, 2, \dots, N$ and $\delta_{N+1}^k(I; I) V_{N+1}(I; I)$, and the random variable return to the investor on his portfolio would be $[\sum_1^N \delta_j^k(0; I) \tilde{D}_j + \delta_{N+1}^k(I; I) I \tilde{\rho}]$.

Suppose instead that the distributional characteristics of investment across firms were I_j , $j = 1, 2, \dots, N+1$ with $I_{N+1} \neq 0$. Consider the portfolio strategy to buy $\delta_j^k(0; I)$ percent of firm j and simultaneously, to sell short $\delta_j^k(0; I) I_j / I_{N+1}$ percent of the new firm $N+1$, for $j = 1, 2, \dots, N$, and then to buy back $\delta_{N+1}^k(I; I) I / I_{N+1}$ percent of firm $N+1$. The contribution of the transaction with respect to the j^{th} firm to the end-of-period return to the portfolio would be: $\delta_j^k(0; I) \tilde{D}_j(I_j) - [\delta_j^k(0; I) I_j / I_{N+1}] \tilde{D}_{N+1}(I_{N+1}) = \delta_j^k(0; I) [(\tilde{D}_j + I_j \tilde{\rho}) - I_j \tilde{\rho}] = \delta_j^k(0; I) \tilde{D}_j$ for $j = 1, 2, \dots, N$; $[\delta_{N+1}^k(I; I) I / I_{N+1}] \tilde{D}_{N+1}(I_{N+1}) = \delta_{N+1}^k(I; I) I \tilde{\rho}$. Hence, this strategy would give exactly the same pattern of returns as for the optimal portfolio chosen when the investment distribution was $I_j = 0$, $j = 1, \dots, N$ and $I_{N+1} = I$. Thus, any pattern of returns for the risky part of the portfolio that can be achieved for a given distribution of investment in the project across firms can be achieved

for any other distribution of investment across firms. Further, any such pattern will be feasible to each investor because he can satisfy his budget constraint by borrowing or lending at the riskless rate. However, to ensure that the same pattern of returns will be optimal for each investor for all distributions of investment, we must show that for each distribution of investment, there exists a set of market values for the firms such that, for a given initial wealth, each investor can hold the same total portfolio (i.e., risky plus riskless assets) for all distributions of investment. To do so, we find a set of prices such that the cost of buying a particular pattern of risky returns will be the same for all distributions of investment across firms. Consider the k^{th} investor in the first hypothesized equilibrium: the cost of achieving the random variable end-of-period returns $\delta_j^k(0;I)\tilde{D}_j$ was $\delta_j^k(0;I)V_j(0;I)$, $j = \underline{1}, \underline{2}, \dots, N$. In the second hypothesized equilibrium, the cost of achieving the same returns would be $\delta_j^k(0;I)V_j(I_j;I)$ less the proceeds from the short sale of $\delta_j^k(0;I)I_j/I_{N+1}$ percent of the $(N+1)^{\text{st}}$ firm, $\delta_j^k(0;I)I_j V_{N+1}(I_{N+1};I)/I_{N+1}$ (i.e., the cost would be $\delta_j^k(0;I)[V_j(I_j;I) - I_j V_{N+1}(I_{N+1};I)/I_{N+1}]$). Similarly, the cost of achieving the returns $\delta_{N+1}^k(I;I)I$ in the first equilibrium would be $\delta_{N+1}^k(I;I)V_{N+1}(I;I)$, while, in the second equilibrium, it would be $\delta_{N+1}^k(I;I)IV_{N+1}(I_{N+1};I)/I_{N+1}$. Clearly, the set of price relationships which keep the cost of each pattern of returns the same for all distributions of investment is:

$$V_j(I_j;I) = V_j(0;I) + I_j V_{N+1}(I_{N+1};I)/I_{N+1}, j=1, 2, \dots, N \quad (5a)$$

$$V_{N+1}(I_{N+1};I) = I_{N+1} V_{N+1}(I;I)/I \quad (5b)$$

Thus, if the $\{V_j(I_j;I)\}$ are chosen to satisfy (5), then, for the same initial wealth, the investment opportunity set available to the investor will not change. Hence, if he was maximizing expected utility with the pattern of returns chosen in the first situation, then the same pattern of returns will maximize his expected utility in the second situation. Therefore, if $\{V_j(0;I), V_{N+1}(I;I)\}$ are equilibrium market values in the first situation, then for the same distribution of initial wealth across investors, the $\{V_j(I_j;I)\}$ as defined in (5) will be equilibrium market values in the second situation.

To prove part (6) of the Theorem, we first rewrite (5) as

$$V_j(I_j;I) = V_j(0;I) + I_j g(I) \quad (6a)$$

$$V_{N+1}(I_{N+1};I) = I_{N+1} g(I) \quad (6b)$$

where $g(I) = V_{N+1}(I;I)/I$. Note, from (6b), that for a fixed aggregate level of investment the market value of a new firm is proportional to the amount of investment by the firm in the project. Further, from (6a), the market value of an "old" firm is equal to the sum of the market values of the old firm if it took no investment in the project plus the market value of a new firm which makes investment I_j in the project. $I_j g(I)$ is the value that the firm would receive if it sold its part of the project to another firm or "spun it off" as a separate firm. Hence, under the hypothesized conditions of the Theorem, risk-independence of the project obtains. Further, the aggregate market value of the project will be $\sum_1^N I_j g(I) + I_{N+1} g(I) = g(I)I = V_{N+1}(I;I)$. Hence, the aggregate market value of the project will be invariant to the distribution of investment across

firms and will only depend on the aggregate amount of investment made in the project. Q.E.D.

So, except for possible wealth distributional effects across investors which depend only upon the initial allocation of ownership across firms prior to any new investment decisions by the firms, the investor will be indifferent to the distribution of investment across firms. The proof of Theorem 1 was independent of any mean-variance or pure competition assumptions, and only depended upon the assumption of perfect correlation of the returns on the same project taken by different firms.

As a corollary to Theorem 1 which is directly relevant to the Jensen-Long analysis, we have that if the hypothesized condition that the distribution of initial wealth among investors remain fixed is replaced with the condition that all investors have constant absolute risk aversion utility functions, then the conclusions of Theorem 1 obtain. To see this, note that it is a well known property of such utility functions that the demands for risky assets in the investors optimal portfolio are invariant to the amount of his initial wealth. Hence, any change in the initial wealth distribution among investors will not affect their choice for an optimal portfolio of risky assets. To get the same result in the mean-variance model, we need only assume that λ , the market price per unit of risk, is invariant to changes in investors' initial wealth distribution.

Hence, we know that in the Jensen-Long case where the price of risk is held constant, the market value of firm j can be written as in (6), where

$$V_j(0;I) = \frac{1}{r}[\bar{D}_j - \lambda(\sigma_{Mj} + I\sigma_{J\rho})] \quad (7)$$

and

$$g(I) = \frac{1}{r}[\bar{\rho} - \lambda(\sigma_{M\rho} + I\sigma_{\rho}^2)] \quad (8).$$

It will throw light on the analysis to compute the value-maximizing solution using the representation for firm value in (6). Namely, to determine the optimal level of investment to take in a new project, the firm picks $I_j = I_j^*$ so as to maximize $[V_j(I_j; I) - I_j]$, which is the market value of the firm net of the cost of inputs. For $I_j = I_j^*$ to be an interior solution, the first-order condition is

$$\frac{d[V_j(I_j; I) - I_j]}{dI_j} = \frac{\partial V_j}{\partial I_j} - 1 + \frac{\partial V_j}{\partial I} \frac{dI}{dI_j} = 0 \text{ for } I_j = I_j^* \quad (9)$$

where

$$\frac{\partial V_j}{\partial I_j} - 1 = \frac{1}{r}[\bar{\rho} - r - \lambda(\sigma_{M\rho} + I\sigma_{\rho}^2)], \text{ independent of } I_j, \quad (10)$$

and

$$\frac{\partial V_j}{\partial I} = -\frac{\lambda}{r} [\sigma_{j\rho} + I_j\sigma_{\rho}^2] \quad (11).$$

If we assume, as Jensen-Long do, that the firm takes investment by other firms in the project as fixed, (i.e., $I' = I - I_j$ fixed) then, $dI/dI_j = 1$ and the value-maximizing solution I_j^* as defined in (4). If instead, the firm takes the aggregate level of investment in the project as fixed (i.e., the firm believes that it cannot affect aggregate investment in the project) as might be suggested by Fama [, p. , assumption (c.6)], then $dI/dI_j = 0$ and the optimal solution for each firm is the same. Namely, if $[\bar{\rho} - r - \lambda(\sigma_{M\rho} + I\sigma_{\rho}^2)] > 0$, the firm(s) will invest nothing; if

$[\bar{\rho} - r - \lambda(\sigma_{M\rho} + I\sigma_{\rho}^2)] > 0$, each firm will be willing to invest an indefinite amount; if $[\bar{\rho} - r - \lambda(\sigma_{M\rho} + I\sigma_{\rho}^2)] = 0$, each firm is indifferent to the amount it invests. Obviously, the only equilibrium solution in this case is

$$I = \text{Max}\left\{0, \frac{1}{\lambda\sigma_{\rho}^2} [\bar{\rho} - r - \lambda\sigma_{M\rho}]\right\} \quad (12)$$

which is the pareto optimum amount of investment as defined in (2). As is usual for standard competitive models, while the aggregate amount of investment in the project is determinate, the scale and hence the number of firms investing in the project is not. Therefore, if firms do not believe that they can affect the aggregate amount of investment in a project, then the number of firms required to achieve a pareto optimal allocation is finite and can be as few as one. The reason that Fama did not arrive at the same conclusion (when he made essentially the same assumption about Firms' beliefs) was that he did not require that the returns on the same project taken by different firms be perfectly correlated.

However, to demonstrate that value-maximization by firms leads to a pareto optimum in a competitive market does not depend on whether firms take I or I' as fixed in making their investment decision, we continue our analysis in the spirit of Jensen and Long assuming that firms act according to (4) or in using (9), take $dI/dI_j = 1$.

We have shown that provided that, at least, one new firm enters, Theorem 1 holds for the Jensen-Long model. We now ask under what condition will no value-maximizing new firm enter? If the aggregate investment by the N "old" firms is I' , then, from (4), noting that the new $(N + 1)^{\text{st}}$ firm has no previous investments, we have that

$$I_{N+1}^* = \text{Max}\left\{0, \frac{1}{2\lambda\sigma_\rho^2} [\bar{\rho} - r - \lambda(\sigma_{M\rho} + I'\sigma_\rho^2)]\right\} \quad (13).$$

Therefore, for a new firm not to enter, $[\bar{\rho} - r - \lambda(\sigma_{M\rho} + I'\sigma_\rho^2)] \leq 0$, or in other words, the condition for no entry by a new firm is that $I' \geq I_w^*$. Thus, a new firm will enter unless the old firms' aggregate investment is at least as large as the pareto optimal amount. Since in all the previous analyses, the concern has been with aggregate investment by old firms being less than I_w^* , for the moment, we will concentrate on the case where $I' < I_w^*$ and at least one new firm enters.

To examine the effect of entry by new firms in determining the equilibrium, we first consider a kind of psuedo sequential-type analysis where firms arrive in the market place in a random fashion and make an investment decision according to (4). However, once they commit themselves, they cannot reverse the decision (disinvest) although they can "come back" to the market place to make more investment. On completion of this analysis, we then consider the more conventional tatonnement process for reaching equilibrium. For simplicity, we assume that old firms do not invest in the project (i.e., $I_j = 0$, $j = 1, \dots, N$) although the analysis would be virtually identical provided that $\sum_1^N I_j^* \leq I_w^*$.

The process for getting to an equilibrium is as follows: a new firm (chosen at random) arrives at the market place and is given the following "macro" information (in addition to the "micro" information which it already has: namely, the distribution for $\tilde{\rho}$): r , λ , and the probability distribution for the market portfolio at the "time" of the firm's arrival. "Time" is counted here as incrementing by one with each arrival of a new

firm in the market place. Thus, if the current new firm making an investment decision is the k^{th} such firm to do so, then the distribution for the market cash flow at time k is denoted by the random variable $\tilde{D}_M(k) \equiv \tilde{D}_M + \sum_1^{k-1} I_{N+t} \tilde{\rho}$ where I_{N+t} is the amount of investment in the project chosen by the t^{th} (new) firm to arrive ($\sum_1^{k-1} I_{N+t}$ is the aggregate amount of investment in the project by all previously-arrived firms) and $\tilde{D}_M(1) \equiv \tilde{D}_M$. By choosing the information provided in this way, the new firm need not know what investments other firms have taken which is more in the spirit of decentralized decisions by firms and an impersonal market place for raising or issuing capital. Since it is assumed that each firm acts fo as to maximize market value, the optimal investment by the k^{th} (new) firm will be

$$I_{N+k}^* = \text{Max}\{0, \frac{1}{2\lambda\sigma_\rho^2} [\bar{\rho} - r - \lambda\sigma_{M\rho}(k)]\} \quad (14)$$

where $\sigma_{M\rho}(k) \equiv \text{Cov}(\tilde{D}_M(k), \tilde{\rho}) = \sigma_{M\rho} + (\sum_1^{k-1} I_{N+t})\sigma_\rho^2$. Formally, (14) and (4) are the same equations. However, (14) shows that direct knowledge of other firms' investment decisions is not necessary. The process of arrivals by firms continues until no new firm would want to enter in which case the market will then be in equilibrium.

To determine the amount of investment taken by the k^{th} new firm, we have that $\Delta_k \sigma_{M\rho}(k) \equiv \sigma_{M\rho}(k+1) - \sigma_{M\rho}(k) = I_{N+k}^* \sigma_\rho^2$ and therefore, $\Delta_k I_{N+k}^* = -\frac{1}{2} I_{N+k}^*$, or

$$\begin{aligned} I_{N+k+1}^* &= \frac{1}{2} I_{N+k}^* \\ &= \left(\frac{1}{2}\right)^k I_{N+1}^* \end{aligned} \quad (15)$$

where $I_{N+1}^* = \text{Max}\{0, \frac{1}{2\lambda\sigma_\rho^2} [\bar{\rho} - r - \lambda\sigma_{M\rho}]\}$. Further, the aggregate amount

of investment taken by the first k firms, $I(k)$, is

$$\begin{aligned} I(k) &= \text{Max}\left\{0, \frac{[1 - (\frac{1}{2})^k]}{\lambda\sigma_\rho^2} [\bar{\rho} - r - \lambda\sigma_{M\rho}]\right\} \\ &= [1 - (\frac{1}{2})^k] I_w^* . \end{aligned} \quad (16)$$

By this process, it actually takes an infinite number of new firms ($k = \infty$) to reach equilibrium which also corresponds to the pareto optimal amount of investment. We did not assume that many new firms, but deduced it as the result of free entry and the assumption that as long as profit opportunities exist, firms will enter. Note, however, that the convergence to equilibrium is rapid: for example, after entry by firms, the aggregate amount of investment taken is 99.9 percent of the pareto optimal amount. Because of the non-tatonnement nature of the approach to equilibrium, there are all sorts of wealth distributional effects if trades actually take place at the "false" interim prices. However, given the assumption of a constant price of risk, we have that the final equilibrium will be unaffected by these transfers of initial wealth, and this final competitive equilibrium will be a pareto optimum (relative to the final distribution of wealth). Finally, it should be noted that the process described of the entering new firms corresponds to the behavior of a perfectly discriminating monopolist which behavior, in the standard theory when the approach is by the conventional tatonnement process, but where we still allow entry by new firms as long as profit opportunities exist. We can write the equilibrium value-maximizing level of investment for the j^{th} firm from (4) as

$$\begin{aligned} I_j^* &= \text{Max}\left\{0, \frac{1}{\lambda\sigma^2} [\bar{\rho} - r \lambda(\sigma_{M\rho} + \sigma_{j\rho} + I^* \sigma_\rho^2)]\right\} \\ &= \text{Max}\{0, (I_w^* - I^*) - \sigma_{j\rho}/\sigma_\rho^2\} \end{aligned} \quad (17)$$

where I^* is the equilibrium aggregate amount of investment in the project and I_w^* is the pareto optimal amount of investment as defined in (2).

To determine the equilibrium, we start by partitioning the firms into three groups: group I contains all firms such that $\sigma_{j\rho} < 0$; group II contains all firms such that $\sigma_{j\rho} = 0$; group III contains all firms such that $\sigma_{j\rho} = 0$. Groups I and II contain only old firms while group III contains all new firms and in addition, any old firms whose $\sigma_{j\rho} = 0$. For notational simplicity and without loss of generality, we assume that there are no old firms in group III. By renumbering firms if necessary, let the firms in group I be numbered 1, 2, . . . , N_1 ; the firms in group II be numbered N_1+1 , . . . , N ; the firms in group III numbered $N+1$, . . . , $N+k$ (where k is the number of new firms which enter).

By inspection of (17), all new firms will choose the same amount of investment, $I_{\text{new}} = I_{N+1}^* = I_{N+2}^* = \dots = I_{N+k}^*$. Further, for I^* to be an equilibrium amount of aggregate investment when free entry is allowed, then there must be no incentive for any firm to change its chosen level of investment. In particular, consider the $(k+1)^{\text{st}}$ (potential) new firm: from (17), it will not enter only if I^* is such that

$$\bar{\rho} - r - \lambda(\sigma_{M\rho} + I^* \sigma_{\rho}^2) \leq 0 \quad (18).$$

Hence, from (18) and (17), we have that the equilibrium amount of investment by all firms in group I will be zero, i.e., $I_1^* = I_2^* = \dots = I_{N_1}^* = 0$. Further, strict inequality in (18) can only obtain if $I_{\text{new}} = 0$ and no new firms enter.

(18) is equivalent to the condition that $I^* \geq I_w^*$ with $I^* > I_w^*$ only if $I_{new}^* = 0$. As in our analysis of the non-tatonnement approach to equilibrium, we have found that free entry ensures that the equilibrium amount of investment in the project will never be less than the pareto optimal amount. A sufficient condition for $I^* = I_w^*$ is that $-\sum_{N_1+1}^N (\sigma_{j\rho} / \sigma_\rho^2) \leq I_w^*$. If the strict inequality holds, then new firms will enter and $KI_{new}^* = I_w^* - [-\sum_{N_1+1}^N (\sigma_{j\rho} / \sigma_\rho^2)]$.

Inspection of the Jensen-Long formula for the equilibrium amount of investment when project returns are perfectly correlated across firms shows that if any new firms enter, an infinite number will (i.e. $K = \infty$).

Hence, as in the previous non-tatonnement analysis, if at least one new firm enters, then the number of new firms entering will be infinite, and the competitive equilibrium will be a pareto optimum. Again, we emphasize that it was not assumed a priori that the number of firms was infinite, but deduced from the assumptions of value-maximization by firms, free entry, and infinitely-fivisible assets. The convergence to the pareto optimum is less rapid for the tatonnement process than in the previous case because entry by a new firm induces previously-entered firms to contract their (contingent) investment. I.e., $1 \geq I^* / I_w^* \geq K / (k+1)$.

Actually, the convergence to the pareto optimum does not even require that all firms are value-maximizers. Essentially, the analysis has shown that as long as the aggregate amount of investment is less than the pareto optimal amount and as long as entry is not restricted, profit opportunities will exist and new firms will enter.

In concluding our discussion of the Jensen-Long analysis, we examine the case where the project's returns are sufficiently negatively correlated with currently existing assets so that the investment taken by firms in group II is greater than the pareto optimum amount. For simplicity, consider the case where group II contains only one firm, the N^{th} one, and the value maximizing $I_N^*(=I^*) > I_w^*$ which implies that $-\sigma_{N\rho} > I_w^* \sigma_\rho^2$.

The change in market value of the k^{th} firm, $k = 1, 2, \dots, N-1$, going from an "equilibrium" with aggregate investment in the project of I_N^* to one with aggregate investment equal to I_w^* (with $I_k^* = 0$ in either case) would be

$$V_k(0; I_w^*) - V_k(0; I_N^*) = \frac{\lambda}{r} [I_N^* - I_w^*] \sigma_{k\rho} > 0, k = 1, \dots, N-1 \quad (19)$$

and the change in value of the N^{th} firm if it divested itself of the investment in the project and (through the entry of new firms) the aggregate investment in the project were I_w^* , would be

$$\begin{aligned} V_N(0; I_w^*) + I_N^* - V_N(I_N^*; I_N^*) &= \frac{\lambda}{r} [I_N^* - I_w^*] [\sigma_{N\rho} + I_N^* \sigma_\rho^2] \\ &= \frac{\lambda}{2r} [I_N^* - I_w^*] [\sigma_{N\rho} + I_w^* \sigma_\rho^2] > 0. \end{aligned} \quad (20)$$

Now, if the total change in the market value of all existing firms were positive by going from I_N^* to I_w^* , then a profit opportunity would exist for a "new" firm to buy out all existing firms at the values associated with the I_N^* "equilibrium" and then to divest the N^{th} firm of its investment in

the project and sell everything out at the new values associated with the I_w^* equilibrium.

By summing (19) from $k = 1$ to $(N-1)$ and adding it to (20), we have that the condition for the sum of the change in the market values of all firms to be positive is

$$\bar{\rho} - r + \frac{\lambda}{2} \sum_1^{N-1} \sigma_{k\rho} > 0 \quad (21)$$

Hence, provided that (21) is satisfied and that tender offers or mergers are allowed, the competitive equilibrium (where firms act so as to maximize their market value) will be a pareto optimum.

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